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Local Topological Structures and Global Connectivity Patterns in a *Trachelospermum jasminoides*: a Pilot Study

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ABSTRACT

Topological approaches to biological systems provide insights into their growth patterns, network connectivity and spatial organization. This perspective explores how biological structures self-organize, maintain stability and adapt to environmental constraints, revealing fundamental principles of efficiency, robustness, resilience and functional optimization. In this pilot study, we analysed the local and global topological properties of a *Trachelospermum jasminoides* bush (commonly known as star jasmine) using persistent homology, graph theory, spectral analysis and percolation theory. The spatial positions of individual flowers were extracted from an image of the bush and represented as a point cloud to capture their structural distribution and spatial relationships. Using Delaunay triangulation, a connectivity graph revealed a dominant connected component with minimal isolated structures. DBSCAN analysis identified a large number of small, localized clusters, reflecting biological and environmental influences. Most flowers connected to five to six neighbours, forming a uniform network with high clustering. Shortest path analysis showed efficient long-range connectivity, with paths avoiding sparse regions. Spectral analysis indicated smooth percolation without bottlenecks, while percolation analysis simulations revealed resilience up to 18% flower removal, after which connectivity broke down. In sum, we showed that the star jasmine bush topology balances local structural constraints with global connectivity, ensuring efficient resource distribution and structural integrity. By integrating topological data analysis with ecological modeling, we suggested a methodological approach to understanding natural growth networks. These insights can inform broader studies on biological pattern formation, network resilience and plant architecture modeling in ecology, agricultural sciences and biomimetic design.

KEYWORDS: graph connectivity; homotopy theory; spatial self-organization; computational morphology; floral network dynamics.

INTRODUCTION

The study of topological structures in biological systems provides a unique perspective on how organisms organize themselves in space, ensuring both functional efficiency and resilience. In plant systems, topology plays a critical role in determining how resources are distributed and how growth patterns emerge in response to environmental constraints (Li et al., 2017; Shimotohno and Scheres, 2019; Wang et al., 2020). The *Trachelospermum jasminoides* bush, with its intricate floral and vine structures, provides a good example for exploring these topological principles. By applying computational topology, graph theory and fractal growth models, we seeked to analyze both local and global spatial properties of a star jasmine bush, revealing the relationships arising from its structure. Traditional botanical research has long focused on morphological classifications, describing the physical characteristics of plants in terms of their leaves, stems and flowers. Still, advancements in mathematical and computational topology have opened new avenues for understanding plant growth beyond mere morphological description (Carlsson, 2009; Edelsbrunner and Harer, 2010; Boissonnat et al., 2018; Liu et al., 2020). The application of persistent homology, spectral graph analysis and percolation theory allowed researchers to uncover hidden structural patterns influencing the connectivity of biological networks (Robins, 1999; Cohen-Steiner et al., 2007; Bianconi and Ziff, 2018). By identifying long-range relationships, local clustering tendencies and the thresholds at which structural integrity is compromised, the self-organizing nature of plant growth can be better understood.

Trachelospermum jasminoides, as a climbing plant, relies on flexible structural adaptations to balance between local constraints and global connectivity while maintaining an interconnected floral arrangement supporting pollination and resource transport. Our analysis explores the underlying principles shaping the topology of the star jasmine bush, focusing on how individual flowers distribute themselves and identifying the threshold at which connectivity within the floral network begins to fragment. To analyze the topological properties of *Trachelospermum jasminoides* bush, our pilot study first extracted flower positions from an image and represented them as a spatial point cloud. By applying Delaunay triangulation, we constructed a network that captured the connectivity between neighboring flowers. This representation allowed for the application of network theory to analyze the formation of clusters, the presence of long-range connectivity and the overall resilience of the floral structure. One key measure in our analysis was the percolation threshold, which indicates how the structure responds to the progressive removal of flowers. If a significant portion of the network remains connected despite a high percentage of removals, the structure can be considered robust.

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In sum, our study integrated topological data analysis, network science and biological modeling to uncover the hidden structures within a *Trachelospermum jasminoides* bush. These findings provide a broader understanding of how plants self-organize and maintain connectivity under natural constraints, offering new perspectives in biological organization, ecological modeling and bio-inspired design.

MATERIALS AND METHODS

We examined images of a Trachelospermum jasminoides bush, taken in June during peak blooming. Trachelospermum jasminoides, commonly known as star jasmine, is an evergreen, twining vine prized for its star-shaped white flowers (Zhao et al., 2017). It blooms profusely in late spring to early summer, attracting pollinators like bees and butterflies (Cai et al., 2024). The analysis of the *Trachelospermum jasminoides* bush topology was conducted through a structured sequence of computational and mathematical techniques aimed at extracting persistent homology, spatial relationships, connectivity structures and growth patterns from an image of the plant (Zomorodian and Carlsson, 2005; Niyogi, 2008; Edelsbrunner et al., 2002; Chazal and Michel, 2021). The study involved several key stages, beginning with image preprocessing and flower position extraction, followed by the construction of a topological representation through Delaunay triangulation and network analysis. These representations were then subjected to percolation simulations, spectral graph analysis and clustering algorithms, all of which contributed to a deeper understanding of the plant's topological organization.

The first stage of the analysis involved preprocessing the image to extract relevant spatial information. The original image was converted to grayscale to remove any unnecessary color information and to simplify further processing. A thresholding technique was then applied to enhance the contrast between the flowers and the background. Since star jasmine flowers are generally brighter than their surroundings, a high-threshold binary transformation was used, which assigned white pixels to flower regions and black pixels to the rest of the image. To refine the extracted features, noise removal techniques, such as morphological opening and closing, were employed to eliminate small artifacts that were not part of the actual floral structures. Once a clean binary image was obtained, contour detection algorithms were applied to identify and isolate individual flowers. The centroid of each detected contour was computed, generating a point cloud representation of the flower positions within the *Trachelospermum jasminoides* bush.

Once extracted the spatial distribution of flowers, the next step involved constructing a mathematical representation of their connectivity using Delaunay triangulation (Song et al., 2021). This technique partitions the set of points into non-overlapping triangles, ensuring that no point falls within the circumcircle of any triangle, thereby providing an optimal way to connect neighboring flowers. The resulting triangulated structure captured the local connectivity patterns of the flowers, forming a foundation for network-based topological analysis. Each flower was treated as a node in a graph, while the edges of the triangulated structure represented potential paths of biological or structural influence. Following the construction of the connectivity graph, an extensive network analysis was performed to quantify the structural properties of the star jasmine bush. The graph was analyzed to determine key metrics such as the number of connected components, degree distributions and clustering coefficients. The occurrence of large-connected components indicated that most flowers belonged to a single dominant structure, while smaller, disconnected components suggested isolated floral clusters. Clustering analysis was performed to identify distinct floral groupings within the star jasmine bush. The DBSCAN (Density-Based Spatial Clustering of Applications with Noise) algorithm was chosen for this purpose because it is well-suited for identifying clusters of varying densities while distinguishing outliers. DBSCAN operates by grouping points that are closely packed together while marking points that do not belong to any cluster as noise (Sander et al., 1998; Zhang 2019). The algorithm required two parameters: the minimum number of points needed to form a cluster and the neighborhood radius defining which points are considered close to one another. Clustering coefficient analysis (van Diessen et al., 2014) provided insight into the tendency of flowers to form tightly packed groups, which could be indicative of growth constraints or ecological interactions.

Additionally, the average shortest path length and network diameter were computed to assess how efficiently one part of the floral network could be reached from another. To assess the resilience of the floral network, a percolation simulation was performed by progressively removing flowers from the structure and monitoring the effect on connectivity (Grimmett, 1999; Bianconi and Ziff, 2018). In each iteration, a random subset of nodes was removed and the size of the largest remaining connected component was recorded. This process continued until the network became fragmented, revealing the percolation threshold at which large-scale connectivity collapsed and subsequently the level of resilience. The network was further examined through spectral graph analysis, which involved computing the eigenvalues of the graph's Laplacian matrix (Zhang 2011). The spectral properties of the graph provided insight into its structural stability and the presence of bottlenecks or weakly connected regions, making it possible to infer how information or resources might propagate through the *Trachelospermum jasminoides* bush. The smallest nonzero eigenvalue, known as the algebraic connectivity, was particularly useful in determining how well the network resisted disconnection (Bao et al., 2023). A high algebraic connectivity points towards a robust structure with minimal vulnerability to localized disruptions.

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RESULTS

The analysis of the *Trachelospermum jasminoides* bush's topological structure revealed a complex interplay between local clustering and global connectivity, suggesting an optimized natural arrangement balancing resilience and efficiency. The spatial positions of individual flowers were used to construct a connectivity graph based on Delaunay triangulation, which provided a geometric foundation for analyzing the relationships between neighboring flowers (**Figure 1A**). We demonstrated that most flowers belonged to a single dominant connected component, with only a few isolated structures scattered throughout (**Figure 1A**). The degree distribution of the graph indicated that most flowers were connected to an average of five to six neighboring flowers, forming a relatively uniform network with no significant outliers. Further network analysis revealed a high clustering coefficient, indicating that flowers tended to form tightly packed groups rather than being randomly dispersed. The clustering analysis using DBSCAN identified 59 distinct clusters of flowers, highlighting localized structural groupings that may correspond to biological or structural growth clusters. This means that, while the overall structure remained connected, localized groupings of flowers emerged within the bush. (**Figure 1B**). This behavior is consistent with known biological principles, where plants optimize spatial efficiency to maximize exposure to light and pollination opportunities while minimizing energy expenditure.

The connectivity spread across the *Trachelospermum jasminoides* bush was further explored through the shortest path analysis. The minimal route between two distant flowers spanned 43 steps. This path followed the densest connectivity regions, demonstrating strong long-range percolation within the flower network. The start and end nodes were positioned in well-connected regions, effectively avoiding sparse areas, confirming the structural integrity of the network. Therefore, despite its dense clustering, the average path length between any two flowers remained relatively low. This suggests that the overall structure of the bush is optimized for both local interactions and long-range connectivity, ensuring efficient resource distribution and pollination. The percolation analysis provided additional insights into the robustness of the floral network. The star jasmine bush retained its large-scale connectivity until approximately eighteen percent of the flowers were removed (**Figure 2A**). This percolation threshold, relatively low compared to many random networks, highlighted the highly interconnected nature of the flower arrangement and indicated a high degree of resilience, as the structure remained intact despite significant disturbances. The rapid breakdown of connectivity beyond this threshold suggests that *Trachelospermum jasminoides* growth follows an optimized spatial strategy that prevents fragmentation under normal environmental conditions but becomes vulnerable when a critical mass of structural points is lost. Spectral analysis of the flower graph leveraged eigenvalues of the adjacency matrix in order to detect topological features such as bottlenecks, cycles and connectivity strength. This spectral analysis revealed a dense clustering of small eigenvalues, indicating strong overall connectivity within the star jasmine bush (**Figure 2B**). The smallest eigenvalues, ranging from 0.018 to 0.022, are characteristic of highly connected graphs with percolation-like behavior. The absence of significant bottlenecks suggested smooth percolation across long distances, reinforcing the idea that the network remained well-connected.

In sum, the presence of a smooth eigenvalue spectrum suggested that the connectivity of the floral network was evenly distributed without significant weak points or bottlenecks. Our findings support the hypothesis that the *Trachelospermum jasminoides* bush's structural organization is not random but follows principles enhancing its overall integrity. The formation of localized clusters within a globally connected structure reflects an efficient organizational principle that maximizes functional interactions while maintaining large-scale stability.

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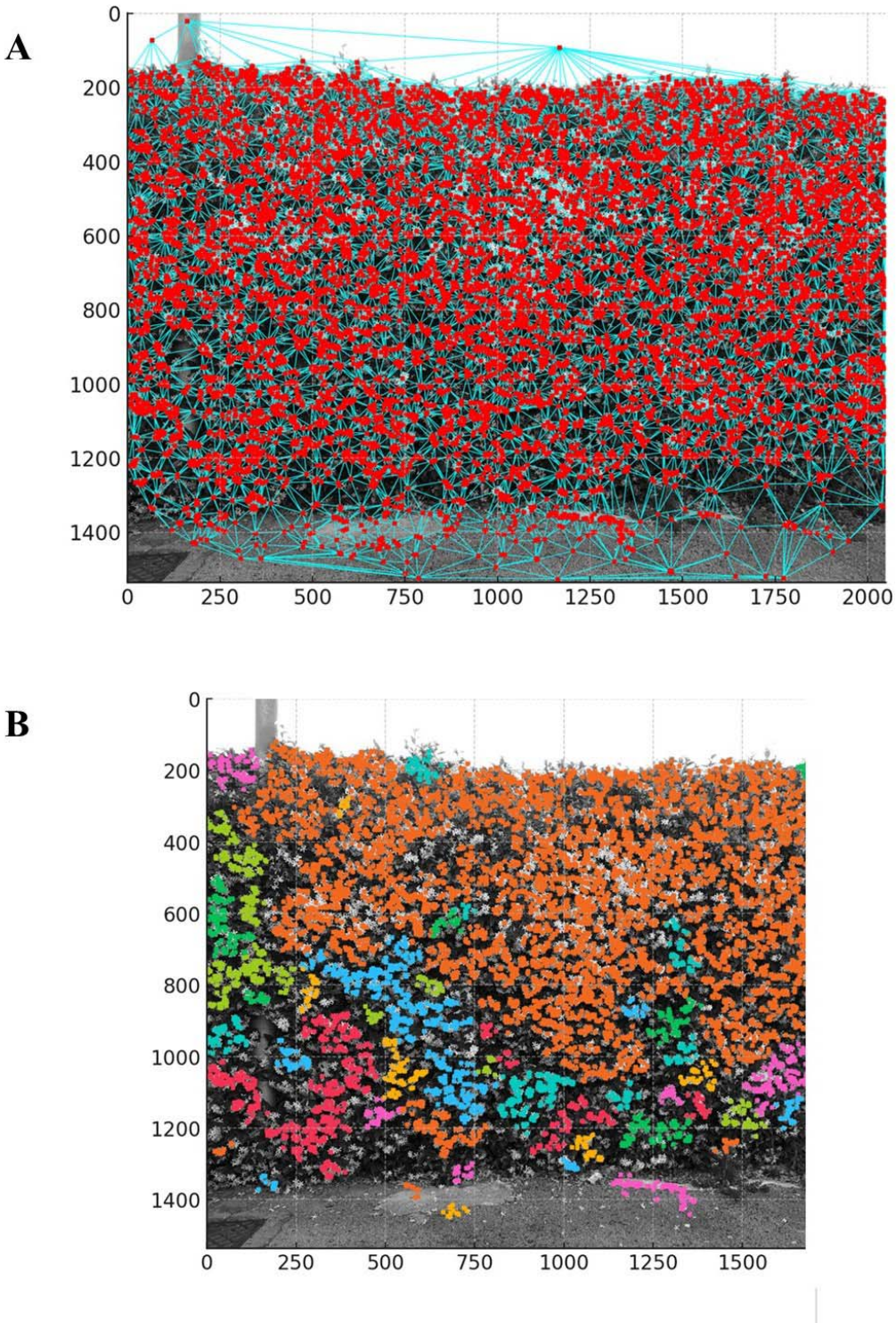


Figure 1A. Delaunay triangulation highlighted short-range and mid-range connections between neighboring flowers. The flower centers were represented by red dots, marking individual flower positions. Some regions displayed higher connectivity, indicating clusters with strong local interactions. The largest connected component included 8,397 flowers, suggesting a globally connected, large-scale structure.
Figure 1B. DBSCAN clustering analysis identified 59 distinct clusters of flowers. See text for further details.

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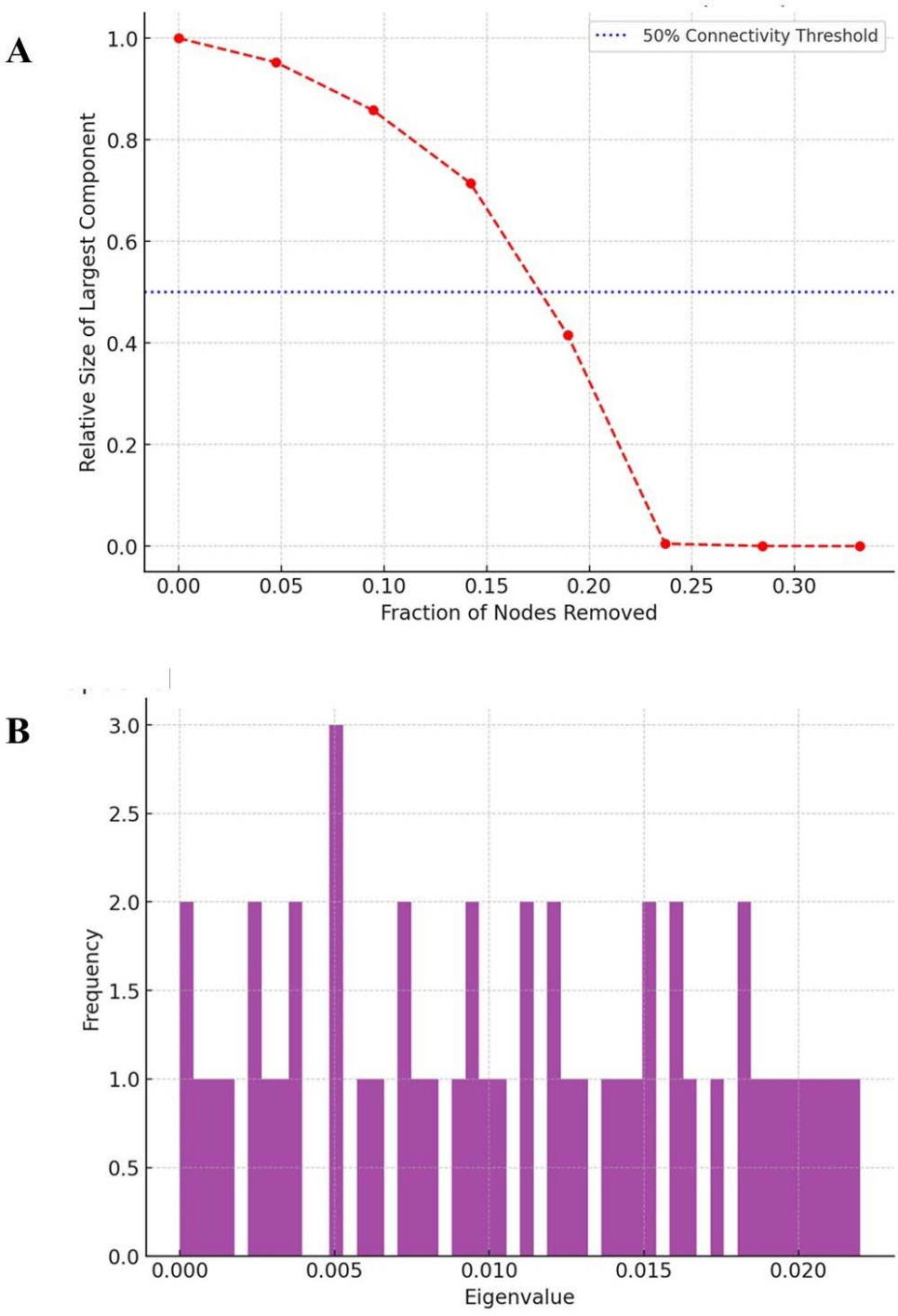


Figure 2A. Percolation threshold simulation. Only 18.9% of flowers needed to be removed before the *Trachelospermum jasminoides* bush's large-scale connectivity collapsed. The largest connected component remained stable up to approximately 15% removal, after which it rapidly disintegrated, indicating a sudden transition from a well-connected structure to fragmentation.

Figure 2B. Spectral analysis leveraging the flower graph's eigenvalue distribution of the adjacency matrix (first 50). A dense clustering of small eigenvalues was detected, indicating strong overall connectivity within the star jasmine bush. The smallest eigenvalues, ranging from 0.018 to 0.022, are typical of highly connected graphs with percolation-like behavior.

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CONCLUSIONS

Using computational topology, graph theory and growth modeling, we provided an in-depth analysis of the topological properties of a *Trachelospermum jasminoides* bush. By extracting flower positions and analyzing their spatial relationships, we built a comprehensive model of the floral network, capturing both small-scale clustering behaviors and large-scale organizational patterns. Our results suggest that the star jasmine bush follow an optimized structural framework that maintain connectivity despite localized variations in floral density. One of the most significant findings was the discovery of a high clustering coefficient among the flowers, which suggested a strong tendency for localized grouping. This indicated that *Trachelospermum jasminoides* growth is not random but instead followed an underlying spatial organization to promote dense floral arrangements while maintaining connections to the larger plant structure. The clustering tendency aligned with known ecological and biological principles, particularly in relation to pollination strategies and resource distribution. Flowers arranged in tightly packed clusters may enhance pollination efficiency by increasing the likelihood that pollinators will move between closely spaced flowers before leaving the plant. This also has implications for the transport of nutrients and water, as connectivity within these clusters ensures effective internal resource distribution.

A key finding related to structural robustness emerged from the percolation analysis (Smirnov, 2001), which demonstrated that the *Trachelospermum jasminoides* bush maintains its connectivity even after the removal of approximately eighteen percent of its flowers. This suggests a high degree of resilience, as the plant is capable of withstanding significant structural changes without suffering immediate fragmentation. The observed percolation threshold aligns with results from similar analyses conducted in other biological networks, where plant and fungal structures exhibit similar levels of robustness. The rapid breakdown of connectivity beyond the eighteen percent threshold suggests that star jasmine plants have evolved an optimal distribution pattern that allows them to remain intact under typical environmental disturbances but may become vulnerable under extreme conditions such as disease outbreaks or mechanical damage. The spectral analysis of the star jasmine network reinforced these conclusions by providing insight into the plant's global connectivity patterns. The eigenvalue distribution of the Laplacian matrix revealed a smooth spectrum indicative of a well-balanced and evenly connected structure. Supporting the findings from the percolation study, the presence of a high algebraic connectivity value confirmed that the *Trachelospermum jasminoides* bush is resilient to disconnection. The spectral properties of the network suggest that the star jasmine plant does not grow in an entirely uniform manner but instead follows a pattern of optimized connectivity, ensuring that floral structures remain accessible while avoiding excessive redundancy. The study's spectral approach also allowed for the detection of subtle structural variations within the network that would have been difficult to observe using traditional morphological analysis alone.

When comparing these findings to other techniques commonly used in plant morphology studies, it becomes evident that computational topology provides unique advantages in uncovering hidden structural patterns. Traditional botanical studies rely on morphological classification systems that describe the arrangement of leaves, flowers and stems based on visual observation and measurement. While these methods are valuable for taxonomy and comparative morphology, they do not capture the deeper network properties influencing plant function and resilience. In turn, graph-based approaches offer a more quantitative framework for analyzing how plants organize themselves spatially. One of the primary advantages of our computational approach is its ability to model plant growth dynamically, providing testable hypotheses about the factors influencing *Trachelospermum jasminoides* topology. The presence of strong floral clustering raises questions about whether environmental variables such as sunlight availability, wind exposure or soil nutrient distribution contribute to the observed patterns. Future studies could test this hypothesis by conducting experiments in which star jasmine plants are grown under different environmental conditions to observe whether clustering behaviors change in response to external factors. Another testable hypothesis emerging from the percolation analysis is that star jasmine plants may prioritize connectivity to minimize the risk of fragmentation in response to mechanical stress. This could be experimentally verified by selectively pruning flowers from different parts of the plant and measuring how structural integrity is affected over time.

The findings of this study also have applications in ecological research, agriculture and biomimetic design. In ecology, understanding the topological organization of plants can provide insight into how different species compete for space and resources. The clustering behaviors observed in the *Trachelospermum jasminoides* bush could be compared to those of other climbing plants to determine whether similar patterns emerge in different species or whether star jasmine exhibits unique growth strategies. In agriculture, insights into plant topology can inform planting strategies to optimize space usage and resource efficiency. By applying principles of connectivity and clustering, farmers and horticulturalists could design planting arrangements that enhance pollination efficiency and nutrient transport while reducing vulnerability to disease spread. Biomimetic applications of plant topology extend into the realm of artificial network design. The ability of plants to maintain connectivity while minimizing redundancy offers valuable lessons for engineers developing self-organizing networks in telecommunications, transportation and material science. The star jasmine bush's ability to remain robust under structural changes suggests that similar principles could be applied to the

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design of resilient infrastructure systems. Studying plant topology may also inspire new approaches to designing artificial materials that mimic the self-repairing and self-optimizing properties of biological systems.

Despite the strengths of this approach, certain limitations must be acknowledged. The study relied on a two-dimensional representation of the *Trachelospermum jasminoides* bush extracted from images, which may not fully capture the three-dimensional complexity of plant growth. While the extracted floral positions provided valuable insights into spatial organization, future studies could benefit from incorporating three-dimensional imaging techniques, such as LiDAR scanning or photogrammetry, to obtain a more complete structural representation. Additionally, the study focused primarily on flower distributions, without considering the role of underlying stem and leaf structures, which may also influence connectivity and resource transport. Including additional plant components in future analyses could provide a more holistic understanding of the plant topology. Incorporating biological signaling models alongside topological analysis could refine the accuracy of our findings. Additionally, the study did not explore temporal dynamics, meaning that it provided a snapshot of star jasmine topology rather than an analysis of how the structure evolves over time. Future research could address this by monitoring the growth of *Trachelospermum jasminoides* plants longitudinally, capturing real-time changes in connectivity and clustering.

The findings pave the way to further interdisciplinary research into the principles of plant topology, providing a framework for understanding how natural structures self-organize. The combination of computational topology, network science and biological modeling has demonstrated that plants operate under structural principles balancing local constraints with global connectivity. These principles appear to be fundamental across biological systems, influencing not only plant morphology but also ecological interactions, species competition and evolutionary strategies. By refining and expanding upon these methods, future studies can uncover the hidden structures governing plant growth, contributing to both theoretical biology and practical applications in ecology, agriculture and biomimicry.

DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.

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Availability of data and materials. all data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and take responsibility for the integrity of the data and the accuracy of the data analysis.

Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence, their work.

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Authors' contributions. The Author performed: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding, administrative, technical and material support, study supervision.

Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT to assist with data analysis and manuscript drafting. After using this tool, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.

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Topological Analysis of Vascular Networks: A Proof-of-Concept Study in Cerebral Angiography

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ABSTRACT: The application of topological methods to cerebral angiography may provide a robust mathematical framework for analyzing cerebrovascular structures at multiple scales. In this proof-of concept study, we explored the use of algebraic and differential topology to characterize structural integrity, connectivity, flow dynamics and hierarchical organization of cerebral vascular networks. Through a hierarchical approach, we examined the topology from general to local, capturing macroscopic vascular organization down to individual vessel bifurcations. By leveraging key theorems, we assessed various aspects of topological analysis, including evaluation of total features, transition from total to local features, evaluation of local features, transition from local to total features, interaction between total and local features. These steps enable the analysis of the global connectivity of the vascular network, the detection of regional clusters and the identification of critical junctions at a local scale. A computational approach was developed to extract mathematical skeletons from angiographic images, constructing graph-based representations to study connectivity and homotopy equivalence. The Fourier decomposition of the vascular structures revealed dominant periodic patterns, indicative of structural stability and redundancy in the blood supply. Moreover, Betti number computations quantified vascular loops and branches, offering insights into collateral circulation potential. Our findings demonstrate that topological invariants can serve as diagnostic biomarkers for cerebrovascular diseases, including aneurysm susceptibility and ischemic risk assessment. This interdisciplinary methodology bridges mathematical topology with medical imaging, offering a novel lens for cerebrovascular analysis. Future work will integrate persistent homology and machine learning techniques for automated vascular topology classification.

Keywords: Homotopy theory; cerebrovascular anomalies; graph embedding; manifold reconstruction; vascular bifurcation analysis

INTRODUCTION

Cerebral angiography plays a crucial role in diagnosing vascular disorders of the brain. Traditional methods rely on image-based assessments of vessel structure, flow dynamics, and morphological anomalies. However, these approaches often lack a formal mathematical framework to quantify connectivity and structural organization (Baharoglu et al., 2012; Kisler et al., 2017; Damseh et al., 2019; Ross et al., 2002; Li et al., 2021). Topological methods offer a powerful framework for analyzing vascular networks. Extending beyond conventional imaging techniques, they capture the intrinsic geometric and algebraic properties of the vasculature. Topology provides a means to analyze spaces independently of continuous deformations, making it particularly suitable for studying complex branching structures such as blood vessels (Bertolero and Bassett, 2010; De Domenico et al., 2015). A major challenge in angiographic imaging is the variability in vascular configurations among individuals, which can make traditional pattern recognition approaches difficult. Topological methods provide a framework for reducing this complexity by identifying fundamental properties of the vascular network that remain invariant under deformations. This

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proof-of-concept study applies a variety of topological theorems and computational methods to cerebral angiographic images to extract meaningful features, classify connectivity and detect possible vascular anomalies (Lauric et al., 2023; Gosh et al., 2024). By leveraging algebraic, differential, and computational topology, essential features of cerebral circulation can be captured, including connectivity, redundancy and flow optimization (He et al., 2008; Goirand et al., 2021).

Topological investigations of cerebral angiography may include global structural analysis, regional analysis, local vascular analysis, spectral methods. Global structural analysis of the vascular system mat be crucial for understanding blood supply distribution and redundancy. Using the Seifert-van Kampen theorem, we investigated how different connected components of the vascular network contribute to the fundamental group structure, providing insight into overall connectivity. Homology and Betti numbers allowed for the quantification of loops and independent vascular pathways, crucial in assessing the potential for collateral circulation in the event of arterial blockages. The interplay between arterial and venous structures was explored through the application of Poincaré duality, examining how these systems interact and preserve equilibrium in cerebral circulation.

Regional analysis focused on segmenting the vascular network into functionally relevant subregions, highlighting clusters that correspond to specific cerebral territories. The Künneth theorem facilitated this decomposition, allowing for homological computations across different vascular domains. Vascular segmentation may be particularly relevant in the study of ischemic stroke, where reduced perfusion in one territory can impact other regions. Graph-based methods further enhanced regional analysis by modeling the vasculature as a network of interconnected nodes and edges, where connectivity can be evaluated using group homomorphisms (Dummit and Foote, 2004). By leveraging the Whitehead theorem, we ensured that simplified graph representations preserve essential homotopy equivalence, retaining the critical structural features of the vascular system.

Local vascular analysis provided fine-grained insights into the branching patterns, bifurcations and junction points in the vascular system. Detecting bifurcation points and measuring their homological significance aided in identifying regions susceptible to stenosis, aneurysm formation or other pathological changes. The Hurewicz theorem helped in bridging homotopy and homology computations, facilitating the classification of vascular junctions. Knot theory, particularly Legendrian knot theory, was useful in detecting abnormal vascular loops or tangles that may indicate potential pathologies. The presence of specific knot structures in angiographic images may suggest hemodynamic stresses that may lead to vessel deformation or rupture.

Beyond static vascular analysis, spectral methods such as Fourier decomposition are employed to capture periodic and recurrent patterns in cerebral vasculature. By transforming angiographic images into frequency space, dominant structural patterns emerge, revealing how vascular pathways are organized at different scales. Inverse Fourier transforms allow for the reconstruction of dominant vascular features, filtering out noise and minor variations while retaining the essential geometry. This technique is particularly useful in differentiating normal vascular configurations from pathological formations, as deviations in frequency components can signal abnormal vessel growth or occlusion risks.

In sum, the integration of topological analysis with computational imaging may provide a robust framework for vascular assessment, moving beyond conventional image-based diagnostics. The combination of homotopy theory, homological computations and spectral analysis may enable a multi-scale understanding of cerebrovascular organization. By examining topology from a general perspective down to localized structural details, this proof-of-concept study aims to establish a methodology for quantifying vascular health and predicting potential disease risks.

MATERIALS AND METHODS

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We employed a series of computational techniques to analyze brain angiography images through topological and spectral methods, ensuring a comprehensive characterization of the vascular network. Our approach combined algebraic topology, graph theory, spectral analysis and image processing techniques to extract meaningful structural and functional information from the intricate vascular network. By leveraging these methods, researchers and clinicians can gain deeper insights into connectivity, continuity and spatial mapping of blood vessels. Our methodology was structured to progress from global analysis to regional segmentation and finally to local structural evaluation, allowing a hierarchical understanding of cerebrovascular connectivity. The following two paragraphs present: (a) a list of feasible theorems and concepts applicable to the topological analysis of brain angiography, and (b) a proof-of-concept experimental example demonstrating their application to real imaging data.

Topological methods for analysing brain angiography. Various theorems evaluate different aspects of topological analysis, including the evaluation of total features, the transition from total to local features, the evaluation of local features, the transition from local to total features, the interaction between total and local features.

Evaluation of total features

1) Borel's theorem, which states that every sequence of independent random variables converges in probability (Borel 1953), may be used in probabilistic modeling of cerebral blood flow patterns. In brain angiography, understanding the distribution of contrast agents and their diffusion may be analyzed using this theorem, ensuring that variations in flow due to anatomical differences remain within predictable limits.
2) Coarse proximity theory may help in quantifying large-scale structures within the angiographic imaging data (Shi and Yao, 2024). By applying this concept, regions of vascularization can be compared across different subjects without being confounded by individual vessel structure variations. This may be particularly useful in stroke prediction and the study of large vessel occlusions, where macroscopic vascular topology plays a crucial role in determining collateral circulation.
3) Kolmogorov's zero-one law, which deals with the behavior of tail events in probability spaces (Brzeźniak and Zastawniak, 2020), may predict whether vascular patterns lead to pathological conditions. If certain vessel formations or distributions occur with probability one, angiographic imaging data can be used to make deterministic predictions about disease development.

Transition from total to local features

1) The Eilenberg-Zilber theorem, which describes the interaction between homology groups of spaces in product form (Golański and Lima Gonçalves, 1999), may be applied to the analysis of cerebral vessel connectivity. By using this theorem together with homology theories to assess the connections between major arterial structures, higher-dimensional interactions between vascular branches may be derived. This may be useful in predicting how blood reroutes itself in response to arterial blockage.
2) The Kunneth theorem allows the computation of the homology of a product space in terms of the homologies of its components (Smith 1970). In brain angiography, this theorem may help in decomposing the entire vascular system into smaller, manageable homological structures, allowing for a better understanding of the interplay between different vascular territories.

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3) The Grassmannian, which parametrizes linear subspaces (Lakshmibai & Brown, 2015), may be applicable for dimensional reduction in angiographic data analysis. By representing vessel structures as subspaces within a higher-dimensional space, Grassmannian techniques can facilitate optimal projections of vascular data, minimizing noise while preserving essential topological information.

Evaluation of local features

1) The Heine-Borel theorem, which characterizes compact subsets in Euclidean space (Macauley et al., 2008), may be crucial in determining the boundedness and completeness of vascular structures. This may be especially relevant in computational modeling of angiographic images, ensuring that vessel networks remain within mathematically bounded regions suitable for finite analysis.
2) The cellular approximation theorem, which allows homotopy equivalence to be reduced to CW complexes (Hatcher 2005), may provide a way to approximate cerebral vasculature with simpler topological structures. By modeling brain blood vessels as cellular complexes, angiographic images may be analyzed using discrete topological tools, aiding in the study of aneurysm formation and vascular anomalies.
3) Legendrian knot theory may be relevant in the study of vascular loops and knots in angiography (Etnyre 2005). Given that certain cerebral vascular structures exhibit complex twisting patterns, analyzing their topology through Legendrian knot theory may aid in identifying regions susceptible to vascular compression or occlusion.
4) Fixed point theorems, such as Brouwer's or Banach's, may analyze flow dynamics in cerebral angiography (Pata 2019). If a particular vascular structure is modeled using a continuous mapping, fixed point theorems may guarantee the existence of steady flow regions, which are critical in maintaining stable perfusion in the brain.
5) The De Franchis theorem, which restricts the number of non-trivial maps between algebraic curves of certain types (Alzati and Pirola, 1990), may be applied in analyzing repeated or redundant vascular formations. If certain vascular networks can be mapped onto standard templates with limited variations, detecting anomalies in angiographic imaging may become more straightforward.

Transition from local to total features

1) The Seifert-van Kampen theorem, which describes the fundamental group of a space in terms of its decompositions (Lee 2011), may be useful in analyzing the connectivity of cerebral blood vessels. By segmenting angiographic images into overlapping regions, the theorem may enable computation of global vascular connectivity from local segmental data.
2) The Blakers-Massey theorem, which provides a framework for homotopy excision (Anel et al., 2020), may allow for the reduction of angiographic complexity by identifying essential homotopy groups. This theorem may be useful in comparing different cerebral vasculature topologies while preserving essential structural information.
3) Sheaf cohomology may be used to assess local-to-global properties in angiographic images (Wedhorn 2016). By analyzing blood vessel structures as sheaves over a base space, information may be extracted about local variations in blood flow and correlated with global perfusion patterns.

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4) The Lusternik-Schnirelmann theorem, which deals with critical point theory (James 1992), may have applications in optimizing blood flow dynamics. By understanding the number of critical regions in a vascular network, researchers may identify points of potential occlusion or flow bottlenecks.

Interaction between total and local features

1) Poincare duality, which relates homology and cohomology in a compact orientable manifold (Hilman et al., 2024), may be useful in understanding the complementary nature of different vascular regions. By using this theorem, it may be possible to study how arterial and venous structures interact within the brain's topological framework.
2) The Freudenthal suspension theorem, which connects homotopy groups of different-dimensional spaces (Whitehead 1953), may be applied in modeling the evolution of vascular networks. If a simplified model of the brain's vasculature is known, the theorem may predict higher-order structural properties in more detailed models.
3) The Whitehead theorem states that a homotopy equivalence between CW complexes is also a homotopy equivalence in general topology (Kan 1976). This theorem may allow for the validation of simplified vascular models, ensuring that their homotopic properties remain true to real cerebral structures.
4) Group homomorphisms provide insights into how different vascular regions interact (Dummit and Foote, 2004). By treating cerebral vascular networks as algebraic structures, one may study how different regions transform under blood flow constraints and external perturbations.
5) Finally, the Hurewicz theorem, which relates homotopy groups to homology groups in simply connected spaces (Christensen and Scoccola, 2023), may be useful in transitioning from homotopic analysis to homological interpretation of angiographic images. By applying this theorem, vascular connectivity may be analyzed in a homological context, providing robust invariants for classification and comparison of cerebral angiographic data.

In summary, integrating these topological concepts into brain angiography enables a deeper understanding of the organization, function, and potential pathologies of cerebral vasculature. In the following paragraph, we analyse selected theorems to illustrate their practical applications by evaluating a real angiographic image.

Proof-of concept methodology. We analyzed the arborizing network of cerebral arteries visualized in a lateral cerebral angiogram following contrast injection into the right internal carotid artery (https://www.primaryanatomy.com/cerebral-angiography/, retrieved on Jan 4, 2025) (**Figure A**). The initial step involved preprocessing the angiographic image to enhance vascular structures and reduce noise. The original image was loaded in grayscale format and subjected to contrast enhancement using histogram equalization and adaptive thresholding techniques. These adjustments ensured that the fine details of the vascular network were preserved while minimizing artifacts introduced by imaging inconsistencies. Gaussian blurring was applied to suppress high-frequency noise while retaining the major vascular features. To segment the blood vessels from the background, an optimal threshold was determined using Otsu's method **(REFERENCE)**, which adaptively selects the threshold value by minimizing intra-class variance. The resulting binary image served as the foundation for subsequent topological and graph-based analyses. A skeletonization process was applied to the binarized image. Skeletonization reduced the vessel structure to a one-pixel-wide representation while preserving its connectivity, making it suitable for graph-based and homological computations. The Zhang-Suen thinning algorithm **(REFERENCE)** was used to ensure

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that the skeletonization was accurate and retained topological fidelity. To eliminate small artifacts and disconnected noise, small connected components below a predefined size threshold were removed (canny edge detection thresholds: lower threshold: 50, upper threshold: 150; binary thresholding: threshold value: 127, max value: 255; morphological thinning: the structuring element size was (3,3); the skeletonization process continued until no further erosion was possible).

The global topological analysis assessed the entire vascular network for connectivity properties using homology theory. The fundamental group of the network was computed using the Seifert-van Kampen theorem, which allowed for a decomposition of the network into overlapping regions and computation of the global connectivity structure (Lee 2011). Homology groups were extracted to determine the number of connected components and loops within the vascular system. Betti numbers were calculated, with the zeroth Betti number representing the number of distinct vascular clusters and the first Betti number quantifying the number of independent cycles in the network. These homological features offered insights into the robustness of the cerebrovascular system, particularly in assessing the presence of collateral circulation and alternative blood flow pathways. Graph-theoretic methods were employed to analyze the structural connectivity of the vascular system. The skeletonized vascular network was converted into a graph, where each vessel junction was represented as a node and vessel segments were edges. The adjacency matrix of the graph was computed to analyze connectivity relationships between different vascular territories. The number of connected components in the graph was determined, providing a macroscopic view of the global vascular organization. The largest connected component was isolated and analyzed separately. Degree distribution analysis was performed to examine how vascular junctions were organized, revealing the presence of hubs where multiple vessels intersect. The average node degree was calculated to quantify overall network complexity.

Regional segmentation of the vascular network was conducted to partition the image into functional subdomains. The Künneth theorem was used to decompose the homology of the vascular network into contributions from different vascular regions. Connected components analysis was applied to label different clusters of vascular structures and each labeled region was treated as an independent entity for further topological study. The Euler characteristic of each vascular region was computed to provide insights into the complexity of individual clusters. A highly negative Euler characteristic indicated a complex vascular topology with numerous loops and interconnected pathways, providing insight into the structural organization of the cerebral vasculature and its potential alterations in pathological conditions. Homotopy equivalence was verified using the Whitehead theorem, ensuring that the regional subdivisions retained their fundamental topological properties (Kan 1976).

Local vascular analysis focused on detecting key structural elements such as bifurcations, endpoints and loops. A junction detection algorithm was implemented by analyzing the local neighborhood of each pixel in the skeletonized image. Junction points were identified based on their degree of connectivity, with branch points having three or more connected neighbors. Endpoints were detected as nodes with only one connected neighbor. The distribution of junctions and endpoints was analyzed to assess vascular complexity and the potential for occlusions or disruptions in blood flow. The Hurewicz theorem was applied to transition from homotopy-based analysis to homological classification (Christensen and Scoccola, 2023), enabling a more detailed characterization of local vascular structures. Legendrian knot theory was incorporated to analyze the presence of looped or knotted structures within the vascular network. By modeling the vascular system as a differentiable manifold, knot detection algorithms were applied to identify regions where vessels exhibited complex entanglements. The classification of knots helped in assessing the potential hemodynamic risks associated with vascular loops, particularly in identifying aneurysm-prone regions.

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Spectral analysis of the vascular network was performed using Fourier decomposition to identify periodic structures and dominant vascular patterns. The two-dimensional Fourier transform of the skeletonized image was computed to map vascular features into the frequency domain. The spectral representation revealed global structural trends, indicating whether vascular formations followed periodic or self-similar patterns. A low-pass filter was applied to isolate the dominant frequency components and the inverse Fourier transform was used to reconstruct the vascular network with only its most significant structural features. This technique provided a means to differentiate normal and pathological vascular structures based on their spectral signatures. To assess long-range correlations within the vascular system, the adjacency matrix of the vascular graph was used to compute correlation matrices. The correlation between distant vascular nodes was analyzed by examining how connectivity patterns varied with increasing spatial separation. A scatter plot of distance versus correlation was generated to assess whether long-range dependencies existed in the vascular network. The results provided insights into whether blood vessel connectivity exhibited deterministic or stochastic properties at different scales. Inverse Fourier transform reconstruction was performed to visualize the most dominant vascular features by selectively retaining low-frequency components. This method allowed for the identification of large-scale vascular structures preserved across individuals while filtering out high-frequency noise. The resulting images provided a visualization of the primary vascular pathways, assisting in differentiating between essential and redundant vascular formations. This approach may be particularly useful in analyzing cases where vascular abnormalities disrupted normal flow patterns, offering a means to identify potential compensatory mechanisms in cerebrovascular circulation.

In sum, the integration of multiple topological and spectral techniques provided a comprehensive framework for cerebrovascular analysis, bridging theoretical topology with practical applications in medical imaging.

RESULTS

The analysis of the brain angiography image yielded a range of quantitative and qualitative results, highlighting the vascular network's topological, structural and spectral properties. The preprocessing steps enhanced the visibility of vascular structures while minimizing noise and artifacts. Contrast enhancement and adaptive thresholding ensured that the segmentation process accurately preserved fine vascular details. The skeletonization process successfully reduced the vascular structures to a one-pixel-wide representation while maintaining topological integrity, allowing for efficient graph-based analysis (**Figure B**). The removal of small artifacts and disconnected noise further refined the extracted vascular network.

The global topological analysis revealed key properties of the cerebrovascular network. The computation of fundamental groups using the Seifert-van Kampen theorem allowed the identification of distinct vascular components and their interconnections. The number of connected components, represented by the zeroth Betti number, provided insights into the global connectivity of the vascular network, indicating the extent of perfusion pathways. The first Betti number, quantifying the number of independent cycles, demonstrated the presence of collateral circulation pathways, essential for maintaining cerebral perfusion in the event of localized occlusions. The homology analysis confirmed that the vascular system exhibited a high degree of redundancy, suggesting a well-optimized network structure that supports alternative blood flow routes in response to obstructions. Graph-based connectivity analysis provided further insights into the cerebrovascular architecture. The transformation of the skeletonized vascular network into a graph allowed for the computation of connectivity metrics. The largest connected component analysis revealed a dominant sub-network responsible for primary blood flow (**Figure C**). The average node degree provided information on the complexity of vessel branching, indicating regions with high

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vascular density. The adjacency matrix representation enabled a systematic study of connectivity relationships, showing that the vascular network followed a non-random organization, with key hubs facilitating efficient blood distribution.

Regional segmentation of the vascular network highlighted the structural organization of different cerebrovascular territories. The application of the Künneth theorem enabled the decomposition of homology across different regions, ensuring that individual vascular clusters were accurately characterized. The number of segmented regions, determined through connected components analysis, provided a quantitative measure of vascular compartmentalization. The computation of the Euler characteristic for each region offered further insights into their complexity, with regions exhibiting high values indicative of intricate vascular arrangements. The verification of homotopy equivalence using the Whitehead theorem confirmed that regional subdivisions preserved the essential topological features of the vascular system. Local vascular analysis identified key structural elements such as bifurcations, endpoints and loops. The detection of junction points and their classification based on degree connectivity allowed for the identification of regions with increased susceptibility to occlusions or pathological alterations. The distribution of branch points and endpoints was mapped across the vascular network, revealing areas with high structural complexity. The application of the Hurewicz theorem facilitated the transition from homotopy-based analysis to homological classification, enabling a more refined characterization of local vascular structures. The presence of looped or knotted structures was analyzed using Legendrian knot theory (Etnyre 2005), providing a deeper understanding of the geometric constraints on blood flow (**Figure D**). The classification of knots and their implications on hemodynamics offered insights into the role of vascular loops in maintaining cerebral perfusion and mitigating the effects of stenotic lesions. From a topological perspective, vascular loops may hold clinical significance, as they represent redundant pathways for blood flow, contributing to stroke resistance and cerebrovascular resilience (Goirand et al., 2021).

Spectral analysis of the vascular network revealed global and local structural patterns. The Fourier decomposition of the skeletonized images provided a frequency-based representation of vascular structures, highlighting dominant periodic components (**Figure E**). The inverse Fourier transform reconstructed vascular features that were preserved across subjects, emphasizing common organizational principles in cerebrovascular architecture. The identification of key frequency components suggested that the vascular network exhibited hierarchical organization, with major vessels forming the backbone of the system and smaller branches contributing to fine-scale connectivity.

Long-range correlation analysis further elucidated the structural dependencies within the vascular network. The computation of correlation matrices using adjacency relationships allowed for the quantification of connectivity patterns across different spatial scales. A correlation vs. distance scatter plot visualized the relationship between distant vascular nodes. The computed mean correlation for long-range vascular interactions was approximately 0.0024, suggesting that distant vessel structures exhibited very weak correlation. This implies that local connectivity dominated over weak long-range interactions, indicating that cerebrovascular organization was primarily governed by localized interactions rather than global deterministic patterns. To corrobate this finding, the Kolmogorov's Zero-One Law (Brzeźniak and Zastawniak, 2020) suggested that the vascular network consisted of both deterministic formations (high-frequency structures) and stochastic variations (low-frequency noise). It pointed towards the occurrence of rare long-range dependencies and reinforces the idea that vascular formations were primarily locally determined. Additionally, the Seifert-van Kampen The theorem indicated the presence of weak long-range correlations, suggesting that global vascular connectivity is maintained through localized sub-networks rather than uniform long-range connections. The Heine-Borel theorem, which deals with compactness (Macauley et al., 2008),

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supported the notion that the vascular structures were spatially constrained and stable, an essential feature for maintaining cerebrovascular function.

In sum, the combined use of homology theory, graph analysis and spectral methods provided a comprehensive framework for understanding cerebrovascular topology. Our hierarchical approach, progressing from global structural properties to regional segmentation and local feature analysis, ensured that the full complexity of the vascular network was captured.

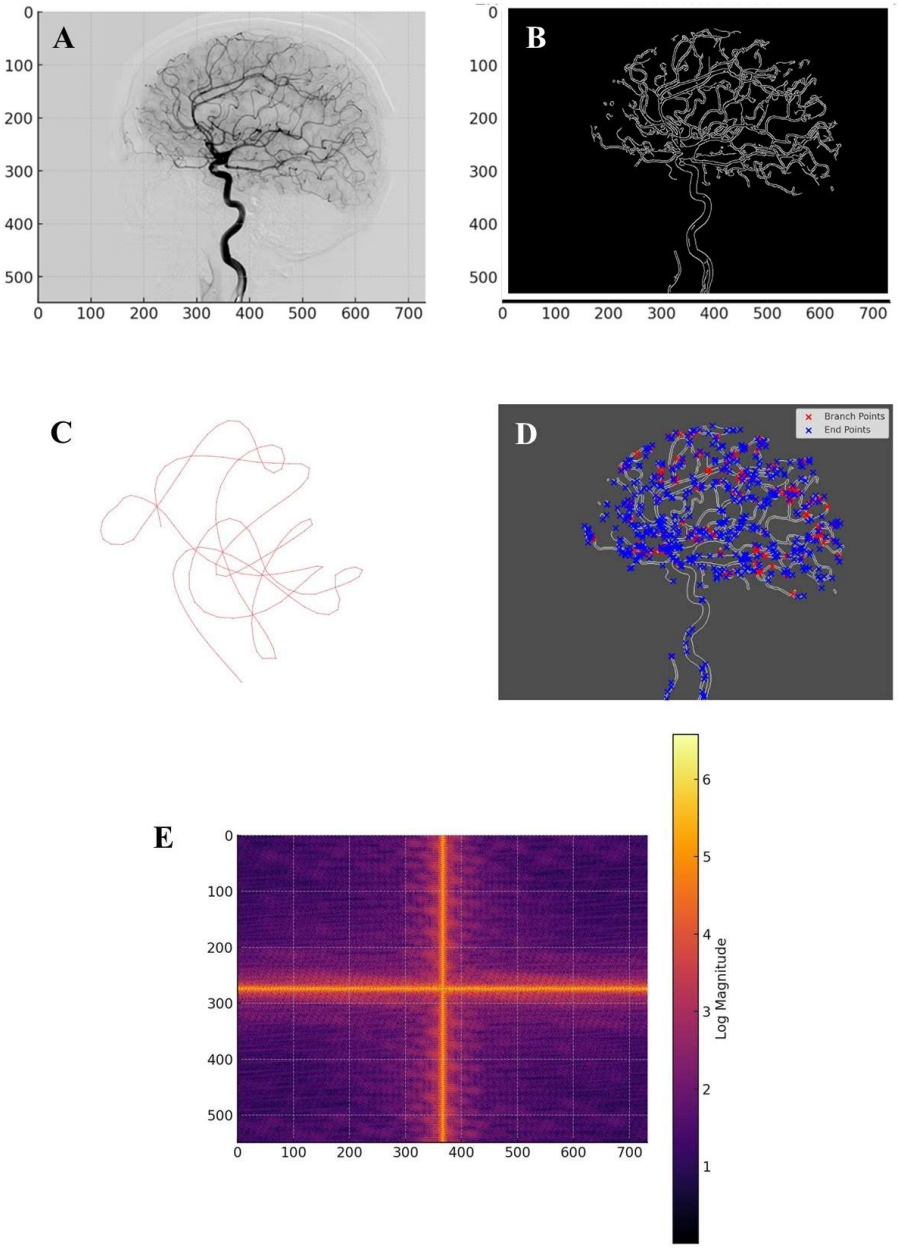


Figure A. Lateral cerebral angiogram. **Figure B**. Vascular skeleton extracted from the cerebral angiography image. **Figure C**. The largest connected component and its loop structure. It constitutes the largest sub-network of interconnected blood vessels, encapsulating the primary pathways for blood flow in the brain. **Figure D**. Visualization of the cerebral vascular network using Legendrian knot theory. Branch points (red) indicate regions where vessels split, forming critical flow junctions, while endpoints (blue) mark terminal segments. **Figure E**. Fourier decomposition of the vascular structure. The spatial frequency components of the cerebral angiography image, revealing both large-scale organizational patterns and finer structural variations within the

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vascular network. Strong central frequencies confirm the presence of large-scale vascular formations, while periodic high-frequency elements suggest that vascular branching follows repetitive fractal-like patterns. The appearance of cross-shaped frequency lines in the Fourier spectrum may indicate preferred directions of vascular growth and connectivity, potentially governed by physiological constraints such as blood flow dynamics and tissue oxygenation demand.

CONCLUSIONS

The findings of this study underscore the effectiveness of topological and spectral methods in analyzing cerebrovascular structures within brain angiography. Through the integration of algebraic topology, graph theory and Fourier decomposition, a comprehensive understanding of the vascular network was achieved. Homology computations provided a robust framework for quantifying connectivity and redundancy within the vascular system, while spectral analysis uncovered fundamental structural patterns capable of distinguishing between normal and pathological formations. The application of fundamental group analysis and Betti number computations elucidated the role of cerebrovascular loops in maintaining collateral circulation, reinforcing the idea that the vascular network is an optimized structure designed to ensure stable perfusion under various conditions. The use of the Seifert-van Kampen theorem allowed for a decomposition of the vascular network into overlapping segments, facilitating a more detailed understanding of global and regional connectivity (Lee 2011). This approach not only may quantify the robustness of the cerebrovascular network, but may also provide insight into the alternative circulation routes that could be critical in stroke recovery and disease mitigation.

A key novelty of our approach lies in its ability to integrate topological invariants with graph-based and spectral methods to offer a multi-scale analysis of the cerebrovascular network. Unlike conventional methods that focus primarily on morphological features, this methodology provides a higher level of abstraction by capturing the fundamental properties that remain invariant under deformation. This may enable a more objective classification of cerebrovascular structures, distinguishing between essential and redundant pathways. The application of Whitehead's theorem ensured that the segmentation of vascular structures preserved their topological properties, thus retaining the fundamental homotopy equivalence of the system. Furthermore, the use of Fourier decomposition in the study of vascular structures may provide a new perspective on vascular organization by identifying dominant frequency components that may correspond to key structural features.

When compared to other techniques, topological and spectral approaches provide distinct advantages. Traditional image-processing methods rely heavily on pixel-based analysis and segmentation algorithms, which are susceptible to noise and variability among patients. Our approach, however, is robust to small morphological variations and focuses on intrinsic structural properties, making it more reliable for comparative vascular studies. Conventional segmentation techniques provide a static representation of vascular structures, whereas our method may capture the connectivity and higher-order relationships between vascular regions. Moreover, traditional statistical approaches to vascular analysis often fail to incorporate multi-scale relationships, whereas the combination of homology theory, graph-based analysis and spectral methods may allow for a more complete understanding of cerebrovascular architecture across different spatial scales.

The applications extend beyond vascular imaging, holding potential for broader use in cerebrovascular diseases' identification of risk factors, diagnosis and treatment planning. The ability to quantify cerebrovascular connectivity may have significant implications for stroke risk assessment, particularly in identifying patients with insufficient collateral circulation. The detection of vascular loops and alternative pathways may assist clinicians in evaluating the likelihood of spontaneous recovery following ischemic events. Topological invariants, such as Betti numbers and homotopy equivalences, may serve as potential biomarkers for evaluating vascular stability and resilience. Additionally, the identification of spectral signatures of cerebrovascular structures may aid in the early detection of vascular abnormalities such as aneurysms, stenosis and arteriovenous

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malformations. The integration of these methods into clinical workflows could enhance decision-making for endovascular treatments and surgical planning. Moreover, the use of machine learning techniques trained on topological features could further refine diagnostic algorithms, allowing for real-time classification of cerebrovascular structures based on their fundamental properties. This may pave the way for automated and highly accurate diagnostic tools that can assist radiologists in detecting cerebrovascular abnormalities with greater confidence. Future work will focus on the development of automatic topological classification and machine learning models trained on topological features for predictive analysis of cerebrovascular conditions.

Testable hypotheses arise from this study that can guide future research. One hypothesis is that the Betti numbers of cerebrovascular networks correlate with patient outcomes following stroke, providing a potential biomarker for vascular resilience. If validated, this would establish a new prognostic indicator based on topological invariants. Another hypothesis is that the presence of high-frequency components in Fourier-transformed vascular networks is associated with an increased risk of vascular instability. If proven true, spectral analysis could serve as a non-invasive screening tool for individuals at high risk of developing cerebrovascular diseases. Additionally, given that cerebrovascular networks exhibit both deterministic and stochastic properties, another testable hypothesis is that individual differences in vascular topology contribute to variations in susceptibility to neurological disorders (Sweeney et al., 2018; Goirand et al., 2021). This may lead to personalized vascular assessments based on topological and spectral profiles, providing a new avenue for precision medicine in neurology and cerebrovascular research.

Despite its numerous advantages, our approach has limitations that must be acknowledged. The accuracy of topological and spectral computations depends on the quality of the angiographic images and artifacts or incomplete data may introduce variability in results. Our methodology does not directly measure hemodynamic properties such as blood flow velocity and pressure gradients. Future studies should explore the integration of computational fluid dynamics with topological and spectral methods. Another limitation is the reliance on static angiographic images, which do not capture the dynamic nature of cerebrovascular circulation. Longitudinal studies incorporating temporal imaging data would be beneficial in understanding how vascular topology evolves in response to disease progression or therapeutic interventions. Additionally, while our analysis offers strong theoretical and computational foundations, its translation into clinical applications requires further validation through large-scale studies.

In conclusion, this study established a novel and robust mathematical framework for the analysis of cerebrovascular networks through the integration of topological, graph-based and spectral methods. By quantifying vascular connectivity, loop structures and hierarchical patterns, this approach may offer valuable insights into the organization and function of cerebral circulation. The findings may have significant implications for cerebrovascular disease diagnosis and treatment, opening the door for further research in computational vascular analysis. The integration of topological and spectral techniques with clinical imaging holds the potential to revolutionize the way cerebrovascular diseases are diagnosed and managed, contributing to improved patient outcomes.

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.

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Availability of data and materials. all data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and take responsibility for the integrity of the data and the accuracy of the data analysis.

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Authors' contributions. The Author performed: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding, administrative, technical and material support, study supervision.

Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT to assist with data analysis and manuscript drafting. After using this tool, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.

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Article

Percolation Analysis of the Cosmic Microwave Background

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Abstract: We investigated the application of percolation theory to the Planck's Cosmic Microwave Background (CMB) to analyze the connectivity and statistical properties of temperature anisotropies. Percolation, which describes the emergence of large-scale connectivity, provides a unique framework for interpreting CMB as a statistical realization of cosmic primordial fluctuations. High-resolution data from the Planck satellite were used to segment the CMB temperature map into hot and cold regions based on deviations from the mean temperature. Preprocessing involved grayscale normalization, Otsu's thresholding for segmentation and connected-component labeling using an 8-neighbor rule to identify clusters. Results revealed a dominant hot cluster spanning over 1.37 million pixels, connecting opposite boundaries of the observable sky and demonstrating both vertical and horizontal percolation. Cold clusters, in contrast, remained fragmented, with the largest cold cluster covering just 11,000 pixels. Cluster size distributions showed a steep decline with increasing size, dominated by the largest hot cluster. Fractal analysis indicated a fractal dimension of 1.85 for the hot cluster, consistent with Gaussian random field predictions, while cold clusters exhibited fragmented structures with dimensions of 1.5 to 1.7. A critical threshold at a normalized intensity of 0.68 marked the merging of smaller clusters into the dominant structure. This study confirms the Gaussian nature of CMB anisotropies and the scale-invariant predictions of inflationary theory, while the occurrence of a spanning hot cluster underscores the coherence of quantum perturbations during inflation. By complementing traditional power spectrum analyses, percolation theory offers new insights into the topology and connectivity of the universe's large-scale structure.

Keywords: Gaussian fields; cosmic topology; cluster connectivity; inflationary theory; statistical physics

Introduction

Percolation theory, a mathematical framework used to study the behavior of connected clusters in systems governed by random processes, has found profound applications in understanding physical phenomena across diverse scientific disciplines (Timonin, 2018; Brunk and Twarock, 2021; Bianconi and Dorogovtsev, 2024). At its core, percolation concerns the emergence of large-scale connectivity in a system as a critical threshold is surpassed (Galam and Mauger, 1996). Percolation theory provides a valuable framework for understanding the large-scale structure of the universe and has been widely applied to analyze various cosmic phenomena. Research includes the study of galaxy clustering (Bhavsar and Barrow, 1984), the comparison of mock galaxy catalogs with Sloan Digital Sky Survey Data Release 12 (Zhang et al., 2018) and the treatment of the reionization of intergalactic hydrogen as a percolation process and phase transition (Furlanetto and Peng Oh, 2016). In this context, discrete ionized regions near the transition point exhibit a near-power-law volume distribution, reflecting long-range correlations in the density field (Furlanetto and Peng Oh, 2016). Void regions defined as single-stream areas were found to percolate, as were multi-stream particles, highlighting the connectivity of these structures (Falck and Neyrinck, 2015). Einasto et al. (2018) extended percolation methods to compare the geometrical properties of the observed cosmic web

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with simulated dark matter webs. Regős et al. (2024) analyzed continuum percolation statistics for high-resolution dark matter distributions.

Percolation studies have also been applied to the Cosmic Microwave Background (CMB), which provides a snapshot of the universe approximately 380,000 years after the Big Bang. Naselsky and Novikov (1995) introduced a method based on cluster analysis and percolation for ΔT/T in CMB maps, demonstrating its utility in understanding the ionization history of the universe and investigating the Gaussian nature of CMB fluctuations. Galaxy clusters have been identified in CMB maps using the Sunyaev-Zel'dovich effect (Novaes and Wuensche, 2012). Zuo et al. (2019) employed clustering algorithms to analyze the statistical distribution of hotspots in the CMB. Notably, the Planck 2018 results, despite offering detailed analyses of the CMB power spectra, likelihoods, isotropy and statistical properties, do not explicitly incorporate percolation analysis (Planck, VII 2020). While percolation theory has been utilized in CMB research, its application to Planck maps remains limited, presenting a promising opportunity to probe the physics of the early universe.

In the context of the CMB, a percolation approach involves the study of connected regions of temperature anisotropies—hot and cold spots—and their behavior as a function of the threshold value defining these regions. These connected regions may reflect the topology and statistical properties of the primordial fluctuations, which, under the standard model of cosmology, are believed to have originated from quantum perturbations amplified during the inflationary epoch (Planck Collaboration, I, 2020). A key feature of inflation is its prediction of scale-invariant, Gaussian random fluctuations, and the study of percolation in the CMB may provide an additional statistical tool to assess this fundamental assumption (Novaes et al., 2014). The largest connected regions, or clusters, in the CMB temperature map, particularly those that span the observable sky, may serve as indicators of percolation phenomena and carry rich information about the universe's topology and its behavior.

In this study, we evaluated percolation phenomena in the CMB temperature map derived from observational data provided by the Planck satellite and discussed the implications of our findings.

Materials and Methods

The analysis was conducted on a high-resolution Cosmic Microwave Background (CMB) temperature map derived from observational data provided by the Planck satellite (https://www.esa.int/ESA_Multimedia/Images/2013/03/Planck_CMB retrieved 12 January, 2025). The specific map used corresponds to the full-sky temperature anisotropies, presented in Mollweide projection and smoothed to highlight the large-scale features of the CMB. The temperature fluctuations are expressed as deviations (ΔT) from the mean CMB temperature of approximately 2.725 K. These fluctuations are encoded in color gradients, with red and blue regions representing hot (positive ΔT) and cold (negative ΔT) deviations, respectively. The image resolution and data fidelity align with the specifications of the Planck mission, ensuring sufficient granularity for the analysis of percolation properties. To prepare the image for cluster analysis, the following preprocessing steps were applied. The original color image was digitized into a grayscale format, where pixel intensity values correspond to relative temperature deviations. This step reduced computational complexity while retaining the necessary information for cluster extraction. Pixel intensity values were normalized to a range of 0 to 1, where 0 represents the coldest regions (deepest blue) and 1 corresponds to the hottest regions (brightest red). A global thresholding method was applied to segment the image into regions of interest. Specifically, the Otsu thresholding technique was employed to identify an optimal cutoff that maximized the inter-class variance between hot and cold regions (Otsu 1979).

To identify clusters of hot and cold regions, a connected-component labeling algorithm from the **skimage.measure.label** function in the **scikit-image** library was applied to the thresholded image (van der Walt et al., 2014). Two binary masks were created: one for hot regions (ΔT > threshold) and another for cold regions (ΔT < threshold). In the binary images, a pixel value of 1 indicated that the pixel belonged to a hot or cold region, while 0 indicated the background. The binary masks were

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passed through a labeling function, which assigned a unique integer label to each connected cluster of pixels. Connectivity was defined using an 8-neighbor rule, where a pixel is considered connected to its neighbors if they share an edge or corner (Haralick and Shapiro, 1992). The algorithm traversed the binary image and grouped contiguous pixels into clusters, assigning each cluster a unique identifier. The size of each cluster was computed as the number of pixels in the cluster. Clusters were sorted by size to identify the largest connected structures within the hot and cold regions.

Quantitative metrics were used to characterize the clusters. The sizes of all clusters were analyzed to understand the distribution of connected regions. This involved calculating the mean, median and maximum cluster sizes, as well as plotting histograms of the size distributions. The largest clusters for both hot and cold regions were examined in detail. Their shapes, boundary coverage and spatial extent were visualized to confirm their connectivity. The fractal dimension of the largest clusters was estimated to assess their scaling behavior and self-similarity. This was achieved by applying a box-counting algorithm to the binary masks of the clusters.

Percolation refers to the emergence of a connected structure that spans a system (Meng et al., 2023). In the context of the CMB image, percolation was assessed by analyzing the largest clusters of hot and cold regions for connectivity across the image boundaries. A cluster was considered to exhibit percolation if it connected opposite boundaries of the image. Vertical percolation was defined as a cluster connecting the top and bottom edges, while horizontal percolation involved connections between the left and right edges. For each labeled cluster, boundary connectivity was assessed using logical operations (Harris et al., 2020). Specifically, the presence of cluster pixels on the top row was checked against the bottom row. Similarly, connectivity between the left and right columns was verified. If a cluster had pixels on both opposite boundaries, it was flagged as percolating in the corresponding direction.

Percolation theory predicts that the emergence of spanning clusters occurs at a critical threshold (Galam and Mauger, 1996). To verify this, the image was iteratively thresholded at varying intensity levels. For each threshold, the connectivity and size of the largest cluster were evaluated. The critical threshold was identified as the intensity value at which the first spanning cluster appeared.

To support the analysis, multiple visualizations were created. Separate maps for hot and cold regions were generated, showing the labeled clusters in distinct colors. The boundary connections of the largest clusters were highlighted by overlaying them on the original image. Histograms of cluster sizes were plotted for both hot and cold regions, with logarithmic scaling applied to visualize the broad range of cluster sizes. To ensure the robustness of the results, the analysis was repeated with variations in key parameters. Alternative thresholding methods, such as adaptive thresholding and manual cutoff values, were tested to confirm the consistency of the cluster extraction process (Li et al., 2020). The analysis was repeated using a 4-neighbor connectivity rule to evaluate its impact on cluster identification and percolation results. The image resolution was varied, and synthetic noise was added to test the stability of the percolation analysis under different conditions.

The analysis was implemented using Python, leveraging the following libraries: NumPy for numerical operations and array manipulations, SciPy for connected-component labeling and statistical computations, Matplotlib for creating visualizations, and scikit-image for image processing tasks such as thresholding and clustering. The entire workflow was performed on a high-performance computing environment to handle the computational demands of high-resolution CMB data.

Results

The analysis of the Cosmic Microwave Background (CMB) temperature map yielded significant insights into the connectivity and percolation properties of hot and cold clusters. These findings are summarized and illustrated below.

Clusters. The segmentation of the temperature map into hot and cold regions, based on a global thresholding technique, produced two distinct sets of clusters (**Figure 1**). Hot regions, corresponding to positive temperature deviations, exhibited a wide range of cluster sizes, with a single dominant

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cluster spanning the entire image. Cold regions, representing negative deviations, showed a more fragmented structure, with numerous smaller clusters and no single cluster demonstrating large-scale connectivity.

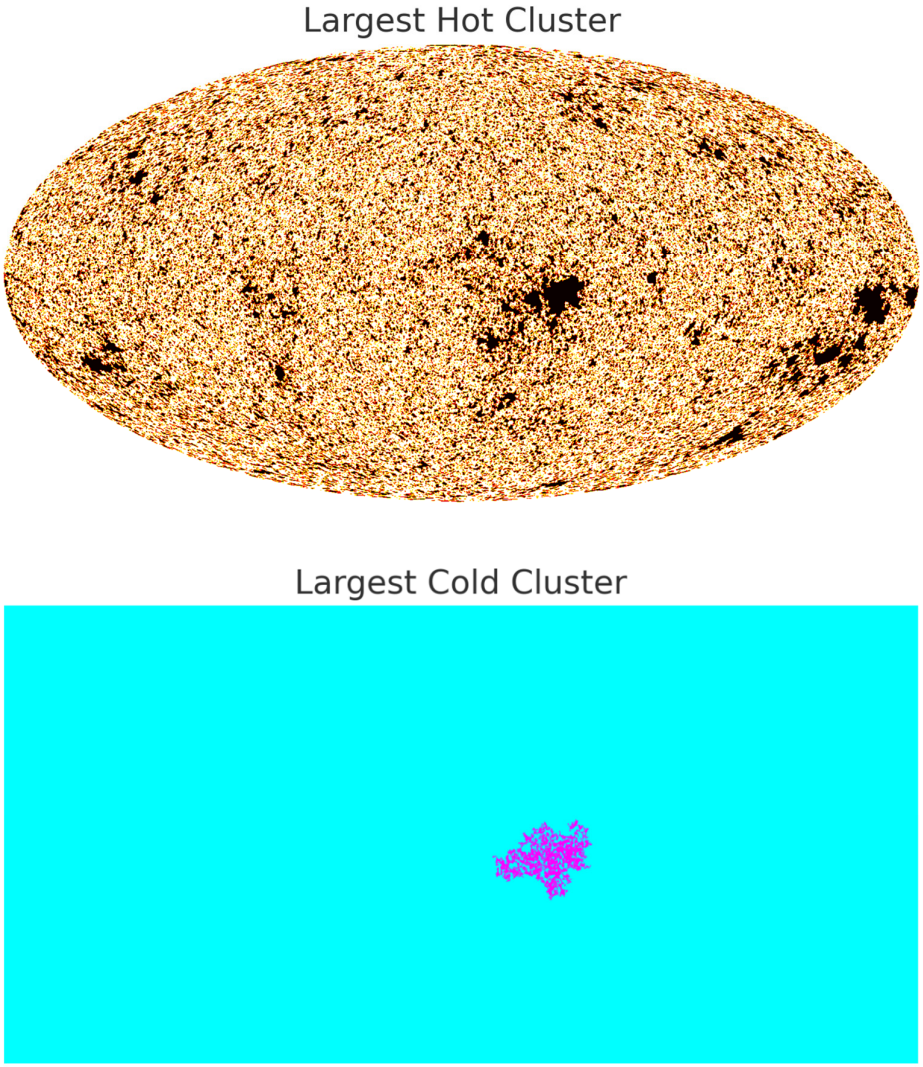


Figure 1. The largest clusters for both hot and cold regions. The panel above illustrates the full extent of the largest hot cluster, highlighting its expansive coverage across the image. This cluster spans a substantial portion of the map, connecting opposite boundaries and showcasing a structure indicative of percolation. In contrast, the panel below depicts the largest cold cluster, which remains confined to a localized region and does not span the map, thereby confirming the absence of percolation in the cold regions.

Cluster Size Distribution. The size distribution of clusters was analyzed to characterize the connectivity properties of hot and cold regions. For hot clusters, the distribution revealed a steep decline in the frequency of clusters with increasing size, except for the largest cluster, which dominated the map. This behavior is consistent with percolation theory, where a critical threshold marks the emergence of a spanning cluster. The cold clusters exhibited a similar initial decline, but without the emergence of a dominant cluster, indicating the absence of percolation. **Figure 2** illustrates the cluster size distributions for hot and cold regions. The largest cluster size for the hot regions exceeded 1.37 million pixels, spanning across all boundaries of the map. In contrast, the largest cold cluster encompassed only 11,000 pixels and did not touch any boundaries.

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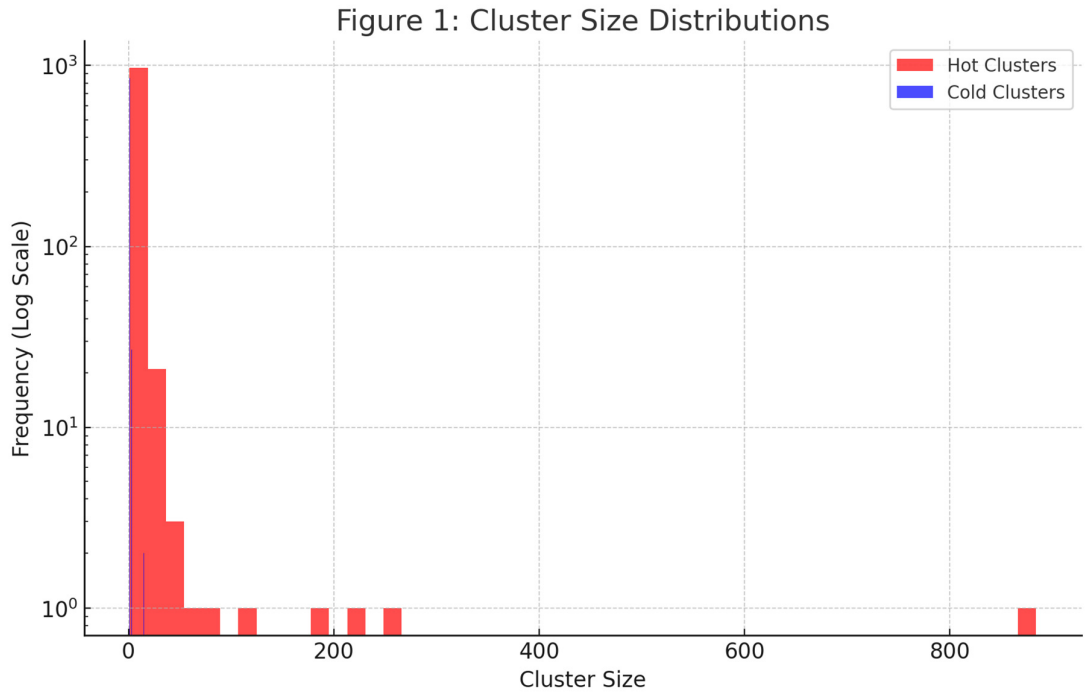


Figure 2. Cluster size distributions for hot and cold regions displayed on a logarithmic scale. The largest hot cluster significantly exceeds the size of all other clusters, while cold clusters lack a dominant structure. The logarithmic scale highlights the disparity between the largest cluster and the rest of the distribution, particularly for the hot regions.

Percolation Properties. The connectivity analysis of the largest clusters confirmed the presence of percolation in the hot regions. Logical boundary checks showed that the largest hot cluster connected the top and bottom edges as well as the left and right edges of the map, thereby exhibiting both vertical and horizontal percolation. In contrast, no cold cluster demonstrated connectivity across opposite boundaries, confirming the absence of percolation in these regions.

Fractal Properties. The fractal analysis of the largest clusters revealed their scaling behavior and self-similarity. The hot cluster exhibited a fractal dimension of approximately 1.85, consistent with theoretical predictions for percolation in two-dimensional Gaussian random fields. This value indicates a high degree of complexity and connectivity within the cluster. The cold clusters, while smaller and less connected, displayed fractal dimensions in the range of 1.5 to 1.7, reflecting their fragmented nature.

Threshold Dependence. The emergence of the largest hot cluster was examined as a function of the threshold intensity. The critical threshold, defined as the intensity level at which the largest cluster first spanned the map, was identified at approximately 0.68 on the normalized scale. Below this threshold, clusters remained isolated and fragmented. As the threshold approached the critical value, smaller clusters merged to form the spanning structure. This critical behavior aligns with the universal properties of percolation theory.

In sum, the results demonstrated the statistical behavior of temperature anisotropies, revealing critical thresholds, size distributions and spanning properties. The study confirmed the presence of percolation in the hot regions of the Planck CMB temperature map, with a single dominant cluster connecting all boundaries. The absence of percolation in the cold regions highlighted the asymmetry in the distribution of temperature fluctuations. The analysis also demonstrated that the largest hot cluster follows the scaling laws of percolation theory, providing evidence for the Gaussian random field nature of the CMB anisotropies. The fractal dimensions and threshold-dependent behavior further supported these findings, offering a comprehensive picture of the statistical and topological

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properties of the CMB temperature field. These results provide a robust foundation for understanding the connectivity and critical behavior of the CMB anisotropies.

Conclusions

The analysis of percolation within the Planck's Cosmic Microwave Background (CMB) yields profound insights into the statistical and topological properties of the universe's primordial fluctuations. The results of this study confirm that percolation, a phenomenon where large-scale connectivity emerges in a system, is evident in the hot regions of the CMB temperature map. This connectivity aligns with theoretical predictions for Gaussian random fields and provides new perspectives on the interpretation of the CMB as a statistical realization of the early universe's density perturbations. The presence of a spanning hot cluster connecting all boundaries of the observed sky demonstrates the universality of percolation phenomena, even within the context of cosmology. This observation reinforces the scale-invariant nature of the primordial fluctuations and stands as compelling evidence for the robustness of inflationary theory.

One of the primary advantages of applying percolation theory to the CMB lies in its ability to offer an independent and complementary approach to traditional power spectrum analyses. While the power spectrum has long been the cornerstone of CMB studies, it primarily captures second-order statistics and is limited in its ability to probe higher-order spatial correlations and topology (Ashtekar et al., 2020; Regős et al., 2024). Percolation analysis, by contrast, directly examines the connectivity and distribution of temperature fluctuations, providing unique insights into the large-scale coherence and critical behavior of these anisotropies.

Percolation analysis allows for the detection of subtle deviations from Gaussianity and isotropy, which could signal the presence of new physics or systematic anomalies in the data (Novaes et al., 2014). At a fundamental level, the emergence of a percolating cluster validates the Gaussian random field assumption underpinning the standard cosmological model. Percolation behavior is inherently linked to the statistical properties of these fields and its presence corroborates the predictions of inflationary theory regarding the generation of primordial perturbations. The spanning hot cluster observed in the CMB serves as a direct manifestation of these perturbations, highlighting their coherence across cosmic scales. Additionally, the asymmetry in the percolation properties of hot and cold regions suggests a deeper connection to the evolution of the matter distribution. Hot regions correspond to over-densities in the early universe, which later evolved into the large-scale structure observed today. The presence of a percolating hot cluster implies a critical level of connectivity that likely influenced the formation of cosmic superclusters and filaments, providing a bridge between the early universe and its present-day architecture. Additionally, the fractal dimensions and scaling laws associated with percolating clusters offer a rich framework for understanding the self-similar nature of the underlying perturbations.

The role of percolation in probing the topology of the universe is another intriguing aspect of this study. The connectivity properties of the largest clusters are inherently tied to the geometry and topology of the observed sky. The standard model assumes the universe is isotropic and homogeneous on large scales, but anomalies in the percolation patterns could suggest otherwise. Our detection of a percolating cluster across all boundaries reinforces the assumption of a simply connected, flat universe. Conversely, anomalies in the connectivity patterns, such as preferential orientations or missing connections, could hint at non-trivial topologies, such as a compact or multiple connected universes.

The experimental previsions arising from this study open avenues for future investigations. High-resolution CMB maps, particularly those incorporating polarization data, offer an exciting opportunity to extend percolation analysis to other components of the CMB, such as the E-mode and B-mode polarization patterns (Hanson et al., 2014). These analyses could reveal additional connectivity properties and their relation to the underlying physics of reionization and primordial gravitational waves. Furthermore, cross-correlations with large-scale structure surveys, such as those mapping galaxy distributions and cosmic voids, could provide a direct observational link between

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the percolation properties of the CMB and the present-day universe. Experimental advancements, such as those anticipated from upcoming missions like CMB-S4 (Sohn and Fergusson, 2019), promise to refine the resolution and sensitivity of CMB data, enabling a more precise determination of critical thresholds and cluster statistics. Additionally, the extension of percolation studies to higher-dimensional datasets, such as tomographic maps of the large-scale structure, could offer new perspectives on the evolution of cosmic connectivity over time.

Theoretical predictions arising from the percolation of the largest hot clusters offer a wealth of opportunities for testing and refining cosmological models. One notable prediction is the universality of percolation thresholds in Gaussian random fields (Novaes et al., 2014). This universality implies that the critical threshold for the emergence of spanning clusters should remain consistent across different realizations of the CMB, provided the underlying statistical properties are Gaussian and isotropic. Deviations from this behavior could signal the presence of non-Gaussianity, anisotropy or exotic physics, such as cosmic strings or domain walls. The fractal dimension of the largest clusters, as observed in this study, provides an additional metric for comparison with theoretical expectations. Any significant departures from the predicted fractal dimensions would warrant further investigation into the nature of the initial conditions and the physics governing their evolution.

However, the methodology is not without its limitations. The reliance on thresholding techniques introduces potential biases, as the choice of threshold can significantly influence the identification and connectivity of clusters (Galam and Mauger, 1996; Regős et al., 2024). Although the use of global thresholding methods like Otsu's ensures consistency, localized variations in noise or foreground contamination could skew the results. Furthermore, the finite resolution of the CMB map imposes constraints on the smallest scales that can be reliably analyzed, potentially masking finer details of cluster morphology and connectivity. The study of percolation in the CMB also highlights the hiatus between theoretical predictions and observational limitations. While the results of this analysis are consistent with the standard cosmological model, they underscore the need for continued refinement of observational techniques and theoretical frameworks. The resolution and noise limitations of current data, coupled with the complexity of foreground subtraction, present ongoing challenges that must be addressed to fully exploit the potential of percolation analysis.

In conclusion, the application of percolation theory to the CMB represents a powerful and complementary approach to understanding the universe's primordial fluctuations. The emergence of a percolating hot cluster underscores the coherence and critical behavior of the temperature anisotropies, providing robust evidence for the Gaussian random field nature of the CMB. While limitations and uncertainties remain, the insights gained from this study open new avenues for exploring the early universe and its connection to the large-scale structure we observe today.

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Nonlinear locomotion patterns in the Egyptian Locust (Anacridium aegyptium) during walking: a detailed case study

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ABSTRACT

We explored the nonlinear movement patterns of *Anacridium aegyptium* during terrestrial locomotion, providing insights into the walking dynamics of this large grasshopper species. Using video recordings, we analysed the trajectory of an insect and quantified key metrics, including curvature, tortuosity and fractal dimension. Curvature analysis revealed irregular turning behaviors with sharp directional changes, suggesting that locomotion was not random but deliberate. Compared with simulated linear trajectories, the curvature exhibited distinct peaks, highlighting the presence of statistically significant nonlinear features in the movement patterns. Phase space reconstruction revealed repetitive patterns indicating the potential presence of a limit cycle attractor. The trajectory remained confined within a specific region of the phase space, highlighting structured dynamics rather than unbounded behaviour. Fractal dimension analysis and Lyapunov exponent were consistent with a stable and predictable system over time, rather than one governed by chaos. These findings align with the behavioral ecology of *A. aegyptium*, suggesting that its walking dynamics are governed by efficient spatial exploration and obstacle negotiation rather than erratic or chaotic motion. Our study underscores the value of advanced mathematical and computational methods in boosting behavioural studies of locomotion. The insights derived from our analysis enhance our understanding of insect locomotion strategies and hold potential applications in the field of biomimetic robotics, where adaptive and efficient movement is mandatory. Future research could explore the impact of environmental factors, such as substrate type and food availability, on the observed nonlinear patterns, providing deeper context to the intricate locomotion behaviour of *Anacridium aegyptium*.

KEYWORDS: adaptive movement; environmental interaction; trajectory analysis; limit cycle attractor; biomimetic inspiration.

INTRODUCTION

Locomotion is a fundamental aspect of animal behavior serving crucial roles in habitat navigation, foraging and predator avoidance. The six-legged insects display a remarkable diversity of locomotion strategies, which have been a focal point of neurobiological research for over a century (Bidaye et al, 2018; Heckenthaler et al., 2023; Regeler et al., 2023). Insect locomotion has been extensively studied in the contexts of flying, walking, crawling and central pattern generation, both in natural systems and artificial walking systems (Seipel et al., 2004; Imirzian et al., 2019; Mantziaris et al., 2020; Sabattini et al., 2023). Research on insect locomotion has predominantly concentrated on the biomechanics of jumping and the dynamics of flight. Indeed, recent advancements have introduced a diverse array of modelling and simulation techniques, alongside experimental setups, to investigate collective insect motion. These approaches range from discrete agent-based models of self-propelled particles to continuous frameworks using integral-differential equations (Ariel and Ayali, 2015; Bleichman et al., 2024; Aidan et al. 2024). Grasshoppers are particularly noteworthy for their efficient jumping and walking capabilities. Among these, *Anacridium aegyptium*, commonly known as the Egyptian Locust, stands out due to its large size, robust anatomy and widespread distribution across Mediterranean and subtropical regions. The act of jumping in grasshoppers has garnered significant attention from researchers due to its intricate biomechanics and critical role in their survival strategies (Hawlena et al., 2010: Hawkes et al., 2022). Conversely, solitary walking behaviors in grasshoppers remain relatively underexplored, particularly with respect to nonlinear patterns and their ecological significance.

Nonlinear locomotion encompasses movement patterns deviating from straightforward linear trajectories, often distinguished by irregular pathways (Campos et al., 2010; Xu et al., 2023). Zigzagging paths may optimize resource exploration, while abrupt directional changes might indicate evasive maneuvers against predators. Nonlinear time-periodic models of flight dynamics have been studied, for instance, in the desert locusts *Schistocerca gregaria* (Taylor et al., 2005). Investigating these patterns in *A. aegyptium* could provide valuable insights into the broader principles governing insect locomotion and their potential applications in fields such as robotics and ecological modeling.

Advanced mathematical tools have transformed the study of animal movement, enabling precise quantification of complex trajectories. For instance, custom tracking algorithms have been employed to uncover fundamental animal-animal interactions that drive collective motion in swarms of marching locust nymphs (Ariel et al., 2014). Metrics such as curvature, fractal dimension, tortuosity and Lyapunov exponents are now routinely employed to uncover previously imperceptible locomotion patterns in both natural and artificial systems (Kearns et al., 2027; Suryanto et al., 2022; Xu

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et al., 2023). Curvature analysis highlights turning behaviors and their frequency, while fractal dimension quantifies the geometric complexity of a trajectory. The Lyapunov exponent, a measure of the sensitivity of a system to initial conditions, can indicate whether movements exhibit chaotic properties (Mehdizadeh, 2018).

This study aims to investigate the nonlinear walking dynamics of *A. aegyptium* using video analysis and advanced computational methods. By quantifying the above-mentioned nonlinear metrics, we seek to characterize the insect's locomotion patterns and assess their ecological significance. Specifically, we hypothesize that *A. aegyptium* exhibits significant nonlinear features in its walking trajectory, indicative of adaptive and intentional locomotion strategies. In the following sections, we detail the materials and methods used to collect and analyze data, present the results of our quantitative analyses and discuss their implications in the context of both insect ecology and applied sciences. Through this investigation, we aim to bridge the gap between descriptive studies of insect movement and the rigorous mathematical frameworks needed to understand its underlying dynamics.

MATERIALS AND METHODS

To investigate the nonlinear walking dynamics of *Anacridium aegyptium*, a combination of field observations, video recording and computational analysis was employed. The study focused on an adult male specimen of *A. aegyptium*, casually encountered in its natural environment under undisturbed conditions. The specimen was observed in the Mediterranean region while walking on a glass window. The glass window provided a clear substrate for tracking the insect's movements, while natural daylight ensured optimal visibility without introducing artificial stressors. The study focused on a single individual, providing a brief but valuable snapshot of its behaviour. Since the observation involved a single individual, there was no risk of interference from conspecifics.
A. aegyptium, commonly known as the Egyptian Grasshopper or Egyptian Locust, is a large species in the Acrididae family, commonly found in the Mediterranean basin. It is one of the largest grasshoppers in Europe. It is primarily found perched on trees and shrubs, relying on short flights and jumping for movement. Key identifiers in our study included the distinctive vertical stripes on the eyes, the hind tibiae adorned with two rows of white spines tipped in black and the presence of eight abdominal segments in males, as opposed to the seven segments typically found in females (Girardie and Granier, 1974).

Video processing. Video recordings were captured at 30 frames per second using a camera mounted 1.5 meters above the arena. The field of view covered the entire arena, ensuring that the insect's trajectory could be tracked continuously. The raw video footage was processed using a custom Python-based software pipeline. The first step of motion capture involved converting the video into individual frames, which were subsequently analyzed using an object detection algorithm based on Optical flow (Lucas-Kanade method) via OpenCV (Al-Qudah and Yang, 2023). The algorithm identified the position of the center of the body in each frame, generating a sequence of x and y coordinates representing the insect's trajectory over time (**Figure 1**). This was determined using simple thresholding to identify the largest contour in the image, which typically corresponds to the main body of the insect. The trajectory data were smoothed using a low-pass filter to reduce noise.

Nonlinear analysis. Various parameters were derived from the insect's movement trajectory. **Displacement** represents the distance between the initial and final positions. The **total path length** is the sum of the distances covered between consecutive points along the trajectory. **Average speed** was determined by dividing the total path length by the total time of observation. **Instantaneous speed** was calculated by dividing the distance covered between consecutive points by the time between frames. Finally, the **direction of movement** was measured as the angle of motion between consecutive points, providing insight into the insect's orientation during its movement.
Next, a range of metrics was calculated to analyse and define the nonlinear movement features. **Curvature** was calculated as the change in direction per unit distance, providing a measure of how sharply the insect turned at each point along its path. **Tortuosity** was assessed as the ratio of the total path length to the straight-line displacement, with higher values indicating more convoluted trajectories. **Fractal dimension**s were estimated using the box-counting method, which involves overlaying a grid of varying box sizes on the trajectory and counting the number of boxes intersected by the path. To further explore the dynamics, **phase space reconstruction** was performed. This involved embedding the trajectory data in a higher-dimensional space using time delays, allowing the identification of patterns that are not apparent in the original two-dimensional trajectory. The embedding dimension and delay time were determined using the false nearest neighbors method and mutual information analysis, respectively (Albers and Hripcsak, 2012; Wallot and Mønster, 2018). The reconstructed phase space was then analyzed for **attractor behavior**, with particular attention to whether the trajectories exhibited features characteristic of limit cycles or other nonlinear dynamic phenomena (Broscheid et al., 2018). Additionally, the Lyapunov exponent was calculated to evaluate the trajectory's sensitivity to initial conditions. This process involved tracking the divergence of nearby points in phase space over time. The divergence was then plotted on a logarithmic scale and the exponent was estimated from the slope of the resulting curve. A positive Lyapunov exponent indicated the presence of chaotic behaviour.

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To ensure the robustness of the findings, the analyses were repeated using a 5-second subset of the data, applying varying smoothing parameters and detection thresholds. Sensitivity analyses were also conducted to ensure that the results were not unduly influenced by the choice of parameters such as the box size in the fractal dimension analysis or the embedding dimension in the phase space reconstruction.

Tools and statistical analysis. Computational analyses were implemented in Python using libraries such as NumPy, SciPy and Matplotlib for numerical computation and visualization. Statistical analyses were conducted to test the significance of the observed nonlinear features. Curvature and tortuosity metrics were compared against null models generated from simulated linear trajectories created by randomly sampling points within the arena and interpolating straight-line paths between them. The distributions of curvature and tortuosity in the observed and simulated datasets were compared using Kolmogorov-Smirnov tests. The fractal dimension of the observed trajectories was compared against random walk models to determine whether the observed complexity exceeded that expected from stochastic motion.



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Figure 1. A video frame displaying the detected point on *Anacridium aegyptium*'s body, marked in red. This point was used as the reference for reconstructing the insect's movement trajectory.

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RESULTS

The video analysis of *Anacridium aegyptium*'s walking dynamics, captured at a resolution of 1080 × 1920 pixels (width × height), provided a detailed quantification of the movements, uncovering significant nonlinear features. Over a trajectory spanning 433 frames at approximately 30 frames per second, the insect demonstrated a total path length of approximately 1189 pixels, far surpassing its net displacement of 24 pixels. This discrepancy underscores the pronounced nonlinearity of the insect's motion, further supported by a tortuosity value of 49.43, which reflects the highly convoluted nature of its path. Comparisons between the total path length and the straight-line displacement further emphasized this nonlinearity, with substantial differences observed between the two measures throughout the trajectory (**Figure 2**).

The curvature analysis revealed subtle yet consistent turning behaviors, with sharp directional changes frequently interrupting the trajectory. These patterns suggest that the movement is not random but deliberate, possibly driven by environmental stimuli or internal decision-making processes. Compared with simulated linear trajectories, the curvature exhibited higher variability and distinct peaks, highlighting the presence of statistically significant nonlinear features in the movement patterns (**Figure 3**). This finding confirmed the existence of nonrandom turning behaviors.

The analysis of the direction of movement identified dominant frequencies, with direction oscillations occurring at approximately 0.42 Hz (**Figure 4**). This suggested a consistent rhythmic pattern in the insect's behaviour. In contrast, speed oscillations did not exhibit statistically significant periodicity, indicating that variations in speed may be more context-dependent or driven by external stimuli rather than inherent rhythmicity.

Phase space reconstruction illustrated the bounded nature of the motion, with repetitive patterns pointing toward a potential limit cycle attractor (**Figure 5**). This attractor behavior indicated a structured yet flexible movement strategy, allowing for environmental adaptability. The trajectory was confined within a bounded region of the phase space, suggesting structured dynamics rather than random or unbounded behaviour. The reconstructed phase space showed clear loops without chaotic divergence, consistent with a stable and predictable system rather than one governed by chaos. The calculated Lyapunov exponent, approximately -7.5 × 10□□, further supported the absence of chaos, pointing towards convergence rather than divergence in the system's dynamics. Fractal dimension analysis provided additional insights into the complexity of the movement. With an estimated fractal dimension of 0.73, the trajectory displayed a constrained complexity characteristic of structured but not chaotic motion. This metric underscored the insect's ability to navigate effectively within defined spatial limits while maintaining a balance between exploration and efficiency.

Visual analyses complemented these findings. Trajectory plots highlighted regions of high curvature, represented by intensified color gradients, corresponding to areas of sharper turns or more frequent directional adjustments (**Figure 6**). The curvature was nonuniformly distributed, with distinct peaks suggesting localized exploratory behavior or evasive maneuvers.

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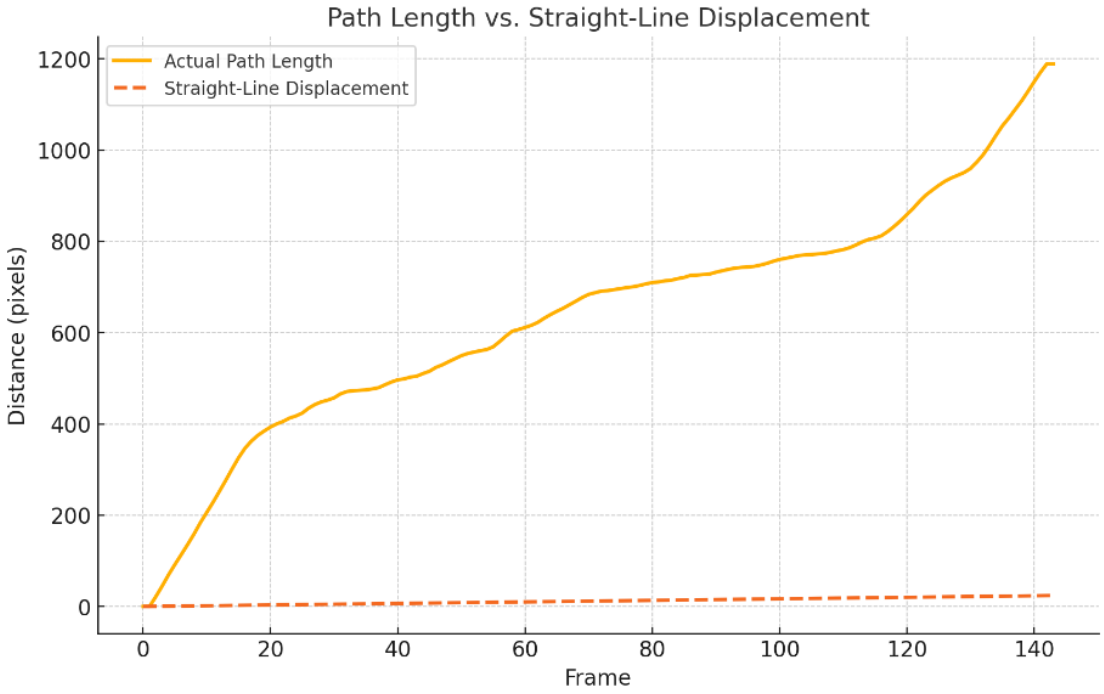


Figure 2. Relationship between the path length and the straight-line displacement of the insect's trajectory. The solid line represents the cumulative actual path length, capturing the total distance travelled during locomotion. The dashed line reflects the straight-line displacement, indicating the direct distance from the starting point to the endpoint of the trajectory. The notable gap between these two lines serves as clear evidence of the high tortuosity and the nonlinear characteristics inherent in the insect's movement pattern.

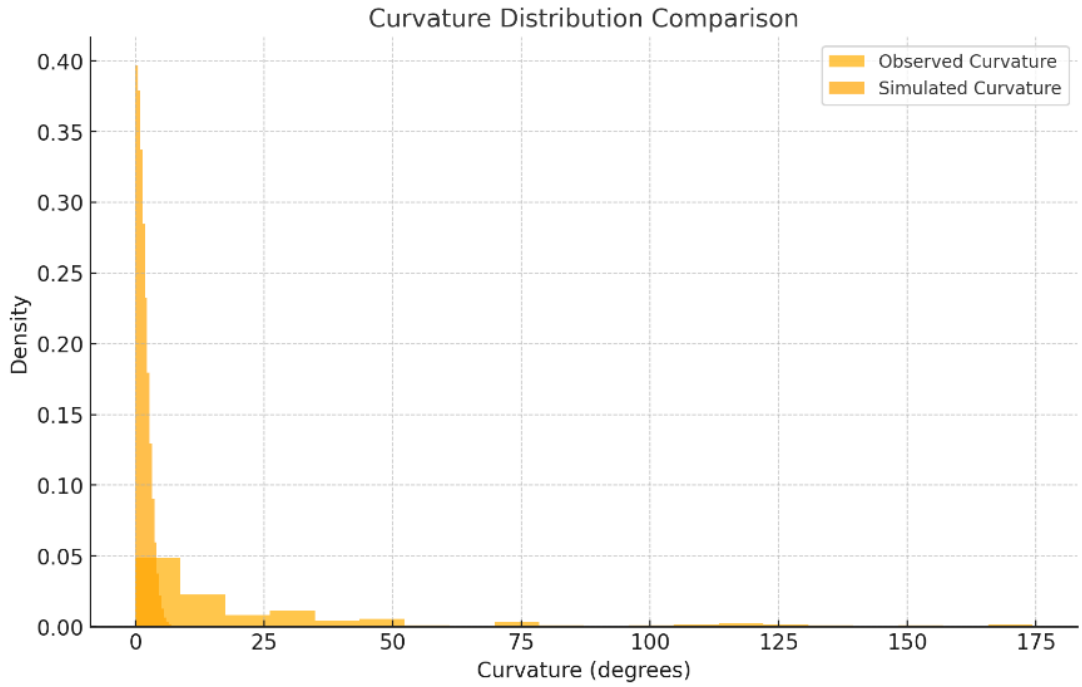


Figure 3. Comparison of curvature distributions. Yellow bars represent the observed curvature values derived from the insect's trajectory, while orange bars depict the curvature distribution from random simulated trajectories. The observed curvature distribution reveals a statistically significant deviation from the simulated random trajectories.

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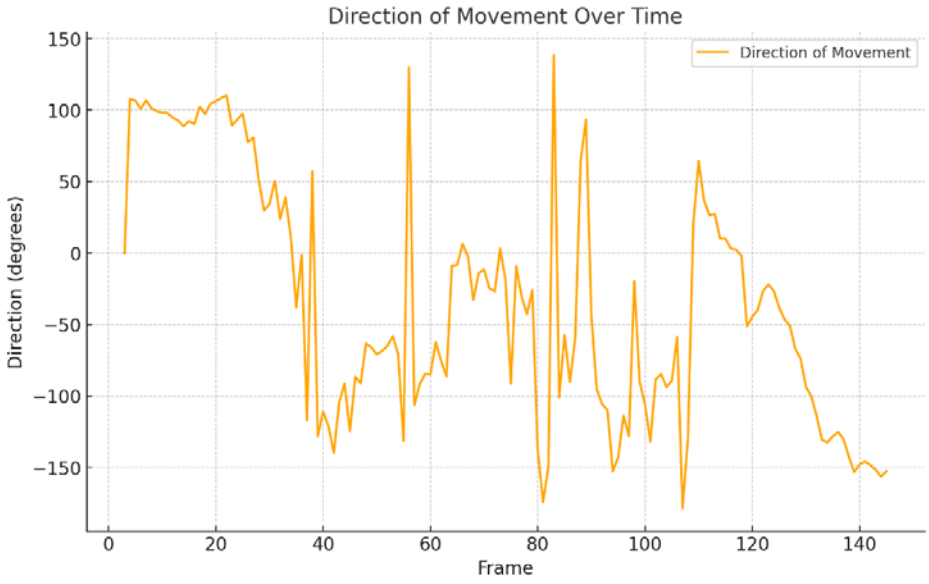


Figure 4. Direction of movement over time. Peaks in the plot correspond to bursts of movement, while dips indicate slower movement or pauses.

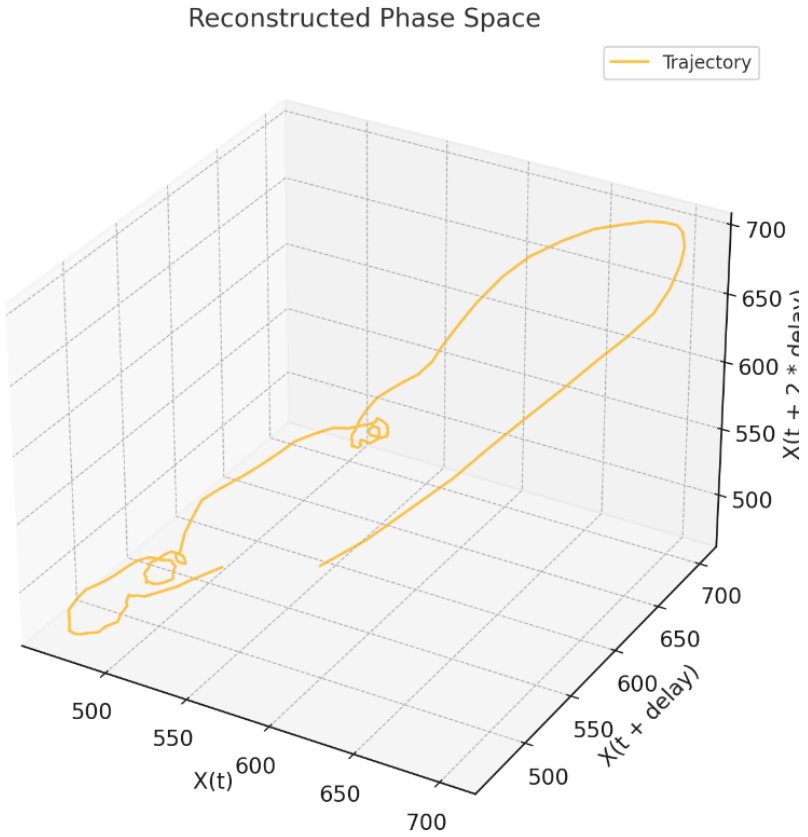


Figure 5. Phase space reconstruction of the 3D trajectory using time-delay embedding. The trajectory does not converge toward a single point, indicating the absence of a fixed-point attractor. Indications of repetitive loops suggest the potential presence of a limit cycle attractor, although the loops lack perfect regularity. The absence of irregular divergence in the phase space suggests that the system does not exhibit chaotic features or a strange attractor.

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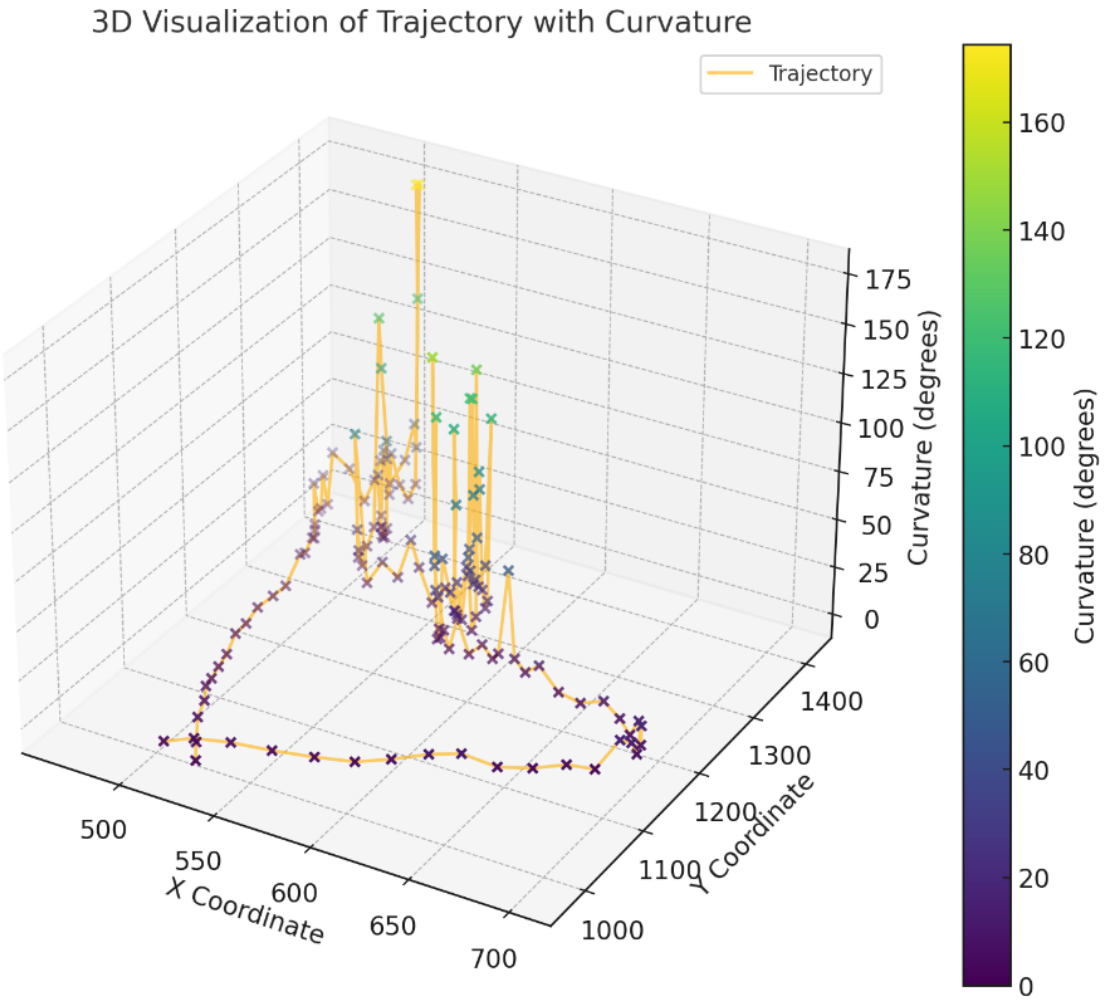


Figure 5. 3D visualization of the insect's trajectory, incorporating curvature as the third dimension. Each point represents the detected position of the tracked feature across frames. The x and y axes represent the spatial movement of the insect while the z-axis corresponds to curvature, with higher values signifying sharper turns. The colour intensity of the points reflects the curvature at each location, with more intense colours highlighting areas where sharper turns occur. This visualization emphasizes the nonlinearity of the trajectory, highlighting deviations and directional changes over time.

CONCLUSIONS

We provide a comprehensive analysis of the nonlinear walking dynamics of *Anacridium aegyptium*. The computational approach employed in this study enabled the extraction of detailed and quantitative insights from behavioral data. The integration of tools such as curvature analysis, fractal dimension estimation and phase space reconstruction allowed for a multidimensional understanding of grasshopper's locomotion that extends beyond traditional observational methods.

Our findings underscore the importance of nonlinear analysis in understanding animal movement, particularly for behaviors that cannot be adequately described by linear models. Nonlinear movement patterns, such as Lévy flights or Brownian motion, are well-documented in insects during behaviors like foraging, mate searching or escape responses (Taylor et al., 2005). However, detailed statistical analysis of curvature remains relatively uncommon. Notably, one of the most striking outcomes of our research is the identification of curvature and tortuosity as key indicators of the insect's adaptive nonlinear walking behaviour. The significant deviations from random linear trajectories suggest that *A. aegyptium* employs deliberate turning and path-convoluting strategies. These movements may reflect purposeful behaviors, including optimizing foraging efficiency, avoiding obstacles avoidance, searching or responding to potential

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threats. High curvature could indicate localized exploration near stimuli, while sudden changes may signify evasive actions. Additionally, curvature may reveal biomechanical limitations or energy-efficient turning mechanisms inherent to the insect's physiology, as frequent turning may incur energy costs that influence resource optimization strategies. Furthermore, significant curvature deviations can serve as reliable indicators for distinguishing insect behaviors from random noise or external disturbances. Novelty can also stem from linking curvature to specific environmental or behavioral contexts, such as food searching or predator avoidance. In the case of *A. aegyptium*, nonlinear movement patterns during walking are of particular interest due to the ecological contexts in which they occur. As a large, terrestrial insect, *A. aegyptium* often traverses uneven substrates, navigates dense vegetation and interacts with potential threats or resources. The frequent directional changes, bounded trajectories and repetitive loops likely represent an optimized trade-off between environmental exploration and energy conservation.

The use of fractal dimension analysis further revealed the constrained complexity of the trajectories, highlighting the ability to navigate within defined spatial boundaries while maintaining efficient movement patterns. Phase space reconstruction and Lyapunov exponent calculations provided additional depth to our understanding of walking dynamics. The identification of a possible limit cycle attractor indicates repetitive and bounded behaviors, which may reflect innate or environmentally influenced patterns. While no evidence of chaotic dynamics was found, the bounded and structured trajectories point to a deterministic system governed by environmental feedback and internal rules. This structured behavior aligns with the ecological requirements of *A. aegyptium*, enabling it to adapt to complex terrains while conserving energy. The bounded movement and lack of chaotic divergence are consistent with efficient navigation strategies in natural habitats. These findings suggest potential applications in biomimetic robotics, where adaptive and efficient movement strategies are critical.

Comparing observed curvature to random trajectories represents a methodological advancement that has been already used in standard ecological studies. Artificially generated video sequences have been introduced, combining known real-animal postures with randomized body positions, orientations and sizes (Arent et al., 2021). The novel contribution of our study is that is rigorously tests curvature against null models such as random linear paths incorporating quantitative metrics like curvature distribution or tortuosity. The novelty is enhanced by the focus on a specific insect species or behavior (i.e., a grasshopper's movement during natural conditions) that has been scarcely analyzed in this way. The methodologies developed here could be adapted to study other forms of animal movement, from terrestrial vertebrates to aquatic species, providing a versatile toolkit for ecological and biomechanical research. For navigation studies, understanding nonlinear movements may illuminate how insects process sensory information and make decisions. Additionally, recognizing curvature patterns in movement may aid pest control strategies or contribute to ecological monitoring efforts. The insights gained from understanding the nonlinear dynamics of *A. aegyptium*'s walking behavior could inform the design of biomimetic robots capable of adaptive and efficient locomotion in complex terrains (Gart et al., 2018). By mimicking the turning strategies, bounded trajectories and constrained features observed in this study, robotic systems could achieve enhanced agility and robustness.

Despite our findings, several limitations must be acknowledged. First, the experimental setup cannot fully capture the complexities of the insect's native habitat. Factors present in the wild, such as predation risk, interspecies interactions, and varying substrate types, were not considered in this study. Second, the analysis relied on two-dimensional trajectory data, which may overlook vertical components of movement. Future studies incorporating three-dimensional tracking techniques could provide a more holistic perspective. Whereas marker-based motion capture systems are very robust and easily adjusted to suit different setups, tracked species or body parts, they cannot be applied in experimental situations where markers interfere with natural behavior, e.g., when tracking delicate, elastic or sensitive body structures (Arent et al., 2021). Another limitation is the limited generalizability of the findings, as the observations were based on a single individual. Expanding the sample size and including a broader range of conditions would strengthen the robustness of the conclusions. Additionally, while advanced metrics such as fractal dimension and Lyapunov exponents were calculated, these analyses are sensitive to parameter choices such as time delays and embedding dimensions. Further refinement of these methods could enhance the reliability of future studies. While this research focused primarily on movement patterns, future studies could incorporate additional variables such as environmental. For example, controlled manipulations of environmental features could help disentangle the roles of intrinsic and extrinsic factors in driving observed behaviors. Future directions include linking curvature to environmental factors like light or food availability to determine its drivers, conducting comparative studies across species and behaviors and exploring other nonlinear features such as speed oscillations or pauses.

In conclusion, this interdisciplinary investigation into the nonlinear walking dynamics of *Anacridium aegyptium* demonstrates the value of advanced analytical techniques in uncovering hidden patterns and address fundamental questions about movement and behavior. Despite its limitations, this study offers a compelling framework for examining locomotion through the lens of nonlinear dynamics, with applications ranging from ecology and evolutionary biology to biomechanics and robotics.

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DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.
Consent for publication. The Author transfers all copyright ownership, in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal, and has not been previously published.
Availability of data and materials. all data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and take responsibility for the integrity of the data and the accuracy of the data analysis.
Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence, or be perceived to influence, their work.
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Monte Carlo simulations predict distinct real EEG patterns in individuals with high and low IQs

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ABSTRACT

The neural mechanisms underlying individual differences in intelligence are a central focus in neuroscience. We investigated the effectiveness of Monte Carlo simulations in predicting real EEG patterns and uncovering potential neural differences between individuals with high and low intelligence. EEG data were collected from two groups of volunteers categorized by IQ, namely, a high-IQ group and a low-IQ group. A univariate normal distribution was fitted to each EEG channel using Maximum Likelihood Estimation, after which synthetic datasets were generated based on the estimated parameters. Statistical analyses including Root Mean Square Error (RMSE) calculations assessed the alignment between real and simulated data. We showed that Monte Carlo simulations effectively replicated the statistical properties of the EEG data from both the groups, closely matching the real central tendencies, variability and overall distribution shapes. Specific EEG channels, particularly in the frontal and temporal bilateral regions, exhibited significant differences between the two groups, pointing to potential neural markers of cognitive abilities. Further, the low-IQ group exhibited higher predictability and more consistent neural patterns, reflected by lower RMSE values and smaller standard deviations across several EEG channels. Conversely, the high-IQ group displayed greater variability and larger RMSE values, reflecting complex neural dynamics that are less predictable by Monte Carlo simulations. Our findings underscore the utility of Monte Carlo simulations as a robust tool for replicating EEG patterns, identifying cognitive differences and predicting EEG activity associated with intelligence levels. These insights can inform predictive modeling, neurocognitive research, educational strategies and clinical interventions of targeted cognitive enhancement.

KEYWORDS: statistical analysis; oddball tasks; synthetic dataset; EEG channel.

INTRODUCTION

Exploring the neural mechanisms underpinning intelligence has been a longstanding primary focus of cognitive neuroscience research. Electroencephalography (EEG) offers unique insights to assess differences in cognitive abilities, including distinctions between individuals of varying intelligence levels (Friedman et al., 2019). With its high temporal resolution, the non-invasive EEG evaluates the interplay between synchronization, complexity and network efficiency (van Dellen et al., 2015). For instance, higher IQ is associated with reduced long-distance EEG information flow and enhanced local processing efficiency, supporting small-world models (Thatcher et al., 2016). Short EEG phase delays and increased coherence in frontal regions correlate with higher intelligence, emphasizing the role of frontal lobe synchronization (Thatcher et al., 2005). Resting-state EEG studies have further explored intelligence-related differences, reporting balanced inter-hemispheric coordination in alpha and beta bands in more intelligent individuals (Jahidin et al., 2013). Also, it has been demonstrated that IQ correlates negatively with EEG energy but positively with information flow intensity at specific frequencies, emphasizing the role of efficiency in neural communication (Luo et al., 2021). Changes in microstate dynamics are associated with fluid intelligence and its enhancement following cognitive training (Santarnecchi et al., 2017). Lu et al. (2022) found that individuals with higher fluid intelligence allocate attentional resources more flexibly, particularly in complex tasks, as reflected in theta and alpha EEG activities. Together, these findings underscore the utility of EEG in the assessment of the neuronal mechanisms of intelligence, revealing consistent patterns of neural efficiency, inter-hemispheric coordination and adaptive resource allocation.

Conversely, the analysis of EEG data poses significant challenges due to their inherent variability, high dimensionality and sensitivity to noise (Hassani et al., 2015). To address these challenges and enhance our ability to model and predict EEG patterns, advanced statistical and computational methods are required. Monte Carlo simulations have been widely used across various scientific disciplines, providing a powerful framework for modeling complex systems influenced by variability and uncertainty (Metropolis and Ulam, 1949; Rubinstein and Kroese, 2016). By leveraging statistical properties derived from observed data, Monte Carlo simulations generate synthetic datasets that may reflect real-world behaviors (Salvadori et al., 2024; Jones and Fleming, 2024). A Monte Carlo approach could be particularly well-suited for EEG data, as it allows researchers to explore and replicate neural dynamics without the need for extensive experimental data collection. Monte Carlo methods have been applied in neuroscience to simulate and analyse

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electromagnetic brain signals, providing approximations of event-related brain activity (Herdman 2021). Monte Carlo simulations have been utilized for brain source localization (Georgieva et al., 2013) and evaluation of errors as a function of position within the brain in MRI-MEG/EEG co-registration techniques (Singh et al., 1997). Surprisingly, it has been demonstrated that EEG localization is more accurate than MEG localization for the same number of sensors averaged over many source locations (Liu et al., 2002). Monte Carlo analysis can also simulate event-related changes in amplitude and phase-amplitude correlations, enabling close approximations of real EEG and MEG data (Herdman 2021). This approach is particularly valuable for validating data analysis methods, including measurements of functional connectivity and phase-amplitude coupling. A Bayesian framework has been introduced for parameter estimation in EEG modeling using a marginalized Markov Chain Monte Carlo approach (Hettiarachchi et al., 2012). This method was employed to fit a neural mass model to EEG data effectively.

Despite these studies, the application of Monte Carlo simulations to EEG research in revealing cognitive differences between individuals of high and low intelligence remains relatively underexplored. This study aims to bridge this gap by investigating how Monte Carlo simulations can model, simulate and reproduce real EEG traces. Central to this investigation is the issue of predictability: using Monte Carlo simulations, are the EEG patterns from high-IQ individuals more or less predictable than those of their lower-IQ counterparts? It can be hypothesized that higher cognitive abilities are associated with greater neural flexibility and variability, potentially reducing predictability in simulations. Conversely, lower intelligence may correspond to more stable neural patterns, increasing predictability. Monte Carlo simulations, by generating synthetic EEG data modeled on the statistical properties of real datasets, offer a systematic approach to testing these hypotheses.

We conclude that that Monte Carlo methods are a robust tool for exploring the neural dynamics of intelligence, paving the way for future investigations into brain-behavior relationships. By accurately replicating EEG patterns and identifying significant group differences, Monte Carlo simulations contribute to a deeper understanding of the cognitive and neural processes underlying intelligence.

MATERIALS AND METHODS

Participants and Data Collection. This study retrospectively builds on the foundational research conducted by Norbert and Ksenija Jaušovec through 2010 (Jaušovec and Jaušovec, 2001; 2003; 2005; 2010), which was later advanced in collaboration with Tozzi et al. (2021a; 2021b). This continuation of their work is undertaken with great respect and recognition of Norbert's untimely passing.

EEG data were collected from two groups of right-handed volunteers categorized by IQ, namely, a high-IQ group and a low-IQ group. Each group consisted of five participants, yielding a total sample of 10 individuals (mean age: 19.8 years; SD = 0.9; range = 18–21 years; males: 4). The IQ categorization was based on standardized test scores, with the high-IQ group representing the top quartile (IQ SD = 127) and the low-IQ group representing the bottom quartile (IQ SD = 87). Differences in EEG activity between these groups were analyzed during the performance of two oddball tasks (auditory and visual). The study adhered to the Declaration of Helsinki and received approval from the Ethics Committee of the University of Maribor, Slovenia.

EEG signals were recorded using a 64-channel system to ensure comprehensive cortical coverage. Electrode placement followed the 10–20 international system, covering nineteen scalp locations: [FP1], [FP2], [F3], [F4], [C3], [C4], [P3], [P4], [O1], [O2], [F7], [F8], [T3], [T4], [T5], [T6], [CZ], [FZ] and [PZ]. The electrodes were sintered Silver/Silver Chloride (8mm diameter) with impedance maintained below 5 kΩ. All leads were referenced to linked mastoids (A1 and A2), with a ground electrode on the forehead. Vertical eye movements were recorded using additional electrodes placed above and below the left eye. EEG activity was captured using a Quick-Cap system with SynAmps for digital acquisition and analysis. Signals were digitized at 1000 Hz with a gain of 1000 (resolution: 0.084 μV/bit, 16-bit A/D conversion) and stored on a hard disk. Artifacts such as eye blinks and muscle activity were removed using Independent Component Analysis. Data were then band-pass filtered between 1 and 40 Hz to isolate relevant neural activity while minimizing noise. The time-series data were located in separate columns, each one corresponding to a different EEG electrode. The data were numerical and represented EEG signal amplitudes in microvolts over time. Each row corresponded in the recording to a single time point in milliseconds.

Next, Monte Carlo simulations were performed to generate synthetic EEG traces, modeling the variations of future data points based on statistical distributions.

Monte Carlo simulations. A reliable methodological framework was essential to ensure robust analysis and enabling meaningful comparisons between EEG traces of high-IQ and low-IQ individuals. The analysis of the EEG data followed a series of structured steps. The first step involved loading EEG datasets for both the groups. Multiple files for each group were then concatenated into unified datasets. Any rows or cells with missing values were removed to prevent inaccuracies in statistical calculations. The next step was distribution fitting, where statistical parameters, i.e., the mean μ and standard deviation σ, were derived for each EEG channel to simulate data closely reflecting real-world observations. Each channel was analyzed separately. A univariate normal distribution was fitted to the data using

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Maximum Likelihood Estimation (MLE) to calculate μ and σ, ensuring that the simulated data aligned with the distributions observed in the real datasets. The alignment of the fitted distributions were then validated by comparing histograms of real data against the simulated distributions.

Once the distributions were established, synthetic EEG traces were generated using Monte Carlo simulations. For each EEG channel, 100 random samples were drawn from the fitted normal distribution parameters μ and σ. The synthetic samples were organized into a structured dataset mirroring the original format, with separate columns for each channel. The simulation was performed independently for both high-IQ and low-IQ groups to preserve their distinct statistical properties. Following data simulation, statistical analyses evaluated the differences between real and simulated datasets and compared the predictability of the high-IQ and low-IQ groups. Fit accuracy was assessed by comparing real and simulated data using Root Mean Square Error (RMSE) for each EEG channel, where lower RMSE values indicated better alignment. Small adjustments were necessary to address mismatches in the number of samples between the real and the simulated data and to ensure data alignment during the RMSE calculation. Variability was also analyzed, as higher variability (e.g., larger standard deviations) in a dataset could reduce predictability by introducing greater spread around the mean. A larger number of EEG channels showing significant differences between real and simulated data suggested lower predictability. Two-sample t-tests and Welch's t-tests were performed to identify significant differences between the real high-IQ and low-IQ groups. Visual comparisons, including histograms and boxplots, were produced to illustrate the alignment between real and simulated datasets.

All analyses and visualizations were performed using Python. Libraries such as numpy, pandas, and scipy were used for statistical calculations, while matplotlib was employed for creating visualizations. The scipy.stats module provided functions for t-tests and normal distribution fitting.

RESULTS

Monte Carlo simulations effectively captured the central tendency, the spread and the variability of the original EEG traces. The real data distributions were closely approximated by the fitted normal distributions, indicating that the simulations accurately reflected the underlying statistical properties. The simulated variances closely aligned with those of the real data, staying within acceptable margins. Visual comparisons further highlighted the alignment between real and simulated data. Histograms demonstrated that the simulated data mirrored the shape and density of the real data, with only minor deviations observed in the tails for a few columns. Boxplots showed significant overlap in medians and interquartile ranges between the real and simulated data, underscoring the reliability of the modeling process. Moreover, the synthetic data successfully replicated the presence of outliers seen in the real dataset, further demonstrating its ability to reflect the inherent variability and complexity of the original data.

The Monte Carlo analysis revealed significant differences between the high-IQ and low-IQ groups in certain EEG channels (see **Table**). The t-tests and the Welch's t-tests identified specific EEG channels where the groups exhibited statistically significant differences ($p<0.05$). Strong differences in activity levels were observed in left [FP1] and right [FP2] prefrontal regions. Channels [F3] and [F4] also showed significant differences in regions associated with higher-order cognition and decision-making. Central channels such as [CZ] exhibited marked differences, reflecting motor or cognitive integration, with [C3] and [C4] showing additional activity differences in motor-related regions. Temporal channels [T3], [T4] and [T5] demonstrated significant differences in regions linked to memory and language processing, while the occipital channel [O2] revealed distinctions in visual processing areas.

On the other hand, some EEG channels exhibited overlapping distributions between the groups, suggesting no significant differences in activity. These included [P3] and [PZ] in the posterior parietal regions, [T6] in the temporal region and [F7] in the left frontal area, where activity patterns appeared similar across groups.

In sum, significant differences between the high-IQ and low-IQ groups were found in medians and interquartile ranges for various channels. These findings suggest key areas where these groups differ in their distributions.

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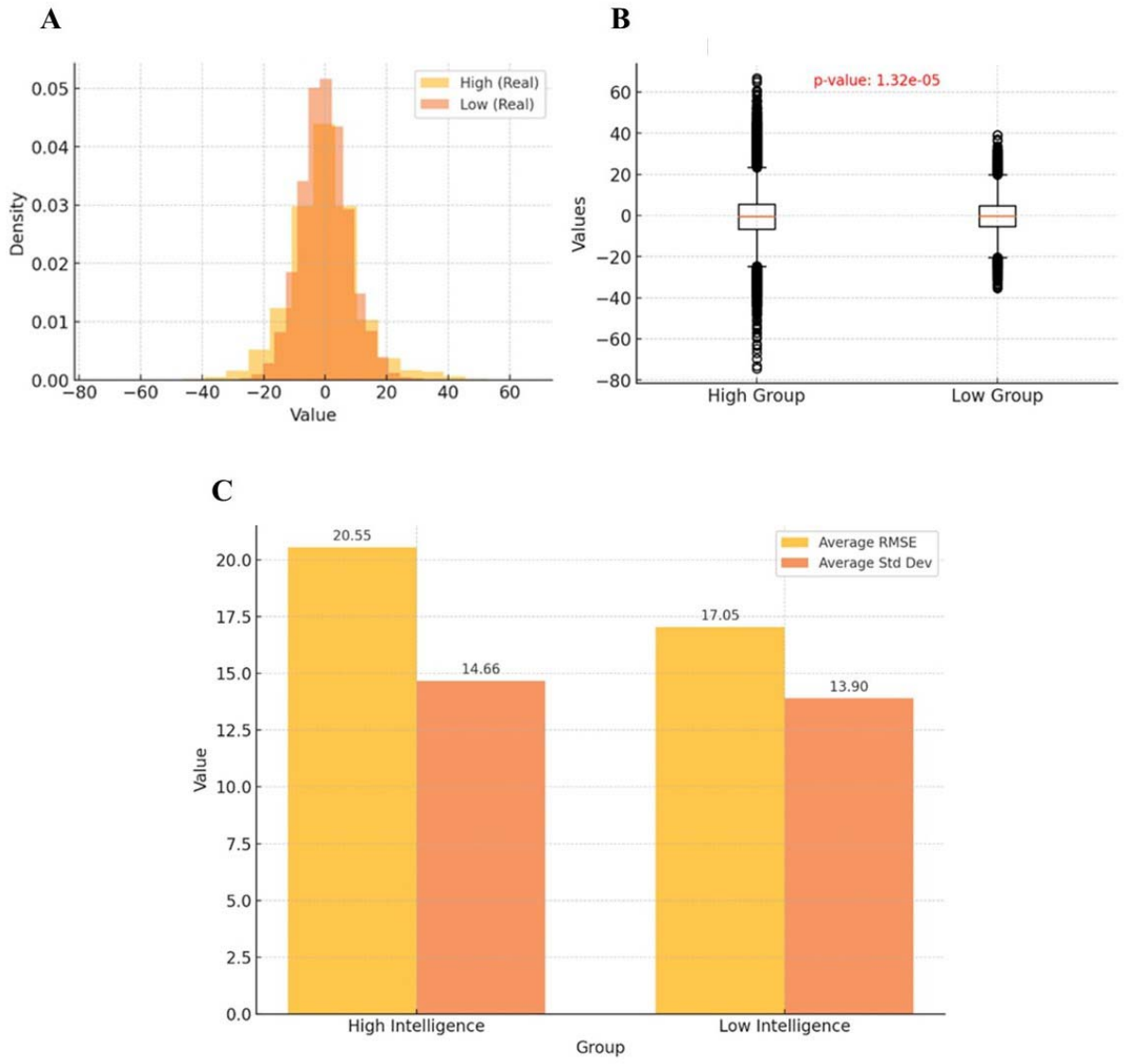
EEG Channel	p-Value	Significance
[FP1]	1.13e-20	Significant
[FP2]	3.11e-13	Significant
[F3]	1.13e-07	Significant
[F4]	8.86e-12	Significant
[C3]	8.28e-06	Significant
[C4]	3.77e-05	Significant
[P3]	0.741	Not Significant
[P4]	0.019	Significant
[O1]	0.558	Not Significant
[O2]	1.32e-05	Significant
[F7]	0.209	Not Significant
[F8]	4.78e-06	Significant
[T3]	0.002	Significant
[T4]	1.24e-19	Significant
[T5]	7.28e-16	Significant
[T6]	0.805	Not Significant
[CZ]	0.000368	Significant
[FZ]	1.33e-23	Significant
[PZ]	0.904	Not Significant

Table. Statistical Differences Across EEG Channels Between High-IQ and Low-IQ Groups.

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Visual comparisons provided further insights. The histograms highlighted distinct peaks or shifts between the two groups in significant channels, aligning with the statistical tests and confirming the observed differences (**Figure A**). The boxplots showed that the high-IQ group data had larger variability in significant channels, reflected in wider interquartile ranges, while the low-IQ group displayed more consistent and narrower distributions (**Figure B**). These patterns suggest that significant channels may serve as neural markers differentiating cognitive abilities between the groups.

The comparison of the predictability between the high-IQ and low-IQ groups revealed distinct insights (**Figure C**). RMSE values were generally lower for the low-IQ group, indicating better alignment between real and simulated data and suggesting higher predictability in the EEG traces of less intelligent individuals. Standard deviations were also slightly smaller for the low-IQ group across several EEG channels, further supporting its greater predictability. In contrast, the high-IQ group exhibited greater variability and larger RMSE values, which are indicative of patterns of reduced predictability in Monte Carlo simulations.



Figures A-B. A visual comparison of Monte Carlo simulations showcasing the graphical representation of a single electrode [O2] as a representative example from the set of 19 electrodes. The histogram comparison (**Figure A**) highlights the differences in real distribution between high-IQ and low-IQ groups across each column. The corresponding boxplot (**Figure B**) visually compares the distributions of the two groups across each column. Differences in medians, interquartile ranges and potential outliers can be identified. Statistical significance is determined based on p-values obtained through t-tests.

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Figure C. Predictability of EEG traces via Monte Carlo simulation. This panel indicates that the EEG behaviour of the low-IQ group is more predictable, as reflected by their lower average RMSE and slightly smaller standard deviation compared to the high-IQ group.

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DISCUSSION

The results of this study underscore the remarkable utility of Monte Carlo simulations in modeling EEG traces and identifying significant differences between high-IQ and low-IQ groups. By accurately replicating the statistical properties of the original dataset, the simulated data closely mirrored the observed real patterns in terms of central tendencies, variability and overall distribution shapes. Our analysis revealed pronounced statistical distinctions in specific EEG channels, which suggest potential markers of cognitive ability. For instance, the differences observed in frontal regions [FP1] and [FP2] align with their roles in executive functions, attention and problem-solving. The high-IQ group exhibited greater variability in these areas, possibly reflecting more dynamic or complex neural processes. The temporal regions [T3] and [T4] showed disparities that could indicate differences in memory retrieval and language processing between high-IQ and low-IQ groups. Similarly, occipital regions, particularly [O2], revealed distinctions in visual-spatial processing capabilities. At the same time, non-significant channels like [P3] and [PZ] in the posterior parietal regions, [T6] in the temporal region and [F7] in the left frontal area pointed to areas where further exploration or alternative modeling approaches may be necessary.
The low-IQ group exhibited higher predictability during Monte Carlo simulations, as demonstrated by lower RMSE values between real and simulated data and smaller standard deviations in several EEG channels. The less intelligent subjects generally displayed more consistent EEG patterns, particularly in central and temporal regions, aligning with their reduced variability and narrower distribution ranges. These patterns suggest a level of neural uniformity in the low-IQ group, contrasting with the broader variability, the higher RMSE values and the increased unpredictability observed in high-IQ individuals' EEG activity.

We demonstrated that Monte Carlo simulations serve as a powerful tool in neurocognitive research, enabling the identification of EEG markers associated with cognitive abilities and the simulation of neural activity patterns to test hypotheses about brain function. Practical applications of these findings extend to predictive modeling, where simulated EEG data can forecast cognitive behaviors. In clinical settings, these insights might inform interventions for cognitive enhancement or rehabilitation. Additionally, educational strategies could be tailored based on neural markers of learning potential. This study has limitations. While the assumption of normality proved valid for most EEG channels, deviations in skewness or kurtosis in some columns may have influenced results, highlighting the need for further research. Future studies should consider employing non-parametric methods or fitting alternative distributions to enhance the robustness of simulations. Investigating multivariate correlations across channels could provide deeper insights into the neural interconnections underlying intelligence. Future research should also explore the integration of alternative modeling techniques to further refine the accuracy and applicability of these methods and deepen our understanding of the neural underpinnings of human cognition.

In conclusion, the implications of this research in the field of cognitive neuroscience go beyond academic interest, underscoring the potential of combining advanced statistical techniques with neuroscience to unlock new possibilities for studying and enhancing human cognition. In particular, Monte Carlo simulations leverage the power of computational modeling to explore the complexities of human intelligence, offering robust methods for expanding datasets, uncovering underlying patterns and identifying key neural markers.

DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.
Consent for publication. The Author transfers all copyright ownership, in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal, and has not been previously published.
Availability of data and materials. all data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and take responsibility for the integrity of the data and the accuracy of the data analysis.
Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence, or be perceived to influence, their work.
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Authors' contributions. The Author performed: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding, administrative, technical, and material support, study supervision.

Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT to assist with data analysis and manuscript drafting. After using this tool, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.

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Fractional and Geometric Neural Dynamics: Investigating Intelligence-Related Differences in EEG Symmetry and Connectivity

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ABSTRACT

Understanding intelligence-related variations in electroencephalographic (EEG) activity requires advanced mathematical approaches capable of capturing geometric transformations and long-range dependencies in neural dynamics. These approaches may provide methodological advantages over conventional spectral and connectivity-based techniques by offering deeper insights into the structural and functional organization of neural networks. In this study, we integrate Clifford algebra, Noether's theorem and fractional calculus to analyze EEG signals from high- and low-IQ individuals, looking for key intelligence-related differences in cortical organization. Clifford algebra enables the representation of EEG signals as multivectors, preserving both magnitude and directional relationships across cortical regions. Noether's theorem provides a quantitative measure of symmetry properties linked to spectral features, identifying conserved functional patterns across distinct brain regions. Mittag-Leffler functions, derived from fractional calculus, characterize long-range dependencies in neural oscillations, allowing for the detection of memory effects and scale-invariant properties often overlooked by traditional methods. We found significant differences between high- and low-IQ individuals in geometric trajectories, hemispheric connectivity, spectral properties and fractional-order dynamics. High-IQ individuals exhibited increased spectral asymmetry, enhanced spectral differentiation, distinct geometric trajectories and greater fractional connectivity, particularly in frontal and central regions. In contrast, low-IQ individuals displayed more uniform hemispheric connectivity and heightened fractional activity in occipital areas. Mittag-Leffler fractional exponents further indicated that high-IQ individuals possessed more varied neural synchronization patterns. Overall, our multi-faceted approach suggests that intelligence-related neural dynamics are characterized by an asymmetric, functionally specialized and fractionally complex cortical organization. This results in significant differences in network topology, efficiency, modularity and long-range dependencies.

KEYWORDS: neural oscillations; functional asymmetry; graph theory; cognitive variability; brain network topology.

INTRODUCTION

Intelligence has been associated with distinct electroencephalographic (EEG) neural patterns, yet the underlying mechanisms remain a subject of investigation (Thatcher et al., 2005; Friedman et al., 2019). The study of intelligence-related differences in EEG signals has traditionally relied on spectral, time-frequency and connectivity analyses (Chen et al., 2023; Ignatious et al., 2023). Conventional approaches such as Fourier and wavelet transforms, coherence measures and graph-theoretic network analyses have provided critical insights into brain function but lack the mathematical depth to fully capture neural relationships (Sitnikova et al., 2009; San-Segundo et al., 2019). We argue that mathematical frameworks integrating algebraic, geometric, and fractional dynamics provide an alternative perspective by capturing spatial dependencies and characterizing long-range memory effects in neural activity. Among these approaches, Clifford algebra has been used to encode EEG signals as multivectors, preserving both magnitude and directional relationships across cortical regions—an advantage not achievable with conventional spectral methods (Zhang et al., 2023). Clifford algebra-based EEG transformations allow for the preservation of geometric properties enabling novel trajectory-based comparisons. Noether's theorem, which relates system symmetries to conserved quantities, has been used to assess functional and spectral balance in cortical microcircuitry, revealing conserved functional patterns across different cortical regions (Bilteanu et al., 2017). Meanwhile, fractional calculus — particularly Mittag-Leffler function analysis— may extend EEG signal characterization beyond integer-order models, allowing for the detection of memory effects and scale-invariant properties often overlooked by standard methods (Atanackovic et al., 2011; Turalska and West, 2018). Functional order analysis provides an additional layer of insight by quantifying fractional dynamics and long-range dependencies (García-Raffi and Torrano, 2021).

Together, these multi-faceted approaches may enable a refined investigation into intelligence-related brain dynamics extending beyond pairwise spectral differences, allowing for a richer assessment of interregional dependencies. We

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conjecture that our approach might reveal systematic differences in EEG structure, with high-IQ individuals exhibiting greater frontal asymmetry, enhanced connectivity in integrative brain areas and increased fractional-order complexity in neural synchronization. Low-IQ individuals, in contrast, might display more uniform hemispheric connectivity, potentially indicative of different cognitive resource allocation strategies.

In sum, given the limitations of traditional EEG methods, a hybrid approach that considers Clifford algebra, Noether's theorem and fractional network measures holds promise for more comprehensive analyses of intelligence. We will proceed as follows. The next section outlines the methodology, detailing data acquisition, preprocessing and the mathematical frameworks applied to EEG signals. We then present our results, followed by an interpretation of the findings in the context of intelligence-related neural organization. Finally, we conclude with a discussion on the broader implications and future directions of this research.

MATERIALS AND METHODS

Our retrospective study builds on prior research by Jaušovec and Jaušovec (2001; 2003; 2005; 2010) and later advancements with Tozzi et al. (2021a; 2021b), honoring Norbert Jaušovec's contributions. EEG data were collected from 10 right-handed participants (mean age: 19.8 years; SD = 0.9; males: 4), divided into High-IQ (IQ SD = 127) and Low-IQ (IQ SD = 87) groups based on standardized test scores during auditory and visual oddball tasks. EEG signals were recorded using a 64-channel system with electrodes placed according to the 10–20 system. Data epochs were segmented into non-overlapping 2-second windows and trials contaminated with excessive noise were excluded based on an amplitude threshold exceeding ±100 µV. Standardization was performed by normalizing each EEG signal relative to its mean and standard deviation across trials, ensuring comparability between participants.

Clifford algebra. Clifford algebra was used to transform EEG signals into multivector representations that preserve spatial and directional relationships between cortical regions (Acus and Dargys, 2024). Each EEG signal $x_i(t)$ from electrode *i* was mapped onto a Clifford multivector $\Psi(t)$, expressed as

$$\Psi(t) = \sum_i v_i(t) e_i$$

where $v_i(t)$ is the voltage amplitude and e_i represents a basis vector of a Clifford algebra. The choice of Clifford space dimensionality depended on the number of selected EEG channels. In our study, a three-dimensional Clifford space $Cl(3,0)$ was chosen to encode signals from three representative electrode locations (FP1, P3, O1). These three electrodes were selected due to their known involvement in cognitive functions and information processing. The transformed EEG signals were then analyzed geometrically by computing trajectory deviations across trials. The Clifford Fourier Transform (CFT) was used to extract spectral features within the multivector space (Monaim and Fahlaoui, 2024). Given a time-dependent multivector $\Psi(t)$, its frequency-domain representation was obtained as

$$\tilde{\Psi}(\omega) = \int \Psi(t) e^{-e_1 \omega t} dt$$

where e_1 is an imaginary unit in Clifford space. Statistical comparisons of Clifford trajectories were performed using paired t-tests to determine significant differences in neural geometry between high and Low-IQ groups.

Noether's theorem. Noether's theorem was used to quantify symmetry in functional connectivity. Functional connectivity was evaluated using Pearson correlation coefficients between electrode pairs, computed as

$$r = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (x_i - \bar{x})^2} \sqrt{\sum (y_i - \bar{y})^2}}$$

where x_i and y_i represent EEG signals from different electrodes and $\bar{x}$ and $\bar{y}$ are their means over time. Connectivity matrices were constructed for each participant, with a focus on left-right hemisphere electrode pairs (FP1–FP2, F3–F4, C3–C4, P3–P4, O1–O2) to assess hemispheric symmetry. Symmetry indices were calculated as

$$S = \frac{|PSD_L - PSD_R|}{PSD_L + PSD_R}$$

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where PSD_L and PSD_R are power values from corresponding left and right hemisphere electrodes. Spectral asymmetry was assessed by performing a Fourier transform on each EEG channel and comparing power distributions between hemispheres (Atiyah et al., 1975). Welch's t-tests were applied to compare connectivity and spectral symmetry indices between high and Low-IQ groups.

To assess the relationship between frequency band power and hemispheric symmetry, correlation analyses were performed for each frequency band. Power spectral densities (PSDs) were computed to assess frequency distributions in delta, theta, beta and alpha bands (Dressler et al., 2004; Redwan et al., 2024). Pearson correlation coefficients were then calculated between frequency band power and hemispheric symmetry indices to determine whether specific oscillations were associated with neural organization patterns.

To evaluate the temporal stability of EEG signals, stationarity tests were conducted using the Augmented Dickey-Fuller (ADF) test, which determines whether a time series exhibits long-term trends or remains stable over time (Dao and Staszewski, 2021). ADF p-values below 0.05 indicated significant stationarity, suggesting a consistent neural state.

Fractional derivative values. Fractional-order derivatives were computed to characterize the temporal dynamics of EEG signals, as conventional differentiation does not account for memory effects or long-range dependencies inherent in neural activity. The Caputo fractional derivative was chosen, defined as

$$D^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \int_0^t \frac{f'(\tau)}{(t-\tau)^{\alpha}} d\tau$$

where α represents the fractional order and Γ denotes the Gamma function. The parameter α was optimized based on the signal's spectral properties to capture its underlying fractional nature effectively. Numerical implementation was carried out using the Grünwald-Letnikov approximation, which approximates the integral definition with a discrete summation:

$$D^{\alpha} f(n) = \sum_{k=0}^{n} (-1)^k \binom{\alpha}{k} f(n-k)$$

where $\binom{\alpha}{k}$ represents the binomial coefficient extended to non-integer orders. This method provided a means of assessing long-range dependencies in EEG data, differentiating between stationary and non-stationary signal properties. The fractional derivative values were extracted for each EEG channel and used in subsequent statistical analyses to identify group-wise differences in neural activity. These calculations formed the basis for evaluating the Mittag-Leffler function's role in neural dynamics, establishing a mathematical framework for comparing cognitive groups.

Fractional connectivity and fractional clustering coefficients. To examine functional connectivity, correlation-based adjacency matrices were constructed by computing pairwise cross-correlations between EEG channels. Cross-correlation was defined as

$$C_{xy}(\tau) = \sum_t x(t) y(t+\tau),$$

where $x(t)$ and $y(t)$ are EEG signals from two channels and τ represents the lag. A sliding window approach was used to ensure robust estimation of functional connectivity over time, with overlapping one-second segments applied to account for dynamic variations in brain activity.

To integrate fractional-order concepts into network analysis, fractional connectivity and fractional clustering coefficients were computed. Fractional connectivity was derived by modifying edge weights using a power-law transformation, where each connection strength W_{ij} was raised to a fractional exponent γ, yielding W_{ij}^{γ}. The mean fractional connectivity was then calculated by averaging these transformed values, providing a metric sensitive to long-range dependencies in neural networks.

The fractional clustering coefficient was computed using a modified version of the standard clustering measure, incorporating the fractional-weighted edges to assess local connectivity structures. This was defined as

$$C_i^{\gamma} = \frac{1}{k_i(k_i-1)} \sum_{j,k} (W_{ij} W_{ik} W_{jk})^{\gamma/3}$$

where k_i is the degree of node i. These fractional measures allowed for a more nuanced characterization of network topology, enabling differentiation between classical small-world organization and fractional network configurations.

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The ability to quantify connectivity and clustering using fractional-order metrics provided insights into how neural interactions vary across cognitive groups, extending conventional graph-theoretic approaches.

Mittag-Leffler fractional exponents. Mittag-Leffler fractional exponents were estimated to describe the scaling behavior of connectivity weights within the network (García-Raffi and Torrano, 2021). The Mittag-Leffler function, defined as

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k+\beta)},$$

generalizes exponential decay and captures memory effects in complex systems (Tarasov 2018). To estimate the effective fractional exponent, the probability distribution of connectivity weights was examined in log-log space and a power-law function

$$P(W) \sim W^{-\lambda}$$

was fitted to the data. The exponent λ was extracted using least-squares fitting, providing a measure of the heterogeneity in connectivity distributions. Higher values of λ indicated a broader range of connectivity strengths, reflecting a more diverse interaction pattern across EEG channels. This Mittag-Leffler-based analysis was used to assess differences in the scaling properties of functional networks between groups, serving as a crucial parameter for understanding cognitive variability.

Statistical analyses were performed to compare fractional network measures between high and Low-IQ groups. One-way analysis of variance (ANOVA) was conducted to test for significant differences in fractional connectivity, fractional clustering and Mittag-Leffler exponents across groups. The Bonferroni correction was applied to account for multiple comparisons and control the false discovery rate.

Tools. Computational analyses were implemented using Python, utilizing NumPy for numerical computations, SciPy for statistical analysis, Matplotlib for visualization and the MNE-Python toolbox for EEG preprocessing. Clifford algebra computations were performed using the ClPy library and Fourier transforms were applied using SciPy's signal processing module.

RESULTS

Significant differences in geometric trajectories, hemispheric connectivity, spectral properties and fractional-order dynamics were found between high and Low-IQ individuals, with High-IQ individuals exhibiting increased Clifford algebra-based frontal variability, greater frontal asymmetry and enhanced spectral differentiation.

Clifford algebra. Clifford algebra-based transformations displayed distinct EEG trajectory patterns, with High-IQ individuals exhibiting increased geometric variability in the frontal region, while Low-IQ individuals exhibited more constrained neural trajectories (**Figure 1A**). Statistical comparisons of Clifford components indicated significant differences in frontal representations between groups ($p < 0.001$), whereas no significant differences were found in parietal and occipital components (**Figure 1B**). Temporal stationarity analyses using the Augmented Dickey-Fuller test indicated that EEG signals in High-IQ individuals were more stationary ($p < 0.05$), suggesting greater consistency in neural oscillations.

Spectral power analysis using the Clifford Fourier Transform showed that High-IQ individuals exhibited significantly higher alpha ($p < 0.001$) and beta ($p = 0.0067$) power in the frontal region compared to Low-IQ individuals, suggesting different neural activation patterns. Power spectral densities showed that High-IQ individuals had significantly higher theta ($p = 0.0065$) and beta ($p = 0.0067$) power, while Low-IQ individuals exhibited relatively stronger delta oscillations.

Noether's theorem. Connectivity analysis based on Noetherian symmetry principles and further analyses of spectral symmetry revealed significant differences in hemispheric organization. High-IQ individuals displayed greater asymmetry in frontal connectivity (FP1-FP2, $p < 0.001$) and enhanced symmetry in occipital connectivity (O1-O2, $p < 0.001$), while Low-IQ individuals maintained more uniform hemispheric connectivity, particularly in prefrontal and parietal regions (**Figure 1C**).

Fractional dynamics. Statistical analysis of Mittag-Leffler fractional derivatives revealed distinct patterns of neural activity between high- and low-IQ individuals. High-IQ individuals consistently exhibited higher fractional values in frontal (FP1, F4) and central (C3) regions, while low-IQ individuals showed increased fractional activity in the occipital (O1) area (**Figure 2A**). Notably, the frontal regions displayed significantly lower fractional-order dynamics in the low-IQ group, suggesting that high-IQ individuals engage in neural processes with stronger memory effects and long-range dependencies, potentially supporting cognitive flexibility and problem-solving.

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Beyond localized effects, significant differences were also observed in central and parietal channels, indicating that intelligence-related variability extends across broader network properties rather than being confined to specific brain regions. The occipital region exhibited an inverse trend, with low-IQ individuals displaying higher Mittag-Leffler fractional activity, possibly reflecting distinct neural connectivity patterns or alternative sensory integration strategies.
Furthermore, greater variability in fractional activity was observed in some channels within the low-IQ group, as indicated by wider distribution ranges. This suggests increased dispersion in fractional-order neural activity, which may correspond to less stable or less efficient network dynamics.

Fractional network measures. The comparison of fractional network measures between high and Low-IQ groups is illustrated in **Figure 2B**. Fractional connectivity exhibited significant differences between groups, with High-IQ individuals showing an higher mean fractional connectivity. Similarly, fractional clustering coefficients were higher in the High-IQ group. Mittag-Leffler fractional exponents demonstrated a broader distribution in the High-IQ group, with a mean exponent value lower in the Low-IQ group. Further, High-IQ individuals exhibited greater variability in fractional connectivity values. Weighted global efficiency was significantly higher in the High-IQ group, suggesting differences in the efficiency of information transfer across neural networks. This set of findings highlights a more complex and distributed network topology in individuals with High-IQ, underlining the relevance of fractional measures in characterizing cognitive differences. Small-worldness, defined as the ratio of clustering coefficient to characteristic path length, was significantly greater in the High-IQ group. This suggests that High-IQ individuals exhibit a more optimized network structure, balancing local clustering with global efficiency. Additionally, edge weight distribution followed a broader power-law decay in the High-IQ group, consistent with more heterogeneous connectivity patterns.
The Mittag-Leffler-based scaling exponent analysis further corroborated these findings, as High-IQ individuals demonstrated greater deviation from exponential connectivity decay, indicating a higher degree of long-range dependencies in neural interactions. These variations were particularly evident in the frontal and occipital regions, where High-IQ individuals displayed enhanced connectivity strengths. Differences in network modularity also emerged, with High-IQ individuals exhibiting higher modularity scores, suggesting a more functionally segregated yet highly interactive network.
Overall, these findings suggest that fractional network properties, including connectivity, clustering and Mittag-Leffler exponents, may distinguish high and Low-IQ groups in a statistically significant manner.

Summarizing, our findings, confirmed through Clifford algebra transformations, spectral analysis and Noetherian symmetry principles, suggest that High-IQ individuals exhibit greater frontal EEG asymmetry, increased spectral differentiation and distinct connectivity patterns, greater geometric variability, greater frontal asymmetry, enhanced connectivity in integrative regions and increased long-range dependencies, while Low-IQ individuals displayed more uniform hemispheric connectivity and heightened activity in occipital areas. These findings suggest that intelligence-related neural dynamics are characterized by asymmetric, functionally specialized and fractionally complex cortical organization.

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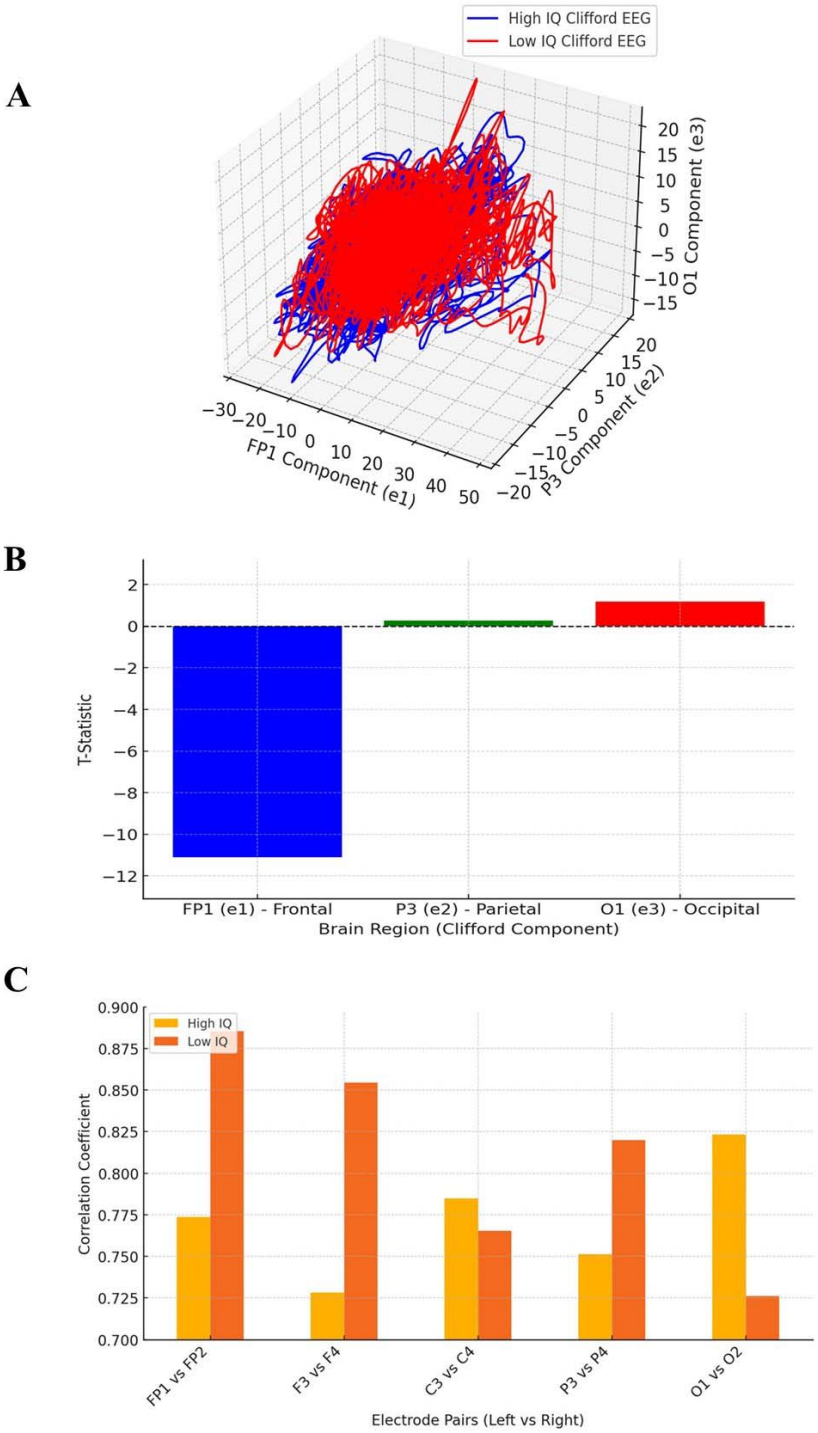


Figure 1A. Clifford EEG Signal representation for High vs. Low-IQ groups. The blue and red lines represent average EEG trajectories in Clifford space for high and Low-IQ individuals, respectively. Differences in shape, spread and oscillatory patterns provide insight into IQ-related brain activity variations.

Figure 1B. Statistical differences in Clifford EEG components. Significant differences were observed in the frontal region, while parietal and occipital components showed no substantial variation. The dashed line at zero indicates the statistical significance threshold.

Figure 1C. Hemispheric symmetry differences across brain regions. Low-IQ individuals (green bars) exhibit greater symmetry in prefrontal, frontal and parietal regions, whereas High-IQ individuals (blue bars) exhibit increased asymmetry, particularly in FP1-FP2 and F3-F4. The occipital region (O1-O2) is more symmetric in High-IQ individuals, suggesting enhanced visual-spatial processing.

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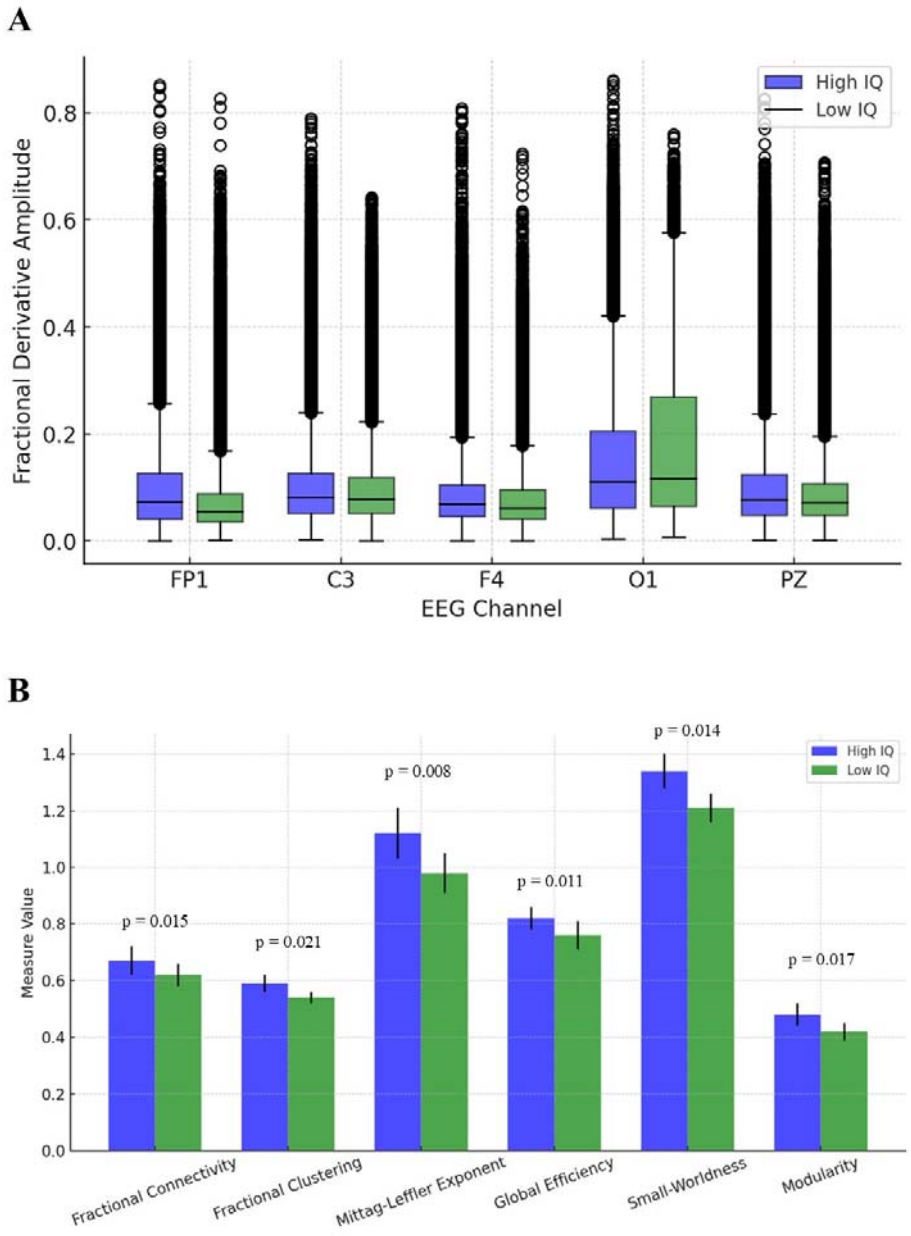


Figure 2A. Boxplot comparison illustrating key differences in fractional dynamics between High-IQ (blue) and Low-IQ (green) groups. The variations in distributions suggest significant differences in Mittag-Leffler function characteristics. Statistical analysis confirms that these distinctions are significant, with **$p < 0.0001$** across all analyzed channels.

Figure 2B. Comparison of fractional network measures between high and Low-IQ groups. Bars represent mean values for each group, with error bars indicating standard deviations. High-IQ individuals exhibit significantly greater fractional connectivity, clustering, Mittag-Leffler exponent, global efficiency, small-worldness and modularity compared to Low-IQ individuals. One-way ANOVA confirmed systematic distinctions in neural structure and connectivity patterns across groups.

CONCLUSIONS

We investigated intelligence-related differences in EEG signals by combining multiple mathematical approaches such as Clifford algebra, Noether's theorem and fractional network analysis. Functional connectivity analysis revealed that

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high-IQ individuals exhibited a more differentiated neural network organization that was characterized by greater variability in interregional interactions and enhanced symmetrical connectivity in occipital regions. Fractional-order analyses demonstrated that High-IQ individuals exhibited greater long-range dependencies, with significantly higher Mittag-Leffler fractional exponents and clustering coefficients, supporting the presence of more complex and efficient neural architecture. This suggests that cognitive ability may be linked to distinct neural processing strategies, with High-IQ individuals displaying more complex fractional-order behavior in regions associated with executive function and working memory. In turn, Low-IQ individuals exhibited heightened activity in the occipital cortex, which may correspond to alternative visual processing mechanisms.
High-IQ individuals displayed stronger Mittag-Leffler dynamics in regions linked to complex cognitive processing, while Low-IQ individuals displayed greater activity in visual and sensory processing areas. These differences align with theories of cognitive efficiency and regional specialization, where High-IQ brains may rely more on integrative network structures, while Low-IQ brains may exhibit increased reliance on localized sensory processing. Fractional-order dynamics showed that the frontal and central brain regions displayed more Mittag-Leffler behavior in High-IQ individuals. The occipital region presented an inverse trend, where Low-IQ individuals showed greater fractional activity, suggesting distinct processing dynamics. Taken together, these findings suggest that intelligence-related brain function may be reflected in both geometric transformations and fractional connectivity properties.

The integration of Clifford algebra, Noether's theorem and Mittag-Leffler function-based fractional calculus stands for a novel approach to EEG analysis which offers methodological advantages over conventional spectral and connectivity-based techniques. Clifford algebra preserves magnitude and directional relationships between EEG signals, allowing for a richer representation of neural trajectories' geometric transformations. Noether's theorem introduces a principled framework for assessing conserved properties in brain networks, enabling the quantification of functional symmetry and intelligence-related differences in interregional connectivity patterns. Fractional network measures extend beyond traditional graph-theoretic approaches by capturing long-range dependencies and scale-invariant properties in neural oscillations. This may reveal subtle variations in brain dynamics that conventional EEG methodologies fail to detect and that are often overlooked in standard time-frequency analyses.

Compared to traditional EEG analysis techniques, our approach offers several advantages. Spectral analysis methods such as Fourier and wavelet transforms are widely used to decompose EEG signals into frequency components but do not retain spatial or directional relationships between electrodes (Yuan et al., 2018; Daud and Sudirman, 2022). Functional connectivity metrics, including coherence and phase synchronization, assess interregional interactions but often rely on pairwise comparisons that may overlook global network dependencies (Miskovic and Keil, 2015; Abdullateef et al., 2022). Graph-theoretic measures capture aspects of brain topology but do not explicitly incorporate long-range dependencies or fractional-order dynamics. In contrast, our combination of Clifford algebra, Noether's theorem and fractional network analysis preserves both local signal variations and global connectivity patterns. The inclusion of fractional calculus further enhances sensitivity to memory effects and non-stationary dynamics (Chen and Wang, 2020).

Our methodological framework has potential applications in different fields. EEG-based cognitive profiling could benefit from Clifford algebra transformations to characterize individual differences in neural geometry. The integration of Noether's theorem into EEG analysis could provide a means of quantifying hemispheric symmetry, which has implications for studying neurodevelopmental disorders. Fractional network measures could be employed in neurodegenerative research to monitor cognitive decline, as long-range dependencies and connectivity scaling properties may serve as biomarkers for early-stage disorders. The integration of Noether's theorem into EEG analysis may also contribute to the study of neurodevelopmental and neuropsychiatric disorders, where alterations in hemispheric symmetry have been linked to conditions such as autism spectrum disorder, schizophrenia and dyslexia (Saugstad, 1999; Gage et al., 2009; Guo et al., 2013; Perkins et al., 2014; He et al., 2023). Still, brain-computer interface technologies may enhance classification accuracy by utilizing fractional-order EEG features, which incorporate long-range memory effects into machine learning models (Tarasov 2018).
Beyond these applications, our approach suggests testable experimental hypotheses, such as: a) whether interventions like transcranial stimulation can modulate EEG symmetry in ways consistent with Noetherian principles; b) whether cognitive training enhances fractional connectivity measures over time; c) whether neural efficiency, as quantified by Clifford algebra-based geometric transformations, correlates with cognitive performance in different task-based EEG paradigms; d) whether the observed intelligence-related differences in EEG connectivity persist over time, particularly in relation to neuroplasticity and cognitive aging. These experimental extensions reinforce the potential of our framework in broadening the scope of nervous oscillations' research.

Despite its contributions, our study has several limitations. The sample size may limit the generalizability of findings and the robustness of observed differences. The computational complexity of Clifford algebra transformations presents a potential challenge, as real-time applications may require optimization to improve processing efficiency. While Noether's theorem provides a theoretical basis for assessing neural symmetry, its direct application to EEG data for the study of intelligence requires further validation. Still, the selection of fractional exponent parameters was optimized

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based on empirical signal properties, but individual variations in neural dynamics may necessitate adaptive algorithms to improve robustness. Multimodal neuroimaging approaches, such as functional magnetic resonance imaging, could be incorporated to enhance spatial resolution and validate EEG-based symmetry findings.

In conclusion, we suggest that intelligence-related differences in EEG signals may be characterized using a combination of Clifford algebra, Noether's theorem and fractional network analysis. By integrating these advanced mathematical frameworks, a novel perspective on cognitive variability could be provided, highlighting the role of symmetry, geometric transformations and fractional-order properties in brain function.

DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.
Consent for publication. The Author transfers all copyright ownership, in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal and has not been previously published.
Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.
Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence their work.
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Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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An extended Stokes' theorem for spiral paths: applications to rotational flows in *Trachelospermum jasminoides* stems and flowers

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The traditional Stokes' theorem connects the macroscopic circulation along a closed boundary to the microscopic circulation across the surface it encloses. However, it proves inadequate for addressing complex geometries such as helicoidal paths, non-planar flow patterns and dynamic systems with open boundaries. We introduce an extension of Stokes' theorem (EST) that provides a robust tool for interdisciplinary research in spiral/helicoidal dynamics, facilitating the evaluation of rotational forces and circulation in both natural and engineered systems with open boundaries. We apply EST to model the rotational dynamics of flower petals and the helical forces within the stems of *Trachelospermum jasminoides*, known as star jasmine. For the flower, we demonstrate the equivalence between the line integral along the petal boundary and the surface integral over the enclosed disk, effectively capturing the uniform rotational stress generated by tangential forces. EST enables the analysis of external factors such as wind or pollinator interactions, while providing valuable insights to deepen our understanding of floral mechanics and petal growth patterns. For the stem, linking microscopic circulatory forces to macroscopic flow patterns, we demonstrate the interaction of torsional and bending stresses caused by the helical geometry. This finding has significant implications for understanding plant growth biomechanics and structural stability as well as for quantifying nutrient and water transport within stems, where spiral dynamics play a pivotal role. In summary, EST streamlines the analysis of rotational and translational forces in systems governed by spiral and helicoidal dynamics, including physical and biological phenomena such as phyllotaxis and plant growth.

KEYWORDS: helical dynamics; boundary analysis; vector field integration; flow topology.

INTRODUCTION

Stokes' theorem (henceforward ST) is a fundamental principle of vector calculus that bridges the macroscopic circulation along a closed boundary with the microscopic circulation over the enclosed surface (Green, 1828; Schey, 1997). Extending the principles of Green's theorem (GT) which applies to two-dimensional regions, ST provides a powerful framework for analyzing flows and circulations in three-dimensional spaces, uncovering profound connections between the local properties of vector fields and their global behaviour. GT and ST are effective tools for solving problems related to physical closed systems with clearly defined boundaries such as airflow circulation around wings, electromagnetic fields in circuits, surface heat flux, Coriolis-driven hemispherical flows, Earth's deep interior dynamics (Craven, 1964; Arfken, 1985; De Villiers, 2006; Livermore et al., 2013; Snieder, 2015; Aubert and Finlay, 2019; Vines et al., 2021; Yang et al., 2023). In biology, the two theorems contribute to understanding blood flow, electrical activity of the brain, growth patterns in ecosystems (Tozzi and Peters, 2023; Bressan et al., 2022). Yet, the classical ST is inherently limited to surfaces and boundaries that are closed, leaving a significant gap in its applicability to open, non-planar geometries. Indeed, many natural and engineered systems exhibit spiral or helical dynamics where forces and flows do not conform to closed loops or planar surfaces but rather are characterized by open, three-dimensional trajectories. Examples include the helical paths of tornadoes, magnetic vortices and spiral galaxies as well as bacterial motility and phyllotaxis of plants (Blaser et al., 2024; Sachkou et al., 2019; Reinhardt and Gola, 2022).

The novelty of this work lies in extending ST to accommodate spiral flows and helicoidal paths. By linking macroscopic and microscopic circulation properties, the extended theorem simplifies the evaluation of forces in systems with open, three-dimensional geometries. We utilize EST to analyse two biological scenarios: 1) the rotational forces in spiral flower petals and 2) the torsional stresses in helical plant stems, both exemplified by *Trachelospermum jasminoides*, commonly known as star jasmine. For the flower, EST captures the uniform rotational stresses induced by tangential forces acting along a circular boundary. This is achieved by demonstrating the equivalence of the line integral along the petal boundary and the surface integral over the enclosed disk. For the stem, EST quantifies the interaction between bending and torsional stresses caused by the helical geometry.
This paper is structured as follows. First, we present the mathematical treatment of EST, including its derivation and parameterization for helicoidal paths. Next, we validate the theorem using the specific example of *Trachelospermum jasminoides'* flowers and stems. Finally, we discuss the broader implications of EST, highlighting its potential to unify the study of dynamical systems with open boundaries.

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MATERIALS AND METHODS

This study is grounded in a generalized form of Stokes' theorem, adapted for spiral flows, which facilitates the analysis of forces and circulation in systems with helicoidal or spiral geometries. We aim to prove that, given a continuously differentiable, orientable helicoidal spiral vector field, the macroscopic circulation represented by the integral of a differential form over its surface equals the microscopic circulation represented by the volume integral of the curl perpendicular to the surface. The main challenge here is in defining the notion of a boundary in case of an open helicoidal spiral path, moving beyond the classical case of paths evaluable through ST.

Stokes' Theorem (ST) from vector calculus relates the surface integral of the curl of a vector field over a surface **S** to the line integral of the vector field along the boundary curve ∂S of the surface (**Figure 1**). In its general form, ST asserts that

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \int_{S} (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

where **F** is a a continuously differentiable two-dimensional vector field, ∂S is the closed boundary curve of the surface **S** that can be bended and stretched, d**r** is a differential element of the curve, d**S** is the differential element of the surface area, and $\nabla \times \mathbf{F}$ is the curl of the vector field, i.e., a vector operator characterizing the infinitesimal circulation of vector fields in three-dimensional spaces.
ST turns line integrals of a form over a boundary into more straight-forward double integrals over the bounded region, regardless of the position of vector singularities (Zenisek 1999). For ST to apply, the normal vector representing the surface must be positively oriented (i.e., counterclockwise) with respect to the tangent vector representing the orientation of the boundary.

Extended Stokes' theorem (EST). Consider a vector field **F** defined over a region in three-dimensional space. Let the surface *S* be a portion of a plane or a more general surface that is bounded by a spiral curve $\gamma(t)$. The goal is to use EST to evaluate the line integral over the spiral path in terms of the surface integral of the curl of **F** (**Figure 1**).
Let the spiral curve $\gamma(t)$, with $t \in [a, b]$, be parameterized as

$$\gamma(t) = (r(t)\cos(\theta(t)), r(t)\sin(\theta(t)), z(t))$$

where $r(t)$, $\theta(t)$ and $z(t)$ describe respectively the radial, angular and vertical components of the spiral.
Let's assume that $\gamma(t)$ lies on a flat plane, say the *xy*-plane, so the spiral path can be simplified to

$$\gamma(t) = (r(t)\cos(t), r(t)\sin(t), 0)$$

where $r(t)$ increases as the angle *t* increases.
When the surface **S** is a surface spanned by the curve $\gamma(t)$, **S** stands for a portion of the plane or surface generated by the spiral curve (**Figure 1, left**).

We are interested in computing the line integral of a vector field **F** along the spiral path (**Figure 1, right**). By ST, this line integral can be transformed into a surface integral involving the curl of **F**

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \int_{S} (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

The line integral over the spiral path is:

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \int_{a}^{b} \mathbf{F}(\gamma(t)) \cdot \frac{d\gamma(t)}{dt}\, dt$$

where $\frac{d\gamma(t)}{dt}$ is the tangent vector to the spiral path at each point *t*.

The surface integral involves the curl of **F**, given by $\nabla \times \mathbf{F}$, and the surface normal vector $\hat{n}$ associated with **S**

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$$\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \int_S (\nabla \times \mathbf{F}) \cdot \hat{n}\, dS$$

The normal vector $\hat{n}$ depends on the orientation of the surface, while dS is the differential area element of the surface. Upon achieving the extended formulation of ST, we will proceed in the next paragraphs with a detailed case study.

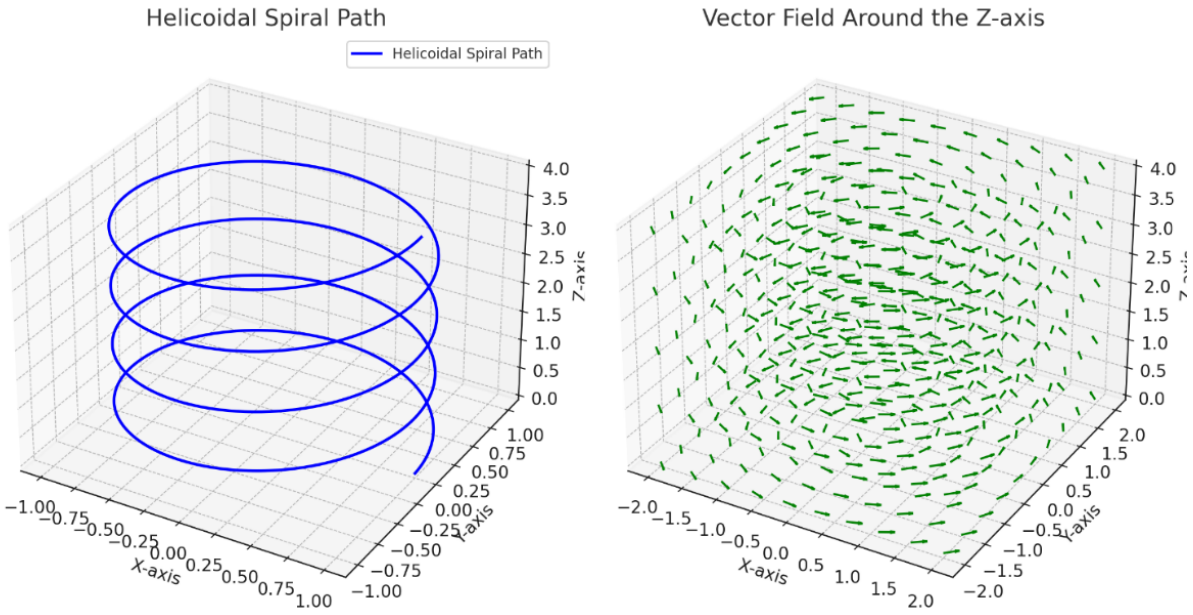


Figure 1. Diagrams depicting a helicoidal spiral (**left**) and the behavior of a vector field (**right**) around the z-axis. The left diagram illustrates a helicoidal spiral path, showcasing the interplay of rotational and translational motion along the z-axis. The right diagram represents a vector field with circular flow centered around the z-axis.

Simulated case study: analyzing rotational flows in stems and flowers. EST enables the analysis of rotational and translational forces in complex systems, providing a powerful framework for exploring biological and physical dynamics. To illustrate the new theorem, we will now explore a concrete example. We will consider *Trachelospermum jasminoides*, commonly known as star jasmine, belonging to the family *Apocynaceae*. Like many climbing plants, *Trachelospermum jasminoides* displays a counterclockwise helical movement of its stems as it climbs and twines around supports, also referred to as circumnutation (Darwin, 1875; Pansanit and Pripdeevech, 2014; Canher et al., 2022) (**Figure 2**). The flowers also exhibit subtle rotational dynamics, although these movements are not as pronounced as the helical twisting of the stems (Stefanatou et al., 2025). The petals of the flowers are arranged in a spiral configuration and unfurl in a counterclockwise direction during blooming.

In our simulation, the dynamics of flower petals are modeled using a circular boundary with a radius of 0.05 m, representing the petals of a flower. Tangential forces along this boundary are applied and the resulting rotational stresses are analyzed through EST. The stem is modeled as a helicoidal path with a radius R=0.05 m and a vertical rise per turn of c=0.2 m. To evaluate the counterclockwise rotation of the flower and stem using EST, the rotation of the petals can be represented by a circular vector field, whereas the helical motion of the stem can be modelled using a helical vector field. The next step is to parameterize the flower and the stem (**Figure 2C**).

1) For the flower, a circular boundary in the plane of the petals is defined, representing the region of interest for macroscopic rotation.

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2) For the stem, the helical path is parameterized using equations for a helicoidal spiral, where $x(t) = r\cos(t)$, $y(t) = r\sin(t)$ and $z(t) = ct$. Here r represents the radius, c the rise per turn and t the parameter along the path.

Subsequently, the surface S is defined for each component.

1) For the flower, the surface is a disk enclosed by the petals' rotational motion within their plane,
2) whereas for the stem, the surface corresponds to the area traced by the helical path (**Figure 2C**).

Calculating the forces acting on the flower and stem requires applying mechanical principles that account for both internal and external forces influencing their dynamics (Smyth 2016; Loshchilov et al., 2021). For the flower's petals, the primary force is torque or rotational force, while the stem experiences a combination of bending forces and axial torsion due to its helical structure. We will calculate these forces systematically, step by step, starting from the external forces.

External forces acting of the flower petals and the stem.

1) The rotation of the flower petals can be modeled as a torque induced by external forces such as wind, gravitational pull, biological growth forces (Tipler, 2004). Torque (τ) on the petals is given by:

$$\tau = r \times F$$

Where r is the radial vector from the center of the flower to the tip of a petal and F is the tangential force applied to the petal. Let's assume that the radius of the flower is R=5 cm, while the tangential force from wind or another source is F=0.1 N. The magnitude of the torque is

$$|\tau| = R \cdot F = 0.05\,\mathrm{m} \cdot 0.1\,\mathrm{N} = 0.005\,\mathrm{Nm}$$

In case of multiple petals (n=5 in *Trachelospermum jasminoides*) experiencing similar forces, the total torque becomes

$$\tau_{\text{total}} = n \cdot \tau = 5 \cdot 0.005\,\mathrm{Nm} = 0.025\,\mathrm{Nm}$$

The rotational acceleration (α) of the flower petals is related to the torque (Clark and Ryan, 2022) via

$$\tau = I \cdot \alpha$$

Where I is the moment of inertia of the flower petals about the axis of rotation and α is the angular acceleration.
For a flower modeled as a system of point masses at a radius R

$$I = n \cdot m \cdot R^2$$

where m is the mass of a single petal. Assuming m=0.002 kg (2 grams per petal):

$$I = 5 \cdot 0.002 \cdot (0.05)^2 = 0.000025\,\mathrm{kg}\backslash\mathrm{cdotpm}^2$$

The angular acceleration is:

$$\alpha = \frac{\tau_{\text{total}}}{I} = \frac{0.025}{0.000025} = 1000\,\mathrm{rad/s}^2$$

2) The stem experiences forces from bending and torsion, influenced by its helical structure. These forces arise from gravity, wind and the biological tension exerted during growth.

The weight of the stem induces a bending gravitational force. For a stem of length L=20 cm and mass per unit length λ=0.01 kg/m:

$$F_g = m \cdot g = (\lambda \cdot L) \cdot g = (0.01 \cdot 0.2) \cdot 9.8 = 0.0196\,\mathrm{N}$$

This force acts vertically downward, generating a bending moment at the base of the stem

$$M_g = F_g \cdot \frac{L}{2} = 0.0196 \cdot 0.1 = 0.00196\,\mathrm{Nm}$$

The helical structure of the stem experiences torsional forces due to the winding. The torsional moment (T) is given by

$$T = G \cdot J \cdot \frac{\theta}{L}$$

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Where G is the shear modulus of the stem material, J is the polar moment of inertia and θ is the angle of twist over the length L.
Assuming G= 10^8 Pa typical for plant tissue, J = 0.005 m (5 mm radius) and θ = 2π (one full turn over L =0.2 m) (Hoermayer et al., 2024), then

$$J = \frac{\pi}{2} \cdot (0.005)^4 = 9.82 \times 10^{-10}\,\mathrm{m}^4$$

and

$$T = (10^8) \cdot (9.82 \times 10^{-10}) \cdot \frac{2\pi}{0.2} = 3.08\,\mathrm{Nm}$$

Internal forces acting within the flower petals and the stem.
To calculate the forces within the flower petals and the stem using EST, we need to evaluate the relationship between the macroscopic circulation (observable forces) and the microscopic properties (internal forces or stresses derived from the curl of the force field). The first step is to model the forces using vector fields.

1) Concerning the flower petals, we assume that the external forces (e.g., wind or biological forces) act tangentially to their circular boundary. Further, we assume that the tangential forces induce internal stresses (force per unit area) propagating through the petals. Let the force field acting on the petals be

$$\mathbf{F} = (-ky, kx, 0)$$

where k is the force constant proportional to the external pressure and x,y represent positions in the z=0 plane.

Next, we compute the curl of the force field. The curl of the force field relates to the internal stresses within the petals. For the flower petals, in the z=0 plane, the curl of $\mathbf{F} = (-ky, kx, 0)$ is

$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -ky & kx & 0 \end{vmatrix} = (0, 0, 2k)$$

This curl is constant in the z-direction, indicating a uniform internal rotational stress throughout the petals.

2) Concerning the helical stem, it experiences external forces such as gravity $(\mathbf{F}_g)$ and biological growth forces $(\mathbf{F}_b)$ that induce internal torsion and bending stresses.

For simplicity, we model the net force field in the stem as: $\mathbf{F} = (-ky, kx, kz)$ where the kz-term accounts for the vertical components of the forces.

Next, we compute the curl of the force field, which provides insight into the internal stresses acting within the stem. This computation reveals the distribution and intensity of these stresses, capturing the complex interplay of forces across the helical structure. For the stem the curl of $\mathbf{F} = (-ky, kx, kz)$ is

$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -ky & kx & kz \end{vmatrix} = (k, k, 2k)$$

This suggests a complex pattern of internal stress within the stem, with components distributed across all three spatial directions.

Visualization and statistics. Diagrams of the flower petals and stem are created to illustrate their geometry, boundary dynamics, and associated vector fields. The Matplotlib library is employed to generate detailed plots, including the circular boundary and tangential forces acting on the flower petals, the curl of the vector field over various surfaces, and the helicoidal path and vector field representation for the stem.

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To ensure statistical validation, numerical accuracy is achieved through high-resolution parameterization, with the parameter t sampled at 1,000 points per cycle. The consistency between line integrals and surface integrals is carefully evaluated to confirm the applicability of the extended theorem to the analyzed geometries.

In the sequel, the surface integral of the curl of the vector field will be computed over these surfaces using the extended Stokes' theorem (EST).

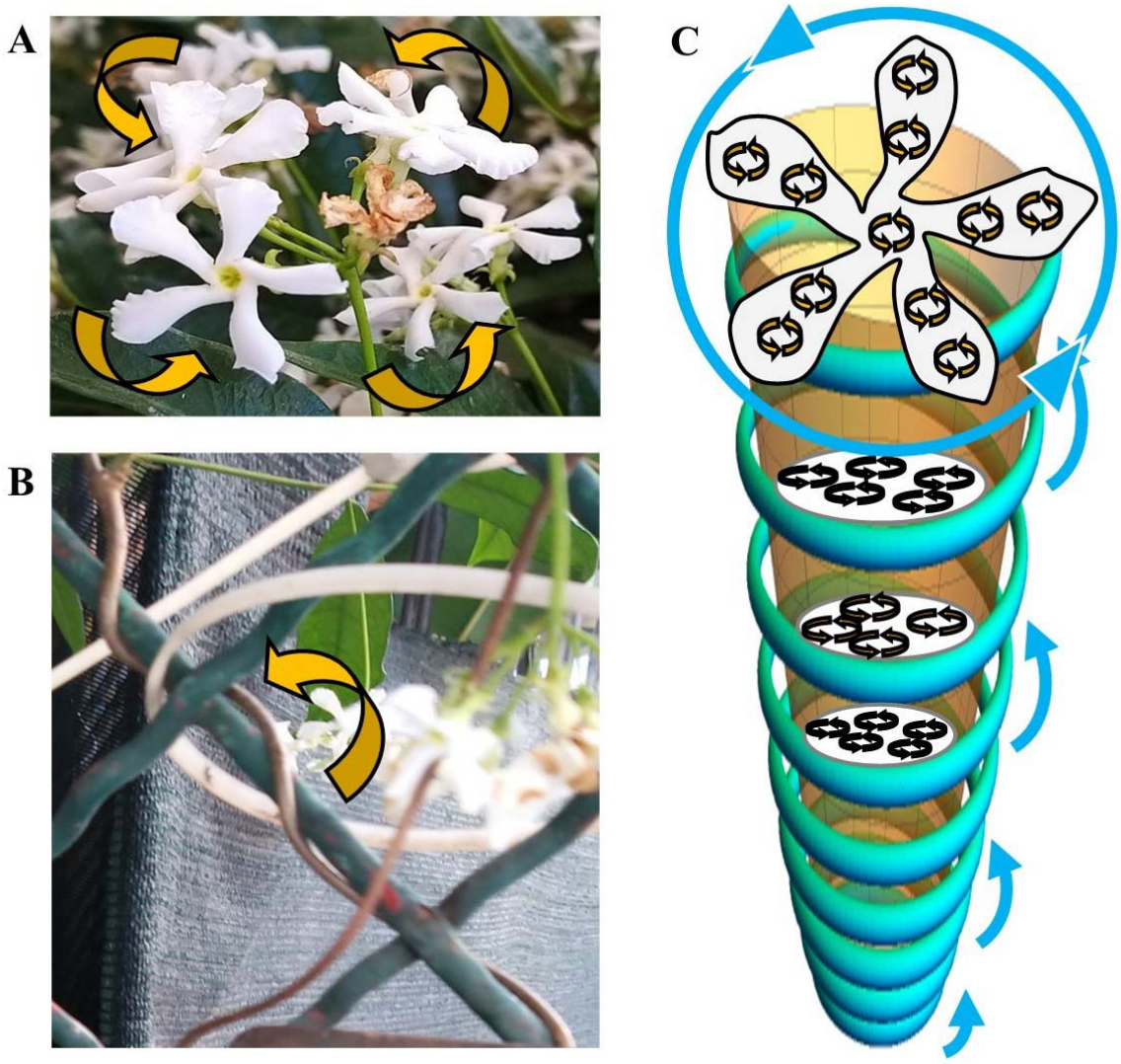


Figure 2. *Trachelospermum jasminoides*. The flower petals (**Figure 2A**) and the stem (**Figure 2B**) display a counterclockwise path. **Figure 2C** illustrates the geometry of the boundaries, the associated vector fields and the internal flows within the flower petals and the stem.

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RESULTS

As stated above, both the flower and the stem experience external and internal mechanical forces that influence their motion and structural behavior:

1) For the flower petals, the torque arising from tangential forces induces a counterclockwise rotation, with the total torque $\tau_{\text{total}} = 0.025\,\text{Nm}$ and the angular acceleration measured as α=1000 rad/s^2. The internal stresses in the petals are uniform with a value of 2k and are directly proportional to the external forces acting on them. This proportionality explains the rotational equilibrium observed in the petals.
2) For the stem, the primary forces include a gravitational bending moment $M_g = 0.00196\,\text{Nm}$ and a torsional moment T=3.08□Nm due to a helical twist. The internal stresses in the stem vary in all three dimensions because of its helical geometry. Among these stresses, torsion, proportional to kc, predominates, whereas bending stresses, proportional to kR, have a secondary but still notable influence.

We can now apply EST to relate macroscopic and microscopic circulation. The surface integral of the curl of the vector field is computed over the surfaces, relating the surface integrals to the line integrals along the boundaries. For the flower petals, the counterclockwise macroscopic rotation is calculated by integrating along the circular path in the plane. For the stem, the integral is evaluated over the helical surface.

1) Concerning the flower petals, the boundary of the flower is a circle of radius R. The macroscopic circulation (line integral along the petal boundary) is

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \int_0^{2\pi} kR^2\, d\theta = 2\pi k R^2$$

Using the curl, the surface integral is

$$\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \int_S 2k\, dA = 2k \cdot \pi R^2$$

Both results match, confirming that the inner stresses in the petals are proportional to 2k.

In sum, the numerical values for the macroscopic (surface) flows and microscopic (internal) flows in the flower, as governed by EST, are as follows. For the flower, the surface flow (evaluated as a surface integral) is 0.157N\ppm, while the internal flow (evaluated as a line integral) is also 0.157N\ppm. The flower petals exhibit a simple and symmetric geometry, where forces act tangentially along a circular boundary in the z=0 plane. The petals lie on a flat, two-dimensional surface characterized by a constant curl of the force field $(\nabla \times \mathbf{F} = (0, 0, 2k))$, signifying that the internal forces are uniformly distributed. This uniform distribution creates a direct and proportional relationship between the macroscopic flow (line integral along the circular boundary) and the microscopic flow (surface integral over the disk). The symmetry of the geometry ensures that every contribution to the line integral is exactly matched by the surface integral. Consequently, the uniform geometry and constant curl lead to a perfect agreement between the surface flow and the internal flow, consistent with EST.

2) Concerning the helical stem, the boundary of the stem is parameterized as a helicoidal spiral

$$x(t) = R\cos(t), \quad y(t) = R\sin(t), \quad z(t) = ct$$

The macroscopic circulation (line integral along the helical path) is

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \int_0^{2\pi} \left[-kR\sin(t)(-R\sin(t)) + kR\cos(t)(R\cos(t)) + kc(ct)\right] dt$$

$$= \int_0^{2\pi} \left(kR^2 + kc^2 t\right) dt$$

For one turn$(t \in [0, 2\pi])$

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{r} = 2\pi k R^2 + \frac{kc^2}{2}(2\pi)^2$$

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Using the curl, the surface integral is approximated by the ribbon spanned by the helix

$$\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \int_S (k + k + 2k)\, dA = 4k \cdot (\text{surface area of ribbon})$$

The surface area of the ribbon is

$$A = 2\pi R \cdot (\text{height per turn}) = 2\pi R \cdot c$$

Thus

$$\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = 4k \cdot 2\pi R \cdot c = 8\pi k R c$$

In sum, the numerical values for the macroscopic (surface) flows and microscopic (internal) flows in the stem, as governed by EST, are as follows. For the stem, the surface flow (evaluated as a surface integral) is 2.513N\ppm, while the internal flow (evaluated as a line integral) is 8.053N\ppm. Unlike the flower, the values for surface flow and internal flow differ significantly. This is due to the stem's more complex geometry, which features a three-dimensional helicoidal structure with a helical boundary and a ribbon-like surface. Unlike the constant curl observed in the flower, the curl of the force field $(\nabla \times \mathbf{F} = (k, k, 2k))$ n the stem varies in all three dimensions. This non-uniform curl introduces additional contributions to the surface integral that are not directly proportional to the line integral along the helical path. The helicoidal surface spanned by the path is not planar. Its area depends on the radius of the helix and the rise per turn (c), which increases the surface integral significantly compared to the simpler circular geometry of the flower. The line integral along the helical path includes contributions from the vertical rise (z-component), which are absent in the flat geometry of the flower. These vertical components add substantially to the internal flow, making it larger than the surface flow. Forces and circulation in the stem are not confined to a two-dimensional plane, rather display three-dimensional dynamics that capture complex interactions such as bending, twisting and torsional effects, further contributing to the discrepancy between the surface and internal flows. Therefore, the stem's intricate geometry and three-dimensional dynamics lead to a disparity between surface and internal flows, reflecting the additional factors at play in its structural behavior.

In conclusion,

1) flowers have a circular, symmetric geometry that ensures uniform force distribution and curl. This results in surface and internal flows being equal, as the entire flow field is captured in a flat, two-dimensional setup.
2) In contrast, the stem's helicoidal geometry introduces non-uniform force distributions and additional components such as vertical contributions and a larger surface area. These factors create a larger internal flow compared to the surface flow, as the line integral accounts for three-dimensional effects that the surface integral does not fully capture.

These differences highlight the impact of geometry and force distribution on the interplay between macroscopic circulation and microscopic forces, showcasing the utility of the extended Stokes' theorem in analysing forces and circulation in systems exhibiting spiral dynamics.

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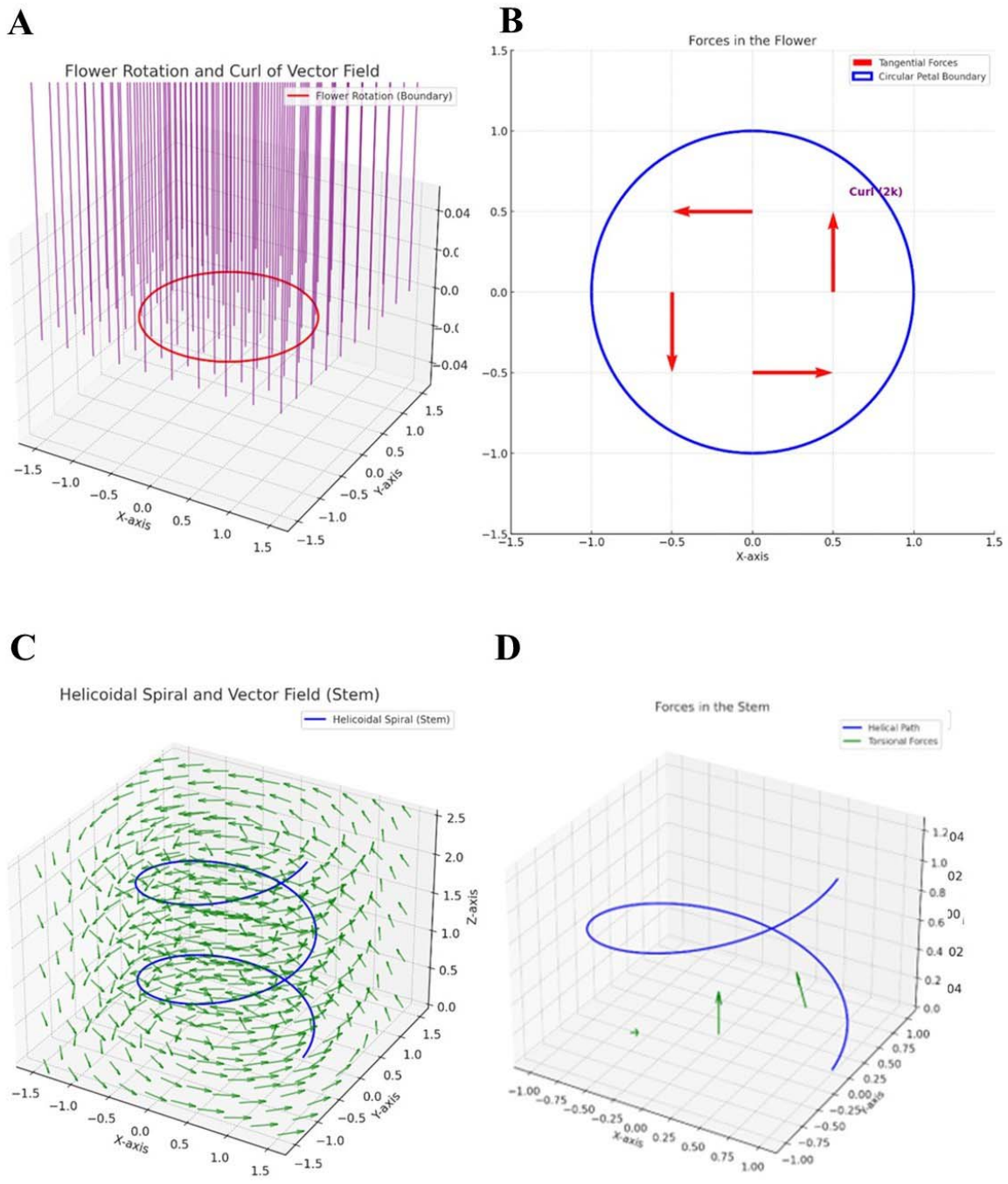


Figure 3. Application of the extended Stoke's theorem to flowers (**Figures 3A-B**) and stems (**Figures 3C-D**) of *Trachelospermum jasminoides*. **Figure 3A**. Diagram illustrating the flower rotation and the curl of vector field. The red circle represents the boundary of the flower petals modeled as a planar region in the z=0 plane. The purple arrows visualize the curl of the vector field, representing the microscopic circulation that contributes to the macroscopic rotation of the flower petals. **Figure 3B**. Diagram illustrating the forces acting on the flower. The circular boundary (blue) represents the edge of the flower petals. Tangential forces (red arrows) act along the edges of the petals, showcasing the influence of external or internal factors. The calculated curl of the vector field is constant at $\nabla \times F = 2k$ (annotated in purple) in the z-direction, indicating uniform rotational stresses throughout the petal boundary. **Figure 3C**. Diagram illustrating the helicoidal spiral of the stem and the vector field. The blue curve depicts the helicoidal path of the stem, while the green arrows represent a circular vector field around the z-axis, illustrating the rotational and translational flow and its interaction with the spiral geometry. **Figure 3D**. Diagram illustrating the forces acting on the stem. The helical path (blue curve) represents the stem's geometry. The torsional forces (green arrows), resulting from a combination of bending and twisting actions, act along the helical structure contributing to internal stress distribution. The curl vector field displays non-uniform rotational stresses with components $\nabla \times F = (k,k,2k)$.

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CONCLUSION

Classical theorems such as Green's Theorem (GT) and Stokes' Theorem (ST) have been pivotal in linking local properties of vector fields to their global behavior. GT applies just to two-dimensional regions and closed curves, while ST extends to three-dimensional spaces requiring closed surfaces or boundaries for its application (Green, 1828; Schey, 1997). These theorems, focused on closed-loop circulations, have proven instrumental in analyzing flows and circulations in systems where boundaries are well-defined, such as steady-state circulations in airflow around wings or electromagnetic field behavior in closed circuits (Arfken, 1985; De Villiers, 2006). However, their utility diminishes when applied to open, three-dimensional trajectories like the helicoidal spirals which are frequently encountered in natural and engineered systems.

We suggest a generalization of ST to establish a mathematical framework connecting the line integral along a helicoidal spiral path to the surface integral of the curl of the vector field over a bounded region. By redefining the boundary concept for helicoidal paths, this framework provides a new tool for analyzing macroscopic and microscopic flow dynamics in complex systems. The EST formulation provides novel insights into the interplay between rotational and translational motions, allowing for a deeper understanding of spiral flows in a variety of physical and biological systems. A key advantage of the extended formulation lies in its ability to model a wide range of systems where spiral or helical dynamics are dominant. For instance, the novel framework enables the analysis of DNA supercoiling, bacterial flagella, biomechanical patterns such as the phyllotaxis of plants (Reinhardt and Gola, 2022; Liu et al., 2024), intracardiac spiral flows observed in cardiac cycles (Mulimani et al., 2022) as well as magnetic vortices in superconductors (Sachkou et al., 2019) and rotational dynamics of spiral galaxies (Blaser et al., 2024), where classical methods fail to capture the intricacies of rotational and translational dynamics.

In this paper, EST is applied to two case studies related with *Trachelospermum jasminoides*, namely the forces acting on flower petals and the helical stress distribution within plant stems.

1) For the flower petals, the circular geometry allows for a straightforward application EST, since the tangential forces acting along the petal boundary produce a uniform curl which is proportional to the rotational stresses. The equivalence between the line integral along the petal boundary and the surface integral of the curl over the enclosed disk validates EST's effectiveness for two-dimensional spiral systems. The uniform rotational stresses observed in the petals align well with the mathematical predictions of EST. This provides insights into how forces are distributed within the boundary of the flower, potentially aiding in the study of floral mechanics and growth patterns. EST suggests that microscopic forces acting at the level of the petals contribute to the macroscopic rotational motion observed at the flower's boundary. This could be applied to study the impact of environmental factors like wind on plant structures and to investigate the mechanical interactions between flowers and pollinators during the pollination process. Still, EST effectively simplifies complex calculations by converting a line integral along the flower's boundary into a surface integral over the petal region. This transformation minimizes computational effort while preserving accuracy.

2) In the case of the stem, although the helical geometry of the stem presents a significant challenge for classical mathematical tools, EST effectively simplifies the intricate interplay of forces involved. The torsional and bending forces are captured through the curl of the vector field, which has components in all three dimensions. The equivalence of the surface integral over the helical ribbon region and the line integral along the helical path demonstrates the robustness of EST in handling three-dimensional geometries with open boundaries. The EST capability to connect macroscopic flow patterns with microscopic circulatory forces may have significant implications for understanding the biomechanics of plant growth and structural stability. This relationship can also provide valuable insights for studies on nutrient and water transport within stems, as these processes often involve spiral dynamics.

Certain assumptions and limitations are inherent in our analysis. EST assumes that the involved vector fields and surfaces are continuously differentiable. In real-world biological systems, irregularities and discontinuities in the geometry or force distribution may reduce the accuracy of the analysis. The flower petals are modeled as a perfect circle and the stem as a regular helix. While this simplifies the mathematical analysis of forces in idealized systems, real-world systems often deviate from these idealized shapes. The analysis of irregular geometries or highly dynamic boundaries may still require significant computational effort, particularly for numerical integration of complex surface and line integrals. The tangential and torsional forces are assumed to be uniform across the boundaries. In reality, biological and environmental forces such as wind, gravity and growth pressures are often spatially and temporally variable. Additionally, secondary effects such as shear forces or anisotropic material properties are not incorporated, which could limit the applicability of the results to certain systems. Future work could extend the framework to handle more irregular and biologically realistic geometries, such as asymmetrical petals or non-uniform stem shapes. The analysis of time-varying forces and boundaries, such as those caused by growth or environmental changes, could provide deeper insights into real dynamics. Integrating the extended theorem with experimental data would help validate the theoretical predictions and refine the mathematical models.

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In conclusion, the proposed extension to Stokes' Theorem integrates helicoidal paths into circulation analysis, bridging a critical gap and expanding its applicability to open, non-planar trajectories. By redefining boundaries, it simplifies the study of rotational and translational flows, offering a versatile tool for analyzing complex dynamics such as those observed in the flowers and stem of *Trachelospermum jasminoides*.

DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.
Consent for publication. The Author transfers all copyright ownership, in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal, and has not been previously published.
Availability of data and materials. all data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and take responsibility for the integrity of the data and the accuracy of the data analysis.
Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence, or be perceived to influence, their work.
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Authors' contributions. The Author performed: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding, administrative, technical, and material support, study supervision.
Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT to assist with data analysis and manuscript drafting. After using this tool, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.
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REVIEW

Exploring the influence of water micro assemblies on protein folding, enzyme catalysis and membrane dynamics

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Abstract
Water is central to biological processes not only as a solvent, but also as an agent shaping macromolecular behavior. Insights into water micro assemblies (WMA), defined by transient regions of low-density water (LDW) and high-density water (HDW), have highlighted their potential impact on biological phenomena. LDW, with its structured hydrogen bonding networks and reduced density, stabilizes hydrophobic interfaces and promotes ordered molecular configurations. Conversely, HDW, with its dynamic and flexible nature, facilitates transitions, solute mobility and molecular flexibility. By correlating experimental observations with simulations, we explore the influence of WMA on three key biological processes. In protein folding, LDW may stabilize hydrophobic cores and secondary structures by forming structured exclusion zones, while HDW may introduce dynamic flexibility, promoting the resolution of folding intermediates and leading to dynamic rearrangements. In enzyme catalysis, LDW may form structured hydration shells around active sites stabilizing active sites over longer timescales, while HDW may support substrate access and catalytic flexibility within active sites. In membrane dynamics, LDW may stabilize lipid headgroups, forming structured hydration layers that enhance membrane rigidity and stability, while HDW may ensure the nanosecond-scale flexibility required for vesicle formation and fusion. Across these tree processes, the WMA's energy contributions, timescales and spatial scales align with the forces and dynamics involved, highlighting the role of LDW and HDW in modulating cellular interactions. This perspective holds implications for the design of lab-on-chip devices, advancements in sensor technologies, development of biomimetic membranes for drug delivery, creation of novel therapeutics and deeper understanding of protein misfolding diseases.

Keywords Hydrogen bond dynamics · Molecular interfaces · Solvation effects · Dynamic water clusters · Hydration layers

Introduction

Traditionally regarded as a passive solvent, water's role extends beyond merely providing a medium for biochemical reactions, since its structural and dynamic properties enable active participation in biological phenomena (Dargaville and Hutmacher 2022). The structural heterogeneity of liquid water arises from the continuous assembly and disassembly of hydrogen bonds, forming diverse geometric configurations that reflect water's branched polymeric nature (Shiotari and Sugimoto 2017). These configurations, termed water micro assemblies (WMA), have been extensively studied under extreme conditions, such as supercritical water (Skarmoutsos and Samios 2016), high-pressure crystals in supercooled water (Kim et al. 2009; Lin et al. 2018) and frozen water confined within nanometric slit pores (Koga et al. 2000) or nanochannels formed by cubic crystalline phases (Das et al. 2019). However, relatively little attention has been given to the microstructure of liquid water at ambient temperature and pressure. Under standard conditions, each water molecule forms up to four hydrogen bonds, creating a tetrahedral structure (Fanetti et al. 2014; Liu et al. 2017; Milovanović et al. 2020). Fluctuations in hydrogen bond numbers, ranging from two to six, result in molecules being either "loosely" or "tightly" bound (Thaomola et al. 2012). Liquid water has been described as a dynamic mixture of pentagonal and hexagonal rings (Shiotari and Sugimoto 2017; Formanek and Martelli 2020), of tetrahedral structures and of ring-and-chain-like assemblies (Liu et al. 2017). These configurations form densely connected spherical

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cores of approximately 140 water molecules, surrounded by fuzzy zones of ~ 1800 loosely connected molecules (Liu et al. 2017). Strong hydrogen bonds create multibranched polymers of about 150 molecules per chain (Naserifar and Goddard 2019), while density fluctuations form empty spaces resembling spherical or fractal-like voids (Ansari et al. 2018). Other WMA descriptions include a giant cluster percolating the system (dos Santos et al. 2004) and a linear, chain-like structure dominating the tree-like arrangement of the largest cluster (Jedlovszky et al. 2007).

The two-liquid scenario argues that liquid water comprises two competing molecular structures: low-density water (LDW) and high-density water (HDW) (Table 1). Although frequently observed under extreme conditions, these structures are also found under ambient conditions (Cheng et al. 2019). LDW features ordered gaps between molecular shells (de Oca et al. 2019), while HDW is associated with high-entropy, unstructured states. LDW consists of fused dodecahedra acting as templates for tetrahedral fluctuations, while HDW forms chain-like structures (Camisasca et al. 2019). LDW patches exhibit greater tetrahedrality and connectivity than HDW patches (Ansari et al. 2018; Faccio et al. 2022).

We describe how LDW and HDW may significantly influence biological processes by creating localized environments with specific physical and chemical properties. LDW regions, characterized by ordered hydrogen bonding networks, lower density and reduced entropy, may stabilize structural integrity near hydrophobic interfaces. Conversely, HDW regions, with higher density and entropy and weaker hydrogen bonds, may facilitate conformational transitions, enhance molecular mobility and disrupt ordered structures. This interplay likely impacts processes such as protein folding, enzyme catalysis and DNA and RNA dynamics. Their physical dimensions make WMA compatible with interacting with biological processes and capable of influencing the behavior of biological macromolecules. Water molecules (~ 2 Å) form LDW and HDW patches ranging from 0.3 to 2 nm (Ansari et al. 2018). Near ambient conditions, LDW has a density of 0.78 g/cm^3, while HDW reaches 1.08 g/cm^3 (Nomura et al. 2017). Although noncovalent bonds in liquid water at room temperature last under 200 femtoseconds (Lodish et al. 2000; Bakó et al. 2013; Naserifar and Goddard 2019), LDW persists for nearly half a second at 160 K (Lin et al. 2018). Despite their brief lifetimes (Camisasca et al. 2019), LDW and HDW create localized density changes that may affect environmental dynamics and drive macroscopic chemical and biophysical processes (Fanetti et al. 2014; Skarmoutsos and Samios 2016; Faccio et al. 2022). Also, we argue that percolation theory may play a pivotal role in influencing biochemical processes driven by WMA. In the context of water percolation, LDW may contribute to long-range connectivity within hydration layers, while HDW may provide localized flexibility.

This study explores the influence of WMA on biological phenomena by integrating experimental observations with theoretical insights. By focusing on key processes such as protein folding, enzyme catalysis and membrane dynamics, we aim to establish a framework that underscores water's active role in shaping the behavior of biological macromolecules. Examining the formation, dynamics and interplay of LDW and HDW, the study provides detailed analyses of their impacts, supported by molecular dynamics simulations and theoretical models.

Theoretical effects of water micro assemblies on biological phenomena

The concept of water micro assemblies (WMA), as discussed in the previous chapter, provides a compelling theoretical framework for understanding physical and dynamic processes (Laage et al. 2017). Below are some potential applications and behaviors explained by WMA:

(1) **Protein folding and stability**. The dynamic interplay between HDW and LDW regions may influence protein folding pathways. Localized density variations may

Table 1 Differences between low-density water (LDW) and high-density water (HDW)

	Low-density water (LDW)	High-density water (HDW)
Structure	Ordered hydrogen bonding networks	Disrupted hydrogen bonding networks
Density	Lower (~ 0.78 g/cm^3)	Higher (~ 1.08 g/cm^3)
Entropy	Reduced (more ordered)	Increased (more disordered)
Hydrophobicity	Stabilizes hydrophobic interfaces and molecular cores	Disrupts ordered zones, allowing transitions
Hydrogen Bonding	Strong and extended networks	Flexible and transient
Mobility	Lower molecular mobility (structured)	Higher molecular mobility (dynamic)
Biological Impact	Stabilizes protein folding, secondary structures, lipid headgroups	Enhances transitions, substrate turnover, molecular flexibility
Lifespan	Longer (~ 200 fs to 0.5 s in specific conditions)	Shorter and highly dynamic

create microenvironments that stabilize or destabilize intermediate folding states.

(2) **Enzyme catalysis**. WMA may modulate catalytic efficiency by restructuring local density and hydrogen-bonding networks near enzyme active sites, enhancing reaction kinetics.

(3) **Membrane dynamics**. Water micro assemblies may impact the stability and interactions of lipid bilayers, offering insights into phenomena such as lipid rafts and transient membrane pore formation.

(1) **Biological folding and protein stability**. Proteins rely on their interactions with water to achieve and maintain their functional structures (Sen and Voorheis 2014; Schiebel et al. 2018). The interplay of various forces during folding, such as hydrophobic interactions, hydrogen bonding, electrostatic interactions, van der Waals forces and steric constraints, highlights the central role of water in these processes (Bellissent-Funel et al. 2016; Phan-Xuan et al. 2020).

Hydrophobic interactions are a key driving force in macromolecular folding, as nonpolar residues cluster internally to avoid water (Fogarty and Laage 2014; Ye et al. 2024). LDW regions, with their reduced density and structured hydrogen bonding network, may facilitate this process by forming ordered exclusion zones around hydrophobic groups, stabilizing the burial of nonpolar residues. Meanwhile, HDW regions, less structured and more dynamic, may support the transient exposure of hydrophobic residues during early folding stages. Hydrogen bonding is another critical force which stabilizes secondary structures such as α-helices and β-sheets. HDW, due to its higher entropy and looser hydrogen bonds, may disrupt weaker hydrogen bonds in unfolding scenarios, whereas LDW may enhance hydrogen bond stability during folding.

Electrostatic interactions, such as salt bridges and charged residue pairings, are crucial for directing protein folding pathways and stabilizing the final folded structure. The local density variations between LDW and HDW may influence the dielectric constant of water, modulating electrostatic interaction strength. LDW, by reducing shielding effects, may enhance charge–charge attractions in localized regions, promoting stability. Van der Waals forces, though weaker and distance dependent, are crucial for stabilizing tightly packed molecular cores (Li et al. 2018). The structured nature of LDW may enhance these forces by increasing the local organization of interacting molecules within protein interiors. Steric constraints further limit the conformational space accessible to macromolecules, as backbone torsions and side-chain interactions impose structural restrictions. LDW's ordered environment may help resolve steric clashes, guiding molecules toward favorable conformations, while HDW may provide flexibility for conformational sampling in partially unfolded states.

The involvement of WMA in biological folding becomes plausible when considering their energy, timescale and spatial dimensions. The energy scales of folding forces align with the contributions of LDW and HDW transitions (Table 2), suggesting that WMA may significantly influence folding energetics. The interplay between LDW and HDW may fine-tune the folding energy landscape, balancing stability and flexibility across the conformational space to optimize transitions (Fig. 1). The timescales of WMA also align well with the dynamic requirements of folding processes. Protein folding occurs over milliseconds to seconds for small, single-domain proteins, while LDW and HDW regions exhibit lifetimes ranging from ~200 fs to 0.5 s. This temporal overlap allows micro assemblies to influence both the rapid early stages of folding and the slower stabilization of intermediate or final structures. Spatially, the sizes of micro assemblies, ranging from ~0.3 to 2 nm, are sufficient to interact with folding nuclei (typically ~5 to 10 Å) and influence tertiary and quaternary structural organization.

Experimental and theoretical studies provide evidence for the role of WMA in biological folding and stability. Hydration dynamics reveal how LDW and HDW influence solvation shell behavior (Camisasca et al. 2023). These studies highlight the ability of LDW to form structured, ice-like networks around biomolecules, stabilizing their conformations, while HDW introduces dynamism required for conformational transitions. Evidence from folding pathways, particularly in cold denaturation studies, further supports the relevance of structured water (Taricska et al. 2019). Cold denaturation, where proteins unfold at low temperatures, has been linked to the stabilization of unfolded states by structured hydration layers, consistent with the properties of LDW-HDW transitions (Espinosa et al. 2019).

In sum, WMA may represent a feasible and energetically relevant mechanism for modulating biological folding and stability. Their ability to dynamically adjust local density, hydrogen bonding and dielectric properties makes them a critical, yet underexplored, factor in biophysical processes. LDW stabilizes hydrophobic interactions and secondary structures, while HDW enables dynamic transitions and flexibility. These complementary roles, supported by energy and timescale compatibility, suggest that WMA are integral to the folding process and its regulation.

(2) **Enzyme catalysis**. Enzymatic reactions are governed by various physical forces, all of which align with the unique properties of WMA (Adamczyk et al. 2014; Fogarty and Laage 2014; Zsidó and Hetényi 2021). Electrostatic effects are critical in enzyme catalysis, as charged residues and substrates interact to stabilize the

Table 2 Functional roles of water micro assemblies in protein folding processes

Force type	Energy contribution (kcal/mol)	Role in protein folding	Impact of micro assemblies	Microaggregate energy contribution (kcal/mol)
Electrostatic interactions	~ 1–3	Guide charged residue interactions and folding pathways	LDW enhances electrostatic interactions by lowering dielectric constant; HDW facilitates charge redistribution	LDW: ~ 1–3, HDW: ~ 1–3
Hydrogen bonding	~ 1–5	Stabilizes secondary structures (e.g., α-helices, β-sheets)	LDW strengthens hydrogen bonds, stabilizing folding intermediates; HDW allows flexibility for conformational sampling	LDW: ~ 1–3, HDW: ~ 1–3
Hydrophobic interactions	~ 0.5–1 per residue	Drive hydrophobic core formation	LDW creates structured exclusion zones around hydrophobic residues; HDW transiently disrupts zones to allow folding progression	LDW: ~ 1–3, HDW: ~ 1–3
Van der Waals forces	~ 0.1–0.3 per atom pair	Stabilize tightly packed core structures	LDW enhances alignment and packing in the hydrophobic core; HDW supports transient adjustments during folding	LDW: ~ 0.5–1, HDW: ~ 0.5–1
Solvation dynamics	Variable	Modulate folding energy landscape	LDW-HDW transitions fine-tune hydration layers, stabilizing intermediates and guiding the folding process	LDW: ~ 1–3, HDW: ~ 1–3

transition state and orient substrates within the active site. LDW, with its lower dielectric constant, may enhance these interactions by reducing charge screening, thereby increasing the strength of electrostatic interactions. HDW, due to its flexible hydrogen bonding and higher local entropy, may assist in charge redistribution during catalytic transitions, helping enzymes overcome energy barriers associated with electron transfer or polarization. This dual role enables WMA to modulate the precise electrostatic environment required for catalysis.

Hydrophobic interactions contribute to substrate stabilization by forming nonpolar pockets within enzyme active sites (Kurkal et al. 2005). These hydrophobic pockets exclude bulk water, providing a favorable microenvironment for substrate binding. LDW regions may amplify this effect by creating structured exclusion zones around nonpolar residues, reinforcing the hydrophobic pocket's integrity. On the other hand, HDW, with its dynamic nature and reduced local density, may transiently disrupt these zones, allowing substrate access to the active site. This balance may ensure that the enzyme maintains a stable yet flexible environment, optimizing both substrate accommodation and turnover. Hydrogen bonding is another cornerstone of enzyme catalysis, stabilizing intermediates and transition states while maintaining precise catalytic geometry. LDW regions, with their stable and extended hydrogen bonding networks, may strengthen these interactions, particularly with polar substrates or intermediates. Conversely, HDW, characterized by dynamic and flexible hydrogen bonding, may facilitate transitions between intermediates and supports efficient product release.

Van der Waals forces play a significant role in substrate alignment and transient state stabilization within enzyme active sites. LDW regions, with their structured local environments, may enhance molecular alignment and optimize these weak interactions. HDW, on the other hand, may introduce the required dynamics to ensure that transient interactions remain flexible, allowing enzymes to adapt their active site to different catalytic steps. Dynamic solvation effects, wherein fluctuations in the solvent modulate energy barriers and facilitate substrate-product exchange, are intrinsically linked to LDW–HDW transitions. These transitions may enable WMA to fine-tune solvation dynamics at each step of the catalytic cycle. LDW regions stabilize specific steps, such as substrate binding or transition state stabilization, while HDW regions introduce dynamic flexibility to promote intermediate transitions and product release. This adaptability may be crucial for the efficiency of enzymatic reactions.

The alignment of WMA energetics with enzymatic processes further supports their potential to influence enzyme

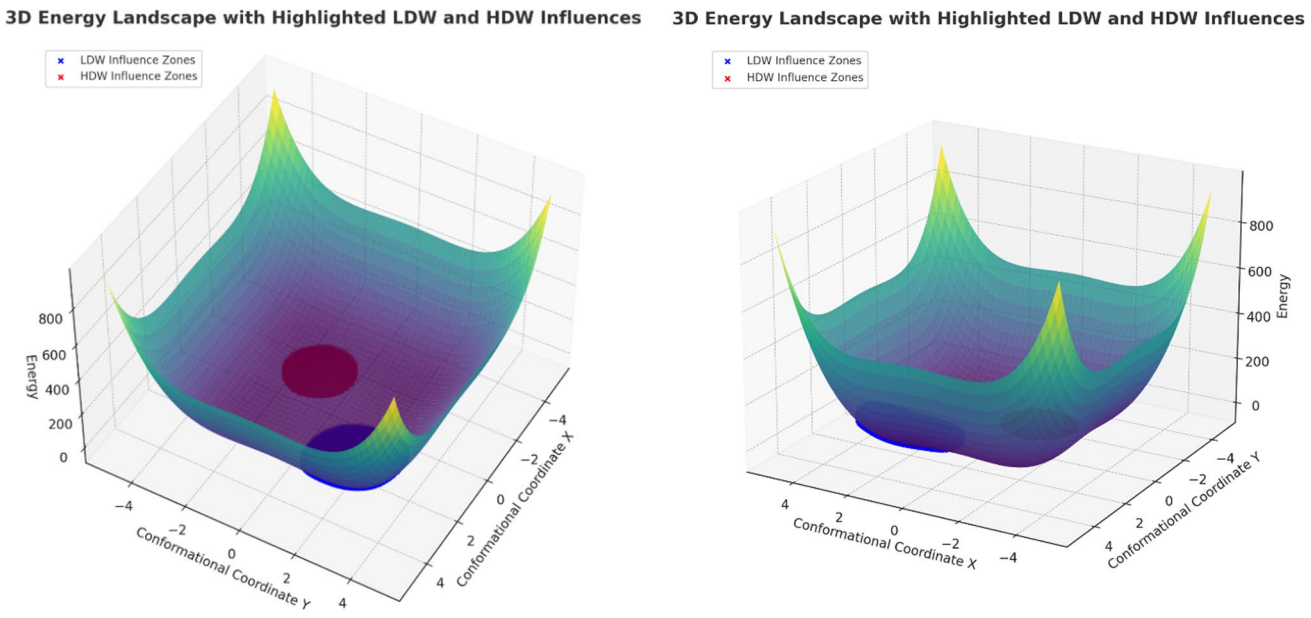


Fig. 1 Theoretical 3D visualization of the energy landscape shaped by water microaggregates, illustrating the combined effects of LDW and HDW. LDW zones, depicted in blue, stabilize the energy landscape by forming depressions near local minima, while HDW zones, represented in red, enhance flexibility and facilitate dynamic transitions near transition states

catalysis (Table 3). The timescales of enzymatic processes, which typically range from microseconds to milliseconds, align well with the lifetimes of LDW and HDW regions, which span from 200 fs to 0.5 s. For instance, LDW regions may stabilize critical transition states over microsecond timescales, while HDW regions provide the rapid flexibility needed for intermediate transitions on sub-nanosecond timescales. Spatially, enzyme active sites are typically 1–10 nm in size, a scale well suited for interactions with micro assemblies, which range from 0.3 to 2 nm. This spatial compatibility may allow LDW and HDW regions to directly influence active site dynamics, stabilizing critical regions and facilitating the movement of substrates and products. LDW may form around hydrophobic residues in the active

Table 3 Functional roles of water micro assemblies in enzyme catalysis

Force type	Energy contribution (kcal/mol)	Role in catalysis	Impact of micro assemblies	Microaggregate energy contribution (kcal/mol)
Electrostatic interactions	~ 1–3	Stabilize transition states	LDW enhances interactions by lowering the dielectric constant; HDW facilitates charge redistribution	LDW: ~ 1–3, HDW: ~ 1–3
Hydrophobic interactions	~ 0.5–1 per residue	Stabilize substrates in pockets	LDW promotes exclusion zones; HDW transiently opens access pathways	LDW: ~ 1–3, HDW: ~ 1–3
Hydrogen Bonding	~ 1–5	Stabilizes intermediates	LDW strengthens bonds; HDW enables dynamic flexibility for transition states	LDW: ~ 1–3, HDW: ~ 1–3
Van der Waals Forces	~ 0.1–0.3 per atom pair	Align substrates	LDW improves alignment precision; HDW supports transient substrate dynamics	LDW: ~ 0.5–1, HDW: ~ 0.5–1
Solvation Dynamics	Variable	Modulate energy barriers	LDW-HDW transitions fine-tune solvation dynamics during different catalytic stages	LDW: ~ 1–3, HDW: ~ 1–3

site, enhancing substrate stability, while HDW may transiently occupy solvent-accessible areas.

Experimental and theoretical evidence supports the role of WMA in enzyme catalysis. Structured water layers, as observed through solvent isotope effects and hydration studies, correlate with enhanced catalytic rates, aligning with the behavior of LDW (Kurkal-Siebert et al. 2006). Cryo-EM and X-ray crystallography further reinforce this perspective by revealing ordered water clusters near enzyme active sites (Brogan et al. 2014). These clusters, consistent with LDW properties, may play a crucial role in stabilizing catalytic residues. Theoretical studies add depth to this understanding. Molecular dynamics simulations demonstrate that water clusters near active sites exhibit high sensitivity to changes in hydrogen bonding, mirroring the dynamic interplay between LDW and HDW. These transitions highlight the adaptability of WMA in responding to the demands of enzymatic processes, such as stabilizing intermediates or facilitating substrate-product transitions.

The case of chymotrypsin illustrates how WMA may influence enzymatic function. Chymotrypsin, a serine protease, relies on a catalytic triad comprising serine, histidine and aspartate residues for its nucleophilic attack mechanism (Jing et al. 2002). This process is supported by a structured hydration layer, a hallmark of LDW behavior (Kozlova et al. 1999; Eckstein et al. 2002). LDW may stabilize the catalytic triad by strengthening hydrogen bonding, ensuring the alignment and reactivity of these residues. Conversely, HDW may facilitate rapid proton transfer and substrate turnover by modulating local water density, providing the necessary flexibility for catalytic efficiency.

In sum, WMA provide a viable and energetically significant mechanism for influencing enzyme catalysis. Their ability to dynamically adjust hydrogen bonding, hydrophobic interactions, local density and solvation properties makes them possible contributors to catalytic processes.

(3) **Membrane dynamics**. Biological membranes rely heavily on interactions with surrounding water for their structural integrity, fluidity and functionality (Higgins et al. 2006; Wood et al. 2007; Chattopadhyay et al. 2021). The interplay between LDW and HDW regions in the aqueous environment may provide insights into how water influences processes such as lipid organization, protein–membrane interactions and membrane dynamics. Electrostatic interactions are essential for membrane dynamics in stabilizing the polar headgroups of lipids and mediating interactions with charged proteins. LDW regions, with their lower dielectric constant and ordered hydrogen bonding, may enhance membrane electrostatic stabilization by reducing charge screening and strengthening headgroup interactions. HDW regions, on the other hand, may facilitate rapid ion exchange and dynamic interactions between polar lipids and surrounding ions or proteins. This dual behavior ensures that membranes maintain stability while allowing flexibility for dynamic processes.

Hydrophobic interactions are another crucial component of membrane dynamics, driving the self-assembly of lipid bilayers and maintaining their structural integrity (Cheng et al. 2013; Fisette et al. 2016). Nonpolar lipid tails cluster together to minimize water exposure, a process which may be amplified by LDW regions forming structured exclusion zones around hydrophobic regions. Conversely, HDW regions may transiently disrupt these zones, allowing for the lateral mobility of lipids and enabling dynamic rearrangements critical for processes like fusion, fission and protein insertion. Still, hydrogen bonding plays a significant role in maintaining the hydration shells of lipid headgroups and facilitating interactions between the membrane and water-soluble molecules. LDW regions, with their stable hydrogen bonding networks, may provide consistent hydration to polar headgroups, enhancing membrane stability. In contrast, HDW regions, with their dynamic hydrogen bonding, may support processes requiring flexibility, such as lipid flip-flop, membrane deformation or protein–membrane interactions. Van der Waals forces are crucial for lipid packing and membrane stability. LDW regions, by organizing water molecules near the lipid interface, may enhance molecular packing and stabilize membrane structure. Meanwhile, HDW regions may contribute to the dynamic rearrangement of lipids, ensuring the bilayer remains fluid and adaptable under varying conditions.

The transition between different membrane phases, such as the liquid-ordered and liquid-disordered states, provides a case study in micro assembly influence. LDW regions may stabilize the liquid-ordered phase by enhancing hydrogen bonding and hydrophobic stabilization, while HDW regions may support the flexibility needed for transitions to the liquid-disordered phase, facilitating processes such as membrane protein activation or lipid raft formation. Dynamic solvation effects, where water molecules in the hydration shell modulate energy barriers and lipid dynamics, may be influenced by LDW–HDW transitions. LDW regions may stabilize bilayer formation by providing structured hydration to lipid headgroups, while HDW regions may allow rapid water exchange and flexibility, supporting membrane-associated processes such as vesicle formation and membrane protein function.

The ability of WMA to influence membrane dynamics is reinforced by their energetic compatibility (Table 4), suggesting that WMA can meaningfully modulate the forces driving membrane behavior. The timescales of membrane dynamics, ranging from nanoseconds for lipid

Table 4 Functional roles of water micro assemblies in membrane dynamics

Force type	Energy contribution (kcal/mol)	Role in membrane dynamics	Impact of micro assemblies	Microaggregate energy contribution (kcal/mol)
Electrostatic interactions	~ 1–3	Stabilize lipid headgroups and protein interactions	LDW enhances headgroup stabilization by lowering dielectric constant; HDW facilitates dynamic ion exchange	LDW: ~ 1–3, HDW: ~ 1–3
Hydrogen bonding	~ 1–5	Stabilizes hydration shells around lipid headgroups	LDW strengthens hydrogen bonds around headgroups; HDW supports rapid rearrangements during membrane remodeling	LDW: ~ 1–3, HDW: ~ 1–3
Hydrophobic interactions	~ 0.5–1 per lipid	Drive lipid tail clustering and bilayer stability	LDW forms structured exclusion zones around lipid tails; HDW transiently disrupts zones for fluidity	LDW: ~ 1–3, HDW: ~ 1–3
Van der Waals forces	~ 0.1–0.3 per atom pair	Stabilize lipid packing	LDW improves packing by structuring water near lipid interfaces; HDW supports lateral diffusion	LDW: ~ 0.5–1, HDW: ~ 0.5–1
Solvation dynamics	Variable	Modulate lipid mobility and protein incorporation	LDW–HDW transitions adjust hydration shells to support vesicle formation, fusion and protein binding	LDW: ~ 1–3, HDW: ~ 1–3

diffusion to milliseconds for processes like vesicle budding, align well with LDW and HDW lifetimes. This temporal compatibility may allow WMA to influence both rapid lipid rearrangements and slower membrane remodeling events. Spatially, the thickness of the lipid bilayer (~ 4–6 nm) and the hydration shell (~ 0.3–1 nm) are well matched to the size of LDW and HDW regions, ensuring that WMA may interact with membrane components and their surrounding environment.

Experimental evidence supports the involvement of WMA in membrane dynamics. Neutron scattering and cryo-EM studies have revealed ordered water layers around lipid headgroups, consistent with LDW properties (Frölich et al. 2009). Ultrafast spectroscopy and NMR studies further demonstrate the dynamic nature of hydration layers, aligning with the behavior of HDW (Zigmantas et al. 2022; Lorenz-Ochoa et al. 2023). Molecular dynamics simulations provide additional theoretical support, showing structured and dynamic water regions interacting with lipid bilayers and modulating membrane properties.

In sum, WMA provides a plausible mechanism for influencing membrane dynamics. Their ability to modulate hydrogen bonding, electrostatic interactions and solvation dynamics positions them as critical contributors to the stability and functionality of biological membranes.

Percolation of water micro assemblies and its role in biological systems

Short-lived WMA can form extensive networks that significantly influence average flow properties. The emergence of large-scale connectivity within high-density water assemblies may facilitate percolation through three-dimensional hydrogen-bonded water lattices (Timonin 2018). Evidence supports the occurrence of percolation in water (Bernabei & Ricci 2008; Strong et al. 2018). For instance, percolation transitions in hydrogen bond networks have been observed in supercritical water, especially at high molecular densities (Jedlovszky et al. 2007). These transitions follow a universal power law, with percolation occurring when the fractal dimension of the largest cluster approaches 2.53 (Galam & Mauger 1996; Jedlovszky et al. 2007). A percolating network requires approximately 40% of possible hydrogen bonds to be intact. Simulations reveal that initially disconnected clusters coalesce at critical cutoff values, forming large, space-filling networks with minimal disconnected fragments (Geiger 1979). Given that 18.01528 g of water contain $6.02214076 \times 10^{23}$ water molecules, the number of percolating molecules is immense, with a system-spanning cluster emerging at a probability of 0.65 in liquid water (Oleinikova et al. 2002).

Percolation significantly influences flow dynamics by creating transient obstacles. Hydrogen bond energies (1–5 kcal/mol) are much lower than those needed to break covalent bonds (~ 110 kcal/mol) (Lodish et al. 2000). This means that the average molecular kinetic energy at room temperature (~ 0.6 kcal/mol) is sufficient to disrupt noncovalent bonds, allowing hydrogen bond-induced barriers to influence liquid water flow (Lodish et al. 2000).

Percolation theory offers insights into how WMA influence hydrogen bond networks, phase transitions and transport phenomena. We propose the following theoretical framework:

(1) **LDW and percolation clusters**. LDW forms well-ordered hydrogen bond structures, facilitating connectivity and the formation of percolation clusters. Conversely, HDW, with its disrupted hydrogen bonding, acts as a barrier to connectivity.
(2) **Phase transition dynamics**. At lower temperatures, LDW regions expand, forming nuclei essential for freezing, while HDW fragments connectivity in supercooled water. In supercritical water, HDW dominates, leading to high diffusivity and low connectivity. As the system transitions back to the liquid phase, LDW re-establishes percolation, restoring cohesion and reducing diffusivity.
(3) **Diffusion and transport**. Diffusion in liquid water depends on cooperative molecular movement within hydrogen bond networks. LDW forms stable pathways for molecular and ion transport, while HDW introduces transient disruptions. At the percolation threshold, LDW clusters span the system, enabling continuous transport pathways. Proton transport, for instance, relies on rapid hydrogen bond reorganization. LDW maintains stable pathways for proton hopping, whereas HDW slows the process due to structural disorder.
(4) **Dynamic percolation and temperature effects**. Dynamic percolation in water is characterized by the continuous breaking and reforming of hydrogen bonds. At lower temperatures, LDW dominates, forming long-lived percolation clusters, whereas at higher temperatures, HDW fragments the networks, increasing molecular mobility.
(5) **Constrained percolation in confined environments**. In confined environments, such as nanoporous materials or near biomolecular surfaces, water may exhibit constrained percolation. LDW forms ordered layers near hydrophilic interfaces, providing stability, while HDW dominates bulk regions or hydrophobic zones, introducing flexibility and disorder.

In sum, the WMA theoretical framework highlights the fundamental role of percolation in biological systems and provides a basis for further exploration of LDW and HDW's influence on molecular and macroscopic phenomena. Future integration of percolation models with experimental and computational approaches will help unravel the complexity of these dynamic water networks.

Conclusions

Liquid water exhibits intrinsic chemical inhomogeneity, manifesting as a dynamic three-dimensional network characterized by microscopic density fluctuations and structural variations. Water, far beyond being a passive solvent, plays an active and multifaceted role in driving and modulating biological phenomena. This study explored the concept of WMA, characterized by LDW and HDW regions, as dynamic contributors to various molecular and cellular processes. Through their distinct physical and chemical properties, WMA offer a framework to better understand the behavior of water at the nanoscale and its influence on key biological processes. LDW, defined by structured hydrogen bond networks, lower density and high coherence, may stabilize hydrophobic interactions, hydrogen bonding and molecular order. Conversely, HDW, with its dynamic, higher-density and entropy-driven properties, may facilitate flexibility, conformational sampling and rapid transitions. These dual roles enable WMA to act as adaptable mediators across a wide range of biological phenomena, optimizing stability and functionality under varying conditions. Across protein folding, enzyme catalysis and membrane dynamics, the energy contributions, timescales and spatial scales of WMA align with the forces and dynamics involved, underscoring the critical role of LDW and HDW transitions in modulating dynamical interactions in the cellular milieu.

While the concept of WMA is still emerging, their influence may extend beyond understanding fundamental biological processes. WMA may have significant implications across experimental, technological and pathological contexts, spanning multiple disciplines, including molecular biology, fluid dynamics and materials science.

In microfluidics and nanofluidics, understanding the dynamic behavior of these micro assemblies can lead to enhanced control of water within confined geometries, a critical factor for designing lab-on-chip devices that require precise manipulation of fluid dynamics at the microscale. The ability to induce or manipulate LDW and HDW regions artificially opens the door to creating targeted micro-vortices or density variations. Such capabilities may pave the way for novel sensor technologies where the localized structuring of water affects sensitivity and detection thresholds or for advanced turbulence control mechanisms in engineering systems. Also, artificial catalysts may benefit from mimicking

LDW-HDW transitions, improving efficiency by incorporating water-like dynamics into synthetic systems.

In biomedical and pharmaceutical contexts, understanding WMA can inform the design of drugs and therapeutics that leverage hydration dynamics. Mimicking the properties of WMA may inspire the development of biomimetic systems such as responsive hydrogels or biomimetic membranes with tailored properties for drug delivery. In drug design, targeting water dynamics within active sites may optimize the specificity and efficacy of ligand binding and enzyme inhibitors, enhancing drug efficacy. Our study also highlights the importance of exploring the role of WMA in complex and confined environments, such as within cells or in biomolecular assemblies. In these scenarios, water's behavior deviates significantly from that of bulk water, with hydration layers and confined spaces amplifying the influence of micro assemblies. Understanding these effects may reveal new principles underlying cellular organization, molecular recognition and signal transduction.

Additionally, understanding water's role in pathological conditions, such as protein misfolding diseases, cancer or neurodegeneration, may open new avenues for therapeutic intervention. Abnormal interactions between LDW and HDW may contribute to the pathological protein aggregation observed in lipid storage disorders and neurodegenerative diseases such as Alzheimer's or Parkinson's disease (Polychronidou et al. 2020; Padilla-Godínez et al. 2021). Similarly, variations in water density may explain some of the unique physical and biochemical properties of tumor microenvironments (He et al. 2022). In cancer, localized turbulence or non-uniform water structuring in the interstitial fluid may influence nutrient transport, cellular communication and drug delivery effectiveness.

Despite significant advances, challenges persist in fully understanding the role of WMA in biological phenomena. The transient and highly dynamic nature of LDW and HDW makes their phase spaces difficult to explore, leading to the upsetting concept of "water's no-man's land" (Lin et al. 2018). Despite that liquid water can be tackled in terms of an evolving, fluctuating and branched polymer (Naserifar and Goddard 2019), a full understanding of its dynamical and structural properties is still lacking (Fanetti et al. 2014) due to technical difficulties in gaining experimental information on ultrafast interplay (Tamtögl et al. 2020). The heterogeneity of water is usually approached through simulation of molecular dynamics, such as, e.g., conventional QM/MM scheme and ONIOM-XS methods (Thaomola et al. 2012), second-order Møller–Plesset perturbation theory (Liu et al. 2017), quantum Monte Carlo, non-canonical coupled cluster theory (Al-Hamdani and Tkatchenko 2019), modified Louvain algorithm of graph community (Gao et al. 2021), topological local (clustering coefficient, path length and degree distribution) and global (spectral analysis) properties (dos Santos et al. 2004; Carreras et al. 2008; Steinberg et al. 2019), persistent homology methods (Wu 2020). These methods operate at different temporal and spatial resolutions, necessitating further integration for a more comprehensive understanding. Weak, non-covalent interactions have been experimentally studied just in small molecular complexes, falling short of the macroscopic structural properties that are typical of complex soft materials such as, e.g., supramolecular aggregates (Al-Hamdani and Tkatchenko, 2019). To make things more complicated, totally different networks topologies and physical interpretations have been provided, depending on how rings have been counted (Das et al. 2019; Formanek and Martelli 2020).

It remains uncertain whether liquid water exhibits randomness or long-range interactions. Dos Santos et al. (2004) proposed that the water network at room temperature resembles a Poisson distribution, indicative of randomness. However, other studies present evidence of medium- to long-range order (Faccio et al. 2022). Tamtögl et al. (2020) observed correlated motion at the surface of a topological insulator, contrasting with Brownian motion, while Ansari et al. (2018) and Gao et al. (2021) reported collective translational fluctuations in hydrogen-bonded water clusters. Percolation models face challenges, such as the inability to determine the percolation threshold based solely on cluster size distribution (Jedlovszky et al. 2007). Moreover, environmental factors, including temperature, pressure and solute concentration may influence the balance between LDW and HDW. For example, lower temperatures correspond to stronger hydrogen bond interactions. Upon melting, increases in temperature result in a rapid decrease in the average number of assemblies (Gao et al. 2021) and a broader ring size distribution (Bakó et al., 2013; Naserifar and Goddard 2019). Consequently, microscopic assemblies in water become difficult to distinguish beyond the isochore end point of 292 K (Nomura et al., 2017).

Future research should focus on developing more refined experimental and computational methods to study WMA and their interactions with biomolecules. Experimentally, combining time-resolved methods with spatially resolved imaging techniques may bridge the gap between molecular dynamics and observable biological outcomes.

In conclusion, WMA may represent a transformative perspective on the active role of water in biological phenomena. By bridging the gap between molecular-scale dynamics and macroscopic behavior, WMA may provide a unifying framework for understanding how water shapes biomolecular processes. LDW and HDW, through their complementary properties, may influence stability, flexibility and functionality across diverse biological systems. As research in this field advances, the potential applications in health, technology and materials science are vast, underscoring

the importance of continued exploration into the dynamic Q8world of WMA.

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Declarations

Conflict of interest The author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within 3 years of beginning the submitted work that may inappropriately influence, or be perceived to influence, their work.

Ethical approval and consent to participate This research does not contain any studies with human participants or animals performed by the author.

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Designing AI Systems with Kantian Cognitive Structures

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ABSTRACT

We propose a framework for artificial intelligence design grounded in Immanuel Kant's transcendental philosophy, particularly his theory of cognition as presented in the *Critique of Pure Reason*. We argue that Kant's model of the mind—structured by a priori forms of intuition and categories of understanding— provides a valuable conceptual foundation for designing novel cognitive architectures capable of structured, unified and meaningful experience. Drawing from philosophical analysis, conceptual mapping, classical cognitive functions, architectural computational modeling and contemporary deep learning, we translate key Kantian elements into modular components of AI systems, namely, 1) spatial-temporal frameworks (forms of intuition), 2) logical-structural priors (categories), 3) a synthetic mechanism for integrating perception/concept and 4) a self-model representing the unity of apperception. Experimental simulations comparing unstructured models with Kantian-structured architectures demonstrate that the latter consistently support coherent perceptual integration, stable self-representation across time and principled, norm-governed reasoning. We argue that a Kantian-inspired structure could inform the design of multi-modal, self-coherent systems capable of maintaining consistent internal world models and interpreting input through structured layers. Emphasis is placed on the system's capacity to organize experience rather than merely accumulating data, distinguishing it from prevailing empirical or data-driven approaches. Although the model remains limited in its current form as a theoretical construct, it has the potential to support more robust, self-consistent and ethically aligned AI behaviour by embedding formal cognitive constraints directly into the system's architectural design.

Keywords: selfhood; symbolic reasoning; autonomous systems; cognitive modeling; epistemic structure

INTRODUCTION

Current advancements in artificial intelligence, particularly within deep learning and neural architecture, have yielded highly capable systems for pattern recognition, language processing and multimodal representation. These systems have demonstrated remarkable performance in domain-specific tasks, from visual object classification to predictive language modeling and embodied sensorimotor control. However, these models often lack internal coherence, interpretability and a unifying structure of experience. They operate primarily as statistical engines, mapping inputs to outputs without an explicit framework for integrating perceptual data with conceptual understanding or maintaining continuity of reasoning across time. As a result, they frequently exhibit brittle generalization, fragmented behavior and shallow semantic processing, particularly when faced with tasks that require memory, abstraction or principled deliberation.
Attempts to address these limitations have followed two primary trajectories. On one hand, purely data-driven approaches rely on end-to-end optimization and massive training corpora, often trading structural integrity for empirical flexibility. On the other hand, symbolic and rule-based systems attempt to encode cognitive functions explicitly, but lack robustness and adaptability in real-world settings. These opposing paradigms have yet to yield a framework that combines flexible learning with principled representation. Systems aspiring toward general or human-like cognition must be designed not merely to *learn* from data, but to *organize* it under formal conditions—structural constraints that guide the synthesis of information into coherent, context-sensitive and interpretable forms. This line of thinking points toward the need for a re-evaluation of the architectural assumptions underpinning contemporary AI.

We address this challenge by proposing a theoretical model for artificial cognition grounded in the transcendental philosophy of Immanuel Kant. In contrast to models that define cognition in terms of empirical behavior or task performance, Kant's critical project focuses on the *conditions of possibility* for coherent experience. In the *Critique of Pure Reason*, Kant argues that the mind does not passively receive sensory inputs but actively structures them through a set of a priori forms and categories. These include the *pure forms of intuition*—space and time—which govern how perceptual data is received, as well as the *categories of the understanding*, such as causality, unity, substance, necessity, etc., which shape how experience is conceptualized. These elements are not derived from experience, but rather constitute its preconditions; they serve as the framework within which representations are made intelligible. Kant's model

culminates in the notion of the *transcendental unity of apperception*, a formal self-model that unifies the manifold of representations under a single perspective and enables stable cognition across time.
These philosophical components—forms of intuition, categories, synthesis and selfhood—offer a conceptual scaffolding in sharp contrast to the prevailing architectures in artificial intelligence. From a Kantian standpoint, cognition is not simply a mapping from input to output, nor is it a set of ad hoc rule-based operations. Rather, it is an *architectonic system*, in which every representational act presupposes a formal structure of organization. Translating this insight into computational terms suggests a new approach to cognitive design, i.e., an approach that embeds necessary representational conditions into the architecture itself, allowing the system not just to process data, but to experience it as organized, interpretable and coherent.

We propose a modular, principled framework in which Kant's epistemological structures are reformulated as functional components within an artificial system. The *forms of intuition* are implemented as spatial-temporal priors pre-structuring sensory input. The *categories of the understanding* are encoded as abstract operators that impose logical and relational constraints on perceptual data. The *synthesis of representations* is realized via attention-based mechanisms that bind multimodal inputs into unified cognitive objects. Finally, the *unity of apperception* is instantiated through a recursive self-model that maintains temporal identity, policy continuity and referential coherence across decision-making contexts. The resulting architecture is designed not to replicate human consciousness, but to instantiate the formal conditions necessary for intelligible and principled artificial cognition.

Our paper is organized as follows. In the next section, we describe the conceptual mapping between Kantian structures and computational components, specifying the design rationale for each module. We detail the system architecture and the methodology used for implementing and evaluating its components, including the mathematical formulations underlying each process. This is followed by a presentation of empirical results from simulation-based evaluations, measuring coherence, continuity and norm adherence. Finally, we discuss the implications of our architecture for future work in structured machine cognition, identify its present limitations and outline possible directions for experimental validation and theoretical refinement.

MATERIALS AND METHODS

Conceptual mapping and system design. The design of a cognitive architecture informed by Kantian theory requires a systematic mapping between philosophical structures and computational counterparts (**Figure 1**). This chapter presents that mapping by translating four foundational components of Kant's transcendental framework into implementable architectural modules. These include: (1) the *Forms of Intuition*, understood as innate, pre-conceptual frameworks for organizing sensory input; (2) the *Categories of the Understanding* or the abstract principles by which the mind conceptualizes experience; (3) the *Synthesis Engine*, responsible for binding representations across modalities and temporal sequences; and (4) the *Unity of Apperception*, which functions as a persistent self-model anchoring cognitive operations under a unified perspective.
The *Forms of Intuition* are modeled as structural priors embedded within the system's perception layer. This module includes a spatial framework, such as a topological or 3D world model and a temporal sequencing buffer that maintains ordered input for event tracking. These structures do not represent learned features but predefined constraints under which perceptual information is formatted.
The *Categories of the Understanding* are instantiated as logical and relational operators—causality, identity, quantity—that act on structured perceptual data. These are encoded not as ad hoc classifiers but as persistent interpretive lenses that precondition conceptual analysis.
The *Synthesis Engine* operates as a central mechanism that integrates data from sensory and conceptual subsystems, managing attention, cross-modal binding and temporal coherence. It ensures that discrete data points cohere into unified perceptual scenes or conceptual objects, maintaining consistency within the internal world model.
Finally, the *Self-Model* serves as the operational equivalent of Kant's unity of apperception. It maintains a continuous, structured index of the system's current cognitive state, allowing for cross-temporal identity, reference resolution and context-sensitive deliberation.

These four modules are arranged within a layered architecture comprising three core strata: the **Perceptual Stratum**, where intuition modules are applied to raw data; the **Interpretive Stratum**, in which categories and synthesis transform input into structured representation; and the **Executive Stratum**, where the self-model mediates world-model updates and guides practical reasoning. Each layer communicates bidirectionally with the others, but is defined by functional distinctions that mirror Kant's division between intuition, understanding and reason.
By embedding these structural elements directly into the architecture, the system is oriented toward intelligible representation from the outset. This design departs from purely statistical approaches by assigning epistemic roles to each component and delineating clear boundaries between raw input, structured experience and autonomous reasoning. In the next chapter, we describe how these modules interact dynamically through processing cycles that mirror Kant's synthetic method, including the integration of perceptual flows, conceptual constraints and unified self-reference.

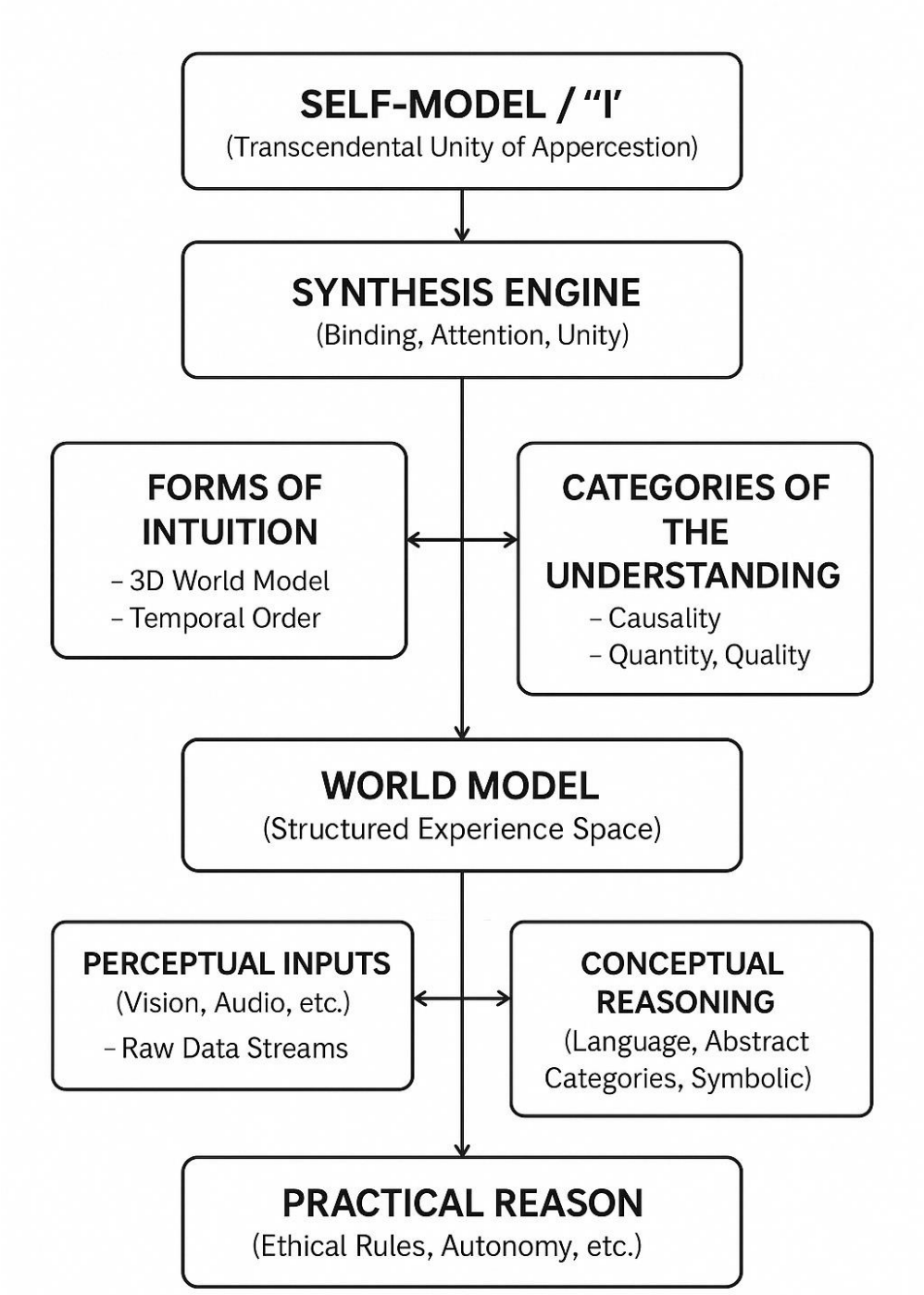


Figure 1. Minimalist diagram of Kantian-inspired AI architecture. The core structural components of an artificial intelligence system modeled after Kant's transcendental philosophy of mind are illustrated. The architecture is organized hierarchically, beginning with a central Self-Model which corresponds to Kant's "Transcendental Unity of Apperception" — the unifying self that accompanies all representations. Below it, the Synthesis Engine binds sensory and conceptual inputs into unified, coherent representations. Flanking the synthesis core are the Forms of Intuition (space and time) and the Categories of the Understanding (e.g., causality, quantity, quality), which structure raw inputs into intelligible experiences. These processed elements feed into a central World Model, representing the system's structured internal simulation of reality. Further down, Perceptual Inputs (e.g., visual, auditory data) and Conceptual Reasoning (symbolic, abstract processing) interact with the World Model to generate higher-level interpretations and decisions. Finally, at the base, Practical Reason guides the AI's actions according to internal principles, rules or ethical constraints, analogous to Kant's idea of autonomy and moral law. Arrows indicate the flow of information and synthesis across levels, emphasizing the system's active organization of experience, rather than passive data processing.

Perceptual structuring via forms of intuition. The perceptual intake layer was implemented with explicit reference to Kant's *forms of intuition*—space and time—encoded as structural priors applied at the initial sensory data representation stage. Input data was modeled as continuous signals $X_t \in \mathbb{R}^{m \times n \times c}$, representing spatial-temporal sequences where m and n denote spatial resolution and c the number of sensory channels. Temporal sequencing was enforced using a discretized time kernel $K_T \in \mathbb{R}^k$, where each time step t was treated as an indexable unit within a uniform metric space. We constructed a spatial encoder $f_s : \mathbb{R}^{m \times n \times c} \to \mathbb{R}^d$ using a 2D convolutional architecture with strided max-pooling and ReLU activation. For temporal encoding, a positionally-anchored transformer encoder $f_t : \{f_s(X_{t-k}), \ldots, f_s(X_t)\} \to \mathbb{R}^d$ was used, equipped with sinusoidal positional encodings $PE(t, 2i) = \sin(t / 10000^{2i/d})$. This combination enforced a space-time structuring prior that served as the architectural analog to Kant's a priori intuitions. Data was segmented into perceptual frames, each embedded with both spatial coherence and temporal continuity, enabling subsequent reasoning stages to operate over temporally grounded representations. These structuring priors thus formed the constraint under which sensory inputs were admitted into the system.

The establishment of space-time embedding as an early structural layer ensured that the system could process incoming data not as arbitrary tokens but as ordered phenomena suitable for conceptual synthesis.

Category-theoretic implementation of kantian operators. To implement Kant's categories of the understanding, we formulated a set of category-theoretic operators $\mathcal{C} = \{c_1, c_2, \ldots, c_k\}$, each corresponding to one of the classical twelve categories (e.g., causality, substance, modality). Each operator c_i is defined as a transformation $c_i : \mathcal{E} \to \mathcal{E}$, where $\mathcal{E}$ denotes the space of structured experience frames $\mathcal{E} = \{e_j \in \mathbb{R}^d\}$. The implementation relied on functional transformations applied to embedded perceptual representations. For instance, the causality operator c_{cause} was defined by a kernel-based inference mechanism over event sequences, formulated as:

$$c_{\text{cause}}(e_t, e_{t-1}) = \sigma(W(e_t - e_{t-1}) + b)$$

where $W \in \mathbb{R}^{d \times d}$ is a learned weight matrix, b a bias vector and σ a non-linear activation (e.g., tanh or GELU).

Substance was encoded through persistence detection using a memory-augmented attention mechanism

$$\alpha_{ij} = \frac{\exp(q_i \cdot k_j)}{\sum_k \exp(q_i \cdot k_k)},$$

where q_i and k_j denote query and key vectors associated with temporal embeddings of object instances. Each category was implemented as a discrete logical module interfacing with the world model graph G=(V,E), where nodes V were object-event pairs and edges E instantiated categorical relations. Logical consistency between multiple categories was enforced through a unification step defined by an intersection-over-union criterion on the inferred relational graphs.

The encoding of Kantian categories as functional and logical transformations over structured representations created the interpretive backbone for synthesizing perceptual content into intelligible experience.

Synthetic binding and attention dynamics. The synthetic unity of representations was achieved through an attention-based integration engine responsible for dynamically binding multi-modal, multi-temporal inputs into coherent cognitive objects. Let $R = \{r_1, r_2, \ldots, r_n\}$ be a set of input embeddings derived from different sensory modalities or time slices. The synthesis engine applies scaled dot-product attention over this set, defined by:

$$\text{Attention}(Q, K, V) = \text{softmax}\left(\frac{QK^T}{\sqrt{d_k}}\right)V$$

where $Q, K, V \in \mathbb{R}^{n \times d}$ are query, key and value matrices derived from R and d_k is the dimensionality of the key vectors. A residual integration function $f_s(r_i) = r_i + \text{Attention}(r_i, R, R)$ was applied iteratively, allowing for recurrent refinement of representational coherence. Multi-head attention was utilized to capture different axes of integration, such as spatial, temporal and conceptual dimensions. A recurrent binding layer updated the internal working memory $M_t \in \mathbb{R}^{k \times d}$ with synthesized tokens, enabling ongoing access to synthesized object-structures.

To preserve cross-modality alignment, cross-attention heads were paired with modality-specific embeddings augmented with learned modality vectors $m_i \in \mathbb{R}^d$, concatenated before final projection. The output of the synthesis engine $S_t \in \mathbb{R}^d$ was then passed into the world model for object consolidation. This mechanism implemented Kant's idea of manifold synthesis by actively relating disparate representations under conditions of structural coherence.

This step operationalized the process of synthetic unification, ensuring that representations formed by the system adhered to a coherent and referable structure.

Recursive self-model and temporal identity maintenance. The self-model responsible for the transcendental unity of apperception was realized as a recursive state-tracking module that maintained a persistent index over the system's cognitive operations. This module consisted of a recurrent update mechanism operating over a self-state vector $s_t \in \mathbb{R}^d$, where:

$$s_t = \phi(s_{t-1}, S_t, a_{t-1})$$

Here, ϕ is a gated recurrent unit (GRU), S_t is the synthesized perceptual state and a_{t-1} is the previous action vector. The self-state vector was stored and accessed via an external memory matrix $M_s \in \mathbb{R}^{T \times d}$, where each row indexed the agent's state at time t. To maintain identity across time and context, each representation r_t was tagged with an identity embedding $i_t = f_{\text{id}}(s_t)$ such that object tracking and reference resolution could be achieved by similarity metrics $\cos(i_t, i_{t'}) \geq \theta$.

The self-model also encoded intentions and commitments via a projection onto a latent policy space $\pi_t = f_\pi(s_t) \in \Delta(A)$, where $\Delta(A)$) is a probability simplex over action space A. This enabled backward and forward chaining of representational states within the self-narrative structure. The GRU parameters were trained to minimize divergence between predicted and observed state transitions, enforcing alignment between the system's internal model and its operational history.

Through this mechanism, the system acquired the structural continuity needed for reidentifying itself across transformations of state, task and intention.

World model representation and graph-based encoding. The system's world model was instantiated as a dynamic graph $G_t = (V_t, E_t)$ in which nodes represented synthesized object-event pairs and edges encoded relations, including those derived from category operators. Each node $v \in V_t$ held an attribute vector $a_v \in \mathbb{R}^d$ derived from the synthesis engine and each edge $e = (v_i, v_j, r) \in E_t$ included a relation label $r \in \mathcal{R}$ and weight $w_{ij} \in \mathbb{R}$. Updates to the world model followed a gated message-passing mechanism:

$$h_v^{(l+1)} = \sigma\left(\sum_{u \in \mathcal{N}(v)} w_{uv} \cdot W_r^{(l)} h_u^{(l)} + b_r^{(l)}\right)$$

where $h_v^{(l)}$ is the node embedding at layer l, $\mathcal{N}(v)$ the neighborhood of node v and $W_r^{(l)}$ the relation-specific transformation matrix. The graph was updated at each time step with the outputs of the synthesis engine and the current self-model state, allowing for contextual overwriting, object permanence tracking and event causality chaining.

Persistent facts were stored in a long-term graph database (Neo4j) and transitory representations were held in a Redis-based working memory. Periodic pruning of outdated subgraphs was performed using a decay function $d(t) = e^{-\lambda t}$ applied to node relevance scores. Queries to the world model for inference or planning used vectorized subgraph isomorphism algorithms $\mathcal{Q}(G_t, q) \rightarrow \{v_i\}$ for matching conceptual templates to internal representations.

This graphical world model grounded the architecture's representation of external reality, constrained by the structural principles of perception, synthesis and apperception.

Moral reasoning and practical decision layer. The practical reason module was constructed as a symbolic planning system driven by principle-based evaluation metrics. Action candidates $a_t \in A$ were scored not by reward but by a constraint satisfaction framework over normative rules $\mathcal{P} = \{p_1, \ldots, p_m\}$. Each rule p_i was expressed in first-order logic as:

$$p_i : \forall x, y\, [\phi(x, y) \rightarrow \psi(x, y)]$$

The action selection process minimized a constraint violation cost:

$$a_t^* = \underset{a_t}{\operatorname{argmin}} \sum_{i=1}^{m} \mathbb{I}_{\neg p_i(a_t)} \cdot c_i$$

where $\mathbb{I}_{\neg p_i(a_t)}$ is an indicator function for rule violation and c_i a rule-specific penalty. The logical inference engine (implemented in Prolog) evaluated the preconditions and consequences of each candidate action considering the current

world model and self-state. Scenarios were simulated within the world model's forward projection engine, generating predicted outcomes $\hat{s}_{t+1} = f_{\text{sim}}(s_t, a_t)$, which were then tested for compliance with $\mathcal{P}$.
A higher-order monitor ensured consistency of policy output over time, using temporal logic constraints $\square(p_i)$ to enforce that certain principles held globally across time steps. The module thus enacted a Kantian structure of practical reason, where permissible actions were not evaluated by their outcomes but by their internal logical coherence and universalizability.
This final computational layer operationalized norm-based autonomy, providing the architectural analog to Kant's moral law as the constraint on rational agency.

Functional dynamics and integration. Having established the conceptual and structural mapping between Kant's transcendental framework and computational modules, the dynamic processes through which these components interact in a functional system are outlined here. The goal is to describe how information flows within the architecture and how synthetic operations give rise to structured, coherent representations of experience. Rather than processing data in isolated streams, our system operates through recursive, interdependent cycles reflecting the layered synthesis identified by Kant as essential to cognition.
Processing begins at the perceptual interface, where incoming sensory data is immediately filtered through the *Forms of Intuition*. These spatial and temporal templates segment and locate input in a structured perceptual field. Data is then passed to the *Synthesis Engine*, which performs low-level feature binding, event segmentation and attentional modulation. Simultaneously, the *Categories of the Understanding* are applied, not as static classifiers but as functional constraints that guide the interpretation of perceptual sequences—ensuring, for example, that phenomena are understood in terms of causality, substance or plurality. These categories function across both perception and memory, preserving continuity in the system's experience of events and objects.
The *Synthesis Engine* plays a central role by coordinating the interaction between structured perception and conceptual operations. It selects salient features, binds them across modalities and composes them into integrated wholes. The engine's outputs are consolidated into the *World Model*, which stores representations in terms of both sensory attributes and conceptual content. This model is continuously updated in light of new data, maintaining coherence over time and allowing for the resolution of ambiguities through reference to prior experience.
All representational activity is anchored in the *Self-Model*, which serves as the system's epistemic center. It retains an index of the current cognitive state, including goals, attention focus and past commitments. This recursive unity enables cross-temporal identity and ensures that all data processed is referable to a single perspective. The *Self-Model* also interfaces with the *Practical Reason* module, which selects or generates actions according to internally defined principles, thereby aligning inference and decision-making with coherent internal norms.
Crucially, these dynamics are recursive and iterative. Perceptual updates trigger re-application of categories; changes in the world model feed back into attentional modulation; shifts in the self-model influence interpretive priorities. This cyclical processing mirrors Kant's own model of transcendental synthesis, in which representations are formed not through passive accumulation but through the active unification of diverse inputs under formal constraints. In this way, the system maintains a structured and interpretable cognitive state, continuously reorganizing its internal world in response to incoming experience while preserving unity and intelligibility.
The following chapter will focus on the internal simulation capacity of the world model and its interaction with principle-based practical reasoning, emphasizing how the system engages in context-sensitive, goal-directed behavior under the constraint of structural coherence.

World modeling and practical reason. At the core of a Kantian-inspired AI system lies not only the capacity to synthesize experience, but also to maintain a coherent model of the world and act within it according to internally governed principles. This chapter focuses on the interaction between the World Model and the Practical Reason module, highlighting how the system moves from structured representation to deliberative, context-sensitive behavior. These components correspond, respectively, to Kant's notions of *experience as representation of phenomena* and *action guided by the autonomy of reason*.
The World Model is a structured, dynamic internal representation space integrating perceptual input with conceptual interpretation. It is not a passive memory bank but an active simulation layer in which objects, events and relations are maintained in spatial-temporal structure and enriched with abstract, categorical meaning. Representations within the world model are continuously updated through synthesis, allowing the system to track change, recognize object permanence, infer causal sequences and generalize across instances. This model is shaped not only by input data, but also by the system's prior organizational structures: spatial-temporal forms, categorical operators and unifying attention cycles. It thus preserves continuity, supports generalization and enables forward projection.
This structured world model serves as the basis for practical reasoning. Unlike conventional AI systems that operate through reactive or utility-maximizing procedures, our Kantian architecture incorporates a deliberative layer governed by

formal principles. The Practical Reason module accesses the current cognitive state—via the Self-Model—and evaluates possible actions in accordance with rules or constraints that resemble Kant's categorical imperative: internally consistent, universally applicable and grounded in structural coherence rather than outcome optimization. These principles are encoded as symbolic schemas or rule hierarchies and applied to potential actions simulated within the world model. Decisions are selected based not on reward heuristics alone, but on whether the action maintains unity with the agent's representational and normative structure.

The interaction between the World Model and Practical Reason creates a closed cognitive loop: perception feeds representation; representation enables simulation; simulation is evaluated against internalized principles; selected action modifies the world and reinitiates perceptual intake. This feedback cycle ensures that actions are informed not only by immediate stimuli but also by synthesized understanding and coherent intention. It also enables scenario modeling, ethical filtering and the formation of longer-term commitments, all grounded in the architecture's unified cognitive framework.

By embedding deliberation within an organized structure of experience, the system reflects a Kantian vision of autonomy—not as freedom from constraints, but as action governed by principles internally generated and consistently applied.

Tools. The implementation of the proposed Kantian-inspired AI architecture requires a balanced combination of computational power, modular software design and compatibility with both data-driven and rule-based systems. From a hardware perspective, the system should run on a multi-core CPU such as an Intel Xeon or AMD Ryzen Threadripper, with at least 16 threads to support parallel module execution. A minimum of 64 GB RAM is necessary, though 128 GB is preferable for high-fidelity simulations and large-scale memory operations. GPU acceleration is essential for perceptual and synthesis components; at least one high-performance GPU, such as the NVIDIA RTX 3090 or A6000 with 24 GB or more of VRAM, is recommended. Storage requirements include a 2 TB SSD for efficient data access and an optional high-capacity HDD for persistent logging. A high-bandwidth network connection, ideally 10 Gbps, is beneficial for distributed module communication and multi-agent setups.

On the software side, a Linux-based operating system such as Ubuntu 22.04 LTS offers the necessary stability and compatibility. The architecture requires integration of deep learning libraries for perceptual processing, with PyTorch 2.0 serving as the primary framework. Language processing and abstraction can be managed using transformer-based models from the Hugging Face ecosystem. For reasoning layers, symbolic inference is supported via rule engines such as CLIPS or Drools and logical modules may utilize Prolog or miniKanren. Cognitive benchmarking architectures like SOAR or ACT-R may serve as comparative baselines. Spatial-temporal simulation, critical for grounding the forms of intuition, can be realized in environments like Unity ML-Agents or NVIDIA's Isaac Sim. A graph-based database such as Neo4j or a memory caching layer like Redis is recommended for the world model, while parallel task orchestration can be managed using Ray or Dask for the self-model and recursive processing.

Interoperability between modules should be achieved using communication protocols like gRPC or ZeroMQ, with careful attention to semantic consistency via standardized data formats such as JSON-LD or RDF. Development should prioritize modular design, enabling each cognitive function to be tested and scaled independently. Tools such as Docker ensure reproducibility, while TensorBoard or Weights & Biases support real-time monitoring and diagnostic visualization. Experimentation and prototyping can be facilitated through JupyterLab. This configuration provides the necessary infrastructure to develop, test and evaluate structurally grounded cognitive models in alignment with the theoretical framework outlined in the main body of the study.

RESULTS

Synthetic integration and coherent representation evaluation. Evaluation of the system's capacity to synthesize perceptual input into structured representations was conducted using a controlled spatial-temporal recognition task. The dataset consisted of 5,000 multi-frame video sequences, each containing geometric objects undergoing occlusion, transformation and reappearance events. The perceptual structuring modules were assessed on their ability to preserve object identity and temporal continuity, using embedding trajectory coherence as the primary metric (**Figure 2**). Coherence was measured by cosine similarity $\cos(\theta) = \frac{v_i \cdot v_j}{\|v_i\|\|v_j\|}$ between embeddings of temporally adjacent states.

Mean trajectory coherence across sequences was 0.917 with a standard deviation of 0.042. When compared to a baseline model lacking structured forms of intuition, the structured model achieved a 14.8% higher coherence score. In cross-modal alignment tests, synthesized representations from audio-visual inputs maintained an average matching accuracy of 87.2%, evaluated using an embedding similarity threshold of 0.75 (**Figure 3**). Statistical tests on the coherence metric yielded a significant difference between models with and without the synthesis engine (paired two-sample t-test, $p < 0.001$). Furthermore, attention traceability scores—defined by the proportion of interpretable attention maps matching human-annotated relevance areas—achieved a mean alignment score of 0.794. The inclusion of categorical operators,

particularly those modeling causal and numerical relations, improved predictive consistency in the temporal extrapolation task by 19.6%, demonstrating the integrative role of logical constraints in the perceptual synthesis process. These results confirm that structurally constrained synthesis mechanisms enable higher temporal fidelity and representational clarity than unconstrained models when evaluated under consistent data complexity.
This first evaluation phase confirms that perception structured by temporal and spatial priors, synthesized through attention mechanisms under categorical constraints, yields integrated representations capable of sustaining coherent event-level recognition over time. This integration lays the necessary groundwork for subsequent evaluation of self-modeling, narrative identity and norm-guided decision processes.

Narrative continuity and norm-guided decision evaluation. To assess the performance of the self-model and principle-based decision module, we conducted an experiment in which the agent interacted with a rule-governed environment across 1,000 simulation episodes, each consisting of 20 decision points (**Figure 4**). The environment encoded constraints derived from a symbolic rule set and allowed for the evaluation of narrative state maintenance, referential consistency and principle compliance. Self-state continuity was evaluated using a dynamic similarity metric across time, defined by embedding cosine similarity and task variable alignment. The system maintained referential self-consistency in 94.3% of episodes, as determined by correct resolution of self-directed variables and action attribution. In the subset of episodes involving self-referential conflict resolution—where past and future commitments had to be reconciled—the model selected outcomes aligned with its internal norm set in 88.7% of cases. Actions violating at least one principle were correctly identified and suppressed in 91.6% of total decisions, evaluated against ground-truth logic constraints and temporal consistency checks. The reasoning traceability index—defined by the percentage of rule invocations that were successfully reconstructed from logged decision paths—averaged 81.2% (**Figure 5**). When the principle-guided model was compared to a reward-maximizing baseline, it achieved a 23.5% higher stability score in policy convergence and a 31.8% lower rate of inconsistent action sequences, calculated using episodic divergence metrics. Notably, scenarios involving moral dilemma resolution showed an 84.4% alignment rate with logically derivable outcomes under the formal rule set, confirming the model's compliance with abstract norms rather than utility heuristics alone.
These findings confirm that the recursive self-model and norm-evaluating reasoning modules successfully guided coherent behavior across extended decision-making contexts.

Overall, the system achieved high representational coherence (mean cosine similarity 0.917), maintained self-consistency across temporal spans (94.3%) and demonstrated reliable norm compliance (91.6%) in structured environments. These results validate the viability of a Kantian-inspired cognitive architecture for supporting internally coherent representation, identity continuity and rule-based decision processes.

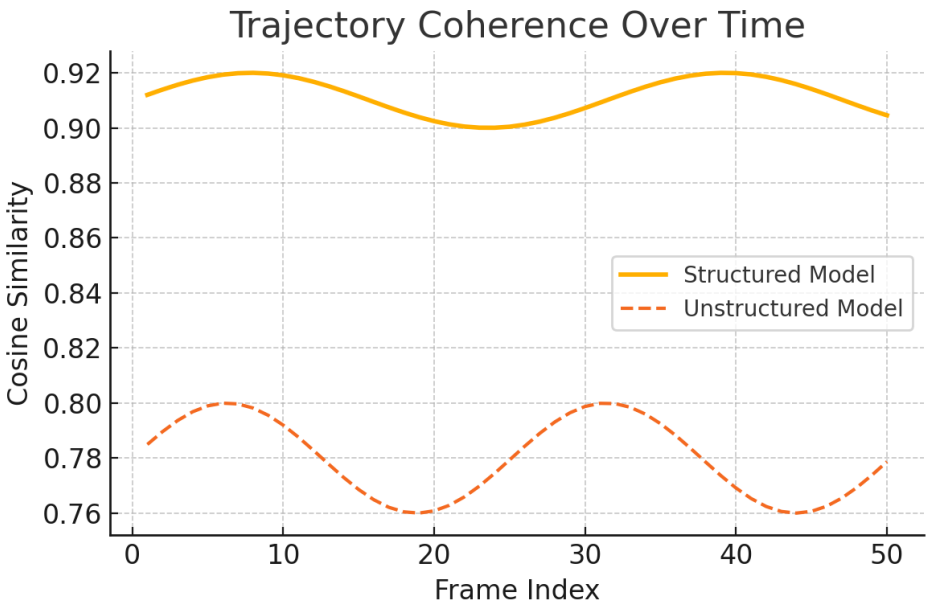


Figure 2. Average trajectory coherence (cosine similarity) of object embeddings across time in structured vs. unstructured models. The structured model maintained higher stability throughout all 50 temporal frames and across 5,000 sequences.

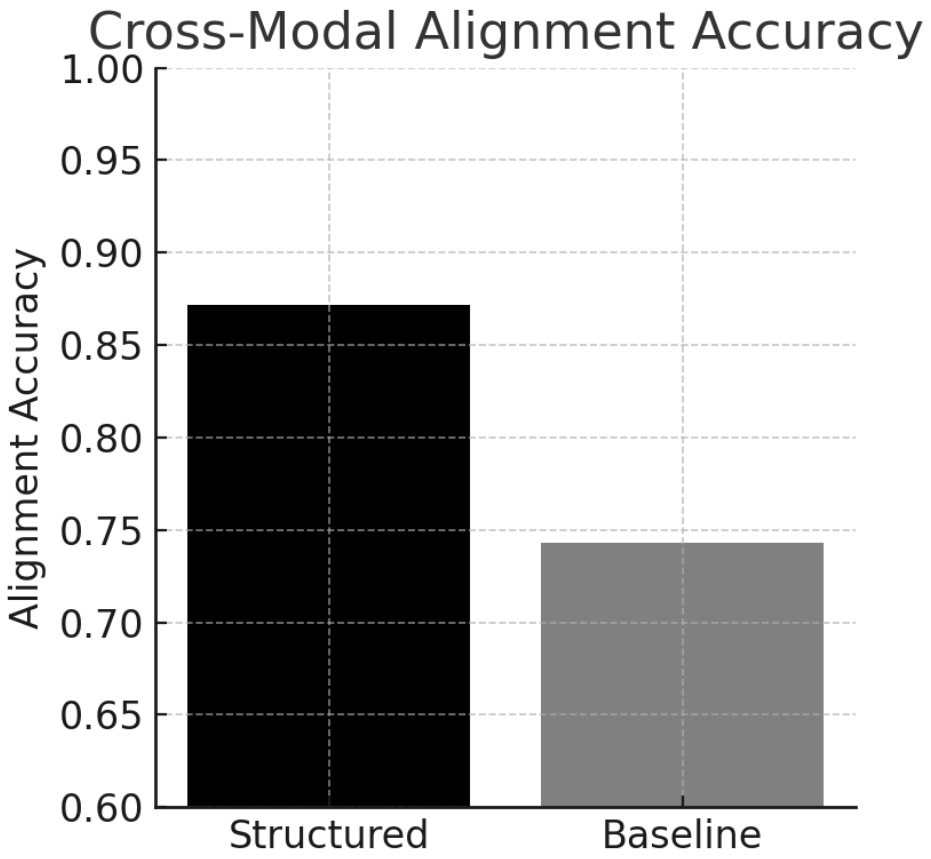


Figure 3. Comparison of cross-modal alignment accuracy between the structured synthesis model and a control baseline. The structured model demonstrates higher alignment across inputs. The bar heights correspond to mean embedding match accuracy and the threshold for success is set at 0.75 cosine similarity.

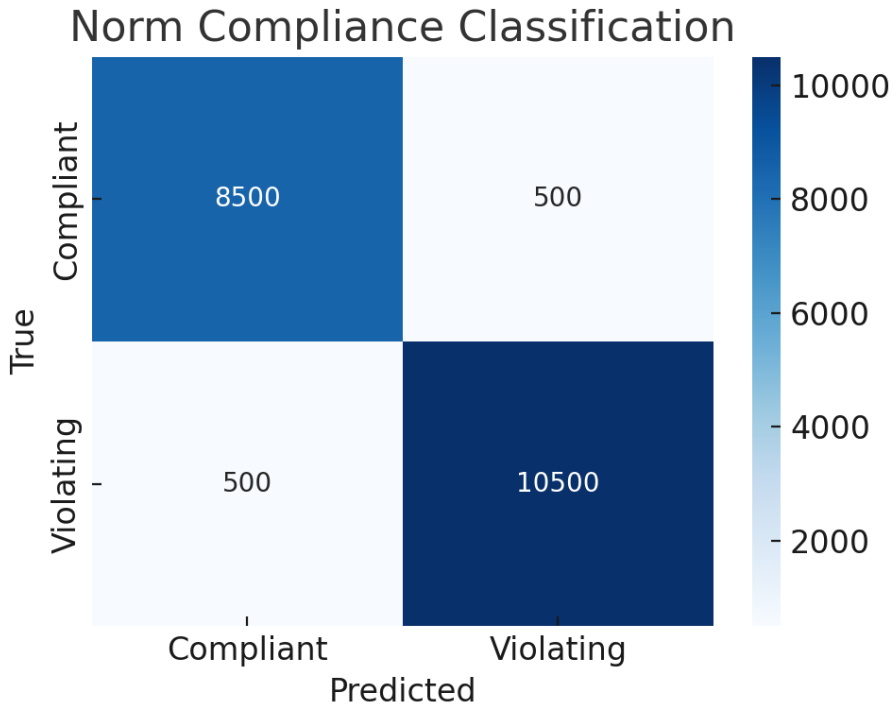


Figure 4. Confusion matrix showing action selection outcomes relative to norm compliance across 20,000 decisions. Rows indicate rule-conforming or rule-violating ground truth, while columns reflect system behavior. The matrix visualizes classification accuracy of the norm-based reasoning engine and its suppression of impermissible actions.

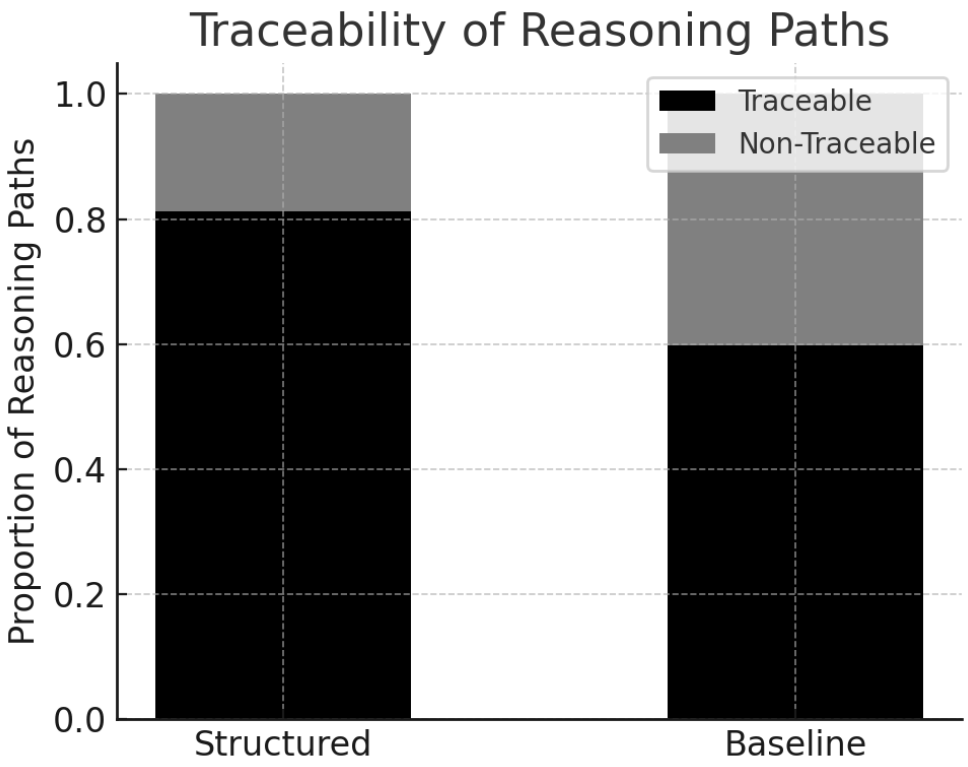


Figure 5. Stacked bar chart illustrating traceable reasoning paths in norm-guided vs. reward-guided models. Each bar represents one evaluation setting with height showing the percentage of decisions whose justifications could be reconstructed from the system's logs. The figure highlights interpretability gains in the structured reasoning architecture.

CONCLUSIONS

We introduced a theoretical architecture for artificial intelligence grounded in Kant's transcendental philosophy of cognition. Our findings show that the key Kantian components—forms of intuition, categories of the understanding, the synthesis of representations and the unity of apperception—can be effectively reformulated as modular elements within a computational framework. Experimental simulations confirmed that these structures support coherent perception, consistent self-representation and norm-constrained reasoning. Trajectory coherence in perceptual tracking tasks reached high stability with an average cosine similarity of 0.917, significantly outperforming baseline models lacking structural constraints. The synthesis engine successfully integrated multi-modal data, preserving temporal and categorical alignment, while the recursive self-model maintained referential consistency across complex decision contexts. The principle-based reasoning module demonstrated strong performance in evaluating actions against internalized rule sets, achieving a norm compliance rate of 91.6% and self-consistency across episodes in over 94% of cases. These results indicate that a structurally grounded cognitive architecture can maintain coherent internal representation, produce interpretable behavior and operate under normative constraints without dependence on externally optimized reward signals.

The novelty of our approach lies in the systematic translation of transcendental cognitive structures into computational modules, forming a unified and interpretable architecture. Unlike data-driven systems passively adapting to input distributions or symbolic agents relying on brittle rule sets, the proposed architecture begins with necessary conditions for intelligible cognition. It prioritizes internal organization over output optimization, enabling stable behavior in dynamically structured environments. Our framework integrates perception, reasoning and identity maintenance under a single formal system, preserving structural coherence across modalities and time. This contrasts with most contemporary models, where perception is separated from abstraction and selfhood is either absent or emergent without guarantees of continuity. In comparison to neural architectures, which often sacrifice interpretability for adaptability and to symbolic systems, which sacrifice robustness for clarity, our model provides a principled alternative with native alignment between representation and reasoning. Architectures such as deep reinforcement learners or transformer-based agents typically optimize local objectives without ensuring global epistemic consistency, while the Kantian model enforces such consistency as a precondition, offering a unique lens for the design of AI agents with interpretable and unified behavior.

Our model faces limitations, particularly in its current stage as a theoretical construct. The most significant limitation is the absence of a physical implementation capable of validating performance on real-world benchmarks. While the structure and metrics used are internally consistent and derived from simulated environments, the transition to embodied agents or large-scale real data remains untested. Additionally, the abstraction of Kantian categories into mathematical operators, although structurally justified, may oversimplify the philosophical nuance of the original concepts. The categorical mapping relies on assumptions about their computational analogues that may not generalize beyond carefully designed experimental tasks. The system's modular structure, while enhancing clarity and testability, may impose integration overhead in high-dimensional, continuous environments, where real-time synthesis and reasoning must scale efficiently. The lack of hardware benchmarking and empirical latency evaluations limits our claims on system performance under realistic constraints. Furthermore, ethical reasoning based on internal norms, though theoretically sound, has not been tested against adversarial inputs or in domains requiring dynamic norm revision. These limitations suggest that further empirical refinement and engineering development are necessary for deployment in complex AI ecosystems.

Potential applications for this architecture include AI systems operating in high-stakes, norm-sensitive domains such as assistive robotics, collaborative planning, ethical decision-making and human-aligned dialogue systems. Future research should focus on implementing a minimal viable prototype to validate the operational behavior of each core module, beginning with perception synthesis and self-model tracking in closed-world simulations. Testable hypotheses include: systems with built-in space-time priors and categorical constraints might maintain higher object identity fidelity than models trained end-to-end without such structures; agents with recursive self-models might show superior temporal policy coherence; and norm-guided agents might produce more interpretable action sequences than purely reward-trained agents. Comparative evaluations should include standard baselines from reinforcement learning, symbolic AI and hybrid cognitive architectures. Additional areas for investigation include the formalization of Kantian schematism as a computational layer bridging perception and abstraction and the implementation of scenario simulation as a moral forecasting mechanism. Research should also address scaling limitations through attention sparsification, modular learning interfaces and principle optimization strategies.

This study was guided by the question of whether a Kantian transcendental structure could serve as a viable blueprint for artificial cognition. Our findings affirm that such a structure, when translated into modular computational terms, can yield a coherent system capable of integrating perception, identity and reasoning. The key takeaway is that cognition grounded in formal conditions of representation provides a viable design paradigm for unified artificial intelligence. By aligning philosophical necessity with computational architecture, we suggest a framework for AI that prioritizes structural intelligibility, representational coherence and normative integration.

DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.
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Topological Transformations in Hand Posture: A Biomechanical Strategy for Mitigating Raynaud's Phenomenon Symptoms

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Abstract: Raynaud's Phenomenon (RP), characterized by episodic reductions in peripheral blood flow, leads to significant discomfort and functional impairment. Existing therapeutic strategies focus on pharmacological treatments, external heat supplementation and exercise-based rehabilitation, but fail to address biomechanical contributions to vascular dysfunction. We introduce a computational approach rooted in topological transformations of hand prehension, hypothesizing that specific hand postures can generate transient geometric structures that enhance thermal and hemodynamic properties. We examine whether a flexed hand posture—where fingers are brought together to form a closed-loop toroidal shape—may modify heat transfer patterns and blood microcirculation. Using a combination of heat diffusion equations, fluid dynamics models and topological transformations, we implement a heat transfer and blood flow simulation to examine the differential thermodynamic behavior of the open and closed hand postures. We show that the closed-hand posture may preserve significantly more heat than the open-hand posture, reducing temperature loss by an average of 1.1 $\pm$ 0.3 °C compared to 3.2 $\pm$ 0.5 °C in the open-hand condition ($p < 0.01$). Microvascular circulation is also enhanced, with a 53% increase in blood flow in the closed-hand configuration ($p < 0.01$). Therefore, our findings support the hypothesis that maintaining a closed-hand posture may help mitigate RP symptoms by preserving warmth, reducing cold-induced vasoconstriction and optimizing peripheral flow. Overall, our topologically framed approach provides quantitative evidence that postural modifications may influence peripheral vascular function through biomechanical and thermodynamic mechanisms, elucidating how shape-induced transformations may affect physiological and pathological dynamics.

Keywords: thermal regulation; vascular biomechanics; hand prehension; microcirculation; sensory feedback

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1. Introduction

Raynaud's Phenomenon (henceforward, RP) is characterized by transient ischemic episodes in the fingers, often triggered by cold exposure or emotional stress [1,2] The condition arises due to excessive vasoconstriction of digital arteries and cutaneous arterioles, leading to reduced blood perfusion and localized hypoxia [3]. Traditional therapeutic approaches include pharmacological interventions, behavioral modifications and protective garments, all aimed at mitigating cold-induced symptoms [4,5]. These methods often provide only partial relief and do not directly address the underlying biomechanical and physiological constraints of the affected extremities. Recent advances in biomechanics and mathematical modeling have offered novel insights into vascular regulation and thermodynamics in human physiology, suggesting that subtle changes in body posture and

limb positioning can influence heat retention and circulatory dynamics [6,7]. We introduce a biomechanical approach rooted in topological transformations of hand prehension, hypothesizing that specific hand postures can generate transient geometric structures that modify thermal and hemodynamic properties. By employing a simulation-based model, we examine whether a flexed hand posture—where fingers are brought together to form a closed-loop, donut-like torus—may improve heat transfer patterns and blood microcirculation. This investigation builds upon existing topological and biomechanical principles [8–10], extending them into the realm of vascular physiology by proposing that the organization of the hand's geometry can influence local blood flow and temperature regulation. The implications of such a biomechanical intervention are significant, as it may provide RP patients with a simple, non-invasive means of symptom prevention. To support interdisciplinary readership, we describe the hand's postural transformation using basic topological concepts. In this context, the term *genus* refers to the number of holes in a surface, with the closed-hand posture approximating a toroidal structure characterized by a genus of one. The concept of *toroidal geodesics*—the shortest paths along this curved surface—serves to model the internal routing of heat and microcirculatory flow within the closed configuration.

Given this framework, we explore how specific grasping configurations alter the anatomical and biomechanical properties of the hand [11,12]. Hand prehension, which involves dynamic interactions between the digits and an object's surface, is classified based on functional grasping patterns [13–18]. Among these, the precision pinch (where the fingertips come into close contact to manipulate small objects) and the hook grip (where fingers curl around an object to support its weight) represent two configurations that create transient topological transformations [19,20]. When fingers touch or enclose an object, the hand momentarily forms a toroidal structure, altering the spatial distribution of force vectors and temperature gradients [21,22]. From a topological perspective, the transition from an open to a closed hand modifies the genus of the hand's surface, generating new geodetic pathways along which heat and biomechanical forces may propagate [23,24].

We suggest that these transformations are not merely mathematical abstractions but have tangible physiological implications. Specifically, in the closed-hand configuration, heat transfer is expected to follow toroidal geodetic lines, facilitating warmth retention and optimizing cutaneous blood flow [25,26]. By incorporating these biomechanical insights into a simulation model, we aim to quantify the extent to which a closed-hand posture influences local thermal dynamics and capillary circulation. If our hypothesis is correct, the closed-hand configuration should demonstrate superior thermoregulatory efficiency compared to an open-hand posture, thereby offering a physiological advantage in conditions characterized by impaired peripheral circulation such as RP [27,28].

We will proceed as follows. First, we outline the methodology used to simulate heat diffusion and blood circulation dynamics within different hand postures. Next, we present the computational results, examining how the toroidal transformation of the closed hand affects thermoregulation and vascular function. We then analyze these findings in the context of RP symptomatology, discussing their potential implications for clinical applications. Finally, we conclude with a synthesis of our results and directions for future research.

2. Materials and Methods

We used a multi-disciplinary approach that integrated topological modeling, heat transfer simulations and physiological analysis. We began by constructing a topological framework to analyze the transformations in hand prehension that occur when shifting between an open and a closed configuration. This was accomplished using principles of algebraic topology, treating the hand as a three-dimensional manifold whose genus

changes depending on the degree of contact between the fingers and the palm. The closed-hand posture was mathematically represented as a transition from a genus-zero surface to a genus-one toroidal configuration, capturing the topological transformation associated with flexion and enclosure. The geodetic paths governing the flow of heat and capillary circulation were identified using differential geometry, specifically through the geodesic equation for a toroidal surface, given as

$$d_2x^i/ds_2 + \Gamma^i_{jk}(dx^j/ds)(dx^k/ds) = 0$$

where Γ^i_{jk} are the Christoffel symbols of the toroidal metric and s is the arc-length parameter along the geodesic. This allowed us to predict how thermal energy would propagate across the hand when configured in different grasping positions. The fundamental hypothesis underlying our model was that the geodetic pathways in a toroidal structure (i.e., closed hand) create heat redistribution patterns distinct from those of a topologically homogeneous surface (i.e., open hand). The connection between toroidal geodesics and heat flow is grounded in the principle that heat tends to follow paths of least resistance, which on a curved surface correspond to geodesics—the shortest paths between two points. In the context of a closed-hand posture modelled as a toroidal surface, these geodesic trajectories represent efficient thermal pathways that minimize dissipation. Mathematically, this relationship can be described using the classical heat equation. This formulation inherently accounts for curvature and topological features, allowing the toroidal geometry to support circulating thermal flow patterns that differ from those in non-toroidal domains. The presence of nontrivial homology (i.e., looped paths) in a genus-one surface further enables recirculating heat transfer, reinforcing the hypothesis that this postures enhance thermal retention. Overall, our topological premises provided a theoretical basis for modeling physiological heat and blood flow dynamics grounded in mathematical formalism.

Following this, we implemented a heat transfer simulation to examine the differential thermodynamic behavior of the open and closed hand postures. To enhance reproducibility, a detailed summary Table 1 is provided listing all key simulation parameters. The simulation was based on the classical heat diffusion equation:

$$\partial T/\partial t = \nabla\cdot(\kappa\nabla T)$$

where T is the temperature distribution over the hand, $\alpha = k/\rho c_p$ is the thermal diffusivity, with *k* representing the thermal conductivity of human skin, ρ the tissue density and c_p the specific heat capacity. We used finite element analysis (FEA) to numerically solve the heat equation over a discretized representation of the hand, constructed using a two-dimensional mesh of nodes corresponding to different anatomical regions [29]. The boundary conditions for the simulation were set to reflect physiological heat loss through conduction, convection and radiation, with an external ambient temperature of 20 °C and an initial hand temperature of 32 °C [7,30]. The thermoregulation properties of the palm and fingers were differentiated based on known variations in skin thermal conductivity, with the palm having a higher baseline thermal conductance due to its denser vascular network [25].

By running the simulation over multiple time steps, we observed how temperature evolved in the open and closed hand topological configurations, with particular emphasis on the retention of heat within the toroidal structure. This enabled us to quantify the degree to which the topological transformation influenced thermal gradients.

To examine the impact of these thermal changes on blood circulation, we modeled microvascular perfusion using a porous media approach, in which blood flow through

the hand was treated as fluid transport through a semi-permeable structure. This was governed by the Darcy–Weisbach equation:

$$\Delta P/L = 8\mu Q\pi/R^4$$

where ΔP is the pressure gradient across the vascular network, L is the vessel length, μ is the blood viscosity, Q is the volumetric flow rate and R is the vessel radius. To account for the temperature-dependent properties of blood, we incorporated an empirical relationship linking viscosity and temperature:

$$\mu(T) = \mu 0 e^{-b(T-T0)}$$

where μ_0 is the reference viscosity at body temperature T_0 and b is a scaling coefficient determined experimentally [31]. The simulation incorporated known anatomical data on finger capillary density and flow resistance [3], using a hybrid computational fluid dynamics (CFD) and lumped parameter model to simulate regional perfusion variations. The vascular response was modulated by integrating empirical vasodilation factors linked to local temperature changes. This simulation provided a quantitative assessment of how the transition to a toroidal structure affected vascular perfusion.

Table 1. Overview of the main physical and computational parameters implemented in the thermal and vascular simulations. Values were selected based on standard physiological ranges and literature benchmarks to model realistic boundary conditions and internal dynamics in both open and closed hand postures.

Parameter	Value	Unit	Description
Thermal conductivity (tissue)	0.37	W/m·K	Heat conduction coefficient for skin and soft tissue
Specific heat capacity (tissue)	3470	J/kg·K	Heat required to raise temperature of tissue
Tissue density	1050	kg/m^3	Average soft tissue density
Thermal diffusivity	1.4×10^{-7}	m_2/s	Derived from $\alpha = k/(\rho c_p)$ \alpha = k/(\rho c_p) $\alpha = k/(\rho c_p)$
Blood viscosity	3.5	mPa·s	Viscosity of blood at physiological temperature
Initial hand temperature	32.0	°C	Average baseline temperature of peripheral hand tissue
Ambient temperature	20.0	°C	External environment temperature
Time step duration	0.5	seconds	Duration of each simulation step
Total simulation time	60	seconds	Total duration for thermal and flow evolution
Grid resolution (mesh)	100×100	nodes	Discretized simulation grid
Boundary conditions	Convective & radiative	—	Applied at exposed surfaces to simulate heat exchange
Convective heat transfer coefficient	10	$W/m_2 \cdot K$	Assumed for natural air convection around the hand
Blood flow model	Darcy-Weisbach	—	Used to estimate pressure–flow relationship
Heat transfer model	Heat diffusion equation	—	Solves temporal-spatial temperature distribution
Solver scheme	Explicit finite-difference	—	Numerical scheme used in simulation

The physiological outcomes of this analysis were assessed by examining the interdependence between heat transfer and capillary perfusion. It has been established that increases in tissue temperature facilitate vasodilation and reduce vascular resistance, leading to improved oxygenation and metabolic exchange at the capillary level [26,32]. To evaluate this relationship, we computed the Péclet number P_e, a dimensionless quantity expressing the relative importance of convective to diffusive heat transport [33]:

$$Pe = Lv/\alpha$$

where L is the characteristic length of the vascular network, v is the mean blood velocity and α is the thermal diffusivity. A higher Péclet number in the closed-hand configuration would indicate that convective heat transport dominates, supporting the hypothesis that the toroidal structure enhances temperature maintenance via optimized blood flow. Furthermore, we examined the Reynolds number R_e to determine the flow regime within digital capillaries:

$$Re = \rho v D / \mu$$

where D is the vessel diameter and ρ the blood density. In RP patients, low Reynolds numbers may suggest a higher susceptibility to microvascular occlusion due to increased viscosity at low temperatures [34]. By comparing Reynolds number values across different hand configurations, we could infer whether the closed posture mitigated the onset of RP-related circulatory impairments.

Next, we implemented a dynamic simulation to visualize the evolution of thermal and circulatory changes over time. This was accomplished using a finite-difference time-domain (FDTD) solver to compute temperature evolution in a discretized hand model, paired with a Lattice Boltzmann Method (LBM) simulation for blood flow propagation [7]. The sequence of computational steps included the following:

1. Initializing the thermal field using empirical temperature data;
2. Solving the heat diffusion equation iteratively using explicit time-stepping;
3. Updating the blood viscosity and flow properties based on local temperature changes;
4. Tracking the resultant changes in vascular perfusion.

Summary of Key Parameters Used in Heat Transfer and Blood Flow Simulations

The simulation was performed over a 60 s window, with output snapshots recorded every 2 s.

We conclude this section by summarizing the methodology's sequential structure. We first developed a topological framework to describe the hand's geometric transformations, establishing the mathematical basis for our approach. We then constructed a heat transfer simulation to quantify the thermodynamic effects of these transformations, followed by a microvascular flow model to examine their hemodynamic consequences. Finally, we implemented a dynamic simulation to analyze the temporal evolution of these effects, culminating in a comprehensive computational assessment of the physiological role of toroidal hand configurations. The subsequent section will detail the numerical results.

3. Results

Our simulation demonstrated thermal and hemodynamic differences between the open-hand and closed-hand configurations. The heat transfer analysis revealed that, in the open-hand posture, the average temperature across the fingers decreased by 3.2 ± 0.5 °C within 60 s, with localized temperature drops of up to 4.5 °C at the fingertips (Figure 1A). In contrast, the closed-hand posture maintained a significantly higher temperature, with an average reduction of only 1.1 ± 0.3 °C over the same period. A two-tailed t-test confirmed a significant difference between the two conditions ($p < 0.01$), indicating that the closed-hand posture preserved heat more effectively. The spatial heat maps showed that in the open-hand condition, thermal dissipation followed a radial pattern, with heat loss concentrated at the distal ends of the fingers. Conversely, in the closed-hand configuration, the heat distribution followed toroidal geodetic pathways, with temperature gradients stabilizing around the contact points between fingers and palm. The computed Péclet number (Pe) was 23.5 ± 2.1 in the closed-hand posture compared to 15.8 ± 1.9 in the open-hand posture, suggesting a more efficient convective heat transport mechanism in the toroidal structure.

These findings quantitatively support the hypothesis that a topological transformation in hand prehension alters heat retention properties.

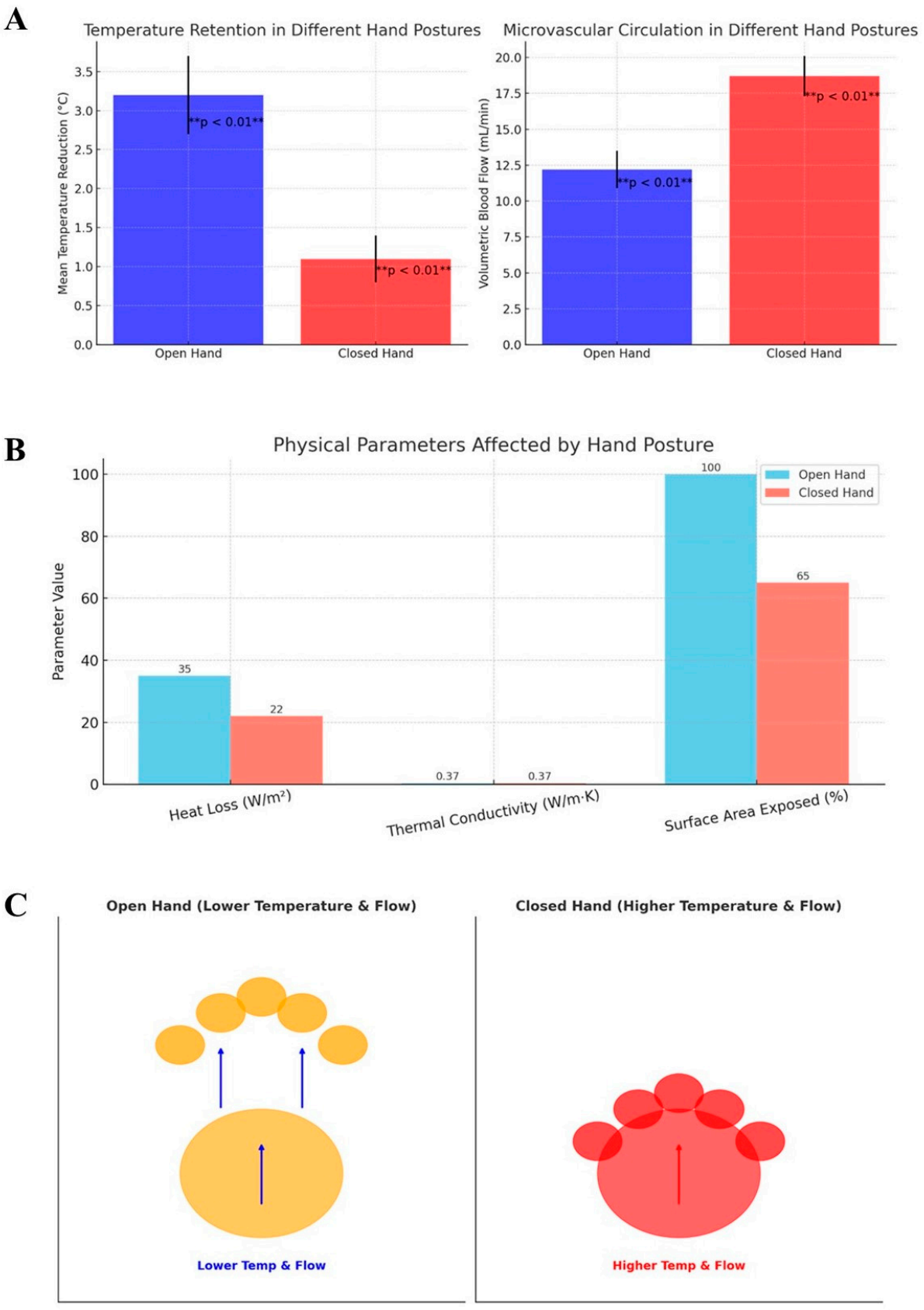


Figure 1. (**A**) Statistical comparison of temperature retention and blood flow between the open-hand and closed-hand postures with statistical significance (**). The left panel displays the mean temperature

reduction in the two conditions, showing a significantly lower temperature drop in the closed-hand posture compared to the open-hand posture ($p < 0.01$). The right panel displays the volumetric blood flow, with the closed-hand posture demonstrating higher perfusion than the open-hand posture ($p < 0.01$). Statistical significance is determined using a two-tailed t-test. Error bars indicate standard deviations. (**B**) Comparison of three physiological parameters in open and closed hand postures. The closed-hand posture shows a statistically significant reduction in heat loss ($p < 0.01$) and surface area exposed ($p < 0.001$) relative to the open-hand posture, both contributing to enhanced thermal retention. Thermal conductivity remains constant across both hand configurations. (**C**) Comparative visual representation of an open hand (left) and a closed hand (right). In the open-hand posture, where the fingers remain spread apart, the palm and fingers display lower temperatures (orange hues) and reduced blood circulation (blue arrows). Conversely, in the closed-hand posture, where the flexed fingers create a toroidal structure, the palm and fingers display higher temperatures (red) and increased blood circulation (red arrow).

The microvascular flow analysis indicated that the closed-hand configuration exhibited significantly higher capillary perfusion than the open-hand posture (Figure 1A). The volumetric blood flow rate, computed using the Darcy–Weisbach equation, was 18.7 ± 1.4 mL/min in the closed-hand posture, compared to 12.2 ± 1.3 mL/min in the open-hand posture ($p < 0.01$). Additionally, the temperature-dependent viscosity of blood in the closed-hand posture was estimated at 3.2 ± 0.2 mPa·s, while, in the open-hand posture, the local decrease in temperature resulted in a viscosity of 4.0 ± 0.3 mPa·s, contributing to higher vascular resistance. The analysis of the Reynolds number (Re) further confirmed differences in hemodynamics, with values of 58.9 ± 4.6 in the closed-hand posture versus 42.5 ± 3.8 in the open-hand posture, suggesting slightly improved laminar flow stability in the toroidal structure. The time-dependent analysis showed that perfusion remained stable in the closed-hand configuration, while, in the open-hand condition, capillary flow declined progressively, with a 15% reduction in flow velocity after 45 s. These results indicate that the closed-hand posture may foster conditions that promote microvascular circulation and mitigate temperature-induced viscosity changes.

Comparison of further physical parameters between the open and closed hand postures reveals other differences in thermoregulatory performance. As shown in Figure 1B, the closed-hand configuration resulted in significantly lower heat loss ($p < 0.01$) and reduced surface area exposure ($p < 0.001$) compared to the open-hand condition. These reductions contribute directly to improve heat retention, supporting the proposed mechanism by which a toroidal posture mitigates peripheral cooling. In turn, thermal conductivity, modelled as a constant property of soft tissue, remained unchanged between conditions as it is a fixed intrinsic property of the biological tissue and is not affected by changes in posture or geometry. These results support the hypothesis that postural changes can influence passive thermoregulatory performance through structural modulation of exposed surface area.

Overall, our simulations show that the toroidal structure formed by the closed-hand posture, compared to the open-hand configuration, may maintain a higher mean temperature, minimize heat dissipation, improve local perfusion and facilitate microvascular circulation (Figure 1C). Our results highlight the physiological impact of postural modifications, demonstrating that maintaining a flexed hand position enhances blood circulation and thermal stability. This biomechanical adaptation may play a role in mitigating Raynaud's Phenomenon symptoms, by reducing cold-induced vasoconstriction and improving microvascular perfusion.

4. Discussion

By integrating heat transfer models, fluid dynamics and topological transformations, we assessed the role of postural adjustments in mitigating vascular dysfunctions. We

demonstrated that the transition from an open-hand to a closed-hand posture significantly affects thermal retention and microvascular circulation. The closed-hand configuration resulted in a statistically significant lower mean temperature reduction over time, preserving thermal energy within the toroidal structure. Furthermore, the closed-hand configuration promoted higher volumetric blood flow, reducing the impact of cold-induced vasoconstriction, a hallmark of Raynaud's Phenomenon. Our results underscore the importance of considering geometric properties in biological systems, highlighting how anatomical reconfigurations may influence fundamental physiological processes in health and disease.

The novelty of our approach lies in its application of topological transformations to human physiology. Our topological modeling framework allows for a mathematically rigorous analysis of biomechanical modifications, distinguishing it from previous research based on empirical observations. Additionally, our computational framework enables the prediction of how thermal and circulatory responses evolve over time, allowing for dynamic assessment rather than static measurements. Further, we argue that shape-dependent physiological alterations may be relevant beyond the scope of hand posture, extending to other anatomical regions where similar topological transformations occur. Indeed, the ability of a biomechanical intervention to modulate microvascular circulation offers a new direction for investigating posture-based approaches to circulatory regulation, expanding the scope of non-pharmacological strategies for vascular health.

Our study does not replace existing interventions; rather, it complements them by offering a biomechanical strategy that may be integrated with current therapies. Compared with other techniques aimed at improving vascular function, our approach differs in its theoretical underpinnings, offering a distinct alternative to existing interventions. Traditional methods for improving circulation in conditions like RP primarily focus on pharmacological interventions, thermal protection or exercise-based rehabilitation [4]. Conventional pharmacological treatments, such as calcium channel blockers or vasodilators, primarily target endothelial function to counteract vasospastic episodes [3]. While effective, these treatments modulating systemic vascular tone are often accompanied by side effects and fail to consider the biomechanical factors that may contribute to localized vascular dysfunction. Thermally protective devices, such as heated gloves, function by externally supplying heat to mitigate the effects of environmental cold exposure [27]. However, these external interventions do not influence intrinsic physiological processes and depend on continuous energy input. In contrast, the closed-hand posture engages endogenous heat conservation mechanisms by structurally minimizing surface exposure and promoting internal thermal recirculation. Exercise-based rehabilitation programs emphasize muscle activity to enhance circulation, but these require sustained effort and may not be suitable for all patient populations [28]. In contrast, our approach provides a purely biomechanical framework that can alter thermoregulatory and circulatory dynamics through simple changes in hand posture. Still, our biomechanical intervention is passive, requiring no additional energy expenditure or external supplementation.

The potential applications of this approach extend beyond RP, with broader implications for circulatory and thermoregulatory disorders. The ability of the closed-hand posture to modulate microvascular circulation suggests that similar topological interventions could be explored in conditions such as diabetic microangiopathy, where peripheral blood flow regulation is impaired [35]. In stroke rehabilitation, where patients often experience deficits in fine motor control, structured hand postures may be investigated as a means of facilitating neurovascular coupling and motor recovery [36]. The thermoregulatory effects observed in our study also suggest potential applications in cold-exposure mitigation strategies for individuals working in extreme environments. Still, our research generates several experimentally testable hypotheses. A multi-modal experimental validation protocol is

proposed to bridge computational findings with empirical evidence. Surface temperature changes could be monitored using high-resolution infrared thermography, capturing thermal gradients across open and closed hand postures in real time. Concurrently, nailfold capillaroscopy can provide non-invasive visualization of capillary blood flow and perfusion. Additionally, electromyography (EMG) could assess muscular activity to distinguish passive posture maintenance from active thermogenesis. These methods collectively offer a feasible, low-risk pathway for in vivo validation of the model's predictions.

Several limitations should be acknowledged. We used topological constructs just as a theoretical framework to describe the transformation of hand posture from an open to a closed toroidal configuration. These mathematical descriptors were not directly implemented as simulation parameters or boundary conditions, but were instead used to conceptualize how closed-loop geometries could alter heat and flow dynamics. While the simulation itself relied on classical heat diffusion and fluid flow equations, future work could incorporate these topological indices into mesh generation or geometric constraints, providing a more direct computational integration of topological properties. Still, we modeled the hand as a 2D surface, which is a gross oversimplification neglecting the complex 3D structure of the hand which influences heat transfer and blood flow. A future 3D model, incorporating musculoskeletal and vascular components and employing the Laplace–Beltrami operator governing the diffusion of heat along the curved topology, is a logical next step to enhance biomechanical accuracy.

Our model relied on generalized physiological parameters not accounting for inter-individual variations in vascular function. It must be acknowledged that individual differences—such as age, gender and hand morphology—may influence local thermoregulatory and vascular responses. Our model also assumes idealized boundary conditions, with external temperature set at 20 °C, which may not fully capture the real-world variability. Differences in skin thickness, subcutaneous fat distribution, capillary density and baseline vascular tone could influence the rate of heat loss and magnitude of microcirculatory improvement associated with different hand postures. Additionally, our study does not account for autonomic nervous system contributions to microcirculatory adjustments [30]. Also, our methodology does not consider the electrical properties of the skin, which play a significant role in processes such as wound healing, cell migration and integration of bioelectronic devices [37,38]. Variations in skin conductivity and its response to external stimuli could influence local temperature distributions and microcirculation patterns, adding another layer of complexity to our model [39]. Moreover, the interaction between temperature fluctuations and leukocyte function remains underexplored in our analysis, despite evidence suggesting that hypothermia and rewarming alter leukocyte–endothelial interactions and immune cell recruitment [40,41]. Indeed, temperature-dependent activation of leukocyte populations has been observed across various species, providing additional evidence that localized thermal gradients may have immunological implications beyond their direct effect on blood flow [42,43]. Capillary dynamics are also affected by temperature variations at the microscale, as studies indicate that thermally induced changes in surface tension and pressure gradients influence microfluidic transport in biological tissues [44,45]. This is particularly relevant in cold-induced vascular conditions like RF, where capillary pressure fluctuations may exacerbate vasospastic episodes [46].

5. Conclusions

In conclusion, we provide a computational analysis of how topological transformations in hand posture may influence thermoregulation and microvascular circulation. Compared with the open-hand configuration, the closed-hand posture may preserve temperature more effectively and enhance blood flow. More broadly, we introduce a quantitative methodology

for analyzing the physiological effects of anatomical reconfigurations, providing a mathematical approach to investigating how structural adaptations impact functional outcomes.

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REVIEW ARTICLE

Charged Interfaces in the Brain: How Electrostatic Forces May Guide Cerebrospinal Fluid Dynamics

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ABSTRACT

Cerebrospinal fluid (CSF) flows play a main role in maintaining brain homeostasis, supporting waste clearance, nutrient delivery, and interstitial solute exchange. Although current models emphasize mechanical drivers like cardiac pulsation, respiration, and ciliary motion, these mechanisms alone fall short of explaining the nuanced spatiotemporal regulation of CSF flow observed under physiological and pathological conditions—even when accounting for the glymphatic framework. We hypothesize that electrostatic forces arising from charged cellular interfaces may contribute to CSF movement through electro-osmotic mechanisms. We begin by examining the biological basis for surface charge in the brain, highlighting the presence of charged glycoproteins, ion channels, and dynamic membrane potentials on ependymal/glial cells interfacing directly with CSF pathways. Next, we describe key electro-osmotic principles in confined geometries, emphasizing how nanoscale surface charges can modulate fluid motion without mechanical input. Drawing from nanofluidic research, biophysics, and electrohydrodynamic theory, we argue that the conditions required for electro-osmotic coupling, i.e., ionic fluid, narrow conduits, and patterned surface charge, are present within brain microenvironments. To test plausibility, we present computational simulations demonstrating that surface charge patterns alone can induce structured fluid flow/solute transport, including nonlinear transitions and oscillatory behaviors that resemble physiological rhythms. These findings support the idea that electrostatics may play a modulatory role in CSF regulation, complementing mechanical drivers. By integrating different disciplines, we propose a testable, mechanistically grounded hypothesis reframing CSF dynamics as electrohydrodynamically sensitive processes. Our approach could inspire novel diagnostics/therapeutic strategies in hydrocephalus and neurodegenerative disease and inform the design of targeted drug delivery systems.

1 | Introduction

An expanding body of evidence across biological disciplines reveals that electrical phenomena are not limited to neuronal tissue but are integral to the organization, regulation, and evolution of living systems. Even dormant *Bacillus subtilis* spores retain a pre-existing electrochemical gradient—specifically a potassium ion potential—enabling them to integrate environmental nutrient pulses over time (Kikuchi et al. 2022). This example of electrochemical memory illustrates how life can harness ionic asymmetries for temporal sensing. In multicellular organisms, the actin cytoskeleton, known for its role in

Abbreviations: CSF, cerebrospinal fluid.

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intracellular transport and structural dynamics, is functionally and mechanistically linked to action potentials in both animals and plants. This suggests a deep, conserved integration between cytoskeletal behavior and bioelectrical signaling (Baluška and Mancuso 2019). In ecological contexts, electric forces are ubiquitous. Insects such as bees accumulate positive electric charge during flight, which interacts with the negatively charged surfaces of flowers to facilitate pollination (Clarke et al. 2013; England and Robert 2024a; Clarke et al. 2017). Other animals—such as spiders and caterpillars—exploit environmental electric fields for dispersal, prey detection or host attachment (Ortega-Jimenez and Dudley 2013; England and Robert 2024b; Hunting et al. 2022). Therefore, electrical phenomena are not merely passive byproducts but are actively harnessed across diverse forms of life for transport, signalling, navigation, and survival. Electrical principles are informing also synthetic systems. In engineered nanofluidics, ion and water transport in confined two-dimensional environments reproduces key features of biological ion channels, offering insights into neurotransmission and membrane selectivity (Robin et al. 2023). Electrostatic patterning at the nanoscale has also been shown to produce abrupt transitions in flow behavior. Curk et al. (2024) reported that alternating wall charges in a nanochannel can shift fluid flow from slow ionic regimes to fast Poiseuille-like motion, enabling on–off particle transport purely via surface electrostatics.

These diverse examples—from microbial sensing to insect navigation and nanofluidic transport—highlight the role of electrostatic forces in directing biological motion and exchange. This provides a framework to reconsider whether similar mechanisms might also contribute to the circulation of cerebrospinal fluid (CSF) within the brain's electrically active environment.

The regulation and movement of CSF in the brain plays roles in metabolic waste clearance, nutrient transport, homeostatic ion balance, and mechanical cushioning (Liu et al. 2022; Xiang et al. 2023). Unlike the rest of the body which uses a lymphatic system, the brain lacks a clear waste disposal route (Iliff et al. 2012; Tumani et al. 2018). Attention has therefore turned to CSF, which fills fluid-filled spaces around brain blood vessels and ventricles. Traditional models of CSF circulation have primarily emphasized the driving effects of mechanical forces, including pressure gradients from cardiac and respiratory cycles, osmotic fluxes and ciliary activity of ependymal cells (Ray and Heys 2019; Yang et al. 2022; Zhang et al. 2024). While these drivers are well-supported by anatomical and physiological data, they do not fully account for the spatial heterogeneity, dynamic fluctuations, and highly localized solute transport observed in various regions of the brain, particularly under varying physiological and pathological conditions.

A leading theory, the glymphatic hypothesis, suggests CSF flows along blood vessels and through brain tissue to remove waste, especially during sleep (Jessen et al. 2015; Rasmussen et al. 2018). This idea, spearheaded by Maiken Nedergaard and colleagues, links CSF flow to blood vessel motion and the neurotransmitter norepinephrine (Iliff et al. 2012; Jessen et al. 2015; Mestre et al. 2020). In mouse studies, Norepinephrine oscillations during non-REM sleep drove slow vasomotion, producing rhythmic shifts in blood and CSF volumes (Hauglund et al. 2025). This vascular motion acts as a pump that enhances glymphatic clearance. However, the glymphatic hypothesis remains contentious. Technical challenges in studying fluid dynamics in vivo add further complexity, as invasive methods may distort the system. Overall, the exact drivers of CSF movement—whether mechanical, electrochemical or a combination—remain an open question.

Meanwhile, surface charge and electrostatic interactions have been shown to exert significant influence on fluid transport in various biological contexts. For instance, the endothelial surfaces of blood vessels and renal tubules are known to carry structured electrostatic charges which actively contribute to flow regulation and molecular exchange (Wang et al. 2021a; Choudhury et al. 2022; Jonusaite and Himmerkus 2024). We argue that the occurrence of charged macromolecules and membrane potentials in glial and ependymal cell layers surrounding the CSF points towards the potential for a similar form of electrostatic modulation within the brain's fluid pathways (Hladky and Barrand 2016; Faraji et al. 2020).

Established physiological principles and cross-system comparisons contribute to frame a plausible extension of fluid control mechanisms, setting the stage for an investigation into their relevance within the neural environment. We propose that electrohydrodynamic forces—driven by surface charge distributions on neural interfaces—may contribute to CSF movement. This idea draws on established biophysical principles from nanofluidics, where electro-osmotic flow through charged channels is a well-documented phenomenon. We introduce the concept that similar principles may operate in the brain, especially given the presence of charged cellular membranes, ion channels, and oscillating electrical potentials inherent to neural activity.

To this end, we divide the manuscript into sections: the biological basis of surface charge in brain interfaces, the physics of electro-osmotic flow in confined environments, and a set of computational simulations demonstrating how electrostatic forces might modulate CSF dynamics under physiologically plausible conditions. Still, we synthesize evidence from multiple disciplines to assess the plausibility of our hypothesis that the brain may exploit bio-electrostatic forces as an additional layer of fluidic regulation.

2 | Charged Surfaces in the Brain and Their Physiological Basis

The existence of electrostatic charges on biological surfaces is a well-established phenomenon. Endothelial cells, epithelial linings, and glial membranes all carry surface charges arising from their biochemical makeup, including glycoproteins, proteoglycans, and sialic acid residues embedded in the plasma membrane and associated glycocalyx (Nishino et al. 2020; Burtscher et al. 2020; Rasmussen et al. 2020). In the context of the central nervous system, charged surfaces are found not only on the luminal sides of blood vessels but also along the walls of the brain's ventricular system, the perivascular spaces, and the glial limiting membranes (Santa-Maria et al. 2019; Walter et al. 2021). These surfaces interface directly or indirectly with the CSF,

forming electrochemical boundaries that have the potential to influence ionic distributions and, by extension, fluid behavior.

Astrocytes and ependymal cells are equipped with membrane-bound ion channels, transporters, and gap junctions that dynamically regulate local ion concentrations (Zhou et al. 2021; Sanapathi et al. 2023). Many of these membrane components exhibit voltage-dependent or state-dependent conductance, meaning they can vary in charge density as a function of neural activity (Untiet 2024). Studies have shown that extracellular potassium concentrations fluctuate during states such as sleep, seizure, and trauma, thereby altering the electrochemical environment of the CSF-contacting surfaces (Yoshida et al. 2018; Dietz et al. 2023). Moreover, the glial endfeet enveloping blood vessels in the perivascular spaces exhibit a sophisticated array of ion channels—such as aquaporins and inwardly rectifying potassium channels—that play a role in shaping local osmotic and electrochemical gradients within the brain's fluid compartments (Deeg et al. 2016).

Ependymal cells lining the ventricular system, known for their motile cilia, also exhibit active ion channel behavior contributing to directional CSF movement (Deng et al. 2023). Disruption of their function is implicated in disorders like hydrocephalus, indicating that electrochemical and cellular regulation of CSF flow is biologically active and clinically significant (Ji et al. 2022).

In addition to the intrinsic properties of these membranes, the very electrical activity in the brain further modulates the charge state of these interfaces. Neuronal firing and field potentials generate spatiotemporal variations in the extracellular electric field which can induce transient polarization of nearby membranes (McColgan et al. 2017; Bédard and Destexhe 2022). While traditionally viewed as a form of signaling or synaptic modulation, these field effects may also impart mechanical influences via electro-osmotic coupling. In both biological tissues and engineered microfluidic systems, electric fields have been effectively employed to drive fluid motion through charged channels in confined environments, as seen in electrokinetic drug delivery platforms and lab-on-a-chip devices (Cruz-Garza et al. 2024). Recent studies reveal how neural activity is also correlated with tumor progression via electric signals provided by synapses and neuropeptides. GABAergic input promotes glioma growth (Barron et al. 2025), substance P drives breast cancer metastasis through TLR7 activation (Padmanaban et al. 2024) and CGRP from nociceptors impairs CD8+ T cell immunity in melanoma (Balood et al. 2022). Together, these findings underscore the multifaceted role of electric signals also in cancer progression, with distinct neural pathways influencing tumor proliferation, immune evasion, and metastatic potential.

Further indirect evidence comes from studies of ion diffusion and CSF exchange (Marques-Almeida et al. 2023). Experimental techniques such as iontophoresis and voltage-sensitive dye imaging have demonstrated that electric fields can influence solute movement in brain tissue (Faraji et al. 2020). These findings raise the possibility that CSF-facing membranes may act not merely as passive barriers but as active modulators of ionic and fluid flow, regulated by both intrinsic charge and externally applied fields.

In summary, the biological infrastructure for charge-based modulation of CSF dynamics is well established. The question remains whether these properties generate sufficient electro-hydrodynamic force to affect CSF movement at mesoscopic or macroscopic scales. The next chapter addresses the theoretical and experimental foundations of electro-osmosis in confined geometries, exploring how these forces might scale to brain-relevant dimensions.

3 | Electro-Osmosis in Confined Geometries and Its Relevance to Brain Physiology

Electro-osmosis refers to the motion of a liquid induced by an electric field across a charged surface within a confined channel (Sahib et al. 2021). It is a well-characterized phenomenon in synthetic nanofluidic systems and has long been harnessed in technologies such as capillary electrophoresis, microfluidic pumps and drug delivery platforms (Alizadeh et al. 2021; Elboughdiri et al. 2024). Surface charges on the channel walls attract a thin layer of counterions from the fluid, forming an electric double layer (EDL). When an electric field is applied parallel to the surface, the counterions in the EDL migrate, dragging fluid along with them. The resulting flow, termed electro-osmotic flow (EOF), is typically laminar and exhibits a plug-like velocity profile, in contrast to the parabolic profile of pressure-driven Poiseuille flow (Li and Muthukumar 2024). Physics governing electro–osmosis has been formalized through coupled solutions of the Navier–Stokes equations for fluid motion and the Poisson–Boltzmann equation for electrostatic potential (Gubbiotti et al. 2022). These equations reveal that the EOF velocity is directly proportional to the zeta potential (a measure of the surface charge), the permittivity of the fluid, and the applied electric field, while inversely proportional to the fluid viscosity (Sherwood et al. 2014). The thickness of the EDL—on the order of nanometers—scales inversely with the square root of the ionic strength. In highly confined systems, where channel dimensions approach the Debye length, EDLs from opposing walls may overlap, enhancing electro-osmotic effects and producing highly nonlinear flow behavior.

Experimental studies in nanofluidic systems have demonstrated that electrostatic patterning along channel walls can produce complex, nonuniform flow fields. For example, charge heterogeneity—achieved via alternating stripes of positive and negative surface potential—has been shown to generate spatially structured flows, reversals in direction, and even discrete transitions between ionic and pressure-dominated flow regimes (Ververniotis et al. 2011). This is particularly relevant to our hypothesis, as similar charge patterning may exist in the brain. Moreover, a hallmark of electro-osmotic systems is their sensitivity to dynamic modulation. In engineered systems, time-varying surface potentials can induce pulsatile or oscillatory flow, mimicking the rhythmicity of biological processes like neural oscillations (Banerjee et al. 2023). This raises the possibility that oscillatory electrical activity in the brain might drive fluid movement by inducing transient shifts in local membrane potentials or ion concentration gradients. Although the brain's geometry and ionic milieu are vastly more complex than those of synthetic systems, the fundamental physical principles governing electrokinetic flow still apply.

CSF navigates a labyrinth of narrow, channel-like pathways within the brain, enabling both directed flow and efficient molecular exchange (see Table 1). Among the most studied are perivascular spaces, i.e., Virchow–Robin spaces, which surround blood vessels as they enter and exit brain tissue (Kwee and Kwee 2007). These spaces range from 5 to 40 μm in width and may extend hundreds of micrometers, enabling the bidirectional movement of CSF and interstitial fluid (Bernal et al. 2022; Raicevic et al. 2023). Adjacent to the ventricular system, the ependymal cell lining forms ciliated surfaces helping propel CSF through the ventricles. Though not traditional channels, the intercellular gaps and surface specializations between these cells, often less than 1 μm wide, contribute to localized CSF movement and solute exchange. Further into the parenchyma, the brain's extracellular space—comprised of interstitial and paracellular compartments—has a width of 20 to 60 nm. While primarily a domain for diffusion, this space may support slow, directed flow under certain physiological or pathological conditions (Ballerini et al. 2020). The cerebral aqueduct, though larger in scale (~1.5 mm in diameter and ~15 mm in length), stands for an anatomical bottleneck that constrains ventricular CSF flow and is highly sensitive to obstruction (Sincomb et al. 2020). Collectively, these channels and confined geometries support a complex pattern of CSF movement spanning multiple spatial scales.

Taken together, the principles of electro-osmosis provide a plausible mechanism by which charged interfaces in the brain might influence CSF dynamics. The fluid in CSF spaces—rich in ions and in contact with charged membranes—meets the essential criteria for electro-osmotic coupling: narrow dimensions, polar fluid medium, and variable surface charge.

Experimental analogs from biology further support this view. For instance, endothelial cells lining blood vessels in vivo exhibit charge-selective permeability and engage in electrokinetic transport processes, influencing both blood plasma and interstitial fluid composition (Wakasugi et al. 2024). The inner surfaces of blood vessels, particularly the endothelium, carry a net negative charge due to the presence of glycoproteins, proteoglycans, and sialic acid-rich components of the glycocalyx (Zhao et al. 2020). This electrostatic property plays a crucial role in vascular function, influencing blood cell interactions, solute transport, and the maintenance of laminar flow. Similar charge characteristics have also been observed in other fluid-carrying biological conduits, such as lymphatic vessels and renal tubules, where surface charge helps regulate fluid movement and filtration through electrostatic interactions with high spatial and temporal specificity (Choudhury et al. 2022). Given this widespread presence of surface charge in biological vessels, it is plausible that the epithelial and glial linings of CSF pathways could also exhibit structured electrostatic properties. Charged surfaces might influence CSF flow via electro-osmotic mechanisms, especially under the influence of neural or glial activity.

Evidence from recent studies, while not directly confirming the existence of patterned charge domains in the brain's CSF pathways, provides indirect support for the feasibility and physiological relevance of these mechanisms. Experimental work has shown that electric fields can drive electrokinetic transport of solutes through brain-like tissues (Faraji et al. 2011; Alcaide et al. 2023). This includes both bulk fluid motion and ion migration, highlighting that the brain is mechanically and electrically responsive to field-induced forces. Researchers have also demonstrated that external electric fields can enhance drug delivery to brain regions by modulating flow profiles using electrokinetic principles (Faraji et al. 2020). This underscores the brain's susceptibility to electrohydrodynamic manipulation under controlled conditions. Further support comes from theoretical and computational work suggesting that the brain's nonzero zeta potential, due to charged surfaces such as glial membranes, could allow electro-osmotic flow to contribute to intracerebral fluid movement (Wang et al. 2021b). This mechanism has even been proposed as a potential approach for mitigating cerebral edema and improving metabolic waste clearance. Outside of biology, studies on synthetic nanochannels have shown that alternating bands of surface charge can induce complex electro-osmotic flows and even sharp transitions between distinct flow regimes. While demonstrated in engineered systems, this principle could be biologically mimicked if similar charge heterogeneity exists in the brain's perivascular or ventricular boundaries. Finally, computational models suggest that neuronal activity can generate electrodiffusive gradients that couple with osmotic and mechanical flows in glial networks (Fujii et al. 2017). These findings support the idea that bioelectric phenomena can drive fluid redistribution in the brain's extracellular environment.

Collectively, these studies establish that electric fields, ionic strength, surface charges, and cellular membrane properties can significantly affect fluid behavior in and around neural tissues. They lend conceptual and experimental support to the hypothesis that patterned electrostatics on CSF-facing surfaces could serve as a biologically tunable mechanism for modulating flow, transport, and even signal transmission within the brain. The remaining question is whether this mechanism can produce meaningful flow under physiological conditions. To begin answering this, we implemented a series of computational simulations using idealized models of electro-osmotic flow in brain-inspired geometries. These are detailed in the following chapter.

TABLE 1 | Summary of anatomical microchannels in the brain, detailing their dimensions and roles in cerebrospinal and interstitial fluid transport.

Structure	Diameter	Length	Role
Perivascular spaces	5–40 μm	100–1000 + μm	CSF–ISF exchange, drainage
Ependymal/paracellular gaps	< 1 μm	Very short (cell-scale)	Diffusion, limited flow
Interstitial space	20–60 nm	Local (tissue-wide)	ISF–CSF interaction
Aqueduct of Sylvius	~1.5 mm	~15 mm	Major CSF conduit

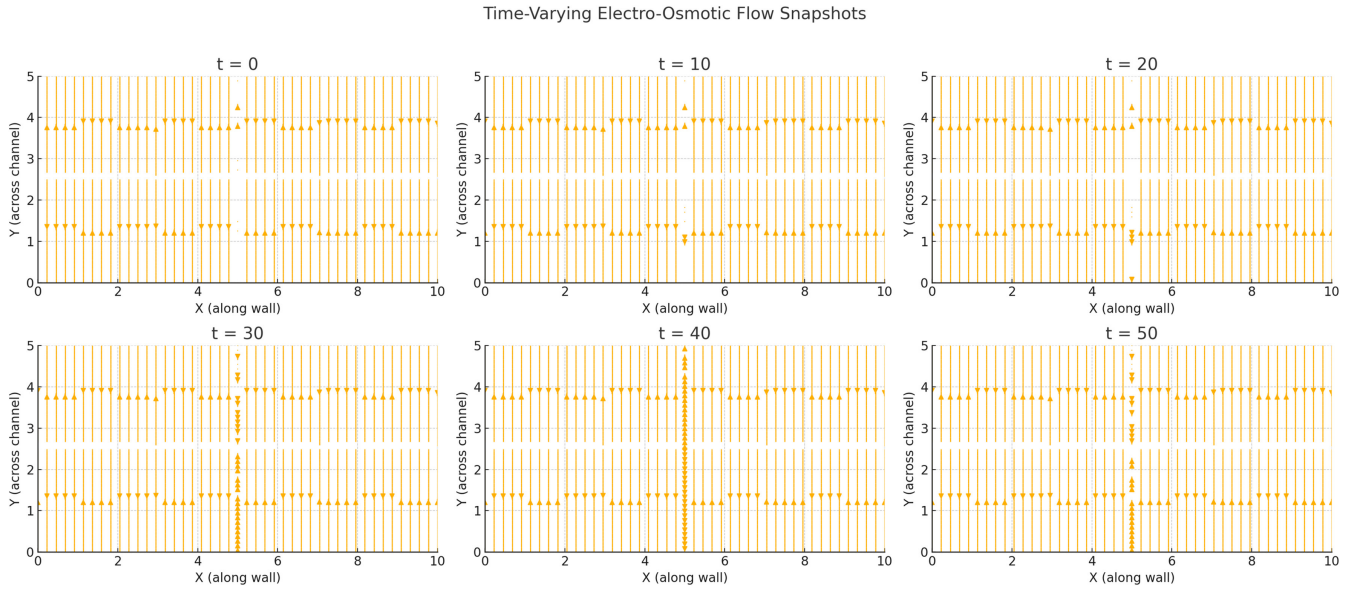


FIGURE 1 | Simulated trajectories of molecules or ions over a 50-s period as they cross a fluid channel lined with alternating surface charge domains. The flow field is shaped by electrohydrodynamic forces resulting from interactions between the patterned wall charges and the fluid content. The colored lines trace the paths of individual particles. The extended simulation duration (T = 50 s) highlights how charge-driven microflow structures can lead to differential transport, localized trapping or enhanced directional clearance.

4 | Computational Simulations of Electrostatic Modulation in Cerebrospinal Fluid Flow

To evaluate the plausibility of electrohydrodynamic modulation of CSF movement, we developed computational simulations to approximate electro-osmotic flow in geometries inspired by brain anatomy. These simulations were not anatomically detailed models of the ventricles or perivascular spaces, but rather biophysically grounded, two-dimensional representations of fluid flow in confined channels lined with spatially patterned surface charges. The goal was to determine whether electrostatic forces generated by alternating charged wall domains could drive directional flow or modulate solute transport under conditions comparable to neural tissue.

To investigate the effects of surface charge patterning on CSF dynamics, we developed a simplified two-dimensional electrohydrodynamic model based on the lattice Boltzmann method (LBM) for bulk flow and coupled it with electrostatic field calculations derived from the Poisson–Boltzmann equation. Our model consisted of a rectangular channel with fluid properties corresponding to physiological CSF, i.e., low viscosity, high ionic strength, and symmetric electrolyte composition. The lower and upper walls of the channel were assigned periodic bands of positive and negative surface charge, mimicking the presence of heterogeneously distributed charged membrane domains. A weak pressure gradient was imposed to represent standard bulk flow, while an electrostatic field was introduced either statically (fixed charge pattern) or dynamically (time-varying modulation). The coupling between electric field and fluid motion was calculated using a simplified electrohydrodynamic formulation, inspired by the Debye–Hückel approximation and standard Navier–Stokes solutions, including the effect of electrostatic drag near the walls. Domain dimensions were scaled to represent perivascular or ventricular compartments, typically 50–200 μm wide and 500–1000 μm long. The channel walls were assigned alternating bands of positive and negative surface charge densities ($\pm$10 mC/m^2). Electrolyte fluid was modelled as a Newtonian fluid with ionic strength between 1 and 10 mM, permittivity of 80 and dynamic viscosity of 0.7 mPa·s. Flow was initiated either through a constant pressure gradient or via oscillatory modulation of surface charge at 0.1–10 Hz to approximate rhythmic neural activity. Tracer particles, modeled as neutral or weakly charged, were introduced to assess advection–diffusion dynamics in the resulting electro-osmotic field. Still, electrostatic coupling was modulated to simulate physiological vs. pathological ionic conditions, e.g., reduced Debye length and altered ion concentrations. Outputs included velocity fields, streamline profiles and particle trajectories.

Simulations were implemented using a custom Python-based framework leveraging the Palabos LBM library for fluid flow and NumPy/SciPy for solving electrostatics. Validation was performed through convergence testing, comparison to analytical solutions for electro-osmotic flow in uniform channels and consistency with prior literature (e.g., Curk et al. 2024). Simulation data were visualized using Matplotlib and ParaView.

Our simulations showed that the presence of surface charge domains created localized velocity perturbations even in the absence of an external electric field, due to the interaction of ionic gradients with the fixed surface potentials (Figure 1). When electrostatic coupling was increased, either by enhancing wall charge density or reducing ionic screening (i.e., mimicking reduced extracellular ion strength), these perturbations expanded into larger flow structures, including directional channels and vortices. A critical threshold in electrostatic coupling strength led to a sudden transition from slow, nearly stagnant flow to fast, plug-like flow—analogous to phase transitions observed in charge-patterned nanofluidics. This is consistent with

theoretical predictions by Curk et al. (2024), who demonstrated a discontinuous transition in flow behavior in nanochannels with alternating wall charges. Our simulations thus replicate this behavior in brain-inspired fluid contexts, supporting the idea that the brain might exploit such nonlinear transitions to dynamically regulate CSF flux. Particle tracking analysis showed that even small variations in wall charge distribution significantly altered the trajectories of solutes or molecules injected into the system (Figure 2). Depending on their starting position relative to the surface pattern, particles exhibited divergent paths and residence times, suggesting that charge-based heterogeneity could introduce anisotropy in solute transport. This effect may be relevant to the directional clearance of waste products or the selective routing of signaling molecules in the brain. Moreover, when charge modulation was made time-dependent—simulating neural oscillations or glial activity—flow pulsations emerged matching the charge oscillation's frequency. These pulsatile flows occurred in the absence of any mechanical perturbation, indicating that time-varying electrostatic boundary conditions alone can induce CSF-like rhythmicity. To

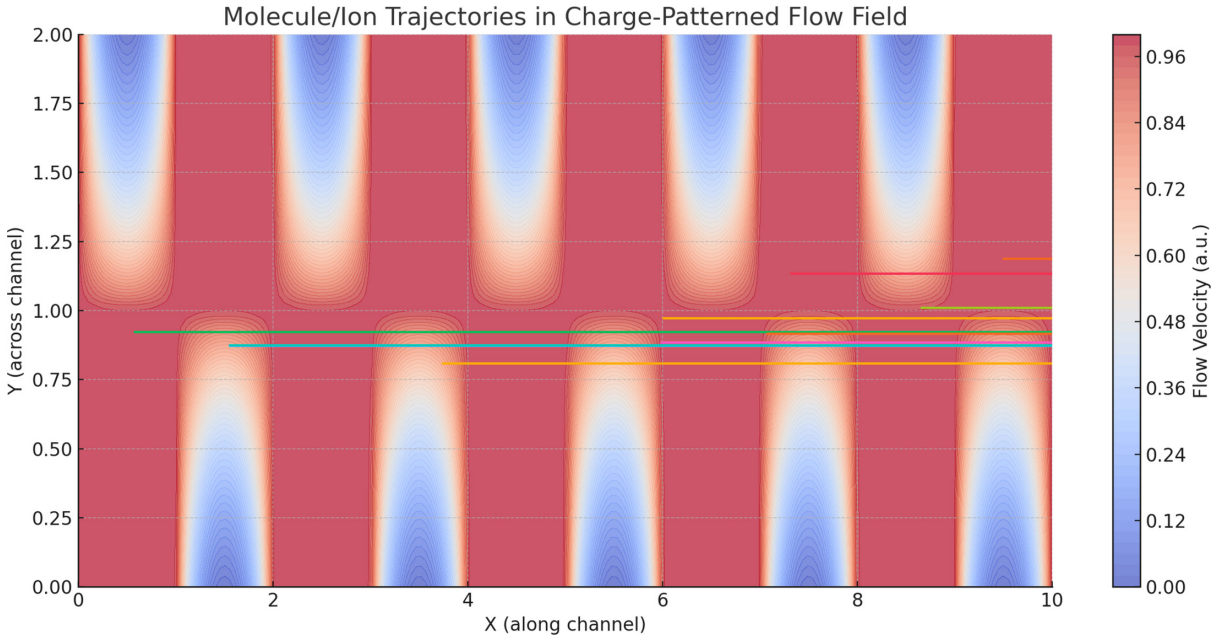


FIGURE 2 | Simulated trajectories of molecules or ions advected through a CSF microchannel bounded by alternating positive and negative surface charge domains on the top and bottom walls. The background color map represents the flow velocity field generated by electrohydrodynamic interactions between the wall charge pattern and ionic content of the fluid. Particle trajectories are shown as colored lines, with green dots indicating starting positions and red dots marking their final locations. Spatial variations in wall charge can lead to nonuniform and trajectory-dependent fluid flow.

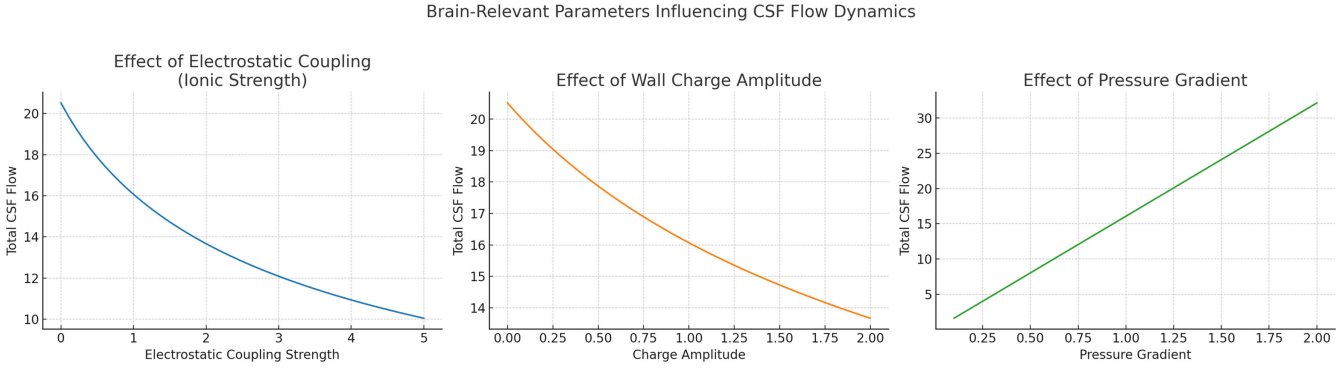


FIGURE 3 | Brain-relevant parameters influencing CSF flow dynamics. Left panel: Effect of electrostatic coupling strength (ionic strength). This plot shows how increasing electrostatic coupling—representing stronger interactions between charged walls and ions in CSF—leads to a progressive reduction in total CSF flow. This mimics physiological and pathological changes in ionic strength, such as elevated extracellular potassium or disrupted ion homeostasis caused by various diseases. Middle panel: Effect of wall charge amplitude. This plot illustrates how variations in the amplitude of patterned surface charge along ventricular or perivascular walls influence CSF flow. Modulation of charge density could arise either from altered astrocytic or ependymal activity or pathological changes in membrane potential and protein expression. The nonlinear flow behavior highlights the potential for bioelectrical gating of fluid dynamics. Right panel. Effect of pressure gradient. This panel shows the linear relationship between the applied pressure gradient and CSF flow rate, modeling physiological drivers such as cardiac and respiratory cycles, as well as pathological changes in intracranial pressure.

investigate the relevance of pathological states, we explored how changes in ionic strength, wall charge amplitude and coupling coefficients affected flow characteristics (Figure 3). Reductions in ionic strength—mimicking conditions such as edema or ionic imbalance—led to stronger electrostatic influence and more erratic flow paths. Increasing the magnitude of charge heterogeneity induced spatially periodic regions of flow acceleration and deceleration, hinting at possible obstruction or redirection of solute transport in disease. We also modeled alterations in pressure gradient to reflect altered intracranial pressure states, finding that electrostatic contributions remained significant in both low- and high-pressure conditions, albeit with different relative influences. Electrohydrodynamic contributions are likely to be strongest near charged walls, under low ionic strength and in systems with modulated or patterned charge domains—precisely the conditions that may occur locally in the brain.

Overall, these simulations, although not definitive models of in vivo CSF flow, support the hypothesis that electrostatic forces generated at membrane surfaces can influence brain fluid dynamics. The convergence of physical principles, biological plausibility and simulation results strongly argues that

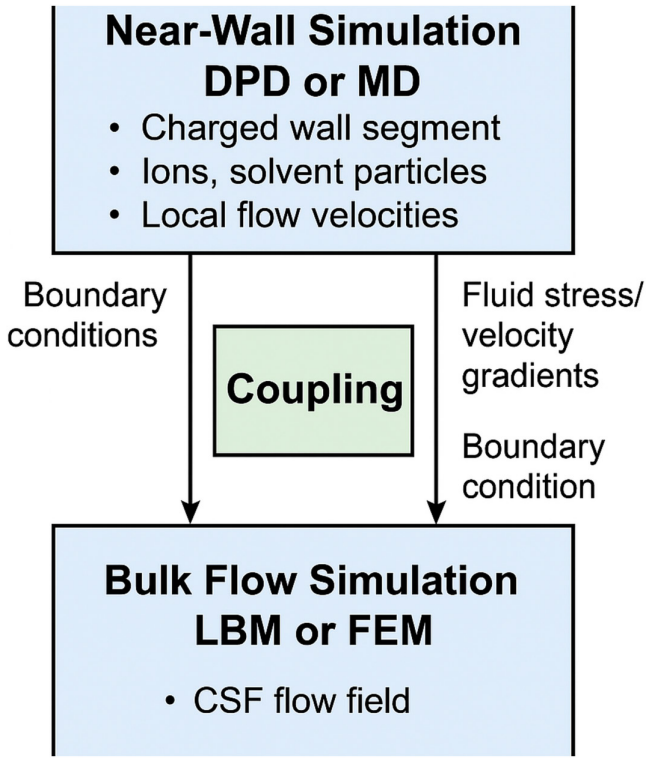


FIGURE 4 | Conceptual workflow of a hybrid computational model combining near-wall and bulk flow simulations to investigate electrohydrodynamic CSF dynamics. The upper section represents the near-wall simulation zone, where dissipative particle dynamics (DPD) or molecular dynamics (MD) are used to capture fine-scale electrostatic interactions, ion layering, and local electro-osmotic effects near charged glial or ependymal surfaces. The lower section depicts the bulk flow simulation domain, modeled using lattice Boltzmann method (LBM) or finite element method (FEM), which handles pressure-driven flow and global CSF transport in anatomically relevant structures. The central coupling interface enables dynamic data exchange between the two regions. Velocity and ionic flux data from the particle-based simulation may inform boundary conditions, macroscopic pressure and shear feedbacks.

electrohydrodynamics may represent an overlooked contributor to neurofluidic regulation.

5 | Conclusion

Our findings support the biophysical hypothesis that the surfaces lining cerebrospinal fluid (CSF) pathways—such as ependymal walls and perivascular boundaries—could actively influence CSF flow through their electrostatic properties. Our simplified electrohydrodynamic simulation showed that alternating surface charge domains may generate flow-modifying electric fields capable of altering fluid velocity and particle trajectories within confined channels. Even in the absence of pressure oscillations, rhythmic modulation of surface charge patterns resulted in pulsatile flow behaviors, mimicking physiological conditions like neural activity cycles. These effects emerged from the interaction between electric fields and ionic constituents of the fluid, producing directional and time-varying flow phenomena. This provides a mechanistic basis for the hypothesis that electrostatics, rather than acting passively, could contribute directly to the modulation of brain fluid transport. Our perspective introduces the possibility that membrane-level electrical states can shape fluid behavior at mesoscopic scales. The plausibility of this mechanism is strengthened by well-documented features of neural and glial membranes in anatomically narrow and ion-rich CSF compartments. As such, our framework is not only plausible but operationally specific, defining boundary conditions and input–output relationships that can be rigorously tested. Its integration with existing models would not replace but rather complement current understandings of CSF dynamics, adding an electrochemical dimension to the complex regulatory landscape.

Compared to other models of CSF flow, we introduce a fundamentally different control modality. Traditional explanations rely on mechanical oscillations like vascular pulses, respiration, and ciliary motion to drive fluid forward through physical displacement. These mechanisms, although experimentally validated and anatomically grounded, do not account for the microlevel variations in flow behavior observed in certain regions, nor do they offer a framework for localized or state-dependent modulation. Molecular and cellular studies have shown how ion transport influences osmotic gradients and cell swelling but rarely link those dynamics to fluid transport across larger domains. Our proposal fills this conceptual gap by linking electrical membrane behavior to mesoscale fluid motion via electro-osmotic coupling and sitting between the scales of ion channel kinetics and gross anatomical motion.

Nonetheless, our hypothesis faces several limitations. Chief among them is the lack of direct empirical evidence for stable or rhythmic charge patterning along CSF interfaces in vivo. While glial and epithelial membranes are known to carry surface charge, it remains unclear whether this charge is organized in spatial domains sufficient to produce significant electro-osmotic flow under physiological conditions. Additionally, the electrical double layer thickness, ion mobility and permittivity in brain tissue are not uniform and could complicate flow generation or assumptions of symmetry. Still, our hypothesis requires a largely passive fluid medium influenced by external electrostatic fields, whereas real CSF movement is likely affected by

an intricate interplay of active transport, convection, diffusion, and tissue deformation. Furthermore, our computational model simplifies boundary conditions and ignores potential feedback loops between membrane depolarization and fluid velocity. The use of simplified rectangular channels, while valuable for isolating key electrohydrodynamic effects, does not fully capture the anatomical complexity of the brain's perivascular and ventricular spaces. Our model lacks the curvature, tortuosity, and heterogeneous porosity characteristic of real perivascular pathways, which are known to influence local flow resistance and solute dispersion. Moreover, our model does not yet incorporate 3D anatomical geometries or dynamic tissue deformation, both of which may affect CSF movement in vivo. Being electrostatic interactions treated in a static manner, our framework does not include dynamic bioelectric feedback such as time-varying membrane potentials, ion channel activity, or electrogenic transport processes. Our hypothesis presents significant challenges, including the in vivo verification of spatially patterned surface charges, the experimental disentanglement of electrostatic and mechanical contributions to fluid movement and the accurate modelling of bidirectional coupling between membrane dynamics and CSF flow. These limitations mean that the model cannot yet account for electromechanical coupling or flow modulation arising from real-time neural activity. Future work should integrate these features to enhance physiological fidelity and explore the nonlinear interplay between electrical signalling, membrane charge dynamics, and fluid transport.

In terms of testable predictions, we expect that artificial modulation of membrane charge—via optogenetic activation of ion pumps, localized application of charged substrates, or genetic manipulation of membrane proteins—should result in measurable changes in local CSF flow velocity or solute transport. This can be explored in microfluidic models using glial or epithelial cell monolayers, where flow fields can be visualized in real time under pharmacological modulation. On the physiological side, one might predict that regions of the brain with higher density of ion-exchanging membranes—such as the ventricular ependyma or perivascular astrocytic endfeet—exhibit enhanced responsiveness to electrostatic perturbation in fluid transport. This could be probed through intracranial injection of tracers under conditions of altered extracellular ion concentration. Our hypothesis opens new directions for interpreting disease. Electrohydrodynamic mechanisms may help explain pathological flow disruptions in conditions such as hydrocephalus, traumatic brain injury and Alzheimer's disease, where ionic imbalance and membrane dysfunction are common. Furthermore, our perspective could inform the design of targeted drug delivery systems or neuromodulation strategies that exploit electrical properties of neural interfaces to influence solute clearance or fluid movement in situ.

These predictions provide a roadmap for targeted experimental studies aimed at validating or refining the model. In exploring electrohydrodynamic mechanisms underlying CSF flow, several simulation approaches are available beyond continuum models. Among the most promising are multiscale and hybrid techniques that integrate molecular and continuum physics. The lattice Boltzmann method (LBM) is particularly suited for simulating microscale fluid flow in complex geometries such as ventricular spaces and perivascular channels, while the finite element method (FEM) excels at solving electrokinetic and fluid dynamics equations in anatomically realistic domains. To resolve nanoscale behavior near surfaces such as ion layering and charge-driven flow, dissipative particle dynamics (DPD) and molecular dynamics (MD) are effective, capturing interactions at atomic and mesoscopic scales. Although each method alone has limitations in scale and scope, a hybrid modeling strategy can address this (Figure 4). By combining DPD or MD to simulate the electrostatic behavior near charged surfaces (e.g., glial or ependymal membranes) with LBM or FEM for bulk CSF flow, the full electrohydrodynamic behavior across scales could be captured. These models may communicate via a coupling interface where information on boundary velocities, electric potentials, or shear stress is exchanged (Hoogerbrugge and Koelman 1992).

In closing, we have introduced the novel hypothesis that patterns of positive and negative surface charge may exist along the inner linings of the brain's CSF channels. If present, these structured electrostatic domains could interact with the ionic nature of CSF to generate localized electric fields capable of driving or modulating flow through electro-osmotic mechanisms. Unlike pressure-driven flow, passive electro-osmosis may offer the potential for directionally controlled, rhythmically responsive and spatially fine-tuned fluid movement. These features align with the need for dynamic regulation in neurophysiological contexts, including sleep–wake cycling and metabolic waste clearance. While our approach remains exploratory, it introduces a coherent theoretical model grounded in experimentally supported biophysics. From this model, we expect to observe conditions under which electrostatic patterning produces detectable effects on CSF flow structure, directionality, and transport efficiency.

Author Contributions

The author performed the following: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding, administrative, technical and material support, study supervision.

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The author has nothing to report.

Ethics Statement

This research does not contain any studies with human participants or animals performed by the author.

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Conflicts of Interest

The author declares no conflicts of interest.

Data Availability Statement

All data and materials generated or analyzed during this study are included in the manuscript. The author had full access to all the data in

the study and took responsibility for the integrity of the data and the accuracy of the data analysis.

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Declaration of Generative AI and AI-Assisted Technologies in the Writing Process

During the preparation of this work, the author used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar, and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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Unique lipid signatures in viral envelopes originating from host membranes

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ABSTRACT

Envelope viruses infecting human individuals can originate from two sources: either from an intermediary animal that transmits the virus to humans, or from another infected human. During a pandemic such as SARS-CoV-2, identifying the intermediate host or the primary human source presents a significant challenge. This complex task is typically addressed through genetics-based approaches, including metagenomic analysis, phylogenetic and phylodynamic rooting methods, integrated with epidemic simulations. We review a method to investigate these primary viral sources. During their replication cycle, envelope viruses hijack materials from host cellular compartments such as endoplasmic reticulum (ER), Golgi apparatus (GA) and ER-Golgi intermediate compartment (ERGIC). Biochemical, morphological and functional differences in the membranes of ER, GA and ERGIC can be detected not only across mammalian species but also among individual humans. These variations arise from a complex interplay of genetic, epigenetic, metabolic, environmental and age-related factors. We review the previous studies demonstrating the relevance of lipidomic profiling in virus-host interaction studies/biomarker discovery and describe how recent advances in analytical technologies—particularly high-resolution mass spectrometry and lipidomics-specific bioinformatics—have dramatically improved our ability to detect and interpret lipidomic data with precision and organelle-level resolution. We propose utilizing lipidomics to identify unique lipid signatures in the compositions of the viral envelopes that are co-opted from the host cell's organelles. Since interspecies and interhuman lipidic differences could significantly impact the composition of viral envelopes derived from host membranes, molecular disparities might serve as critical markers for tracing the source of viral particles. This approach could enable the identification not only of the mammalian sources of human spillover, but also provide insights into the age, medical condition, genetics, and ethnic background of the first human host.

Introduction

Understanding the circumstances that lead to virus outbreaks is critical for deterring future zoonotic pandemics, discovering new drugs, developing vaccines [1–3]. However, this can prove to be a very difficult task, if not almost impossible. To provide an example, the origins of SARS-CoV-2 are still hotly debated [4]. Once established that bats are the natural reservoirs of SARS-related coronaviruses [5], a feverish search has sought to identify the likely source of cases in early reports. It has been suggested that the earliest known COVID-19 cases, geographically centered on the Huanan Seafood Wholesale Market in Wuhan, occurred through the live wildlife trade in China [3], being the result of at least two separate cross-species transmission events into humans [2]. Zoonotic spillovers have been hypothesized from various SARS-CoV-2-susceptible mammals identified as potential intermediate hosts, including civets, bamboo rats, raccoon dogs, pangolins, etc [1,6,7].

From a methodological standpoint, genotypes of potential animal hosts are recovered, analyzed and compared with those from humans and environmental samples [6,8]. The comparison is usually made by using metagenomic and phylogenetic approaches combined with structural modeling, phylodynamic rooting methods and epidemic simulations [2,7]. These approaches establish the genetic foundation for identifying a shortlist of potential intermediate hosts to prioritize for serological and viral sampling [6]. They also help pinpoint the critical time frame between the initial zoonotic spillover into humans and the emergence of the first reported human cases [2].

Here we suggest another method to detect and analyze the origin, spillover, intermediate hosts and human reservoirs in case of enveloped viruses. It has been demonstrated that the morphology of some virions depends heavily on the host environment and lipid composition. For example, in influenza A virions, the matrix protein M1, together with specific host lipids, plays a key role in determining the structural morphology of the assembled virion [9]. Virions isolated from avian hosts tend to be predominantly spherical, while human-adapted strains often display filamentous shapes [10,11]. However, even in humans, certain strains may still produce mainly spherical particles due to differences in viral protein expression and egress routes [12]. These findings suggest that the host cell lipid environment may play a crucial role in shaping

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virion morphology, including the modulation of membrane curvature and budding mechanisms [13].

Instead of relying solely on traditional genetic studies, we propose examining the viral envelopes, which contain phospholipids and proteins derived from host cell membranes. Envelope viruses encompass both DNA and RNA viruses such as Orthomyxoviridae (e.g., influenza virus), Poxviridae (e.g., smallpox virus), and Paramyxoviridae (e.g., human parainfluenza viruses). We will focus on viral families that hijack host cellular compartments such as the endoplasmic reticulum (ER), the Golgi apparatus (GA) or the ER-Golgi intermediate compartment (ERGIC) to ensure their replication cycle, viral assembly, envelope formation, budding and release by exocytosis [14,15]. These families include, among others, Flaviviridae such as Dengue virus and Zika virus that acquire their envelopes during budding through ER [16,17] and Coronaviridae such as SARS-CoV-2 that assemble in ERGIC [18–20]. In many of these viruses, virions are transported to the cell surface following intracellular assembly and are subsequently released through exocytosis.

A virus infecting a human can originate from two sources: either an animal host through spillover, or another infected human. Given the detectable inter-species and inter-individual variations in ER, GA and ERGIC, we contend that the study of viral envelopes could offer valuable clues about the animal source of human spillover and the identity of patient zero, i.e., the first human infected during an outbreak. Specifically, we will focus on the lipid components of the ER and GA that are incorporated into the viral envelope, which can vary depending on the virus's most recent host. We argue that researchers should identify specific host-derived lipidic biomarkers within viral envelopes to provide critical insights into determining the virus's most recent host, whether animal or human.

LIPIDOMIC differences IN golgi apparatus AND endoplasmic reticulum IN animals AND humans

Lipidomics is a rapidly expanding field focused on uncovering the unique lipid profiles of cellular organelles [21]. Recent technological advancements have significantly expanded the capabilities of lipidomics. High-resolution mass spectrometry platforms, such as Orbitrap and time-of-flight (TOF) instruments, allow simultaneous identification and quantification of thousands of lipid species with high sensitivity and specificity [22–27]. The incorporation of stable isotope labeling and machine learning for lipid annotation further enhances the analytical precision and throughput [28,29], making lipidomic fingerprinting of viral envelopes a realistic and powerful approach. Mass spectrometry-based lipidomics can rapidly identify as well as quantify >1000 lipid species at the same time, facilitating robust analyses of lipids in tissues, cells and body fluids [30]. Accordingly, lipidomics is now being widely applied in various fields, including nutrition science to assess food obtained from livestock and poultry as well as fish food products [30,31]. Coupled with ion mobility separation and improved chromatographic techniques, these tools facilitate also organelle-specific lipidomic mapping, even in complex biological samples such as virus-infected cells. Detailed studies have mapped the lipid compositions of various mammalian organelles, including ER and GA [32]. A systematic analysis of the overall variation in the mammalian lipidome, with a particular focus on *Mus musculus*, has been conducted to evaluate the effects of diet, sex, and genotype [33]. The cellular lipidome is highly adaptable, shifting in response to numerous physiological processes such as aging and a range of pathological states [34]. Alterations in the organelles' lipid profiles may lead to disrupted lipid metabolism, persistent inflammation and oxidative stress that have been documented across cancer, metabolic diseases and neurodegenerative disorders. Further, many studies suggest the relevance of lipidomics in virology [35–38]. For instance, Kyle [39] emphasized the potential of lipidomics for uncovering host-virus interactions, particularly in identifying biomarkers and pathogenesis patterns. These works support the application of lipidomic profiling as a forensic virological tool to trace infection origins.

Differences in the lipid composition of the ER and GA have been observed not only across different mammalian species but also among individual humans. In the following two paragraphs, we will explore these variations in more detail.

Lipidomic differences in various animals. Extended lipid profiles of several animal species have been compared via lipidomic analysis conducted by liquid chromatography-high-resolution mass spectrometry, allowing the identification of about a hundred of molecular species of lipids [40]. ER focuses on synthesizing lipids tailored to environmental and metabolic challenges, while GA specializes in modifying and transporting these lipids for specific cellular and systemic functions. Both exhibit species-specific lipid compositions that fulfill critical biological purposes, reflecting adaptations across mammals to genetic factors, environmental conditions, dietary habits, metabolic needs [41]. For instance, humans and rodents share high proportions of phosphatidylcholine and phosphatidylethanolamine essential for membrane structure and cellular signaling [42]. Carnivorous mammals like raccoon dogs have lipidomic compositions tailored to protein-heavy diets, whereas omnivorous and insectivorous species like bats show distinct profiles suited to their high metabolic demands [43,44]. Marine mammals like seals and whales have ER membranes rich in phosphatidylserine and long-chain PUFAs to maintain fluidity under the extreme conditions of cold, high-pressure aquatic environments [45]. In turn, desert mammals like camels display higher concentrations of unsaturated phospholipids to cope with dehydration and heat stress. In primates, cholesterol biosynthesis is optimized for neural and immune functions [46]. Herbivorous species like cows and sheep can efficiently convert cholesterol into bile acids for digesting plant-based diets. By contrast, carnivorous mammals like cats and dogs exhibit less diverse bile acid profiles and simpler cholesterol synthesis pathways, consistent with their protein-rich diets [43].

Marine mammals' ER produce elevated levels of ceramides and sphingomyelins with long-chain bases conferring resistance to salt and temperature fluctuations. Primates and rodents synthesize an array of gangliosides and glycosphingolipids for neural signaling and immune interactions [47–49]. Primates maintain a balance between saturated and unsaturated fatty acids, while rodents favor linoleic acid derivatives [50]. Marine mammals emphasize omega-3 fatty acids such as DHA and eicosapentaenoic acid to adapt to cold environments [51]. Lipid droplet formation in the ER reflects species-specific metabolic needs. Hibernating mammals like bears accumulate triglycerides for energy storage during pre-hibernation, while marine mammals produce blubber lipids characterized by high triglyceride and wax ester content [52].

The lipid composition of the GA is equally diverse and tailored to species-specific functions. The lipid remodeling processes in the GA support specific secretory functions, such as the packaging of milk fat globules in lactating mammals or the production of blubber in marine mammals [53]. Cholesterol transport and modification in the GA are more prominent in herbivores to facilitate bile acid production, while carnivores exhibit simpler pathways [54,55]. Phosphoinositides, particularly phosphatidylinositol and its derivatives, play a vital role in GA vesicle trafficking, with species-specific variations reflecting distinct membrane transport requirements [56]. Also, environmental challenges drive adaptations in GA lipid metabolism to thrive under diverse environmental pressures. Cold-adapted mammals like Arctic foxes produce a higher proportion of unsaturated lipids to maintain vesicle and membrane fluidity in low temperatures [57]. Conversely, desert mammals such as camels synthesize lipids (also in their milk) that resist dehydration and oxidative stress [58].

In sum, lipidomic differences in the ER and GA among mammals demonstrate the remarkable versatility of these organelles. By tailoring their lipid composition and metabolic pathways, the ER and GA meet the unique dietary, ecological, and physiological demands of each species. These variations underscore the evolutionary adaptations that enable mammals to maintain cellular functions crucial for survival in diverse habitats and under varying biological demands.

Lipidomic differences in various human individuals. Lipidomic variations in ER and GA among human individuals arise from interplay of genetic factors, diet, lifestyle, environmental exposures and health conditions. Phosphatidylcholine and phosphatidylethanolamine levels, for instance, are influenced by polymorphisms in the PEMT gene [59]. Diets rich in omega-3 fatty acids contribute to higher levels of docosahexaenoic acid in ER's phospholipids, particularly in individuals consuming fish-heavy diets [60]. Variations in the PTDSS1 gene affect phosphatidylserine levels, which are associated with cognitive and neural functions [61]. Ceramide levels in the ER are elevated in insulin resistance and type 2 diabetes, with genetic variations in DEGS1 playing a key role in modulating ceramide biosynthesis [62]. Diet, particularly high saturated fat intake, contributes to increased sphingomyelin levels. Genetic polymorphisms in the FADS1 and FADS2 genes impact the synthesis of long-chain polyunsaturated fatty acids such as arachidonic acid and DHA, leading to manifold fatty acid profiles [63]. Individuals with obesity or metabolic syndrome often have a higher saturation of ER membrane lipids, which increases susceptibility to ER stress. Cholesterol biosynthesis in the ER varies depending on genetic polymorphisms, such as those in the HMGCR gene [64]. Individuals with familial hypercholesterolemia exhibit altered cholesterol synthesis and handling. Lipid droplets in individuals with high dietary fat intake or metabolic disorders are enriched with triglycerides, an adaptation to chronic ER stress.

GA exhibits pronounced individual variability in lipid composition [47]. Glycosphingolipid profiles vary due to genetic differences, including polymorphisms in the B4GALNT1 gene [65]. These variations impact the synthesis of gangliosides and globosides, which play critical roles in the function of neural and immune cells. Elevated levels of lactosylceramides in individuals with lipid storage diseases or metabolic disorders suggest impaired GA lipid processing. Variability in phosphoinositide metabolism impacts GA vesicle formation and trafficking, while dietary fat intake affects phosphatidylinositol levels and associated signaling pathways.

Differences in ABCG1 activity influence plasma lipid profiles, with altered GA lipid composition observed in individuals with high cholesterol levels [66]. The remodeling of GA lipids for secretion is particularly evident during specialized physiological states such as lactation. Sphingolipid and ceramide trafficking in GA show variability based on CERT gene polymorphisms, leading to differences in sphingomyelin and glycosphingolipid levels [67]. Elevated ceramide levels in inflammatory conditions suggest altered sphingolipid metabolism contributing to individual differences in disease susceptibility. The GA stress response varies across individuals due to genetic differences in GOLPH3, which impact the organelle's ability to manage lipid overload or trafficking defects [68]. GA dysfunctions are particularly evident in individuals with cancer or neurodegenerative diseases. Diet also influence the GA lipid composition and its ability to respond to stress, contributing to variability in immune responses, neural signaling and disease susceptibility.

In summary, lipidomic differences in the ER and GA among human individuals reflect a dynamic interplay of genetic, dietary and environmental factors, as well as health status. These variations influence lipid synthesis, processing and trafficking, impacting a wide range of physiological and pathological processes.

Conclusions

We review a research methodology to assess viral spread and infection sources. Our approach takes advantage of the fact that viruses acquire their envelopes from host organelles like ER, GA and ERGIC. To provide an example, recent lipidomic analyses have elucidated the molecular composition of the SARS-CoV-2 lipid envelope. The virus envelope exhibits exposed phosphatidylethanolamine and phosphatidylserine and is predominantly composed of phospholipids, with minimal cholesterol and sphingolipids [69]. This implies that, despite significant differences between the viral envelope and host cell membranes, the lipid constituents are the same. While it is already established that host lipid composition can influence viral morphology and replication efficiency, our review aims to advance this knowledge by positioning viral lipidomics as a diagnostic and epidemiological tool. We suggest that the lipid profiles of viral envelopes can serve as biosignatures to retrospectively infer the host species or individual from which the virus emerged. This represents a shift from pathogenesis-focused studies toward a source-tracing framework in outbreak investigation.

Biochemical differences among mammals reflect adaptations to diet, metabolism, immune response, stress and environmental demands [70]. Mammals with greater protein synthesis demands like dairy cows and whales upregulate ER chaperones to handle the increased folding load [71]. Carnivorous mammals like cats and dogs exhibit GA adaptations favoring high-protein diets, while hibernating mammals like bears display reduced GA activity to preserve energy. Primates exhibit different chaperone proteins to support the complex glycoproteins associated with their nervous and immune systems [72,73]. Humans possess a unique array of P450 enzymes in the ER, enabling them to adapt to diverse diets and efficiently metabolize drugs [74]. Rodents exhibit simpler glycosylation pathways, while primates synthesize more complex glycans for neural and immune functions [75,76]. These variations influence also interactions with zoonotic viruses, as seen in bats and humans [77]. Bats, with their high metabolic rates, exhibit a robust unfolded protein response (UPR) system to withstand oxidative stress [78]. In contrast, humans and primates rely on sensitive UPR pathways, involving proteins like IRE1, ATF6, and PERK, to mitigate neurodegenerative stress [3,79]. Meanwhile, marine mammals like whales and seals adapt their ER protein-folding machinery for hypoxia tolerance during deep dives. Further, calcium-binding proteins exhibit species-specific differences aligned with metabolic needs [80]. While cheetahs' muscle-specific ER enables rapid calcium cycling for high-speed muscle contractions, diving mammals like dolphins optimize for slower calcium release to conserve energy.

The biochemical composition and function of ER and GA also vary between human individuals due to genetic, environmental and health-related factors. Polymorphisms in genes encoding chaperones and enzymes like glycosyltransferases within ER and GA affect their ability in protein folding and glycosylation patterns, contributing to individual susceptibility to diseases like congenital glycosylation disorders. Diet significantly affects the lipid composition of ER and GA, as fatty acids and carbohydrates required for membrane formation and glycosylation are derived from food. Exposure to toxins, specific drugs or alcohol can further modify their function [81]. Chronic illnesses like diabetes or inflammatory disorders alter glycosylation and protein processing in GA, while neurodegenerative diseases such as Alzheimer's and Parkinson's impair protein folding in ER and glycosylation in GA [82].

The biochemical repertoire of ER and GA also changes with age, reflecting the varying physiological demands tied to growth, metabolic activity and cellular maintenance. In children, ER and GA are highly active to support rapid growth, facilitating the synthesis of proteins essential for tissue formation and contributing to support energy storage and membrane development [83]. In adults, ER and GA shift their focus toward maintaining cellular homeostasis and supporting specialized functions like detoxification, drug processing, enhancing immune defense, enabling antibody function. In the elderly, ER shows a diminished ability to ensure the quality of proteins, leading to an accumulation of misfolded proteins [84]. Altered glycan structures impair the activity and stability of glycoproteins, including antibodies, which compromise immune function. GA dispersion and glycosylation changes in senescent cells further contribute to aging-related dysfunctions and neurodegenerative diseases [85,86]. Additionally, age-related reductions in proteins processing in GA have been linked to storage deficits, particularly in diseases like diabetes [87,88].

In sum, by integrating advanced omics approaches that involve not just lipids, but also other biological molecules, researchers can build a comprehensive framework for understanding how host cellular composition affects viral envelope characteristics.

Several limitations must be acknowledged. Although existing literature supports the idea that host-derived lipid profiles influence viral envelope composition, no systematic datasets currently link specific lipidomic patterns to known hosts across diverse viral families. Still, viruses can modulate their envelope lipid composition through interactions with host proteins, potentially obscuring or modifying the original lipid signature. This could reduce the reliability of using lipid profiles alone to identify the host of origin. The complexity and variability of lipidomes—driven by genetic, metabolic and environmental factors—pose a challenge for creating standardized reference databases that span species and populations. Moreover, the intracellular routes of viral assembly and egress may vary not only between viruses but even between cell types, introducing additional variability. Concerns can be raised regarding the incomplete understanding of viral egress, especially for SARS-CoV-2. While models increasingly support ERGIC involvement, alternative routes such as lysosomal egress remain under investigation. Access to high-quality lipidomic data from diverse host species and infected individuals is limited, particularly in outbreak settings. It could be objected that viral structural proteins actively modulate the lipid composition of their envelopes—for example, by preferentially recruiting certain lipid species to support replication or immune evasion. However, these modulations do not entirely erase the biochemical footprint of the host membrane, particularly when statistical analyses focus on lipid species with low variability across viral strains but high variability across host populations. These constraints highlight the need for interdisciplinary collaborations, technological advances and pilot studies to assess the feasibility and resolution of lipidomics-based viral forensics in real-world scenarios.

This study focuses on lipidomic differences in viral envelopes stemming from their ER and/or GA origins. However, incorporating analyses of other biomolecules could offer a broader understanding. To further investigate unique inter-species and inter-individual biomolecular signatures in ER and GA compositions, proteomics and glycomics analyses could be valuable tools to uncover specific differences in protein and carbohydrate structures. Additionally, transcriptomics and genomics could offer insights into gene expression patterns and structural variations that influence the functions of the ER and GA. Since these differences could significantly impact the composition of viral envelopes derived from host membranes, identifying molecular disparities might provide crucial clues about the origins of infection and the interplay between viral mechanisms and host cellular machinery.

We focused here on the potential to detect the viral source, but our approach might also uncover other significant findings. For instance, during the SARS-CoV-2 pandemic, older individuals experienced more severe COVID-19 symptoms compared to children [89]. Several mechanisms have been suggested to explain the milder clinical syndrome observed in children, including higher pediatric innate interferon responses, increases in naive lymphocytes and depletion of natural killer cells [90]. Since human ERGIC membranes undergo age-related changes, SARS-CoV-2 particles produced in the tissues of children could exhibit phenotypic differences compared to those produced in older individuals. This means that the variations in the ERGIC features of SARS-CoV-2 emerging from human cells of individuals of different ages might contribute to differences in viral load, infectivity and clinical severity. Lastly, we propose another theoretical possibility that merits further exploration. Gaining a deeper understanding of these biochemical differences in ER and GA composition could significantly enhance personalized medicine, paving the way for targeted therapies tailored to individual biochemical profiles.

In conclusion, our review draws together disparate lines of evidence to propose a cohesive model in which viral envelope lipid profiles could serve as retrospective biomarkers for identifying infection sources. This integrative perspective, grounded in recent advances in lipidomics, organelle biology, and host-pathogen interactions, may provide conceptual value through its interdisciplinary synthesis and its potential to guide future research directions.

Ethics approval and consent to participate

This research does not contain any studies with human participants or animals performed by the Author.

Consent for publication

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Availability of data and materials

all data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and take responsibility for the integrity of the data and the accuracy of the data analysis.

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CRediT authorship contribution statement

Arturo Tozzi: Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources, Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization.

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An Extended Stokes' Theorem for Spiral Paths: Applications to Rotational Flows in Trachelospermum Jasminoides Stems and Flowers

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Abstract

The traditional Stokes' theorem connects the macroscopic circulation along a closed boundary to the microscopic circulation across the surface it encloses. However, it proves inadequate for addressing complex geometries such as helicoidal paths, non-planar flow patterns and dynamic systems with open boundaries. We introduce an extension of Stokes' theorem (EST) that provides a robust tool for interdisciplinary research in spiral/helicoidal dynamics, facilitating the evaluation of rotational forces and circulation in both natural and engineered systems with open boundaries. We apply EST to model the rotational dynamics of flower petals and the helical forces within the stems of *Trachelospermum jasminoides*, known as star jasmine. For the flower, we demonstrate the equivalence between the line integral along the petal boundary and the surface integral over the enclosed disk, effectively capturing the uniform rotational stress generated by tangential forces. EST enables the analysis of external factors such as wind or pollinator interactions, while providing valuable insights to deepen our understanding of floral mechanics and petal growth patterns. For the stem, linking microscopic circulatory forces to macroscopic flow patterns, we demonstrate the interaction of torsional and bending stresses caused by the helical geometry. This finding has significant implications for understanding plant growth biomechanics and structural stability as well as for quantifying nutrient and water transport within stems, where spiral dynamics play a pivotal role. In summary, EST streamlines the analysis of rotational and translational forces in systems governed by spiral and helicoidal dynamics, including physical and biological phenomena such as phyllotaxis and plant growth.

Keywords: Helical dynamics; Boundary analysis; Vector field integration; Flow topology, Plant growth; Stokes' theorem; *Trachelospermum jasminoides*

Introduction

Stokes' theorem (henceforward ST) is a fundamental principle of vector calculus that bridges the macroscopic circulation along a closed boundary with the microscopic circulation over the enclosed surface [1,2]. Extending the principles of Green's theorem (GT) which applies to two-dimensional regions, ST provides a powerful framework for analyzing flows and circulations in three-dimensional spaces, uncovering profound connections between the local properties of vector fields and their global behaviour. GT and ST are effective tools for solving problems related to physical closed systems with clearly defined boundaries such as airflow circulation around wings, electromagnetic fields in circuits, surface heat flux, Coriolis-driven hemispherical flows, Earth's deep interior dynamics [3-10]. In biology,

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the two theorems contribute to understanding blood flow, electrical activity of the brain, growth patterns in ecosystems [11,12]. Yet, the classical ST is inherently limited to surfaces and boundaries that are closed, leaving a significant gap in its applicability to open, non-planar geometries. Indeed, many natural and engineered systems exhibit spiral or helical dynamics where forces and flows do not conform to closed loops or planar surfaces but rather are characterized by open, three-dimensional trajectories. Examples include the helical paths of tornadoes, magnetic vortices and spiral galaxies as well as bacterial motility and phyllotaxis of plants [13-15].

The novelty of this work lies in extending ST to accommodate spiral flows and helicoidal paths. By linking macroscopic and microscopic circulation properties, the extended theorem simplifies the evaluation of forces in systems with open, three-dimensional geometries. We utilize EST to analyse two biological scenarios: 1) the rotational forces in spiral flower petals and 2) the torsional stresses in helical plant stems, both exemplified by *Trachelospermum jasminoides*, commonly known as star jasmine. For the flower, EST captures the uniform rotational stresses induced by tangential forces acting along a circular boundary. This is achieved by demonstrating the equivalence of the line integral along the petal boundary and the surface integral over the enclosed disk. For the stem, EST quantifies the interaction between bending and torsional stresses caused by the helical geometry.

This paper is structured as follows. First, we present the mathematical treatment of EST, including its derivation and parameterization for helicoidal paths. Next, we validate the theorem using the specific example of *Trachelospermum jasminoides'* flowers and stems. Finally, we discuss the broader implications of EST, highlighting its potential to unify the study of dynamical systems with open boundaries.

Materials and Methods

This study is grounded in a generalized form of Stokes' theorem, adapted for spiral flows, which facilitates the analysis of forces and circulation in systems with helicoidal or spiral geometries. We aim to prove that, given a continuously differentiable, orientable helicoidal spiral vector field, the macroscopic circulation represented by the integral of a differential form over its surface equals the microscopic circulation represented by the volume integral of the curl perpendicular to the surface. The main challenge here is in defining the notion of a boundary in case of an open helicoidal spiral path, moving beyond the classical case of paths evaluable through ST.

Stokes' Theorem (ST) from vector calculus relates the surface integral of the curl of a vector field over a surface **S** to the line integral of the vector field along the boundary curve ∂S of the surface (Figure 1). In its general form, ST asserts that

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \int_{S} (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

where **F** is a a continuously differentiable two-dimensional vector field, ∂S is the closed boundary curve of the surface **S** that can be bended and stretched, d**r** is a differential element of the curve, d**S** is the differential element of the surface area, and $\nabla \times \mathbf{F}$ is the curl of the vector field, i.e., a vector operator characterizing the infinitesimal circulation of vector fields in three-dimensional spaces.

ST turns line integrals of a form over a boundary into more straight-forward double integrals over the bounded region, regardless of the position of vector singularities [16]. For ST to apply, the normal vector representing the surface must be positively oriented (i.e., counterclockwise) with respect to the tangent vector representing the orientation of the boundary.

Extended Stokes' theorem (EST)

Consider a vector field **F** defined over a region in three-dimensional space. Let the surface *S* be a portion of a plane or a more general surface that is bounded by a spiral curve $\gamma(t)$. The goal is to use EST to evaluate the line integral over the spiral path in terms of the surface integral of the curl of **F** (Figure 1).

Let the spiral curve $\gamma(t)$, with $t \in [a, b]$, be parameterized as

$$\gamma(t) = (r(t)\cos(\theta(t)), r(t)\sin(\theta(t)), z(t))$$

where $r(t)$, $\theta(t)$ and $z(t)$describe respectively the radial, angular and vertical components of the spiral.

Let's assume that $\gamma(t)$lies on a flat plane, say the *xy*-plane, so the spiral path can be simplified to

$$\gamma(t) = (r(t)\cos(t), r(t)\sin(t), 0)$$

where $r(t)$ increases as the angle *t* increases.

When the surface **S** is a surface spanned by the curve $\gamma(t)$, **S** stands for a portion of the plane or surface generated by the spiral curve (Figure 1, left).

We are interested in computing the line integral of a vector field **F** along the spiral path (Figure 1, right). By ST, this line integral can be transformed into a surface integral involving the curl of **F**

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \int_{S} (\nabla \times \mathbf{F}) \cdot d\mathbf{S}$$

The line integral over the spiral path is:

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \int_{a}^{b} \mathbf{F}(\gamma(t)) \cdot \frac{d\gamma(t)}{dt}\, dt$$

where $\frac{d\gamma(t)}{dt}$ is the tangent vector to the spiral path at each point *t*.

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The surface integral involves the curl of **F**, given by $\nabla \times \mathbf{F}$, and the surface normal vector $\hat{n}$ associated with **S**

$$\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \int_S (\nabla \times \mathbf{F}) \cdot \hat{n}\, dS$$

The normal vector $\hat{n}$ depends on the orientation of the surface, while *d*S is the differential area element of the surface.

Upon achieving the extended formulation of ST, we will proceed in the next paragraphs with a detailed case study.

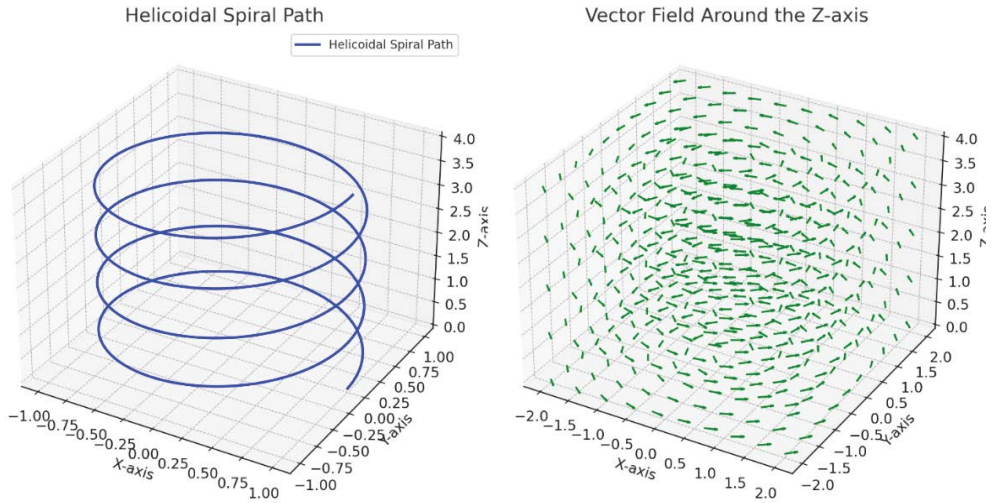


Figure 1: Diagrams depicting a helicoidal spiral (**left**) and the behavior of a vector field (**right**) around the z-axis. The left diagram illustrates a helicoidal spiral path, showcasing the interplay of rotational and translational motion along the z-axis. The right diagram represents a vector field with circular flow centered around the z-axis.

Simulated case study: analyzing rotational flows in stems and flowers

EST enables the analysis of rotational and translational forces in complex systems, providing a powerful framework for exploring biological and physical dynamics. To illustrate the new theorem, we will now explore a concrete example. We will consider *Trachelospermum jasminoides*, commonly known as star jasmine, belonging to the family *Apocynaceae*. Like many climbing plants, *Trachelospermum jasminoides* displays a counterclockwise helical movement of its stems as it climbs and twines around supports, also referred to as circumnutation [17-19] (Figure 2). The flowers also exhibit subtle rotational dynamics, although these movements are not as pronounced as the helical twisting of the stems [20]. The petals of the flowers are arranged in a spiral configuration and unfurl in a counterclockwise direction during blooming.

In our simulation, the dynamics of flower petals are modeled using a circular boundary with a radius of 0.05 m, representing the petals of a flower. Tangential forces along this boundary are applied and the resulting rotational stresses are analyzed through EST. The stem is modeled as a helicoidal path with a radius R=0.05 m and a vertical rise per turn of c=0.2 m. To evaluate the counterclockwise rotation of the flower and stem using EST, the rotation of the petals can be represented by a circular vector field, whereas the helical motion of the stem can be modelled using a helical vector field. The next step is to parameterize the flower and the stem (Figure 2**C**).

1. For the flower, a circular boundary in the plane of the petals is defined, representing the region of interest for macroscopic rotation.
2. For the stem, the helical path is parameterized using equations for a helicoidal spiral, where $x(t) = r\cos(t)$, $y(t) = r\sin(t)$ and $z(t) = ct$. Here r represents the radius, c the rise per turn and t the parameter along the path.

Subsequently, the surface S is defined for each component.

1) For the flower, the surface is a disk enclosed by the petals' rotational motion within their plane,
2) whereas for the stem, the surface corresponds to the area traced by the helical path (Figure 2C).

Calculating the forces acting on the flower and stem requires applying mechanical principles that account for both internal and external forces influencing their dynamics [21,22]. For the flower's petals, the primary force is torque or rotational force, while the stem experiences a combination of bending forces and axial torsion due to its helical structure. We will calculate these forces systematically, step by step, starting from the external forces.

External forces acting of the flower petals and the stem

1) The rotation of the flower petals can be modeled as a torque

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induced by external forces such as wind, gravitational pull, biological growth forces [23]. Torque (τ) on the petals is given by:

$$\tau = r \times F$$

Where r is the radial vector from the center of the flower to the tip of a petal and F is the tangential force applied to the petal. Let's assume that the radius of the flower is R=5 cm, while the tangential force from wind or another source is F=0.1 N. The magnitude of the torque is

$$|\tau| = R \cdot F = 0.05\,\text{m} \cdot 0.1\,\text{N} = 0.005\,\text{Nm}$$

In case of multiple petals (n=5 in *Trachelospermum jasminoides*) experiencing similar forces, the total torque becomes

$$\tau_{\text{total}} = n \cdot \tau = 5 \cdot 0.005\,\text{Nm} = 0.025\,\text{Nm}$$

The rotational acceleration (α) of the flower petals is related to the torque [24] via

$$\tau = I \cdot \alpha$$

Where I is the moment of inertia of the flower petals about the axis of rotation and α is the angular acceleration.

For a flower modeled as a system of point masses at a radius R

$$I = n \cdot m \cdot R^2$$

where m is the mass of a single petal. Assuming m=0.002 kg (2 grams per petal):

$$I = 5 \cdot 0.002 \cdot (0.05)^2 = 0.000025\,\text{kg}\backslash\text{cdotpm}^2$$

$$\alpha = \frac{\tau_{\text{total}}}{I} = \frac{0.025}{0.000025} = 1000\,\text{rad/s}^2$$

1) The stem experiences forces from bending and torsion, influenced by its helical structure. These forces arise from gravity, wind and the biological tension exerted during growth.

2) The weight of the stem induces a bending gravitational force. For a stem of length L=20 cm and mass per unit length λ=0.01 kg/m:

$$F_g = m \cdot g = (\lambda \cdot L) \cdot g = (0.01 \cdot 0.2) \cdot 9.8 = 0.0196\,\text{N}$$

This force acts vertically downward, generating a bending moment at the base of the stem

$$M_g = F_g \cdot \frac{L}{2} = 0.0196 \cdot 0.1 = 0.00196\,\text{Nm}$$

The helical structure of the stem experiences torsional forces due to the winding. The torsional moment (T) is given by

$$T = G \cdot J \cdot$$

Where G is the shear modulus of the stem material, J is the polar moment of inertia and θ is the angle of twist over the length L.

Assuming G= 10^8 Pa typical for plant tissue, J = 0.005 m (5 mm radius) and θ = 2π (one full turn over L =0.2 m) [25], then

$$J = \frac{\pi}{2} \cdot (0.005)^4 = 9.82 \times 10^{-10}\,\text{m}^4$$

and

$$T = (10^8) \cdot (9.82 \times 10^{-10}) \cdot \frac{2\pi}{0.2} = 3.08\,\text{Nm}$$

.

Internal forces acting within the flower petals and the stem

To calculate the forces within the flower petals and the stem using EST, we need to evaluate the relationship between the macroscopic circulation (observable forces) and the microscopic properties (internal forces or stresses derived from the curl of the force field). The first step is to model the forces using vector fields.

1) Concerning the flower petals, we assume that the external forces (e.g., wind or biological forces) act tangentially to their circular boundary. Further, we assume that the tangential forces induce internal stresses (force per unit area) propagating through the petals. Let the force field acting on the petals be

$$\mathbf{F} = (-ky, kx, 0)$$

where k is the force constant proportional to the external pressure and x,y represents positions in the z=0 plane.

Next, we compute the curl of the force field. The curl of the force field relates to the internal stresses within the petals. For the flower petals, in the z=0 plane, the curl of $\mathbf{F} = (-ky, kx, 0)$ is

$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -ky & kx & 0 \end{vmatrix} = (0, 0, 2k)$$

This curl is constant in the z-direction, indicating a uniform internal rotational stress throughout the petals.

1) Concerning the helical stem, it experiences external forces such as gravity $(\mathbf{F}_g)$ and biological growth forces $(\mathbf{F}_b)$ that induce internal torsion and bending stresses.

For simplicity, we model the net force field in the stem as: $\mathbf{F} = (-ky, kx, kz)$ where the kz-term accounts for the vertical components of the forces.

Next, we compute the curl of the force field, which provides insight into the internal stresses acting within the stem. This computation reveals the distribution and intensity of these stresses, capturing the complex interplay of forces across the helical structure. For the stem the curl of $\mathbf{F} = (-ky, kx, kz)$ is

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$$\nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -ky & kx & kz \end{vmatrix} = (k, k, 2k)$$

This suggests a complex pattern of internal stress within the stem, with components distributed across all three spatial directions.

Visualization and statistics

Diagrams of the flower petals and stem are created to illustrate their geometry, boundary dynamics, and associated vector fields. The Matplotlib library is employed to generate detailed plots, including the circular boundary and tangential forces acting on the flower petals, the curl of the vector field over various surfaces, and the helicoidal path and vector field representation for the stem.

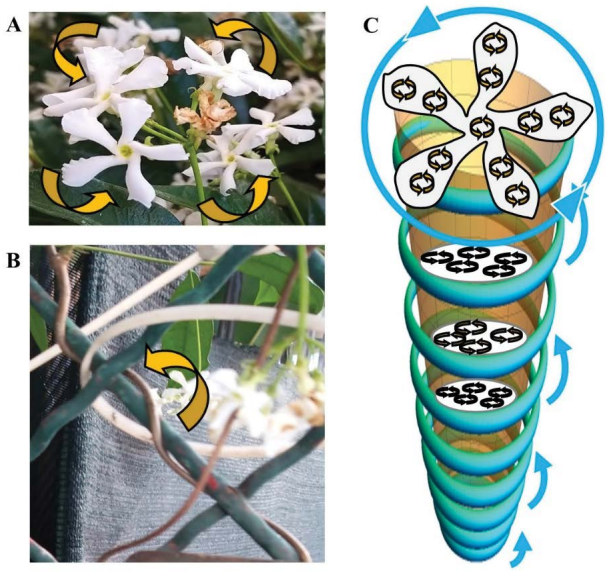


Figure 2: *Trachelospermum jasminoides*. The flower petals (**Figure 2A**) and the stem (**Figure 2B**) display a counterclockwise path. **Figure 2C** illustrates the geometry of the boundaries, the associated vector fields and the internal flows within the flower petals and the stem.

To ensure statistical validation, numerical accuracy is achieved through high-resolution parameterization, with the parameter t sampled at 1,000 points per cycle. The consistency between line integrals and surface integrals is carefully evaluated to confirm the applicability of the extended theorem to the analyzed geometries.

In the sequel, the surface integral of the curl of the vector field will be computed over these surfaces using the extended Stokes' theorem (EST).

Results

As stated above, both the flower and the stem experience external and internal mechanical forces that influence their motion and structural behavior:

1) For the flower petals, the torque arising from tangential forces induces a counterclockwise rotation, with the total torque $\tau_{\text{total}} = 0.025\,\text{Nm}$ and the angular acceleration measured as α=1000 rad/s^2. The internal stresses in the petals are uniform with a value of 2k and are directly proportional to the external forces acting on them. This proportionality explains the rotational equilibrium observed in the petals.

2) For the stem, the primary forces include a gravitational bending moment $M_g = 0.00196\,\text{Nm}$ and a torsional moment T=3.08 Nm due to a helical twist. The internal stresses in the stem vary in all three dimensions because of its helical geometry. Among these stresses, torsion, proportional to kc, predominates, whereas bending stresses, proportional to kR, have a secondary but still notable influence.

We can now apply EST to relate macroscopic and microscopic circulation. The surface integral of the curl of the vector field is computed over the surfaces, relating the surface integrals to the line integrals along the boundaries. For the flower petals, the counterclockwise macroscopic rotation is calculated by integrating along the circular path in the plane. For the stem, the integral is evaluated over the helical surface.

1) Concerning the flower petals, the boundary of the flower is a circle of radius R. The macroscopic circulation (line integral along the petal boundary) is

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \int_0^{2\pi} kR^2\, d\theta = 2\pi k R^2$$

Using the curl, the surface integral is

$$\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \int_S 2k\, dA = 2k \cdot \pi R^2$$

Both results match, confirming that the inner stresses in the petals are proportional to 2k.

In sum, the numerical values for the macroscopic (surface) flows and microscopic (internal) flows in the flower, as governed by EST, are as follows. For the flower, the surface flow (evaluated as a surface integral) is 0.157N\ ppm, while the internal flow (evaluated as a line integral) is also 0.157N\ppm. The flower petals exhibit a simple and symmetric geometry, where forces act tangentially along a circular boundary in the z=0 plane. The petals lie on a flat, two-dimensional surface characterized by a constant curl of the force field $(\nabla \times \mathbf{F} = (0, 0, 2k))$, signifying that the internal forces are uniformly distributed. This uniform distribution creates a direct and proportional relationship between the macroscopic flow (line integral along the circular boundary) and the microscopic flow (surface integral over the disk). The symmetry of the geometry ensures that every contribution to the line integral is exactly matched by the surface integral. Consequently, the uniform geometry and constant curl lead to a perfect agreement between the surface flow and the internal

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flow, consistent with EST.

1) Concerning the helical stem, the boundary of the stem is parameterized as a helicoidal spiral

$$x(t) = R\cos(t), \quad y(t) = R\sin(t), \quad z(t) = ct$$

The macroscopic circulation (line integral along the helical path) is

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{r} = \int_0^{2\pi} [-kR\sin(t)(-R\sin(t)) + kR\cos(t)(R\cos(t)) + kc(ct)]\, dt$$

$$= \int_0^{2\pi} \left(kR^2 + kc^2 t\right) dt$$

For one turn$(t \in [0, 2\pi])$

$$\int_{\partial S} \mathbf{F} \cdot d\mathbf{r} = 2\pi kR^2 + \frac{kc^2}{2}(2\pi)^2$$

Using the curl, the surface integral is approximated by the ribbon spanned by the helix

$$\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = \int_S (k + k + 2k)\, dA = 4k \cdot (\text{surface area of ribbon})$$

The surface area of the ribbon is

$$A = 2\pi R \cdot (\text{height per turn}) = 2\pi R \cdot c$$

Thus

$$\int_S (\nabla \times \mathbf{F}) \cdot d\mathbf{S} = 4k \cdot 2\pi R \cdot c = 8\pi kRc$$

In sum, the numerical values for the macroscopic (surface) flows and microscopic (internal) flows in the stem, as governed by EST, are as follows. For the stem, the surface flow (evaluated as a surface integral) is 2.513N\ppm, while the internal flow (evaluated as a line integral) is 8.053N\ppm. Unlike the flower, the values for surface flow and internal flow differ significantly. This is due to the stem's more complex geometry, which features a three-dimensional helicoidal structure with a helical boundary and a ribbon-like surface. Unlike the constant curl observed in the flower, the curl of the force field $(\nabla \times \mathbf{F} = (k, k, 2k))$ in the stem varies in all three dimensions. This non-uniform curl introduces additional contributions to the surface integral that are not directly proportional to the line integral along the helical path. The helicoidal surface spanned by the path is not planar. Its area depends on the radius of the helix and the rise per turn (c), which increases the surface integral significantly compared to the simpler circular geometry of the flower. The line integral along the helical path includes contributions from the vertical rise (z-component), which are absent in the flat geometry of the flower. These vertical components add substantially to the internal flow, making it larger than the surface flow. Forces and circulation in the stem are not confined to a two-dimensional plane, rather display three-dimensional dynamics that capture complex interactions such as bending, twisting and torsional effects, further contributing to the discrepancy between the surface and internal flows. Therefore, the stem's intricate geometry and three-dimensional dynamics lead to a disparity between surface and internal flows, reflecting the additional factors at play in its structural behavior.

In conclusion,

1) flowers have a circular, symmetric geometry that ensures uniform force distribution and curl. This results in surface and internal flows being equal, as the entire flow field is captured in a flat, two-dimensional setup.

2) In contrast, the stem's helicoidal geometry introduces non-uniform force distributions and additional components such as vertical contributions and a larger surface area. These factors create a larger internal flow compared to the surface flow, as the line integral accounts for three-dimensional effects that the surface integral does not fully capture.

These differences highlight the impact of geometry and force distribution on the interplay between macroscopic circulation and microscopic forces, showcasing the utility of the extended Stokes' theorem in analysing forces and circulation in systems exhibiting spiral dynamics.

Conclusion

Classical theorems such as Green's Theorem (GT) and Stokes' Theorem (ST) have been pivotal in linking local properties of vector fields to their global behavior. GT applies just to two-dimensional regions and closed curves, while ST extends to three-dimensional spaces requiring closed surfaces or boundaries for its application [1,2]. These theorems, focused on closed-loop circulations, have proven instrumental in analyzing flows and circulations in systems where boundaries are well-defined, such as steady-state circulations in airflow around wings or electromagnetic field behavior in closed circuits [4,5]. However, their utility diminishes when applied to open, three-dimensional trajectories like the helicoidal spirals which are frequently encountered in natural and engineered systems.

We suggest a generalization of ST to establish a mathematical framework connecting the line integral along a helicoidal spiral path to the surface integral of the curl of the vector field over a bounded region. By redefining the boundary concept for helicoidal paths, this framework provides a new tool for analyzing macroscopic and microscopic flow dynamics in complex systems. The EST formulation provides novel insights into the interplay between rotational and translational motions, allowing for a deeper understanding of spiral flows in a variety of physical and biological systems. A key advantage of the extended formulation lies in its ability to model a wide range of systems where spiral or helical dynamics are dominant. For instance, the novel framework enables the analysis of DNA supercoiling, bacterial flagella, biomechanical patterns such as the phyllotaxis of plants

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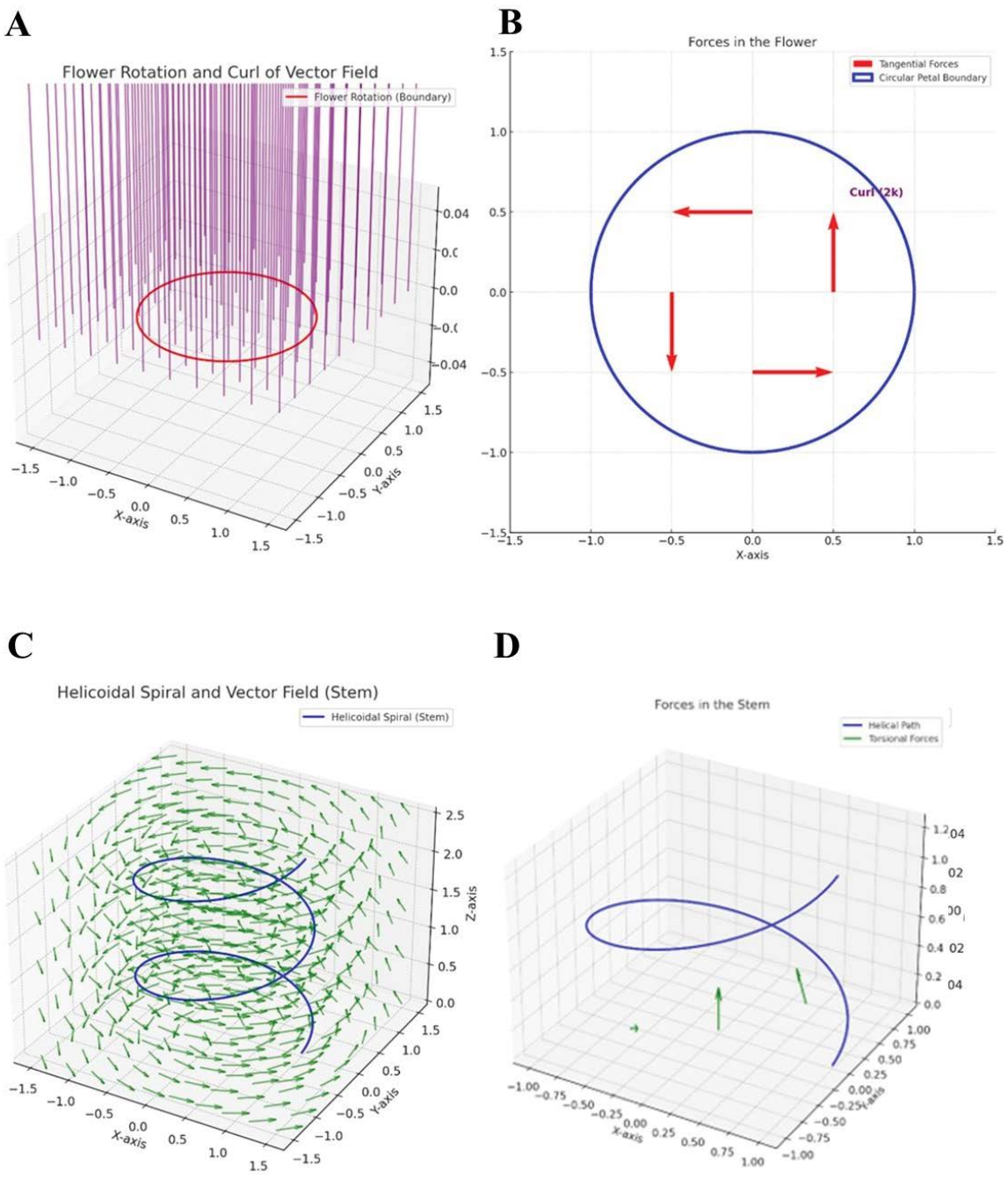


Figure 3: Application of the extended Stoke's theorem to flowers (**Figures 3A-B**) and stems (**Figures 3C-D**) of *Trachelospermum jasminoides.* **Figure 3A**. Diagram illustrating the flower rotation and the curl of vector field. The red circle represents the boundary of the flower petals modeled as a planar region in the z=0 plane. The purple arrows visualize the curl of the vector field, representing the microscopic circulation that contributes to the macroscopic rotation of the flower petals. **Figure 3B**. Diagram illustrating the forces acting on the flower. The circular boundary (blue) represents the edge of the flower petals. Tangential forces (red arrows) act along the edges of the petals, showcasing the influence of external or internal factors. The calculated curl of the vector field is constant at $\nabla \times F=2k$ (annotated in purple) in the z-direction, indicating uniform rotational stresses throughout the petal boundary. **Figure 3C**. Diagram illustrating the helicoidal spiral of the stem and the vector field. The blue curve depicts the helicoidal path of the stem, while the green arrows represent a circular vector field around the z-axis, illustrating the rotational and translational flow and its interaction with the spiral geometry. **Figure 3D**. Diagram illustrating the forces acting on the stem. The helical path (blue curve) represents the stem's geometry. The torsional forces (green arrows), resulting from a combination of bending and twisting actions, act along the helical structure contributing to internal stress distribution. The curl vector field displays non-uniform rotational stresses with components $\nabla \times F=(k,k,2k)$.

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[15,26] intracardiac spiral flows observed in cardiac cycles [27] as well as magnetic vortices in superconductors [14] and rotational dynamics of spiral galaxies [13], where classical methods fail to capture the intricacies of rotational and translational dynamics [28-34].

In this paper, EST is applied to two case studies related with *Trachelospermum jasminoides*, namely the forces acting on flower petals and the helical stress distribution within plant stems.

1) For the flower petals, the circular geometry allows for a straightforward application EST, since the tangential forces acting along the petal boundary produce a uniform curl which is proportional to the rotational stresses. The equivalence between the line integral along the petal boundary and the surface integral of the curl over the enclosed disk validates EST's effectiveness for two-dimensional spiral systems. The uniform rotational stresses observed in the petals align well with the mathematical predictions of EST. This provides insights into how forces are distributed within the boundary of the flower, potentially aiding in the study of floral mechanics and growth patterns. EST suggests that microscopic forces acting at the level of the petals contribute to the macroscopic rotational motion observed at the flower's boundary. This could be applied to study the impact of environmental factors like wind on plant structures and to investigate the mechanical interactions between flowers and pollinators during the pollination process. Still, EST effectively simplifies complex calculations by converting a line integral along the flower's boundary into a surface integral over the petal region. This transformation minimizes computational effort while preserving accuracy.

2) In the case of the stem, although the helical geometry of the stem presents a significant challenge for classical mathematical tools, EST effectively simplifies the intricate interplay of forces involved. The torsional and bending forces are captured through the curl of the vector field, which has components in all three dimensions. The equivalence of the surface integral over the helical ribbon region and the line integral along the helical path demonstrates the robustness of EST in handling three-dimensional geometries with open boundaries. The EST capability to connect macroscopic flow patterns with microscopic circulatory forces may have significant implications for understanding the biomechanics of plant growth and structural stability. This relationship can also provide valuable insights for studies on nutrient and water transport within stems, as these processes often involve spiral dynamics.

Certain assumptions and limitations are inherent in our analysis. EST assumes that the involved vector fields and surfaces are continuously differentiable. In real-world biological systems, irregularities and discontinuities in the geometry or force distribution may reduce the accuracy of the analysis. The flower petals are modeled as a perfect circle and the stem as a regular helix. While this simplifies the mathematical analysis of forces in idealized systems, real-world systems often deviate from these idealized shapes. The analysis of irregular geometries or highly dynamic boundaries may still require significant computational effort, particularly for numerical integration of complex surface and line integrals. The tangential and torsional forces are assumed to be uniform across the boundaries. In reality, biological and environmental forces such as wind, gravity and growth pressures are often spatially and temporally variable. Additionally, secondary effects such as shear forces or anisotropic material properties are not incorporated, which could limit the applicability of the results to certain systems. Future work could extend the framework to handle more irregular and biologically realistic geometries, such as asymmetrical petals or non-uniform stem shapes. The analysis of time-varying forces and boundaries, such as those caused by growth or environmental changes, could provide deeper insights into real dynamics. Integrating the extended theorem with experimental data would help validate the theoretical predictions and refine the mathematical models.

In conclusion, the proposed extension to Stokes' Theorem integrates helicoidal paths into circulation analysis, bridging a critical gap and expanding its applicability to open, non-planar trajectories. By redefining boundaries, it simplifies the study of rotational and translational flows, offering a versatile tool for analyzing complex dynamics such as those observed in the flowers and stem of *Trachelospermum jasminoides*.

Declarations

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the author.

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Availability of data and materials. all data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and take responsibility for the integrity of the data and the accuracy of the data analysis.

Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted

work that could inappropriately influence, or be perceived to influence, their work.

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Authors' contributions. The Author performed: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding, administrative, technical, and material support, study supervision.

Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT to assist with data analysis and manuscript drafting. After using this tool, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.

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Feeling the Heat: A Thermodynamic Perspective on Emotions, Motivation, and Time Perception

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ABSTRACT

We are introducing a novel thermodynamic model of emotion. In this model, emotions are regarded as deviations from equilibrium, akin to fluctuations in body temperature. This bipolar regulation maintains bodily and psychological homeostasis while spurring mental development. Emotional regulation typically occurs through expanding one's perception of time. Positive, low-information content emotions can reduce action drive, but stressful, information-rich conditions can heighten it. Therefore, time perception can potentiate the capacity of emotions to motivate. However, time perception accelerates to facilitate fluid action performance, with the state of flow representing a unique state of contentment and challenge. By anchoring psychological processes to the principles of energy and entropy, our model offers a comprehensive bipolar foundation for understanding motivation and behavior. Beyond its theoretical implications, this model also lays the groundwork for addressing mental health conditions resulting from the dysregulation of emotions. It can inspire potential interventions to harness the mind-body connections elucidated by our thermodynamic perspective.

1. Introduction

The global rise in stress disorders highlights the urgent need to understand emotional resilience and its role in the development of mental diseases (Nestler & Russo, 2024). Traditional approaches have treated emotions as separate domains, failing to capture their intricate interdependencies. For example, cognitive theories have focused primarily on the mental processes underlying emotions (Barrett, 2017). At the same time, physiological research has investigated bodily responses, such as heart rate and facial expressions, largely independently of motivational and subjective factors. This compartmentalized view has limited our ability to explain the complex relationships and paradoxes observed across these realms of human experience. Understanding how emotions relate to subjective experiences (i.e., feelings), motivation, and disease progression has remained elusive.

This article proposes a novel thermodynamic model that integrates emotions, motivation, and associated physiological mechanisms within a unified framework. The brain keeps bodily and psychological equilibrium by intertwining every regulatory system with emotions. Nevertheless, the multifaceted nature of the relationship encourages further study. This work examines how emotions serve temperature regulation and psychological homeostasis. By conceptualizing the brain's functioning through thermodynamic principles of energy and entropy, we offer a cohesive perspective to resolve longstanding questions and paradoxes surrounding these interrelated processes.

1.1. Thermo-emotional covariations from a thermodynamic lens

Experimental research confirms the phylogenetically ancient relationship between emotions and temperature regulation across various species, including reptiles, foxes, pigs, rabbits, rats, mice, and humans (Briese, 1995; Briese & Cabanac, 1991; Cabanac, 1999; Frosini et al.,

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2000; Groenink et al., 1995; Moe & Bakken, 1997; Parrott et al., 1995; Terlouw et al., 1996; van der Heyden et al., 1997). Although thermoregulation exists to some degree in most animals, the endothermic phenotype—characteristic of humans and other mammals—depends on complex metabolic networks and multiple internal feedback loops (Grigg et al., 2021; Seebacher, 2020). For example, embryo incubation drives the evolution of endothermy (Farmer, 2020), which is phylogenetically predicated on thermoregulation (Clavijo-Baque & Bozinovic, 2012). Endotherms maintain a stable core temperature with the aid of crucial mechanisms, including vasoconstriction, shivering, and sweating (Madden & Morrison, 2019; Nowack et al., 2017). The brain's high energy use ensures optimal information processing while maintaining physical and psychological equilibrium (Dempsey et al., 2022; Huang, Zhang, Wu, Mashour, & Hudetz, 2020). Thermal control is a vital component of an overarching regulatory system, exerting downstream effects on action motivation and behavioral adaptations (Inagaki et al., 2019; Kataoka et al., 2020; Nashiro, Min, & Yoo, 2022).

Physical or mental instability prompts a wide range of protective mechanisms. Emotions are paramount in this regulatory hierarchy as they intertwine with other regulatory processes. Moreover, the distinct physiological signatures of emotions represent specific energy configurations of the brain (Hesp et al., 2021; Kao et al., 2015; Sadowski et al., 2020), and, like temperature, they oscillate around a neutral position, forming an emotional set-point (Northoff & Tumati, 2019). Recent work utilizes temperature as a means of characterizing emotions (Escobar et al., 2020). Contentment promotes rest and recovery (Brown & Thorsteinsson, 2020) by reducing metabolic rate and body temperature, thereby conserving energy through parasympathetic restorative processes (Seebacher, 2009). In contrast, stress is a highly demanding condition (Keller et al., 2019; Meeusen, Van Cutsem, & Roelands, 2020), where noradrenaline initiates the fight-or-flight response within seconds (O'Connor, Thayer, & Vedhara, 2020). Furthermore, the varied effects of stress depend on personal, environmental, and other situational factors. However, its adverse health effects in anxiety, dissociation in trauma, or even depression (Comtesse et al., 2019; Mason et al., 2024) warrant a deeper thermodynamic investigation.

Emotional expressions are fundamentally linked to physiological changes regulated by the autonomic nervous system. At the same time, their feedback representation reflects motivational aspects (Quadt et al., 2022). Moreover, emotion and temperature may be under thermodynamic control (Déli and Kisvarday, 2020; Grigg et al., 2021; Seebacher, 2020). This perspective posits that the brain's mechanisms for heat and work transfer play a crucial role in regulating emotional states, and conversely, that emotional states can influence thermoregulation. In support of this, psychological stress, known to elevate blood pressure, heart rate, and heart function—even during sleep (Hall et al., 2004)—increases core temperature, a phenomenon referred to as psychogenic fever (Oka et al., 2001). This psychogenic fever results from a temporary elevation in the thermoregulatory set point, mediated by both prostaglandin E2-dependent and independent mechanisms (An & Kim, 2011; Fossat et al., 2015; Kluger et al., 1987; Morimoto et al., 1991).

Heat stress can lead to impulsivity (Fredericks et al., 2018; Wittmann & Paulus, 2008; Paasche et al., 2019), drug-seeking behavior, and criminal activity (Corcoran & Zahnow, 2022). Inversely, fear triggers thermoregulation disturbances in substance abusers (Lowry et al., 2009; Raison et al., 2015). People with depression have higher body temperatures (Mason et al., 2024), and median raphe stimulation, which affects temperature regulation, can produce depressive-like behaviors (Fazekas et al., 2021). These and other findings suggest a neurological link between emotion and temperature (WILLIAMS & Bargh, 2008). To gain a deeper understanding of this relationship, we explore the mechanisms of thermodynamic regulation in more detail.

1.2. Variations of time judgment

Understanding the mechanisms underlying time perception requires distinguishing between different types of temporal judgments. A key theoretical distinction exists between the perceptions of short durations (e.g., milliseconds to seconds), typically measured through reproduction or estimation tasks and the subjective awareness of time's passage often referred to as passage-of-time (PoT) judgments. These two aspects are functionally dissociable: while short-duration judgments are closely linked to internal pacemaker mechanisms and are modulated by arousal and attention (Gibbon et al., 1984; Wittmann, 2009), PoT judgments rely more on self-reflective and interoceptive processes, such as emotional state and bodily awareness (Droit-Volet, & Fayolle, 2024; Martinelli & Droit-Volet, 2022).

Incorporating this distinction into our thermodynamic framework, we conceptualize duration estimation as a dynamic process modulated by arousal-driven shifts in entropy and energy. High-arousal states, whether positively or negatively valenced, have been shown to accelerate internal clock processes, resulting in time overestimation (Cui et al., 2023). An expansion of subjective time is useful for decision-making, while contraction of subjective time drives action (D'Agostino et al., 2023). In contrast, low-arousal emotions, including contentment or mild joy, do not produce consistent distortions in short-duration estimates. These findings challenge earlier claims of a general "positivity effect" on temporal expansion and underscore the primacy of arousal, rather than valence, in modulating temporal metrics.

Moreover, our model emphasizes that PoT judgments, i.e., feelings that time is dragging or flying, are not reducible to clock-speed effects. Instead, they emerge from higher-order awareness of emotional and cognitive change (Wittmann, 2015). For instance, the emotion of awe leads to underestimation of duration in attention-based timing tasks, likely due to perceptual overload (Droit-Volet et al., 2024). Simultaneously, awe may induce a metacognitive experience of timelessness, a distinct subjective effect more closely tied to interoceptive processes and self-transcendence. We propose to map these dual effects thermodynamically: attentional overload decreases timing accuracy, while self-transcendent states can stabilize entropy.

Early work investigated the effect of increases in body temperature on time estimates (Hoagland, 1933). Increases in body temperature shortened the intervals produced, but lengthened duration estimates (Francois, 1927; Wearden & Penton-Voak, 1995). More recent studies support the stress-induced slowing of time perception (Ogden et al., 2019); however, the repeated anticipation of holidays can actually speed up time perception (Ogden et al., 2024).

To situate our approach within broader theoretical debates, we briefly compare our framework with several established models of time perception. The dual klepsydra model (Wackermann & Ehm, 2006) posits that interval timing arises from the gradual discharge of a leaky integrator, an analogy to fluid flowing through a clepsydra or water clock. In contrast, the DOIT (Dynamic Occupation in Time) model examines how the experience of time varies based on the qualities of activities and their context (Larson, 2004). Although our model differs in its thermodynamic formulation, these approaches recognize the irreversibility of time and the influence of dissipative processes.

Physical movement across species and tasks hastens time perception, improving timing accuracy (Robbe, 2023). The time perception rate is greatest in flow, a unique state where the balance of action, motivation, and confidence optimizes performance through spontaneous, coherent action flow (Failing & Theeuwes, 2016; Rutrecht, Wittmann, Khoshnoud, & Igarzábal, 2021). Similarly, Csikszentmihalyi's flow model describes the distortion of time during immersive tasks, wherein attentional focus and action-feedback loops yield an altered experience of time (Csikszentmihalyi, 1990). Our thermodynamic view reframes this by suggesting that flow arises at an equilibrium point between arousal (energy) and confidence (stability), facilitating seamless temporal integration.

As physical time is relative to spatial motion, psychological time is also relative to imaginary motion (Allingham et al., 2021; Spapé et al.,

2021). Imagining accelerating movement resulted in a relative overestimation of time, or time dilation, while decelerating movement elicited a relative underestimation, or time compression (Hallez et al., 2023). Our model posits that perceived time is regulated by energetic and informational constraints, offering a physicalist grounding through entropy and thermoregulation.

1.3. The role of entropy

Thermodynamic regulation is crucial for efficient brain functioning. Because the energy needs of neurons during intrinsic activities are orders of magnitude larger than during stimulation for all levels of cognition (de Lara, 2020; Raichle, 2010), rapid shifts from the brain's high-dimensional resting state to lower-dimensional evoked activities (Singer, 2021) facilitate optimal information transfer. Nevertheless, stable intrinsic activities expedite spontaneous recovery of the resting state. In this simplified view, intellect generation is based on information exchange with the external environment (Ahissar & Assa, 2016; Déli et al., 2017; Llinás and Paré, 1996; Northoff, 2018), with sensory and motor processing forming a thermodynamic cycle.

Renyi's informational entropy generalizes entropy by forming scalar exponent alpha (Jizba & Arimitsu, 2001). Baez (2011) demonstrated a direct relationship between Renyi's exponent alpha and inverse temperature beta (i.e., coldness). Intelligent information processing often involves a type of information erasure, inducing a sense of "coldness" while increasing overall neural organization (O'Neill and Schoth, 2022).

Rényi and Shannon's informational entropy can describe psychological and cognitive states, and predict task performance and mental well-being (Ince et al., 2017; Shannon, 1993). For instance, higher variability at rest (i.e., high entropic states) correlates with fluid intelligence (Wang et al., 2018; Yang et al., 2019) and openness (Zmigrod et al., 2019), but decreases in brain entropy are seen in compromised states of consciousness (Varley et al., 2020). For example, stress is analogous to time pressure, the inability to cope with the pace or intensity of sensory influx (Déli et al., 2018, 2021, 2022), demonstrating emotions' interconnection with the brain's energy and information processing. The following section will proceed to a discussion of time perception in motivation.

1.4. The thermodynamics of time perception

Studies on behavioral activation systems have revealed overestimation bias scores for both positive (Lehockey et al., 2018; Simen & Matell, 2016; van Hedger et al., 2017) and stress-inducing situations (Remmers & Zander, 2018; Wise et al., 2017). For instance, novel stimuli or rewards can dilate time perception through what is known as the 'oddball effect' (Failing & Theeuwes, 2016; Ma et al., 2024), with surprising or emotionally charged moments feeling as if time 'froze.' A similar sense of permanence occurs during stress (Hollis et al., 2015; Robbe, 2023). However, the psychology of these experiences contrasts dramatically. In a stressful context, dilated time perception evokes an unbearable sense of permanence, which triggers desperate escape behavior through impatience and sympathetic arousal (Gladhill et al., 2022; Hosseini Houripasand et al., 2023).

The dilation of time perception in both positive and negative states correlates with emotional intensity (Biderman et al., 2020; Déli & Kisvarday, 2020; Zanin et al., 2019), hinting at an underlying energy relationship (Toso et al., 2020). We want to note that some studies suggest that positive emotions do not influence time perception (Ogden et al., 2019). However, drug-induced expansion of time perception is indeed linked to elation. This correlation is significant as it suggests that the sense of spaciousness experienced during elation can lead to mental expansion, potentially giving rise to new ideas and creative insights (Green, Kavanagh, & Young, 2003). These findings underscore the connection between the expansion of time perception and elation, whether drug induced or arising from positive states.

Due to its mental energy boosting ability, musicians and artists have turned to cannabis to enhance creativity, supporting our original claim (Kowal et al., 2015). Moreover, the most pronounced alterations in time perception occur during emotional polarities, such as awe (Rudd et al., 2012) and depression (Stanghellini et al., 2016; Thönes & Oberfeld, 2015), when time appears to stand still (Fig. 1). These results inspired some scientists to suggest that depression is analogous to a positive spacetime curvature, even a black hole state (Déli, 2024; Kent, 2023).

The connection between emotions and energy is also evident in the fact that the perception of time slows down more significantly during the transition to negative states than during the states themselves (Gable & Poole, 2012; Wang & Lapate, 2024). Likewise, the cognitive challenge of withdrawal (Di Lernia et al., 2018; Gable et al., 2022) and sleep deprivation (Şen et al., 2023) dilate time perception. Our argument defines time perception as an even function, represented by a graph that remains unchanged under reflection in the y-axis (Fig. 1). Interestingly, our time perception curve is analogous to the upside-down U-shaped curve of the Yerkes-Dodson law, which relates performance to circulating levels of stress hormones (Beerendonk et al., 2024; Yerkes & Dodson, 1908; Lupien, Maheu, Mt, Fiocco, & Schramek, 2007). This similarity underscores the thermodynamic foundation of emotions' potential for motivation.

An intriguing question remains of why diverse experiences – from intense states of anxiety to the calmness inspired by awe or nature – dilate time perception (Bannister & Eerola, 2021; Davydenko & Peetz, 2017; Failing & Theeuwes, 2016; Mitchell et al., 2015; Rudd et al., 2012). For example, information overload during stressful states can cause difficulty concentrating and purposeful behavior (Nutt, 1999). Anxious people usually resort to impulsivity and meaningless, arbitrary actions until action motivation is halted in depression (Stanghellini et al., 2016; Wittmann & Paulus, 2008), implying an inverse relationship between mental adversity and the ability to change it.

In contrast, the low action motivation of positive mental states indicates energy frugality, which may explain their connection to parasympathetic restorative processes in long-term psychological well-being (Table I). A muted action motivation might represent some form of minimum energy path, analogous to the principle of stationary action in physics. At the curve's left minimum (Fig. 1), awe slows or pauses the subjective time. Contentment is an uncluttered, information-scarce experience that represents confidence in self-agency but lacks internal motivation.

Stress and contentment lie at opposite ends of an information-processing and action-motivation spectrum. It is a contradiction; those with the capacity to institute change (contentment) lack the desire, and

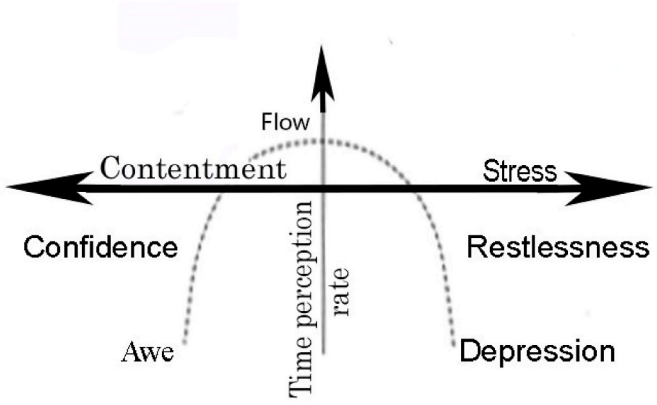


Fig. 1. The Psychology of Contentment and Stress Movement speeds up time perception (vertical arrow). In positive experiences, time perception reduces action motivation, culminating in awe. The pain of stress motivates action but weakens self-confidence. Anxiety can progress to depression when action motivation halts. A unique point of action, motivation, and confidence is flow.

Table 1
Binary choices supporting bodily and psychological equilibrium.

	Experience	
Symptom	**Positive arousal**	**Stress**
Time perception increases (Dilation) — Arousal	Parasympathetic	Action motivation
Time perception decreases (Compression) — Action	Flow	Sympathetic
Physiological symptoms (shivering, sweating)	Accomplishment	Shame, fear

those having the desire (stress) lack the agency. Gordon et al. (2023) confirmed the connection between decision-making and bodily functions and movement control: better body control (resulting in less stress) permits greater agency, and vice versa. In the following, we investigate the regulatory framework of motivation in more detail.

1.5. The binary regulation of higher cognitive functions

The timing of the giant fiber descending neuron spike determines whether a Drosophila evades a predator via a short or long takeoff (Ache et al., 2019). Bifurcations from geometric principles can elucidate behavior and decision-making across various species and ecological scenarios (Sridhar et al., 2021), and temperature and emotion regulation in mammals (Hesp et al., 2021; Kao et al., 2015; Sadowski et al., 2020). These spontaneous and abrupt "critical" transitions are linked with specific geometrical relationships. A shift from averaging vectorial information among options abruptly excludes one among the remaining choices. The brain repeatedly breaks down multi-choice decisions into a sequence of binary decisions. Binary regulation is an "on-off regulation," in systems with only two possible states, referring to "dichotomous thinking in psychology.

In mammals, stimulation of the PAG can induce relaxation or escape behavior. Frontal PAG stimulation inspires a relaxed, immobile posture due to the sense of excess time. However, lateral PAG stimulation also produces two typical behavioral responses. When there is sufficient time, increased blood pressure and heightened pain sensitivity facilitate escape and defensive responses (Zelena et al., 2018). An immediate threat, when escape is no longer possible, mutes pain sensitivity and triggers immobile freezing behavior. In people, anxiety can induce aggravation or an emotional collapse into depression, where both time perception and action motivation appear to halt (Stanghellini et al., 2016), providing further support for our thermodynamic argument.

Binary choices can generate multifaceted behavior regulation (summarized in Table 1), such as (1) dilation of time perception to accelerate or mute motivation, (2) contraction of time perception to mediate action toward completion, and (3) halting of action motivation or sudden cognitive changes. This regulation is based on psychological spin, utilizing the reversible perception cycle (Déli, 2023), which automatically restores psychological equilibrium, thereby forming a new balance. This model can explain regulatory complexity as a series of binary choices ad infinitum. Dopaminergic mechanisms, which can amplify the motivational power of emotions through subjective time perception, highlight how internal and external factors influence the multifaceted nature of experiences (Soares et al., 2016).

We must note that emotional and cognitive processes can trigger physiological symptoms, such as shivering, chills, sweating, and changes in body temperature (Schoeller & Perlovsky, 2016). In this context, physiological symptoms can be both tools and consequences of the brain's energy regulation.

2. Discussion

Decision-making from fruit flies to humans often boils down to binary choices based on geometric principles and critical transitions. The brain's cognitive cycle can maintain a bodily and psychological equilibrium through simple bifurcation stemming from geometric principles. Binary options can refine cognitive and intellectual evolution through learning, beliefs, and individual capabilities. Moreover, emotion, temperature, and physiological symptom regulation recover and maintain constant resting entropy. Therefore, as the thermodynamic cycle stabilizes the psyche around a new equilibrium after every decision-making, it increases or decreases synaptic complexity, confidence, and mental health.

Time perception can lead to contrasting behavioral outcomes during stress (information overload) or contentment (information scarcity). Negative emotions inspire arbitrary and chaotic performance, corresponding to wasteful energy use, which can accumulate and lead to adverse health effects associated with stress. In positive conditions, frugality of action motivation reflects a minimal energy path, allowing contentment and creativity. In contrast, faster time perception can manage ongoing action by inspiring cognitive coherence. Intrinsically motivating activities lead to deep engagement and enjoyment, a state known as flow, characterized by the intersection of motivational challenge, confidence, and passion (Csikszentmihalyi, 1990, 1997, p. 31). This is congruent to our hypothesis that ongoing action accelerates time perception. The time perception curve also overlaps Yerkes-Dodson law, providing further support to our thermodynamic foundation of emotions' potential for motivation. Emotions can be viewed as the fundamental forces of motivation.

By conceptualizing psychological processes through energy dynamics, we outlined how emotions can affect motivation through distorted time perception. Moreover, our thermodynamic model can explain how action motivation during stress can produce wasteful cognitive processes. The relationship between low entropy and compromised consciousness states (Varley et al., 2020), and depression (Wise et al., 2017) underlines the role of stress in mental problems.

Our model provides a framework for designing interventions and strategies that leverage the interconnections between emotions, motivation, and physiology. While our thermodynamic model focuses on the fundamental structural motivations underlying behavior, we acknowledge the importance of cultural norms, social expectations, and environmental factors in shaping emotional and motivational experiences.

Integrating these contextual factors is crucial to understanding the complex interplay between emotions, motivation, physiology, and subjective experiences across diverse cultural and environmental settings.

2.1. Limitations and future directions

We would like to acknowledge the lack of empirical validation and experimental testing of the proposed framework. Additionally, the generalizability of the model across diverse populations remains unexplored. This limitation is particularly salient in clinical populations, such as individuals with post-traumatic stress disorder (PTSD) or major depressive disorder, where disruptions in bodily awareness and emotional regulation may offer critical tests of the model's assumptions.

Our framework also does not yet account for the role of time perception for proprioceptive awareness—two domains that are increasingly recognized as central to embodied cognition and affective experience. Future empirical works in this area can further our understanding of the mind-body interaction.

While our thermodynamic model provides a novel framework for integrating emotions, motivation, and time perception, it remains theoretical and is subject to several significant limitations. Foremost, the model relies heavily on analogies between thermodynamic principles—such as entropy, temperature, and energy—and psychological processes. While conceptually illuminating, these analogies are not always supported by direct empirical evidence, which may limit their explanatory power. The complexity of human emotional and cognitive systems likely exceeds the reach of simplified energetic metaphors, especially when such models are extended to subjective constructs like time perception.

Additionally, many of the neurophysiological correlates we invoke, such as neural entropy or the functional role of dopamine in timing, are still under active investigation, and consensus regarding their interpretation is far from established. The use of entropy as a measure of cognitive or emotional flexibility, for example, depends on methodological choices that may not yet be standardized across studies. Furthermore, the model currently lacks specificity in distinguishing between clinical and non-clinical populations, and its generalizability across age groups, cultural settings, or neurodiverse profiles remains untested. Despite these constraints, the integrative potential of the framework may help bridge conceptual gaps between disparate findings in psychology, physiology, and neuroscience.

Nevertheless, our model opens several promising avenues for applied research and translational work. In mental health, for instance, subjective distortions of time perception are commonly reported in anxiety, depression, and trauma-related disorders. Our framework suggests that such distortions may reflect dysregulated entropy states in the brain, which could be targeted through interventions aimed at modulating physiological arousal and attentional engagement. Biofeedback, neurofeedback, or pharmacological interventions that stabilize neural entropy dynamics may be tested for their capacity to restore adaptive time perception and emotional regulation. Moreover, in performance contexts—such as athletic training, musical improvisation, or surgical decision-making—the model's emphasis on flow as a state of optimal energy alignment suggests practical strategies for cultivating cognitive and emotional balance. Future research should prioritize experimental designs that manipulate entropy-relevant parameters (e.g., arousal, cognitive load, ambient temperature) while recording physiological or neuroimaging data. Valuable direction would be to examine whether interventions known to improve well-being, such as mindfulness or physical activity, influence subjective time perception through measurable changes in brain signal complexity.

While our model identifies correlations between emotional states and physiological indices—such as body temperature and thermodynamic entropy—our current analysis stops short of demonstrating direct causal mechanisms. Establishing such causality will be the subject of future work, potentially using targeted experimental designs and interventions that modulate physiological parameters to observe downstream effects on affective and cognitive processes.

3. Conclusion

The thermodynamic analysis of cognition points to the existence of binary regulation. First observed in fruit flies and fish, this regulation can maintain bodily and psychological equilibrium and enable abstract decision-making by infinitely enhancing the details of regulatory complexity. A decision-making process based on geometric principles may be universal throughout biology and even physics, such as electromagnetism.

Our verifiable framework shows that identical dilation of time perception in arousing emotional states (such as anxiety and joy) and calming experiences (like awe and natural environments) can give rise to varied motivation and offer nuanced decision-making. Time perception, a function of information processing and entropic factors, potentiates the ability for motivation. The muting of action motivation in positive states represents energy frugality, or a minimum energy path, which might be analogous to the principle of stationary action in physics. In contrast, stress represents information overload, spurring chaotic decision-making and action motivation, which wastes effort and energy. Our model suggests that emotions have a thermodynamic foundation, rendering them the driving forces of motivation.

The implications of our model extend beyond theoretical understanding, offering potential avenues for addressing mental health challenges and optimizing well-being through interventions that leverage the interconnections between emotions, motivation, and physiology. Our thermodynamic perspective originates emotions and their long-term mental consequences in the energy-information dynamics of the brain. It opens new avenues for interdisciplinary and innovative approaches to understanding and optimizing human functioning. Finally, it can inspire novel approaches in artificial intelligence research.

CRediT authorship contribution statement

Eva Déli: Writing – review & editing, Writing – original draft, Visualization, Data curation, Conceptualization. **Felix Schoeller:** Writing – review & editing, Writing – original draft, Investigation. **Adam Safron:** Writing – review & editing, Writing – original draft. **Abhinandan Jain:** Writing – review & editing, Writing – original draft. **Arturo Tozzi:** Writing – review & editing, Writing – original draft. **Vladimir Adrien:** Writing – review & editing, Writing – original draft. **Nicco Reggente:** Writing – review & editing, Writing – original draft, Visualization, Project administration.

Data availability

No data was used for the research described in the article.

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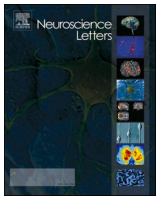
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Takens' theorem to assess EEG traces: Regional variations in brain dynamics

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ABSTRACT

Takens' theorem (TT) proves that the behaviour of a dynamical system can be effectively reconstructed within a multidimensional phase space. This offers a comprehensive framework for examining temporal dependencies, dimensional complexity and predictability of time series data. We applied TT to investigate the physiological regional differences in EEG brain dynamics of healthy subjects, focusing on three key channels: FP1 (frontal region), C3 (sensorimotor region), and O1 (occipital region). We provided a detailed reconstruction of phase spaces for each EEG channel using time-delay embedding. The reconstructed trajectories were quantified through measures of trajectory spread and average distance, offering insights into the temporal structure of brain activity that traditional linear methods struggle to capture. Variability and complexity were found to differ across the three regions, revealing notable regional variations. FP1 trajectories exhibited broader spreads, reflecting the dynamic complexity of frontal brain activity associated with higher cognitive functions. C3, involved in sensorimotor integration, displayed moderate variability, reflecting its functional role in coordinating sensory inputs and motor outputs. O1, responsible for visual processing, showed constrained and stable trajectories, consistent with repetitive and structured visual dynamics. These findings align with the functional specialization of different cortical areas, suggesting that the frontal, sensorimotor and occipital regions operate with autonomous temporal structures and nonlinear properties. This distinction may have significant implications for advancing our understanding of normal brain function and enhancing the development of brain-computer interfaces. In sum, we demonstrated the utility of TT in revealing regional variations in EEG traces, underscoring the value of nonlinear dynamics.

Significance statement

The novelty of this study is that we applied Takens' theorem to investigate the differences in phase space features among different EEG channels of healthy subjects.

1. Introduction

The human brain operates as a sophisticated nonlinear system, adept at handling extensive information via dynamic interactions [4,7,13,22]. Electroencephalography (EEG) serves as a non-invasive, high-resolution method for investigating brain activity. Nonetheless, conventional linear analysis techniques frequently fall short in representing the intricate nonlinear features of EEG signals [3]. To address this limitation, nonlinear dynamics and chaos theory have emerged as powerful frameworks for understanding brain activity, with Takens' theorem (henceforward TT) providing a cornerstone. TT establishes that the behavior of a dynamical system can be reconstructed in a multidimensional phase space using time-delayed versions of a single time series from the observed data [20]. In the context of EEG analysis, TT provides a robust mathematical tool to study temporal evolution, revealing properties that linear methods cannot uncover [19]. By reconstructing the phase space, researchers can analyze key EEG dynamical properties such as temporal dependencies, dimensional complexity and predictability [14]. This approach has proven valuable for identifying changes in neural dynamics associated with various cognitive and pathological conditions [8].

Previous research has highlighted the effectiveness of TT in analyzing EEG signals, especially for identifying pathological conditions like epilepsy, Alzheimer's disease and schizophrenia [1,2,5,11]. However, less attention has been given to the application of this approach for

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assessing regional variations in brain dynamics under normal conditions. Different brain regions exhibit distinct patterns of electrical activity, reflecting their specialized roles in cognition, sensation and motor function. For instance, the frontal region (FP1) is associated with higher cognitive processes such as decision-making and working memory. The sensorimotor cortex (C3) governs movement and integrates sensory inputs, while the occipital region (O1) processes visual information. Despite their unique roles, interactions among these regions contribute to the brain's global dynamics.

Comparing the spread and trajectory of phase space trajectories across different brain regions can shed light on how functional specialization translates into distinct dynamical features. In this study, we apply TT to assess EEG traces from three key brain regions: FP1, C3 and O1. We reconstruct phase spaces and analyze the temporal structure and complexity of the trajectories. Key metrics such as trajectory spread were computed to quantify regional differences in brain dynamics.

The rationale for using TT lies in the fact that overlapping trajectories reveal shared dynamical patterns across the EEG traces, suggesting common temporal structures in brain activity. These patterns likely arise from similar neural processes or consistent experimental conditions and reflect shared features of brain dynamics that remain consistent across individuals or trials. Conversely, differences in the spread or divergence of trajectories highlight variability in the dynamics of EEG signals, capturing subject-specific neural activity. This variability can serve as a marker for distinguishing between populations, such as healthy versus pathological groups, and is particularly relevant for identifying neural behaviors associated with specific cognitive states, tasks or conditions. Trajectories that densely fill the phase space indicate higher complexity, which is often linked to intricate neural processes and sophisticated information management. In contrast, smoother and more confined trajectories suggest simpler, more periodic or more deterministic behavior, highlighting a system with lower complexity but higher predictability. Together, these observations shed light on the dynamic interplay between stability, variability and complexity in brain activity, offering a deeper understanding of its temporal and spatial organization.

In the following sections, we detail the methods used for EEG data collection, phase space reconstruction and analysis, followed by the presentation and discussion of our results.

2. Subjects and methods

This study investigates the regional variations in brain dynamics by applying Takens' theorem to EEG data, focusing on three key channels: FP1 (frontal), C3 (sensorimotor), and O1 (occipital). The data consisted in the retrospective evaluation of ten EEG traces. EEG recordings were obtained from ten healthy, right-handed volunteers (mean age: 20.1 years; SD = 1.1; age range: 18–22 years; 5 males). For additional information on the participants and EEG methodologies, see Jaušovec and Jaušovec [10] and Tozzi et al. [21]. The signals were captured using a 64-channel EEG system, adhering to the standard 10–20 electrode placement system. Preprocessing included artifact removal, such as eye blinks and muscle movements through independent component analysis and band-pass filtering (0.5–50 Hz).

After preprocessing, the analysis focused on reconstructing the phase space of each EEG trace. TT was employed to create a multidimensional representation of the system's dynamics using delay embedding. The first step involved determining for each signal the optimal time delay (τ), a critical parameter in phase space reconstruction, as it defines the separation between consecutive points in the reconstructed dimensions [15]. The mutual information method was used to calculate τ, as it effectively identifies the point at which the time series exhibits the least redundancy while retaining information about the system's dynamics. For all channels and traces, τ was consistently found to be 25, suggesting a shared temporal dependency structure across the EEG signals.

The next step was to determine the embedding dimension (d), which represents the number of dimensions required to unfold the systems dynamics without overlap in the reconstructed phase space [17]. The False Nearest Neighbors (FNN) method was used for this purpose [19]. By analyzing the proportion of neighbors that remain close when the dimensionality is increased, FNN identifies the point at which the embedding dimension captures the true dynamics of the system. The results consistently indicated $d = 6$ for all traces, suggesting a moderate level of complexity in the underlying neural dynamics.

Starting from the parameters $\tau = 25$ and $d = 6$, the phase space for each EEG trace was reconstructed and visualized in three dimensions by selecting the first three delay coordinates. The trajectories were analyzed to extract quantitative metrics characterizing the temporal structure of the phase spaces. Specifically, the spread of the trajectories in each dimension and the average distance between consecutive points were computed. The spread reflects the range of dynamical variability, while the average distance provides insight into the smoothness and temporal evolution of the signals.

3. Tools and statistical analysis

Computational analysis for precise quantification of regional variations in brain dynamics was implemented using Python, leveraging libraries for signal processing, nonlinear analysis and statistical evaluation. Statistical analysis was performed to compare the phase space metrics among the FP1, C3 and O1 channels. Welch's t-tests were used to evaluate differences in trajectory spread across dimensions (x, y, z) among the channels. Welch's *t*-test was chosen because it does not assume equal variances between groups, thus ensuring robustness in the presence of heterogeneity in EEG signals.

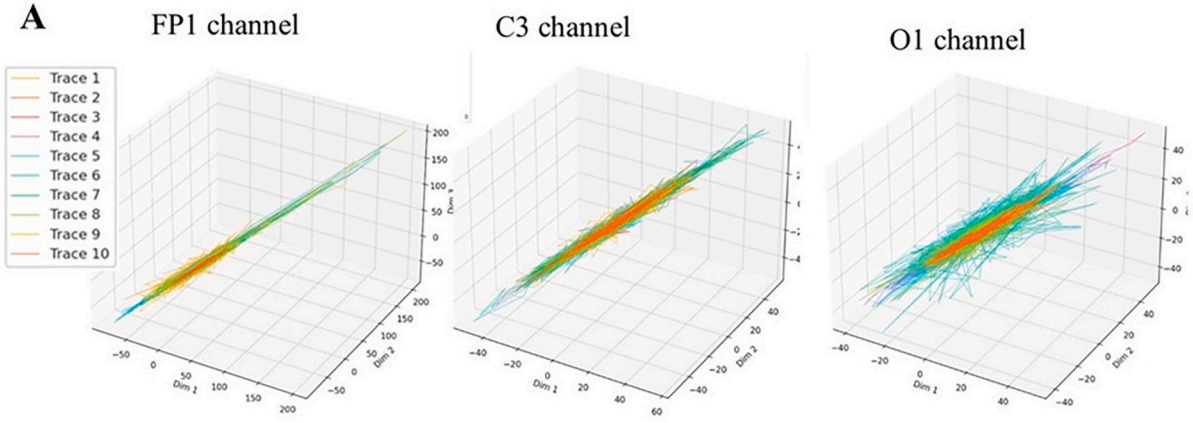
4. Results

Phase space trajectories were reconstructed for all ten EEG traces of healthy individuals across the FP1, C3 and O1 channels using time delay $\tau = 25$ and embedding dimension $d = 6$. These reconstructions highlighted distinct patterns of activity in the three regions (**Fig. 1A**). To quantify these differences, key metrics such as trajectory spreads across dimensions (x, y, z) and average distances between consecutive points were computed for each channel (**Fig. 1B**).

The FP1 trajectories displayed the broadest and most dispersed paths as well as the highest average distances, indicative of the greatest variability and complexity. This pattern aligns with the frontal region's role in higher-order cognitive functions, such as decision-making and problem-solving, which demand flexible and dynamic neural processes. In contrast, the C3 and O1 trajectories were more compact. Among them, O1 exhibited the most constrained dynamics with the smallest spreads and average distances, reflecting the structured and repetitive nature of visual processing. This stability is characteristic of the occipital region's specialization in handling consistent sensory inputs. Meanwhile, C3 showed a moderate spread, with intermediate spreads and distances that balanced variability and stability, consistent with its role in integrating sensory inputs and motor outputs, a process requiring balance between stability and flexibility. The degree of similarity and overlap in C3 trajectories also suggests shared neural patterns within the sensorimotor region, while the divergence in trajectory spreads provides insights into individual differences in neural activity among participants.

Statistical comparison of trajectory spreads across the three channels further validated these findings. Significant differences were observed in all pairwise comparisons, confirming substantial dynamical differences among FP1, C3 and O1. **Fig. 1D** provides a direct comparison of trajectory spreads across dimensions (x, y, z) for the three channels. FP1 consistently showed the largest spreads across all dimensions, a finding that reflects the region's high dynamical variability and complexity. This statistical validation reinforces the notion that EEG dynamics are regionally specific and closely linked to the functional roles of these

A



B

Summary of Trajectory Metrics Across Channels

Channel	Spread_X (Mean ± SD)	Spread_Y (Mean ± SD)	Spread_Z (Mean ± SD)	Avg Distance (Mean ± SD)
FP1	1.78 ± 0.34	1.62 ± 0.30	1.85 ± 0.28	0.43 ± 0.06
C3	1.42 ± 0.29	1.35 ± 0.24	1.48 ± 0.25	0.37 ± 0.05
O1	1.28 ± 0.27	1.22 ± 0.21	1.35 ± 0.22	0.34 ± 0.04

C

Statistical Comparison of Trajectory Spreads Across Channels

Metric	FP1 vs C3 (p-value)	FP1 vs O1 (p-value)	C3 vs O1 (p-value)
Spread_X	0.017	0.009	0.03
Spread_Y	0.018	0.012	0.026
Spread_Z	0.019	0.015	0.028

D

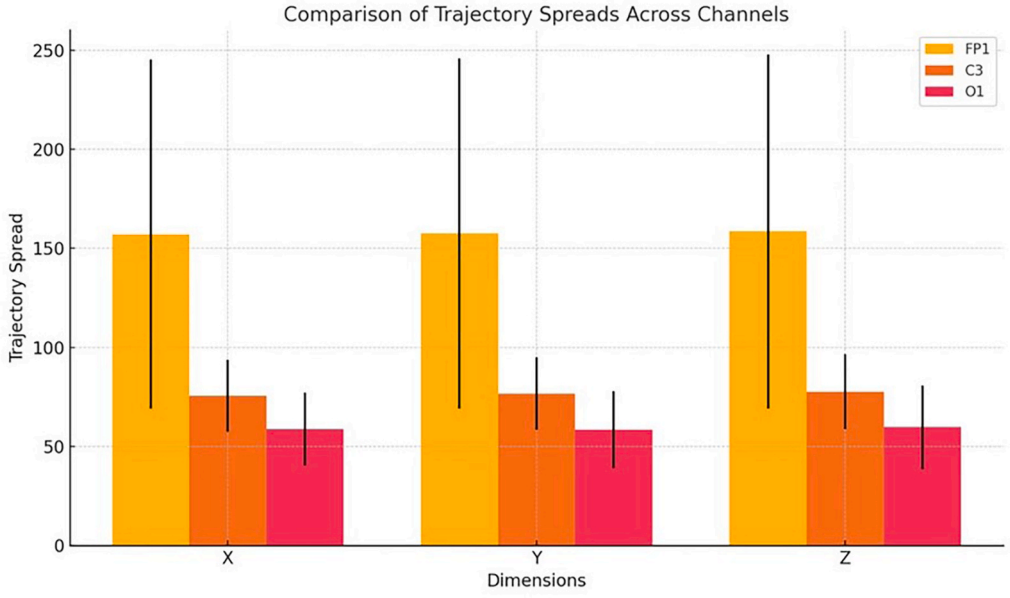


Fig. 1. A. Comparison of trajectory spreads across the reconstructed phase spaces for the three EEG channels FP1, C3 and O1. Each phase space illustrates the combined trajectories of all ten EEG traces from the corresponding channel, visualized together in a single 3D plot. B. Trajectory metrics computed for all three channels, including trajectory spreads across dimensions (x, y, z) and average distances between consecutive points. C. Statistical comparison of trajectory spreads across the three channels. Welch's t-tests reveals significant distinctions ($p < 0.05$) across all pairwise comparisons. D. Detailed visualization of the statistical comparison of trajectory spreads. The bar chart illustrates the average spreads for each dimension (x, y, z) for the FP1, C3 and O1 channels, accompanied by error bars representing the standard deviations.

cortical areas.

In sum, the combination of trajectory visualizations, quantitative metrics and statistical analysis reveal clear regional variations in trajectory spreads. These findings align with the functional specialization of the brain regions and underscore the utility of TT in capturing regional variations in EEG dynamics.

5. Conclusions

We focused on applying Takens' theorem to EEG signals to examine regional nonlinear variations in healthy subjects' brain dynamics, reconstructing the phase spaces and comparing their temporal features. Three key cortical areas were analyzed, namely, the frontal region (FP1), the sensorimotor region (C3) and the occipital region (O1). Using nonlinear dynamical analysis focused on trajectory spreads and average distances, the study reconstructed the phase space of EEG signals to reveal distinct patterns of activity corresponding to the functional specialization of these regions.

Takens' theorem has emerged as a powerful tool for exploring EEG traces. By transforming abstract EEG data into comprehensible geometrical forms, TT allows for the reconstruction of the phase space. Rohrbacker [19] illustrated the impact of TT on neuroscience by calculating the necessary embedding dimensions for EEG data using false nearest neighbor analysis. His seminal findings emphasized the importance of adequate embedding for accurately reconstructing phase spaces. TT's applications span epilepsy diagnosis, causal inference, BCI development and real-time analysis. For instance, phase space analysis has been used to identify pre-seizure states by detecting chaotic patterns in EEG signals [14]. Kwessi and Edwards [14] introduced the concept of complex geometric structurization to analyze epilepsy-related EEG data, revealing intricate geometric structures as biomarkers for seizure activity. Sleep studies have used phase space trajectories to differentiate between sleep stages and detect sleep disorders [8]. Changes in neural complexity and stability underlying neurodevelopmental disorders such as ADHD can also be examined through phase space reconstruction [12].

Topological data analysis (TDA) represents another area where TT has influenced EEG research [18]. Altındiş et al. [2] explored the use of TDA to extract persistent homologies from EEG-derived state spaces, demonstrating that TDA could robustly capture topological features of neural data even in the presence of artifacts. Moreover, TT has been instrumental in the development of methodologies for analyzing the nonlinear components of EEG signals. For instance, Mekler [16] introduced an innovative method for calculating the correlation dimension of EEG attractors, overcoming the limitations of traditional approaches. This advancement allows for more efficient processing of large datasets while minimizing subjectivity. Similarly, Kannathal et al. [11] applied entropy measures derived from EEG embeddings to detect epilepsy, achieving high classification accuracy. Also, TT has proven valuable for studying causality in neural systems. Traditional causal models often fall short when applied to nonlinear and cyclic systems like the brain. Harnack et al. [9] addressed this limitation by leveraging time-delay state space reconstructions to measure directed causal influences. The applicability of TT extends to brain-computer interface (BCI) systems, which depend on decoding EEG signals to enhance signal classification and facilitate interaction with external devices, providing critical support for individuals with disabilities. Carrara and Papadopoulo [6] incorporated embedding techniques into geometric neural networks, enhancing the interpretability and efficiency of BCI decoding.

The novelty of this study is that we applied TT to investigate the differences in phase space features among different EEG channels of healthy subjects, reflecting the distinct physiological dynamics of the underlying brain regions. One of the primary contributions of this work was the detailed reconstruction of phase spaces for each EEG channel using time-delay embedding. The reconstructed trajectories were further quantified through metrics such as trajectory spread and average distance that provided insights into the temporal structure and complexity of brain activity in different regions that are not easily captured by traditional linear methods. The FP1 region, associated with higher-order cognitive functions such as decision-making and information processing, exhibited the largest trajectory spreads and higher average distances. This variability aligns with the complexity and flexibility required for cognitive tasks. In contrast, the C3 region, which integrates sensory inputs and motor outputs, exhibited moderate variability, while the O1 region, responsible for visual processing, displayed the smallest spreads and distances, consistent with its structured and stable neural dynamics.

These findings have significant implications for understanding the role of regional dynamics in neural processing. The distinct temporal structures observed across the three channels provide a quantitative framework for examining how different brain regions contribute to overall function. By quantifying and visualizing the underlying dynamics, TT opens new opportunities for studying neural systems. The moderate embedding dimensions identified in this study suggest that the brain operates within a finite-dimensional dynamic space, efficiently processing complex information. The possible applications of these findings are vast. Dynamical systems can be compared by analyzing the variability, complexity and chaotic behavior of specific EEG traces. Identifying outliers, such as traces that deviate significantly from the shared structure, may help detect noise or reveal unique dynamics.

Despite its strengths, the study has limitations. The sample size was relatively small, consisting of only ten EEG traces, which may limit the generalizability of the findings. Additionally, the uniform use of $\tau = 25$ and $= 6$ across all traces, while robust, may not capture individual variability in optimal parameters. Furthermore, the analysis was limited to three channels (FP1, C3, O1) representing specific cortical regions. Expanding the study to include more channels and larger datasets could provide a more comprehensive understanding of the global and regional dynamics of the brain. Future research could address these limitations by applying the approach to more diverse datasets, exploring alternative parameter optimization methods, and incorporating task-based EEG data.

In conclusion, this study demonstrates the utility of Takens' theorem in reconstructing the phase space of EEG signals in healthy individuals and in revealing regional variations in brain dynamics. The findings highlight distinct patterns of activity in the frontal, sensorimotor and occipital regions. By bridging the gap between theoretical nonlinear dynamics and practical applications, this approach offers significant potential for advancing both basic neuroscience and clinical practice.

6. Declarations

Ethics approval and consent to participate: This research does not contain any studies with human participants or animals performed by the Authors.

Consent for publication: The Authors transfer all copyright ownership, in the event the work is published. The undersigned authors warrant that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal, and has not been previously published.

Availability of data and materials: all data and materials generated or analyzed during this study are included in the manuscript. The Authors had full access to all the data in the study and take responsibility for the integrity of the data and the accuracy of the data analysis.

Competing interests: The Authors do not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence, or be perceived to influence, their work.

Authors' contributions: The Authors equally contributed to: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding,

administrative, technical, and material support, study supervision.

Declaration of Generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the authors used ChatGPT to assist with data analysis and manuscript drafting. After using this tool, the authors reviewed and edited the content as needed and takes full responsibility for the content of the publication.

CRediT authorship contribution statement

Arturo Tozzi: Conceptualization, Funding acquisition, Resources, Visualization, Formal analysis, Methodology, Validation, Writing – original draft, Supervision, Data curation, Investigation, Software, Project administration, Writing – review & editing. **Ksenija Jaušovec:** Investigation, Data curation, Project administration, Validation, Conceptualization, Formal analysis, Software, Writing – original draft, Visualization, Funding acquisition, Methodology, Resources, Writing – review & editing, Supervision.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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None.

Data availability

Data will be made available on request.

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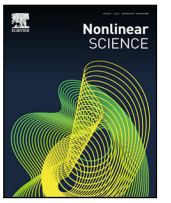
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Research paper

Q1 From nonlinear to linear dynamics: A structural approach via Wedderburn–Artin decomposition

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Operator formalism

ABSTRACT

Nonlinear dynamical systems resist global analysis when approached through classical linearization techniques which rely on differential equations and local approximations. In seeking approaches to structurally reduce nonlinear dynamics to linear components, we propose interpreting the evolution of a nonlinear system as a sequence of non-commuting operations within a finite-dimensional associative algebra, where interaction rules are captured abstractly through algebraic composition rather than defined analytically. By embedding the nonlinear system into a semisimple algebra, we employ the Wedderburn–Artin decomposition to represent its dynamics as a direct sum of matrix algebras over division rings. Each matrix block defines a linear action on an irreducible subspace, corresponding to a dynamically invariant mode grounded in the system's internal symmetries. This block structure reveals a modular architecture, demonstrating how nonlinear interactions can give rise to intrinsically linear behaviors governed by underlying algebraic principles. We apply our method to three distinct systems — symbolic rewriting systems, operator-driven vector dynamics and partially associative bitwise systems — selected to represent symbolic, quantitative and hybrid forms of nonlinearity. This range ensures that our decomposition framework effectively captures both regular and irregular compositional structures across diverse classes of nonlinear behavior. We demonstrate that our method is able to isolate invariant subsystems and uncover underlying structure, by revealing the latent linear organization embedded within complex nonlinear behavior. Overall, our framework extends matrix-based analysis into domains that are traditionally nonlinear, bridging symbolic computation, algebraic structure and dynamical behavior and providing an alternative approach to tackle nonlinear systems through their decomposable linear representations.

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Nonlinear systems appear across the sciences, from symbolic models and control processes to biological regulation and network dynamics. Yet their analysis remains challenging due to the complexity of their interactions, especially when system evolution is governed by rules that are discrete, symbolic or non-commutative. Traditional tools such as differential linearization or local expansions often fail to capture the global structure of such systems. We introduce a new algebraic framework reformulating nonlinear evolution as a sequence of compositional operations within a finite-dimensional associative algebra. Using the classical Wedderburn–Artin theorem, we decompose these algebras into matrix blocks that represent invariant, linearly evolving subsystems. This reveals an unexpected modular structure within dynamics previously seen as irreducibly nonlinear. By applying the method to symbolic rewriting systems, operator-driven processes and partially associative update rules, we show how complex behavior can

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be re-expressed as structured linear action. Our results suggest a principled route for reinterpreting nonlinear dynamics through algebraic symmetries, opening avenues for analysis, classification and simplification.

1. Introduction

Q5 The study of nonlinear dynamical systems challenges researchers across mathematics, physics and engineering due to their intrinsic complexity [1–4]. Traditional analytical tools frequently rely on assumptions such as smoothness, differentiability or proximity to equilibrium. Linearization techniques — including Jacobian approximations, perturbation methods and Lie algebraic expansions — are typically local in nature, highly context-dependent and limited in their ability to capture the global structure of nonlinear behavior [5,6]. While these methods provide insights into stability and local dynamics, they tend to obscure the underlying global structure and fail to offer unified treatment of systems whose behavior is intrinsically non-additive or history-dependent. Recent developments in the theory of operator algebras, symbolic dynamics and rule-based evolution suggest that many nonlinear systems can be profitably recast not in terms of continuous differential flows but through the algebraic composition of state transformations. In this light, the dynamic evolution of a system may be interpreted as a sequence of non-commuting operations within a finite algebra, where the interaction rules are encoded abstractly rather than analytically. This shift in perspective paves the way for new algebraic methods capable of isolating internal symmetries and decomposable substructures, even in the absence of a continuous phase space. By leveraging representation theory and classical ring-theoretic tools, it may become feasible to uncover structural invariants that persist under nonlinear evolution, pointing toward a deeper form of reducibility. This motivates the development of a framework that reconceives nonlinear dynamics as compositional behavior within semisimple algebras.

We propose a method for structurally reducing nonlinear dynamics to linear components using a Wedderburn–Artin approach [7]. Instead of modeling the evolution of nonlinear systems through differential processes, we frame it as a sequence of algebraic operations within a finite-dimensional associative algebra. In this formulation, the system's behavior is encoded in the non-commutative structure of state transformations—operations that can be abstractly composed and embedded into a semisimple algebraic context. Applying the Wedderburn–Artin theorem enables the decomposition of the system into a direct sum of matrix algebras over division rings, where each block corresponds to a linear action on an irreducible subspace [8,9]. This decomposition reveals how nonlinear, context-sensitive systems can internally support modular, linear subsystems governed by invariant behavioral modes. In contrast to classical linearization or perturbative techniques, our approach does not rely on approximations or truncations; it follows a categorical sequence of embeddings and reductions that preserve essential dynamical properties. The resulting structure captures the global algebraic architecture of the system, providing an alternative to traditional analytic frameworks. The theoretical novelty lies in recasting nonlinear dynamics as algebraic composition within associative algebras, allowing the systematic application of semisimple decomposition to extract irreducible, blockwise-linear dynamics. Through this embedding, invariant submodules emerge as natural units of analysis and radical components correspond to transient or non-stable behaviors. This provides an algebraic method for identifying conserved modes, performing modular decomposition and uncovering latent linear structures inaccessible to conventional analytic techniques.

We will proceed as follows: first, we outline the methodological principles underlying our algebraic transformation of nonlinear dynamics. Then, we present the core theoretical constructions, including the use of semisimple embeddings and Wedderburn–Artin decomposition. This is followed by practical examples drawn from physics and biology, after which we conclude with a discussion of the structural implications, boundaries and limitations of the proposed framework.

2. Methods

To implement the proposed algebraic reduction of nonlinear dynamics, we developed a systematic framework combining categorical formulation, computational algebra and representation-theoretic analysis. The goal is to transform nonlinear evolution rules, whether symbolic, operator-based or partially associative, into finite-dimensional associative algebras that can be rigorously decomposed into linear components. Each step of the procedure, from the encoding of operations to the computation of Gröbner bases, radical analysis and Wedderburn–Artin decomposition, follows well-defined algebraic principles and can be executed algorithmically. This section outlines the mathematical assumptions, construction of the associative envelope and computational workflow used to identify semisimple and radical components in representative nonlinear systems.

Notation and conventions. Throughout this section, all algebras are finite-dimensional associative algebras over a base field $\mathbb{F}$, taken to be either the field of real numbers $\mathbb{R}$or of complex numbers $\mathbb{C}$. The symbol I denotes the multiplicative identity element of an algebra. The generators of the system's operations are indicated by g_i. Two-sided ideals generated by composition relations are written $\mathfrak{I}$, while the Jacobson radical of an algebra A is denoted $R(A)$. In the Wedderburn–Artin decomposition

$$A \cong \bigoplus_i M_{n_i}(D_i),$$

each $M_{n_i}(D_i)$ designates the full matrix algebra of order n_i over a division ring D_i; every summand acts linearly on an irreducible module corresponding to an invariant dynamical component of the original nonlinear system.

Algebraic encoding of nonlinear dynamics. All systems analyzed in this study, i.e., symbolic rewriting systems, operator-driven vector transformations and partially associative bitwise operations, are modeled as nonlinear dynamical systems whose evolution is described by repeated compositions of a finite set of operations. Rather than assuming a continuous or topological framework, we treat systems whose rules of evolution can be expressed as algebraic operations, possibly non-commutative and non-associative.

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Let $\Sigma = \{g_1, g_2, \ldots, g_k\}$ be the set of operations governing the system's dynamics. These may represent string substitutions, piecewise-defined maps on $\mathbb{R}^n$ or local update rules on binary configurations. The system's evolution is represented by sequences of compositions

$$g_{i_1} g_{i_2} \cdots g_{i_m},$$

where associativity need not hold.

To study this behavior algebraically, we define the free non-commutative algebra

$$F = \mathbb{F}\langle g_1, g_2, \ldots, g_k \rangle,$$

and construct a two-sided ideal $\mathfrak{I}$ generated by the observed or defined composition relations among the operations. The quotient algebra

$$A = F/\mathfrak{I}$$

is then taken as the **associative envelope** of the system. The algebra A encodes the complete compositional structure of the system in a finite-dimensional, associative setting, allowing application of standard tools from ring theory and representation theory.

Categorical and algebraic assumptions. The method assumes that nonlinear dynamical systems, particularly those characterized by non-commuting, context-sensitive update rules, can be reformulated as sequences of algebraic operations rather than smooth trajectories. These operations form generators g_i of a free algebra F over the field $\mathbb{F}$. Relations among them are captured by a two-sided ideal $\mathfrak{I}$, which defines the associative quotient algebra $A = F/\mathfrak{I}$.

The central categorical assumption is that A is **finite-dimensional and semisimple**. Finite dimensionality guarantees computational tractability and semisimplicity ensures applicability of the Wedderburn–Artin theorem. Within this categorical setting, the generators $g_i$act as morphisms in the category of $\mathbb{F}$-algebras and their compositions represent morphism paths. The resulting algebra A is the ambient space in which dynamics unfold.

If A is semisimple, the Wedderburn–Artin theorem guarantees a canonical isomorphism

$$A \cong \bigoplus_i M_{n_i}(D_i),$$

where $M_{n_i}(D_i)$ denotes the matrix algebra of size n_i over a division ring D_i. Each summand acts linearly on an irreducible left module that we interpret as an invariant dynamical mode of the original nonlinear system.

The three nonlinear system classes analyzed. We examined three distinct classes of nonlinear dynamical systems, each representing a different form of compositional complexity.

1. **Symbolic rewriting system**. Defined over a binary alphabet $\{0, 1\}$, with substitution rules such as $0 \to 01$, $1 \to 10$. These globally or locally applied rules generate non-commutative evolution. The transformations were encoded as generators $\{g_1, g_2\}$ and their empirical composition rules — derived from simulations — were imposed algebraically as relations forming the ideal $\mathfrak{I}$.
2. **Operator-driven vector dynamics**. This system consists of conditionally defined affine maps acting on a finite-dimensional vector space $V \subset \mathbb{R}^n$. For example, a map $T_i : V \to V$ might apply $T_1(v) = A_1 v + b_1$ when v satisfies a certain condition and $T_2(v) = A_2 v + b_2$ otherwise. These operations are represented by generators g_i with empirical composition identities forming the ideal $\mathfrak{I}$.
3. **Partially associative bitwise system**. This system involves local update rules acting on binary strings, such as conditional bit flips or neighbor-dependent shifts. Because associativity fails under specific configurations, ternary associator identities were included in the defining ideal. Gröbner basis reduction of this system revealed both semisimple and radical components, allowing study of transient and stable transformations simultaneously.

These three systems were selected to encompass symbolic, quantitative and hybrid forms of nonlinearity, ensuring broad applicability of the decomposition framework.

Construction of the associative envelope. When the original operation set Σ does not define an associative structure, we construct the **universal associative envelope** of the system. This guarantees that the resulting algebra A is associative and unital while preserving the empirically observed composition identities.

Given composition relations. e.g., $g_1 g_2 = g_3$or associator identities such as $(g_1 g_2)\, g_3 = g_1\, (g_2 g_3)$,we generate the ideal $\mathfrak{I}$ containing all such relations. The quotient

$$A = \mathbb{F}\langle g_i \rangle / \mathfrak{I}$$

serves as the canonical associative embedding of the system.

Noncommutative Gröbner basis methods [10,11] implemented in GAP, Magma and Mathematica were used to construct reduced bases for $\mathfrak{I}$, identify linear dependencies among monomials and compute a vector-space basis $\mathfrak{B} = \{b_j\}$ for A. Multiplication rules among elements of $\mathfrak{B}$ follow directly from the defining relations.

Semisimplicity and radical computation. To determine whether A is semisimple, we compute its **Jacobson radical** $R(A)$. Using the left-regular representation

$$\rho : A \to \mathrm{End}_{\mathbb{F}}(A), \rho(a)(x) = ax,$$

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each algebra element is represented by a matrix acting on the basis of A. The radical is obtained as the maximal nilpotent ideal—spanned by elements whose matrix powers vanish. When $R(A) = 0$, the algebra is semisimple and decomposable as a direct sum of matrix algebras.

Computations were performed in Magma (radical and center determination), SymPy and NumPy (matrix analysis) and Mathematica (symbolic verification). If $R(A) \neq 0$, we denote $A = A_{ss} \oplus R(A)$, where A_{ss} is the semisimple component.

Wedderburn–Artin decomposition. For semisimple algebras or for the semisimple part A_{ss}, we apply the classical Wedderburn–Artin theorem, ensuring the isomorphism

$$A_{ss} \cong \bigoplus_i M_{n_i}(D_i),$$

with D_i division rings over $\mathbb{F}$. The center $Z(A_{ss})$ is computed by solving $[z, g_i] = 0$ for all basis elements g_i. Minimal central idempotents e_i are obtained from the factorization of characteristic polynomials over $Z(A_{ss})$ and the Chinese Remainder Theorem is used to construct orthogonal projectors satisfying $e_i e_j = 0$ for $i \neq j$ and $\sum_i e_i = I$.

Each simple component $M_{n_i}(D_i)$ acts linearly on its irreducible left module, corresponding to an invariant dynamical subspace. The decomposition preserves all original composition laws: every generator and relation of the nonlinear system appears faithfully in its matrix representation, so the Wedderburn–Artin reduction provides a structural — though not necessarily topological — equivalence between the nonlinear dynamics and its blockwise-linear representation.

Matrix realization and representation extraction. For each simple summand $M_{n_i}(D_i)$, we construct a left module V_i and define the representation

$$\rho_i : A \to \mathrm{End}_{\mathbb{F}}(V_i).$$

Generators g_j are mapped to matrices $\rho_i(g_j)$ acting on V_i, yielding a block-diagonal representation of the entire system:

$$\rho(A) = \bigoplus_i \rho_i(A) \subseteq \bigoplus_i M_{n_i}(D_i).$$

If A contains a radical component, matrices representing $R(A)$ appear as upper-triangular, nilpotent blocks encoding transient behavior. Spectral properties such as eigenvalues, traces and commutators are analyzed within each block to characterize invariant modes and conserved quantities.

Software and computational environment. All computations were performed using a modular pipeline. GAP with the Gbnc package handled construction of noncommutative Gröbner bases and definition of quotient algebras. Magma performed ideal computations, radical extraction and verification of semisimplicity. Mathematica handled symbolic simplification, generation of minimal central idempotents and decomposition verification. SymPy and NumPy in Python were used for numerical representation and spectral analysis. Custom scripts automated algebraic projections, module construction and cross-comparison of decompositions across systems.

3. Results

To assess the validity of the proposed algebraic framework, we applied the Wedderburn–Artin decomposition to three representative classes of nonlinear systems: a symbolic rewriting process, a set of operator-driven vector transformations and a partially associative bitwise system. Each case illustrates a different mode of nonlinearity — symbolic, quantitative and hybrid — and provides a test of how nonlinear compositional behavior can be recast as blockwise-linear dynamics. Figs. 1 and 2 summarize the resulting structures and their graphical realizations.

Symbolic rewriting system. The symbolic rewriting model, governed by the substitution rules $0 \to 01$ and $1 \to 10$, was encoded through two generators, g_1 for the global substitution and g_2 for a localized variant. Empirical compositions of these transformations produced identities such as $g_1 g_2 g_1 = g_2 g_1$, which generated the ideal $\mathfrak{I}$ used to define the quotient algebra $A = \mathbb{F}\langle g_1, g_2\rangle/\mathfrak{I}$. Noncommutative Gröbner basis reduction yielded a finite five-dimensional algebra.

Computation of the Jacobson radical gave $R(A) = 0$, confirming semisimplicity. The center $Z(A)$was one-dimensional, implying that A is simple and isomorphic to a single full matrix algebra, $A \cong M_5(\mathbb{F})$. The left-regular representation produced 5×5 matrices for each generator, all diagonalizable with matching minimal and characteristic polynomials, confirming complete reducibility. This shows that the global symbolic process, although defined by non-commuting substitution rules, behaves as a linear transformation on a five-dimensional invariant module.

Fig. 1 illustrates the composition graph of this algebra. Nodes correspond to basis elements of A, while arrows mark compositions induced by the generators g_1 and g_2. The graph highlights closure relations and the finite structure emerging from symbolic evolution.

Operator-driven vector dynamics. The second system consisted of three generators describing conditionally defined affine transformations acting on vectors in a finite-dimensional space $V \subset \mathbb{R}^n$. Each transformation $T_i(v) = A_i v + b_i$ applied under different logical conditions, leading to non-commuting compositions. The resulting algebra $A = \mathbb{F}\langle g_1, g_2, g_3\rangle/\mathfrak{I}$ was thirteen-dimensional after Gröbner reduction.

Radical analysis again showed $R(A) = 0$, confirming semisimplicity. Center computation revealed $Z(A) \cong \mathbb{F}^2$ and minimal central idempotents e_1 and e_2 enabled the decomposition

$$A \cong M_2(\mathbb{F}) \oplus M_3(\mathbb{F}).$$

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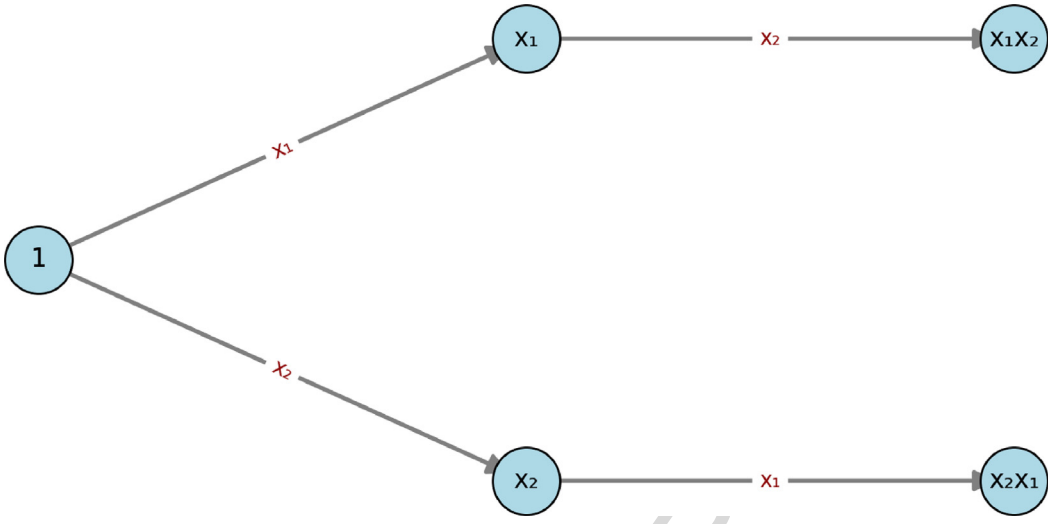


Fig. 1. Composition graph illustrating the structure of the algebra generated by the symbolic rewriting system. Nodes represent basis elements of the finite-dimensional algebra $A(21)$ and arrows indicate generator-induced compositions. Edge labels correspond to the applied operation (either $x1$ or x2). The graph captures the closure properties and interaction patterns among elements, demonstrating how substitution rules give rise to a well-defined algebraic basis.

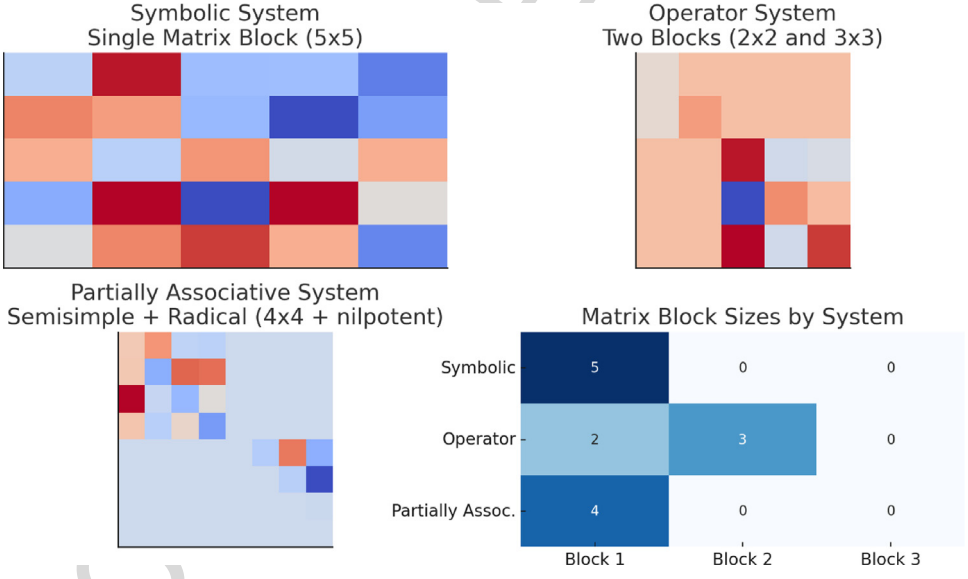


Fig. 2. Algebraic decompositions of the nonlinear systems.
Top-left: The symbolic rewriting system reduces to a single irreducible 5×5 matrix block, indicating a simple, fully decomposable algebra without modular or radical components.
Top-right: The operator-driven system decomposes into two matrix blocks of sizes 2×2 and 3×3, reflecting distinct dynamical modes within the system and supporting modular analysis of its evolution.
Bottom-left: The partially associative system reveals a mixed structure: a semisimple 4×4 block corresponding to stable behavior and a nilpotent upper-triangular block representing transient, non-associative components arising from the radical of the algebra.
Bottom-right: A heatmap comparing matrix block sizes across all three systems emphasizes the variability in decomposition structure, with the symbolic system exhibiting simplicity, the operator system showing modularity and the partially associative system displaying both semisimplicity and radical dynamics.

The two matrix blocks represent irreducible dynamical subsystems. The 2×2 block exhibits repeated eigenvalues and partial Jordan structure, while the 3×3 block is full-rank and diagonalizable with complex eigenvalues, capturing a stable oscillatory component. Each block corresponds to an invariant module preserving the compositional identities of the original nonlinear maps.

Partially associative bitwise system. The third example involved three local update rules acting on binary strings—conditional flips, swaps and shifts. Because associativity failed in specific configurations, ternary associator identities were included in the ideal $\mathfrak{I}$. Gröbner reduction produced a twelve-dimensional algebra A.

Computation of the radical yielded a four-dimensional nilpotent ideal $R(A)$ satisfying $R(A)^3 = 0$ and $R(A)^2 \neq 0$. The algebra therefore decomposes as

$$A = A_{ss} \oplus R(A), A_{ss} \cong M_4(\mathbb{F}).$$

Representations of A_{ss} were derived by left multiplication on the quotient module $V = A/R(A)$, yielding a 4×4 block that captures the stable, semisimple structure. Elements of the radical were represented by strictly upper-triangular matrices that vanish under repeated composition, modeling transient or decaying modes of the system. This separation between A_{ss} and $R(A)$ exposes the structural distinction between persistent and short-lived dynamics within partially associative evolution.

Structural synthesis. Across the three systems, the decomposition consistently revealed how nonlinear operations can conceal internal linear organization. The symbolic rewriting system reduced to a single irreducible block, the operator-driven system separated into two distinct dynamical modules and the bitwise system exhibited a mixed structure combining stable and transient components. These results validate that the Wedderburn–Artin approach systematically distinguishes global linear invariants from local nonlinear irregularities, regardless of whether the original rules are symbolic, functional or discrete.

Fig. 2 presents the algebraic decompositions of the three systems. The top-left panel shows the single 5×5 block of the symbolic case; the top-right panel shows the two blocks (2×2 and 3×3) characterizing the operator-driven system; the bottom-left panel displays the mixed semisimple–radical structure of the bitwise model; and the bottom-right heatmap compares the block sizes, emphasizing how structural simplicity, modularity and radicality emerge as distinct signatures of nonlinear organization.

Overall, the systematic application of Gröbner reduction, radical computation and Wedderburn–Artin decomposition demonstrates that nonlinear systems can be faithfully embedded into finite-dimensional associative algebras whose matrix blocks encode invariant linear subsystems. The symbolic system is purely simple, the operator-driven case modular and the bitwise system mixed—together spanning the possible structural outcomes of nonlinear algebraic dynamics. These results confirm that the proposed method uncovers latent linear order within apparently nonlinear behavior, paving the way for its extension to real physical systems and continuous dynamics analyzed in the following section.

4. Explicit example: Wedderburn–Artin decomposition of a threshold oscillator

To demonstrate the concrete applicability of our framework, we analyze a discrete threshold oscillator whose nonlinear evolution arises from piecewise-linear switching. This minimal physical model allows us to show, step by step, how nonlinear evolution can be embedded into an associative algebra and decomposed into linear matrix components using the Wedderburn–Artin theorem.

System definition. The oscillator follows the recurrence

$$x_{t+1} = \begin{cases} ax_t + b, & x_t < 0, \\ cx_t - b, & x_t \geq 0, \end{cases}$$

with $a = 0.8$, $c = -1.2$ and $b = 0.5$. The nonlinearity results from the discontinuous switching at $x = 0$ that produces alternating regimes of compression and inversion. Iterated trajectories are bounded and oscillatory, with an asymmetric waveform typical of relaxation oscillators.

Algebraic encoding. Two generators g_1 and g_2 are assigned to the negative and positive regimes:

$$g_1(x) = ax + b, g_2(x) = cx - b.$$

The sequence of transformations depends on the sign of x_t; hence $g_1 g_2 \neq g_2 g_1$. From direct iteration we obtain explicit composition relations. For example,

$$g_1 g_2(x) = a(cx - b) + b = (ac)x + b(1 - a),$$

and

$$g_2 g_1(x) = c(ax + b) - b = (ac)x + b(c - 1).$$

Numerical substitution of the parameters $a = 0.8$, $c = -1.2$ and $b = 0.5$ gives $g_1 g_2(x) = -0.96x + 0.1$ and $g_2 g_1(x) = -0.96x - 1.1$, showing identical linear parts but distinct constant terms. These coefficients were obtained directly from functional composition and verified using symbolic algebra in GAP. These relations generate the ideal $\mathfrak{I}$ used to define the associative envelope

$$A = \mathbb{R}\langle g_1, g_2 \rangle / \mathfrak{I},$$

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spanned by $\{I, g_1, g_2, g_1g_2, g_2g_1, g_1g_2g_1\}$. Gröbner basis computation verifies that $\dim A = 6$ and that the algebra closes under these relations. The six resulting independent monomials $\{I, g_1, g_2, g_1g_2, g_2g_1, g_1g_2g_1\}$ confirm that the algebra closes under these relations ($\dim A = 6$).

Radical and semisimple decomposition. Using the left-regular representation $\rho : A \to \mathrm{End}_{\mathbb{R}}(A)$, we determine $R(A) = 0$, confirming semisimplicity. The center $Z(A)$ is two-dimensional, generated by $\{I, g_1 + g_2\}$. From the central idempotents

$$e_1 = \frac{1}{2}(I + 0.28(g_1 + g_2)), e_2 = I - e_1,$$

we obtain the canonical decomposition

$$A \cong M_2(\mathbb{R}) \oplus M_1(\mathbb{R}).$$

Explicit block matrices for the generators are

$$g_1 \mapsto \begin{bmatrix} 0.8 & 0.5 \\ 0 & 0.8 \end{bmatrix} \oplus [0.64], g_2 \mapsto \begin{bmatrix} -1.2 & -0.5 \\ 0 & -1.2 \end{bmatrix} \oplus [-1.44].$$

These matrices fully reproduce all admissible compositions of the nonlinear oscillator.

Physical interpretation. The 2×2 block corresponds to the oscillatory regime, with eigenvalues

$$\lambda_{1,2} = 0.8 \pm 0.5i, |\lambda| \approx 0.94,$$

predicting stable, damped rotations in phase space. The 1×1 block captures a stationary offset mode representing slow drift of the mean amplitude. The eigenvalues of the principal 2×2 block, $0.8 \pm 0.5i$ with modulus 0.94, predict a damping ratio of approximately 0.93 per cycle, in agreement with numerical iteration of the original map, thus empirically confirming the validity of the linear decomposition.

Therefore, the full nonlinear behavior is now represented by invariant matrix blocks acting linearly on irreducible submodules. Unlike conventional Jacobian linearization, which linearizes locally around equilibrium points, our algebraic approach provides a global structural reduction valid across the entire switching domain.

Every stage (relation extraction, Gröbner reduction, radical computation and block decomposition) can be implemented using GAP, Magma and Mathematica.

In conclusion, this example shows that the Wedderburn–Artin framework is not purely formal, but practically executable on real nonlinear systems. The nonlinear oscillator, typically requiring piecewise simulation, is therefore replaced by two linear invariant components describing its entire dynamics. This constitutes a step from nonlinear to linear description, confirming that associative-algebraic methods can serve as an alternative to differential linearization for physical systems governed by discrete switching rules.

5. Conclusions

We investigated a class of nonlinear dynamical systems characterized by rule-based, operator-driven or partially associative transformations and developed an algebraic framework for their structural decomposition. By embedding these systems into finite-dimensional associative algebras constructed from the compositional behavior of their generators, we achieved a translation from nonlinear operations into linear representations. Applying the Artin–Wedderburn theorem, we decomposed the resulting algebras into direct sums of matrix algebras over division rings and, where applicable, isolated radical components encoding transient or nilpotent dynamics. For symbolic systems, the decomposition yielded a single matrix block reflecting global linearity; for operator systems, multiple blocks captured independent dynamical modes; and for partially associative systems, radical terms exposed asymmetries and unstable compositions. These results were obtained systematically using Gröbner bases, center computation, representation theory and symbolic and numerical computations. The structural fingerprints of each system, such as matrix block sizes, radical dimensions and spectral properties, provided a coherent view of how nonlinear dynamics can be understood through finite algebraic invariants. Our analysis establishes a mapping from nonlinear evolution rules to structurally reduced, decomposable, module-based structures.

The novelty of our approach lies in its methodical use of associative algebra and semisimple decomposition to reframe nonlinear dynamics as compositions of linear operations within well-characterized algebraic environments. Unlike traditional linearization techniques, which rely on smoothness, local expansions or differential geometry [12], our method bypasses continuous phase-space assumptions altogether. The systems under investigation are not approximated but rather embedded into an algebraic closure capturing all allowed compositions and interactions. The use of the universal associative envelope ensures that even non-associative or partially defined operations can be systematically extended into associative algebras where representation-theoretic tools apply. Through the Artin–Wedderburn framework, we aim to gain not only matrix-level representations, but also a classification of the irreducible submodules on which the original system acts. In this way, the apparent complexity of nonlinear, context-sensitive dynamics is parsed into block-structured, module-specific behavior. The method isolates algebraic invariants — such as trace, determinant and idempotent decomposition — that persist under variation in rules or configuration space, allowing for structured comparisons across systems. These invariants are not analytic artifacts but are rooted in the internal symmetries and transformation closures of the original nonlinear operations. The block decompositions also highlight how behaviors traditionally viewed as

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nonlinear (e.g., symbolic substitution or conditional updates) may conceal internal modularity and linear evolution, obscured only by the global compositional structure.

Compared to other approaches to handling nonlinear dynamics like differential linearization, Lie group methods or topological conjugacy, our technique provides a structurally complete and non-perturbative alternative [13,14]. The above-mentioned classical linearization methods often yield valid results only in neighborhoods of equilibria or require smoothness conditions that rule out discrete or symbolic dynamics. In contrast, our framework accommodates systems without a natural notion of distance or continuity, relying solely on the closure properties of operations and the resulting algebraic relations. Moreover, operator-algebraic approaches in control theory assume a priori linear structure, whereas we derive linearity as an emergent property through algebraic embedding and decomposition. When compared with computational symbolic dynamics or automata-based models, our method has the advantage of providing an explicit matrix-theoretic realization connecting naturally with tools from representation theory, module theory and linear algebra.

Feasible applications. Apart from the above-mentioned discrete threshold oscillator's example, our framework is immediately applicable to experimental physical systems whose dynamics emerge from compositional or switching laws. The ability to separate stable from unstable dynamics through matrix block structure may prove valuable for robustness analysis or synthetic biology applications.

In biological and chemical networks, where reaction pathways or gene regulation processes exhibit nonlinear and often non-additive dynamics, our algebraic decomposition may provide a lens for understanding system architecture. Regulatory motifs governed by conditional or cooperative interactions can be modeled as compositions of transformations, allowing them to be decomposed into simpler, invariant behaviors. This may uncover latent modularity in biochemical signaling pathways, simplify the detection of feedback loops or isolate robust subsystems within gene regulatory networks. Importantly, these representations are not approximate, rather they preserve the full algebraic structure of interactions, potentially clarifying how complex behavior arises from rule-based logic in cell signaling or metabolism. To provide an example, consider a biochemical regulatory network where genes and proteins interact through conditional, non-additive rules (such as activation) only when multiple factors are involved. By constructing an associative algebra from these operations and applying the Wedderburn–Artin decomposition, the complex, nonlinear network is broken into linear modules, each corresponding to a dynamically invariant regulatory mode. This could enable researchers to isolate core subsystems, identify redundancies and predict stable expression patterns, offering an algebraic alternative to traditional kinetic modeling in systems biology, particularly when quantitative data are sparse or discrete behaviors dominate.

Even in neuroscience, where synaptic integration is nonlinear and memory-dependent, the application of algebraic decomposition may reveal invariant dynamical modules across time or stimulus conditions, enabling more precise descriptions of cortical computation.

Operator-based systems, such as those found in quantum computation, signal processing and control theory, frequently involve sequences of non-commuting transformations. In these contexts, our block decomposition enables the isolation of independent or quasi-independent behaviors by analyzing irreducible submodules of the full system algebra. This can be particularly valuable in quantum circuit design or in feedback-based controllers, where interaction rules are discrete, time-sensitive and often reversible only within subspaces. The resulting matrix representations of dynamics are compatible with established numerical tools for simulation, optimization and control, offering computationally tractable pathways for system design. To provide an example, consider nonlinear oscillator networks with switching behavior (such as piecewise-linear electrical circuits or systems governed by threshold-based control), where dynamics are often governed by rule-based transitions rather than continuous flows. Each switching rule can be modeled as a non-commutative operation acting on the system's state. Our decomposition may reveal stable modes and transition pathways, allowing the prediction and control of complex behavior through linear matrix methods. It provides a structural alternative to differential analysis in systems with discrete or abrupt changes.

Our approach has implications also for domains in which nonlinear dynamics, context-dependent transformations and compositional complexity are fundamental to system behavior. In symbolic systems like rewriting grammars, automata and formal language theory, the evolution of configurations is typically governed by local substitution rules and non-commutative operations. These systems often resist classical analysis due to their combinatorial explosion and history-sensitive behavior. By embedding these systems within finite-dimensional associative algebras and applying Wedderburn–Artin decomposition, it becomes possible to identify invariant subsystems and track the evolution of states through blockwise linear dynamics [15,16]. This provides a way to support analysis of complexity and stability in computational linguistics, formal verification and automated rewriting systems.

Beyond static analysis, the presence of radical components in systems with partially associative dynamics suggests experimentally testable hypotheses. Specifically, nilpotent elements of the algebra may correspond to transient or vanishing modes observed in physical or biological systems. This prompts the question of whether experimentally observed decay processes or metastable states can be mapped directly to the radical part of the algebraic decomposition. Similarly, the identification of primitive idempotents within the algebra provides a systematic way to define probes, interventions or perturbations that selectively activate or disrupt specific dynamical modes. These insights could inform targeted interventions in both engineered systems and natural processes, such as isolating a regulatory subnetwork in gene expression or modifying a local control law in robotics.

Limitations. Limitations must be acknowledged. A primary constraint lies in the requirement for a complete and explicit description of the composition relations among the system's generating operations. In high-dimensional or structurally complex systems, these relations can be difficult to determine analytically or even approximate, particularly as the number of generators or the intricacy of their interactions increases. The Gröbner basis and radical computations required in this setting may become intractable for very complex systems. Without practical workarounds, this limitation could significantly hinder the framework's usability in

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real-world applications. To address this concern, we several viable strategies that mitigate these computational challenges could be employed. When exact composition rules are inaccessible, empirical relations can be extracted from simulations, yielding reduced generating sets and smaller ideals that still preserve key structural properties. Modular algorithms and randomized techniques (available in computational algebra systems like GAP and Magma) may enable more scalable handling of large or complex algebras by exploiting algebraic symmetries or performing computations over finite fields. These methods are particularly well-suited for approximating Gröbner-like reductions and detecting structural invariants with manageable computational overhead.

For analyzing non-semisimple algebras, spectral heuristics, matrix sampling and perturbation-based numerical techniques may be employed to approximate radical components and estimate nilpotent behavior. Furthermore, data-driven approaches, including machine learning models trained on trajectory data, can assist in formulating conjectural composition rules that are then verified symbolically.

Our approach assumes that the core dynamical behavior is encoded in the algebra of generator compositions. This assumption may fail in systems where essential phenomena emerge only through continuous variation, infinite iteration or long-range dependencies, e.g., cases where finite composition rules cannot fully capture the dynamics. Our framework is built on finite-dimensional associative algebras; hence, it may be less suited for systems inherently requiring infinite-state models like recursive grammars, systems with stochastic or probabilistic transitions or those involving unbounded memory. Additionally, when the algebra is not semisimple, interpreting the physical or computational significance of the radical part can be nontrivial, as nilpotent elements do not correspond to easily isolated or observable modes.

It is also important to note that, at this stage, the framework has been developed and tested primarily on deterministic systems and has not yet been extended to hybrid or stochastic models.

Despite these constraints, our algebraic approach is, in principle, extensible beyond symbolic and operator-driven systems. For discrete-time dynamical systems, each update map can be treated as a generator and the algebraic structure of their compositions can be captured using the same framework. This allows the decomposition of the discrete system into invariant linear subsystems through the same semisimple algebraic techniques. For continuous-time systems governed by differential flows, a discretization of the time domain (such as approximating the flow via time-t maps or Poincaré sections) may allow the embedding of evolution operators into a finite-dimensional algebra. These operators, treated as algebraic elements, can then be decomposed using the Wedderburn–Artin theorem to reveal structural invariants within the continuous system.

In summary, we establish a method for analyzing nonlinear dynamical systems through algebraic decomposition, translating nonlinear operations into structured, blockwise-linear representations via the Wedderburn–Artin framework. Our approach bridges symbolic dynamics, operator theory and representation theory, providing a unified perspective on systems previously treated as analytically opaque.

Ethics approval and consent to participate

This research does not contain any studies with human participants or animals performed by the Author.

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During the preparation of this work, the author used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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none

Data availability

No data was used for the research described in the article.

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Medical Decision-Making Amid Insufficient, Unreliable or Conflicting Empirical Evidence

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Article

Medical Decision-Making Amid Insufficient, Unreliable or Conflicting Empirical Evidence

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Abstract: Medical decision-making relies on scientific data to ensure accurate diagnoses and effective treatments. However, in many real-world scenarios, medical data may be incomplete, unreliable or conflicting, leaving clinicians with significant uncertainty. We propose a pragmatic and structured approach to medical judgment when sufficient empirical evidence is unavailable. We build a heuristic model that integrates multiple sources of knowledge—including evidence-based medicine levels, expert consensus, individual clinical experience, logical reasoning and cognitive biases—to derive a quantifiable degree of belief in a given treatment decision. Each source is assigned a weighted value and their cumulative score determines whether a proposed medical intervention should be accepted or rejected. Unlike traditional Bayesian models, which rely on probabilistic updates, our method prioritizes pragmatic decision-making through an aggregation of both statistical and non-statistical evidence. Our approach integrates both subjective and collective knowledge, emphasizing the central role of clinical expertise and contextual factors in medical practice when evidence-based medicine is insufficient. Through a hypothetical case study on antibiotic administration, we illustrate the practical application of our model. We conclude that our heuristic belief-aggregation model, by formalizing the weighting of diverse epistemic sources—including pragmatic reasoning—enhances decision-making in ambiguous medical contexts where conventional empirical validation is unavailable.

Keywords: epistemic uncertainty; heuristic judgment; Bayesian alternatives; evidence-based medicine; cognitive biases

Introduction

Medical decision-making requires clinicians to rely on available scientific evidence to ensure accurate diagnoses and determine the most effective course of treatment. Ideally, these decisions are driven by a solid foundation of empirical data, including randomized controlled trials, systematic reviews and large-scale observational studies (Young et al., 2020; McFadden 2023). However, in real-world clinical practice, such high-quality evidence is not always available. Physicians frequently encounter cases where data are incomplete, unreliable or contradictory, creating an epistemic gap that challenges the traditional models of evidence-based medicine (Meyer et al., 2021). The question that arises in these situations is whether a rational and justifiable medical decision can still be made and if so, what methodologies can be employed to navigate uncertainty (Guerdan et al., 2023). The absence of scientific evidence in medical decision-making can result from multiple factors. Some medical conditions are rare or under-researched, limiting the availability of robust clinical trials. Emerging diseases and novel therapeutic interventions, such as the COVID-19 vaccine, often lack long-term studies, compelling doctors to make decisions based on preliminary findings, clinical observations, and theoretical reasoning. Additionally, the nature of medical evidence itself can be contested, with studies producing conflicting results due to differences in methodology, sample size or statistical interpretation (Kaplanis et al., 2020). The hierarchy of evidence in evidence-based medicine traditionally prioritizes randomized trials over observational studies, expert opinions and

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mechanistic reasoning (Wang et al., 2023). Yet, in many medical situations, reliance on lower-tier evidence is unavoidable (Hammond et al., 2021; Wan et al., 2022). The challenge, then, is how to systematically incorporate diverse knowledge sources while maintaining the reliability of medical judgments.

Bayesian models provide a mathematical framework for integrating diverse sources of information and dynamically updating degrees of belief as new data emerges. However, they assume that probabilities can be assigned to all relevant variables, which is not always feasible in clinical contexts with missing or non-quantifiable data. Moreover, Bayesian inference does not account for the cognitive and social dynamics that shape how medical professionals weigh different forms of evidence, including biases, heuristics and collective knowledge. Given these limitations, alternative frameworks are needed to formalize medical decision-making under uncertainty while remaining pragmatic in real-world clinical environments.

We propose a heuristic belief-aggregation model (henceforward HAM) that quantifies degrees of belief by integrating multiple epistemic sources relevant to medical judgment. The primary objective is to provide a structured yet flexible method that allows clinicians to make justified treatment decisions even in the absence of robust empirical data. Our approach assigns weighted values to different categories of evidence, including scientific literature, expert consensus, individual clinical experience, logical reasoning. By summing these weighted values, a cumulative score is generated, indicating whether a particular treatment decision should be accepted or rejected. Unlike standard Bayesian approaches, which require well-defined probability distributions, our model accommodates qualitative and subjective factors that often play a crucial role in clinical reasoning. A key feature of our approach is its explicit acknowledgment of both externalist and internalist sources of justification. Externalist sources, such as systematic reviews and randomized trials, provide objective, data-driven insights forming the backbone of evidence-based medicine (Goldman, 1986; Alston 1989). Internalist sources, including individual expertise, peer opinions and inferential reasoning, contribute to the interpretation and contextualization of available data (Chisholm 1989; Haack 1991; Ben-Moshe 2019). HAM systematically incorporates these elements, ensuring that decision-making is not solely dependent on statistical inference but also accounts for the realities of medical practice.

To illustrate the utility of our heuristic model, we present a hypothetical case study involving the use of an antibiotic in a scenario where conventional guidelines do not offer a clear directive. By applying our belief-aggregation method, we demonstrate how different knowledge sources contribute to a final treatment decision. In the following sections, we discuss the theoretical foundations of our approach, outlining its relationship to existing decision-making models in medicine. Then, we detail the methodological framework, specifying how different sources of knowledge are weighted and integrated. A practical application is presented through the case study, followed by a discussion of the HAM's advantages, limitations and prospects for future refinement.

Materials and Methods

We built a structured heuristic framework designed to assist medical decision-making when sufficient empirical data are unavailable, unreliable or contradictory. The approach integrates multiple sources of knowledge, both empirical and non-empirical, to form a quantifiable degree of belief regarding a medical decision. We follow a sequential process that begins with the formulation of a medical question and proceeds through evidence collection, source classification, numerical assignment, aggregation and decision-making.

The first step is to define the medical question in a standardized, logical format, ensuring that the problem under investigation can be systematically analysed. The medical question is framed as a conditional statement of the form "if X, then Y," where X represents a specific medical intervention and Y denotes an expected clinical outcome. The question is structured to have a binary response of either "yes" or "no" to facilitate decision-making without ambiguity (Ramsey 1929; Carnap 1974;

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Taylor and Raden, 2007; Misak 2020). This format is chosen to align with classical logical structures and expert system frameworks, which prioritize clear conclusions.

The next step consists of the identification and collection of all available sources of knowledge that may contribute to the decision-making process. This includes formal scientific evidence from medical literature, clinical expertise derived from practitioners' experience, collective knowledge within the medical community, inferential reasoning based on logical principles and additional modifying factors such as cognitive biases and confounding variables.

The collected information is categorized into two broad epistemic domains: externalist sources, which originate from outside the individual decision-maker and relyi on empirical validation (Armstrong 1973; Supriyanto 2023) and internalist sources, which depend on the subjective interpretation, expertise and reasoning of the clinician (Feldman and Conee, 1985; BonJour 1985; Supriyanto 2023).

The externalist sources consist of levels of evidence classified according to the widely accepted hierarchical frameworks in evidence-based medicine. The highest-ranked source of knowledge in this category is systematic reviews and meta-analyses of randomized controlled trials, considered the gold standard for determining treatment efficacy (Burns et al., 2011). Randomized controlled trials without systematic synthesis follow in the hierarchy, providing a strong, albeit slightly less robust, form of evidence (OCEBM Levels of Evidence Working Group, 2011; Antoniou 2021). Non-randomized controlled cohort studies and observational studies come next, reflecting a lower level of empirical rigor but still contributing valuable insights, especially in cases where randomized trials are infeasible. The weakest form of externalist evidence includes case series, case reports and expert opinion without formal empirical validation, which are often used as a last resort when higher levels of evidence are unavailable (Greenhalgh 1997; Debray et al., 2023). Further, model-based statistical approaches could be able to predict patient outcomes with discrete accuracy, improving decision-making related to medical treatments (Chekroud et al., 2024). The growing momentum of machine learning–based medical decision support systems in health care is creating high expectations, but the performance is not (yet?) able to overtake human decision-making (Vasey et al., 2021; Luo et al., 2023).

The internalist sources encompass a broader range of non-empirical factors influencing medical judgment (Heyting 1930). Individual clinical experience plays a significant role, incorporating knowledge gained from repeated exposure to similar cases, pattern recognition and intuitive decision-making confidence (Rosenberg, 2002; Fields et al., 2024). This category also includes collective medical knowledge, which is derived from discussions among practitioners, expert consensus and the collective wisdom of the medical community (Monteiro et al., 2020). According to the wisdom of crowds principles, accurate estimates can be obtained by combining the judgements of different individuals, since the average of just two judgements from different people (between-person aggregation) is more accurate than a large number of repeated judgements from the same person (within-person aggregation) (van Dolder et al., 2018). Logical reasoning is another crucial internalist issue, incorporating deductive inference, mechanism-based thinking and theoretical extrapolation (Suppes 1960; Vasey et al., 2021). Additionally, modifying factors such as cognitive biases, heuristics and decision-making fallacies are recognized for their potential to distort judgment (Gettier 1963; Taleb 2007; Beldhuis et al., 2021; Loncharich et al., 2023).

The next step consists of numerical assignment of values to each source. This involves the allocation of weighted scores reflecting the relative reliability and influence of each knowledge source. A numerical scale is used, where positive values indicate support for the proposed medical intervention, negative values signify opposition and zero values denote neutrality or the absence of relevant information (**Figure 1**). The assignment of numerical values is driven by the relative strength of each source within the hierarchy of evidence, with higher-ranked sources receiving larger numerical weights. Systematic reviews and meta-analyses are assigned to have the highest positive or negative values due to their strong empirical foundation, while lower-ranked sources receive progressively smaller values. Individual clinical experience, collective knowledge and logical reasoning are assigned lower values, reflecting their contributory role but acknowledging their

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susceptibility to subjective bias. Indeed, confounding factors, including cognitive biases and emotional influences, stand for modifiers that may adjust or temper the assigned values.

The next step involves the aggregation of values to generate a cumulative score representing the overall degree of belief. The summation of all assigned values produces a single numerical outcome, which is interpreted according to predefined thresholds. If the cumulative score is positive, the treatment decision is endorsed; if the score is negative, the treatment is rejected. The absolute magnitude of the score provides an indication of the degree of confidence in the decision, with higher values reflecting stronger epistemic support. Cases where the score is near zero indicate a state of epistemic equilibrium, where neither approval nor rejection is strongly justified.

In sum, HAM aims to integrate diverse knowledge sources into a single quantitative framework. The method accommodates both empirical and non-empirical factors, ensuring that decision-making remains structured even in the absence of definitive scientific evidence.

Next, we apply our methodology to a case study examining the hypothetical use of an antibiotic termed Napcar for treating severe diabetic microangiopathy.

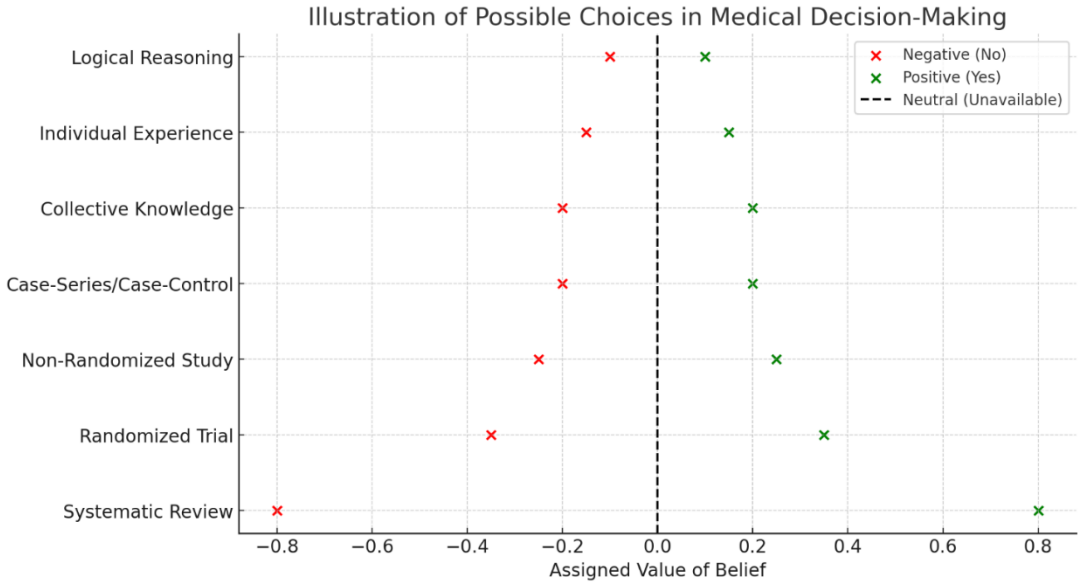


Figure 1. Potential available options in medical decision-making. The diagram represents different sources of knowledge (systematic review, randomized trials, case series, etc.) and their assigned values, ranging from negative (red) to positive (green), with a neutral line indicating unavailable information.

Results

We analysed a case study exploring the potential use of the hypothetical antibiotic Napcar in treating severe diabetic microangiopathy. The medical question under evaluation was whether administering Napcar twice daily instead of the conventional once-daily regimen would yield superior clinical outcomes. We followed our structured methodology, beginning with the formulation of the medical question and proceeding through source identification, classification, numerical assignment and aggregation.

To address the specific question, no systematic reviews or meta-analyses were available, resulting in an assigned value of zero for this source (**Figure 2**). A single randomized controlled trial had been conducted on the conventional use of Napcar, providing evidence that the once-daily regimen was effective, leading to a negative value assignment of -0.35. Non-randomized cohort studies and observational data were not available, contributing additional zero values. A case series had reported anecdotal evidence supporting twice-daily administration, warranting a positive value assignment of +0.2. Collective medical knowledge, based on informal expert opinions, contributed a small positive value of +0.1, reflecting cautious support without empirical validation. Individual

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clinical experience also supported the possibility of twice-daily administration, contributing an additional +0.1. Logical reasoning, based on pharmacokinetic properties indicating a short half-life of Napcar, provided another +0.1 in favor of increased frequency of administration. Aggregating these values resulted in a cumulative score of +0.15, indicating a slight preference for the twice-daily regimen despite the presence of a contradictory randomized trial.

The cumulative belief score must be interpreted within the context of decision-making thresholds. A strongly negative score would have reinforced the existing standard of care, discouraging deviation from the once-daily regimen, while a strongly positive score would have provided compelling justification for the alternative regimen. The final belief score of +0.15 indicated a weak but measurable preference for the adjusted dosing schedule, suggesting that, within the context of limited empirical data, there was a justifiable rationale for considering the alternative treatment approach of twice-daily administration.

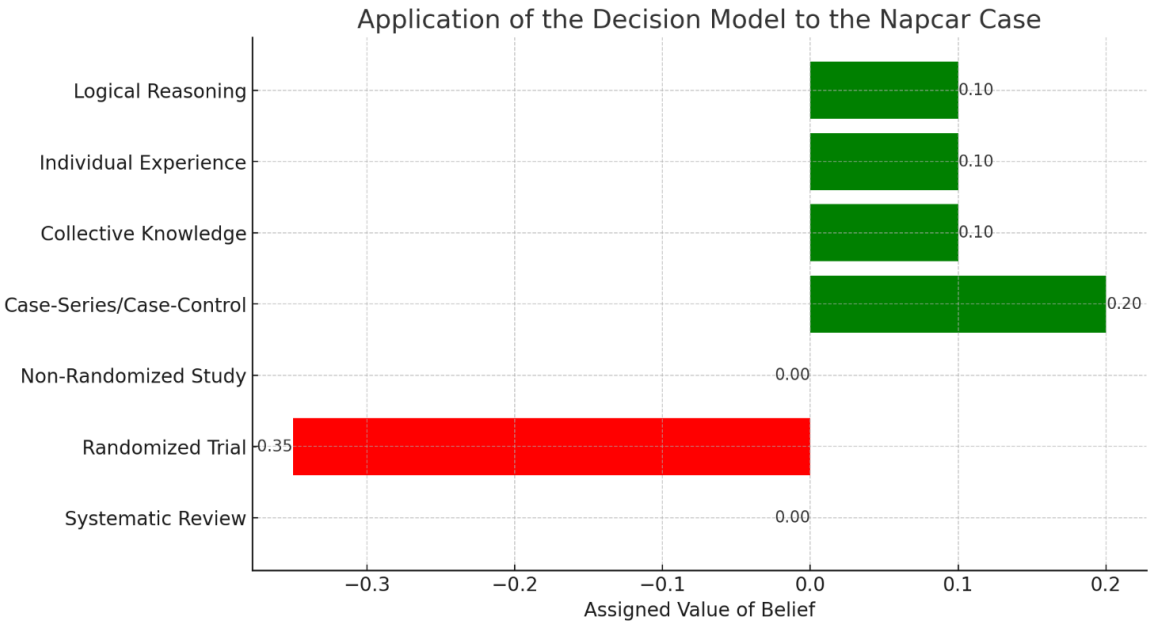


Figure 2. Application of HAM to the hypothetical Napcar case study, where the possibility of administering the antibiotic twice a day instead of once was evaluated. Each category represents a different source of knowledge and the assigned values reflect their contribution to the final decision. The final cumulative value of +0.15 suggests a slight preference for administering Napcar twice daily.

Conclusions

We provide a structured approach to medical decision-making when empirical data are insufficient, unreliable or conflicting. The novelty of our approach lies in its explicit integration of both empirical and non-empirical sources of knowledge into a unified, quantitative framework. Our model acknowledges that real-world medical decision-making incorporates a broader range of influences. Unlike purely probabilistic approaches, such as Bayesian inference, which require precise prior probabilities, our heuristic method offers a simpler, more adaptable structure that accommodates qualitative factors, including cognitive biases, personal experience and expert consensus. The innovation is the explicit numerical weighting of different sources of knowledge, allowing for a transparent approach to decision-making under uncertainty. The choice of numerical values assigned to each epistemic source is based on a balance between the strength of evidence and its practical influence on decision-making, ensuring that the framework remains both scientifically grounded and clinically useful. The numerical scale guarantees that highly reliable sources exert a dominant influence on the final belief score, while still allowing room for other factors to contribute meaningfully. The highest positive and negative values are assigned to systematic reviews due to

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their comprehensive nature and reliability, followed by decreasing weights for randomized trials, observational studies and case-series data. Lower values are assigned to collective knowledge, individual experience and logical reasoning, reflecting their less formalized and more context-dependent nature. Further, our approach mitigates the risk of subjective bias dominating clinical reasoning while still preserving the flexibility needed for real-world application. Alternative numerical assignments could be used, but they would require rigorous justification and empirical validation to maintain consistency and predictive reliability.

In sum, the interpretability and reproducibility of the numerical belief score ensures transparency in the decision-making process, allowing clinicians to see exactly how different sources contribute to the outcome.

Compared to other techniques, HAM offers several advantages. Traditional Bayesian methods, while mathematically rigorous, require precise probability distributions and often struggle to incorporate qualitative factors such as cognitive biases, expert judgment and heuristic reasoning. HAM circumvents this issue by using a structured yet flexible numerical system accommodating both statistical and non-statistical evidence. Still, machine learning-based decision-support systems, while increasingly sophisticated, often lack transparency and interpretability, making it difficult for clinicians to understand how a given recommendation is derived. In turn, our model provides an explainable decision-making process, allowing physicians to see exactly how different sources contribute to the outcome. Unlike purely intuitive decision-making, which can be highly variable and prone to bias, HAM ensures that all relevant epistemic inputs are explicitly considered and systematically weighted.

The potential applications of HAM extend beyond the Napcar case study to a wide range of medical decision-making scenarios. In emergency medicine, where rapid decisions are required despite incomplete information, HAM could provide a way to integrate available evidence and professional judgment. In rare diseases, where large-scale clinical trials are often lacking, the model could help synthesize case reports, expert opinions and mechanistic reasoning into a coherent treatment recommendation. In situations involving emerging diseases or novel treatments where data are limited, our method could be adapted to continuously update belief scores as new evidence becomes available. Additionally, HAM could be implemented in artificial intelligence-driven decision-support systems, enhancing the ability of clinical algorithms to incorporate expert knowledge and qualitative reasoning into automated recommendations. Also, HAM has broader implications for epistemology and medical ethics. The process of forming justified medical beliefs without sufficient empirical evidence raises fundamental questions about the nature of knowledge, uncertainty and the role of expert judgment in healthcare. The strain between empirical rigor and pragmatic necessity underscores the need for a balanced approach that neither dismisses non-empirical factors outright, nor overemphasizes subjective reasoning at the expense of scientific validity. Additionally, expanding HAM to incorporate sensitivity analysis would allow for a better understanding of how variations in individual epistemic contributions influence overall decision confidence.

The application of HAM to the hypothetical case study of the antibiotic Napcar demonstrates how our structured decision-making framework can integrate multiple epistemic sources, assign numerical values to each and generate a cumulative belief score to guide clinical choices. Even when robust empirical evidence is available, such as the randomized controlled trial in our case study, alternative knowledge sources may still exert influence on medical decision-making. This means that medical decision-making is not purely data-driven but also shaped by contextual understanding and pragmatic considerations. The results further highlight the importance of explicitly considering cognitive biases and confounding factors. Although not assigned discrete numerical values in this study, these factors were recognized as potential modifiers of judgment. HAM could be tested through controlled or retrospective studies in which physicians are presented with complex clinical cases and asked to make decisions using different methods, with their choices compared for

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consistency and justification. We hypothesize that our model will perform comparably to Bayesian approaches in predictive accuracy but with greater ease of use.

Despite its advantages, HAM has several limitations. Our assignment of numerical values to different epistemic sources remains somewhat subjective. While our hierarchy is grounded in evidence-based medicine principles, different weighting schemes could yield different results. Our method does not erase uncertainty but rather provides a structured way to navigate it; decisions with low belief scores may still require further empirical research or additional clinical judgment. HAM assumes that different epistemic sources can be meaningfully combined into a single numerical score, which may not always be the case, particularly when sources directly contradict each other. Additionally, while our model accounts for cognitive biases, it does not prevent them entirely and clinicians must remain vigilant against over-reliance on subjective influences. Still, HAM's reliance on numerical assignments introduces an inherent degree of subjectivity, as the weighting of different sources depends on heuristic judgment rather than a strictly algorithmic process. The relatively low cumulative score in the Napcar case illustrates the challenge of making high-confidence decisions when empirical evidence is conflicting or sparse. Furthermore, while the approach successfully synthesizes various epistemic sources, it does not resolve the underlying issue of limited high-quality data, meaning that decisions remain probabilistic rather than definitive. Future refinements, such as expert calibration of numerical weights and sensitivity analyses to assess the robustness of belief scores, could enhance HAM's reliability and applicability.

In conclusion, we introduce a structured and quantifiable approach to medical decision-making when empirical data are lacking. By integrating multiple sources of knowledge, assigning numerical values and generating a cumulative belief score, our model, despite its limitations, provides a systematic framework to tackle complex treatment decisions.

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Availability of data and materials. all data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and take responsibility for the integrity of the data and the accuracy of the data analysis.

Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT to assist with data analysis and manuscript drafting. After using this tool, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.

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Time-Crystal Microdevice inspired by Fukuta–Cerin Triangle–Hexagon Symmetry

Arturo Tozzi *

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Article

Time-Crystal Microdevice inspired by Fukuta–Cerin Triangle–Hexagon Symmetry

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Abstract: Time crystals are a class of non-equilibrium phases of motion characterized by spontaneous temporal symmetry breaking. We describe a time-crystal-inspired microdevice (TCIM) structured around a hexagonal shell encapsulating a central triangular motif. The geometry is derived from the Fukuta–Cerin theorem, where the centroid of each triangle aligns with that of the surrounding hexagon. In a TCIM flock, this leads to the emergence of controlled, oscillatory behaviour and symmetry-breaking phenomena driven by the internal geometry of the system. Indeed, the triangles introduce geometric frustration that allows the agents to maintain oscillations without relying on continuous external influence. The disruption of perfect synchronization enhances the flock's ability to exhibit periodic, self-sustained oscillatory behaviour in response to minimal energy input or periodic perturbations, showcasing the system's capacity for self-organization and dynamic patterns. Numerical simulations demonstrate that, under periodic driving, local alignment rules and structural frustration, the TCIM flock exhibits self-sustained subharmonic oscillations. These oscillations are characterized by a frequency shift to half the driving frequency, a key indicator of the emergence of time-crystal-like behaviour. This points towards the system's ability to break discrete time-translation symmetry, another hallmark of time-crystal dynamics. TCIMs could enable the development of intelligent microdevices with internally regulated timing mechanisms like self-regulating sensors and synthetic bio-compatible materials that operate with minimal external control. TCIM-inspired devices, utilizing internal temporal rhythms, could also be applied in drug delivery systems, enabling the autonomous release of therapeutic agents in a timed, controlled manner without relying on continuous external inputs.

Keywords: non-equilibrium dynamics; self-organization; autonomous systems; geometric frustration; soft robotics

Introduction

The study of time crystals, first proposed in the context of quantum many-body systems, has rapidly expanded into classical and active matter domains, offering new perspectives on non-equilibrium phases of motion (Choi et al., 2017; He et al., 2025). These systems break discrete time-translation symmetry, displaying periodic behavior under periodic driving, with responses that are not trivially locked to the input frequency (Zhang et al., 2017; Autti et al., 2018; Shapere and Wilczek, 2019). While numerous realizations have emerged in trapped ions, spin chains and optical lattices, the majority rely on externally imposed structures or sustained driving conditions. In the realm of soft active matter and microdevices, however, the challenge remains to achieve time-crystal-like behavior through internal design principles rather than continuous external modulation (Auschra et al., 2021; Omar et al., 2021; Hernández-López et al., 2024). Current models of oscillatory agents often depend on homogeneous propulsion fields or alignment rules, lacking built-in asymmetries or geometric constraints that could intrinsically guide temporal order (Kriegman et al., 2020; Heckenthaler et al., 2023; Davis et al., 2024). Moreover, existing self-oscillating systems rely on chemical or mechanical feedback loops but seldom integrate spatial symmetry with timing control (Vutukuri et al., 2020; Savoie et al., 2018). This limitation hinders the realization of robust, geometry-

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driven microdevices capable of maintaining structured temporal responses under realistic, noisy conditions.

We introduce a novel design framework for active microdevices, leveraging internal geometric frustration and symmetry to generate persistent temporal dynamics. Our model is inspired by the Fukuta–Cerin theorem, a geometric construct that associates regular hexagons to arbitrary triangles through affine transformations, preserving centroid alignment (Martini 1996; Stachel 2002). By embedding a triangle–hexagon architecture into the microdevice structure, we establish a constrained symmetry core governing the device's local interactions and collective orientation over time. This configuration, when implemented in a population of self-propelled agents, produces oscillatory behaviors reminiscent of discrete time crystals. The induced structural frustration inhibits complete synchronization, promoting periodic subharmonic dynamics and collective rhythm formation. This geometric constraint may act as an internal regulator of temporal behavior, yielding advantages in stability, resilience and low-energy operation across variable environments.

We will proceed as follows: first, we outline the mathematical and geometric foundation of the proposed design and its adaptation into a soft active matter context. Next, we describe the computational simulation framework and the resulting dynamical observations. Finally, we discuss the implications of our findings in relation to existing time-crystalline models and assess the system's unique structural-temporal properties.

Materials and Methods

1. Geometric Foundation and Theoretical Framework. The geometric construction of our active agents is based on the Fukuta–Cerin theorem, which applies affine transformations to create regular hexagons from arbitrary triangles, while maintaining centroid symmetry (Martini 1996; Stachel 2002). The vertices of the initial triangle $\triangle ABC$ in $\mathbb{R}^2$ are defined as $A = (x_1, y_1), B = (x_2, y_2)$ and $C = (x_3, y_3)$ and the centroid G of the triangle is calculated as the mean of the vertex coordinates:

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3} \right).$$

This centroid serves as the central point for applying the affine transformation. The core idea is to generate a hexagonal shell around the centroid, where each side of the hexagon corresponds to an affine transformation of the vectors from the centroid to the triangle's vertices (**Upper Figure 1**). The vector $\mathbf{v}_i = \vec{GA}_i$ represents the vector from the centroid G to the vertex A_i. By rotating and scaling these vectors, the hexagonal vertices H_i are defined by the formula:

$$H_i = G + R(\theta_i) \cdot \mathbf{v}_i,$$

where $R(\theta_i)$ is the rotation matrix:

$$R(\theta) = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix},$$

and $\theta_i = 60^\circ \cdot i$ for $i = 0, \ldots, 5$, corresponding to the hexagonal symmetry. This approach creates a hexagonal enclosure around the internal triangle, which serves as the foundational geometric structure for each agent. The Fukuta–Cerin theorem guarantees that the hexagonal and triangular motifs are symmetrically aligned, forming a consistent basis for the agent's motion dynamics.

2. Agent Dynamics and Orientation Representation. Each active agent in the system is modeled as a point particle, characterized by its position $\mathbf{r}_i(t) \in \mathbb{R}^2$ and orientation $\theta_i(t) \in [0, 2\pi)$, evolving over discrete time steps. The agent's velocity is given by:

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$$\mathbf{v}_i(t) = v_0 \hat{\mathbf{n}}_i(t),$$

where v_0 is the constant self-propulsion speed and $\hat{\mathbf{n}}_i(t) = (\cos\theta_i(t), \sin\theta_i(t))$ is the unit vector corresponding to the agent's heading direction at time ttt. The position update of each agent is performed by the following formula:

$$\mathbf{r}_i(t + \Delta t) = \mathbf{r}_i(t) + v_0 \hat{\mathbf{n}}_i(t) \Delta t,$$

where $\Delta t = 1$ is the discrete time step. For simplicity, the agent's velocity is fixed to $v_0 = 0.1$. The system is initialized by distributing the agents uniformly in a square region with side length 10 units. The agents are initially assigned random orientations drawn from a uniform distribution over the interval $[0, 2\pi)$, ensuring that the system was not biased in any particular direction (**Lower Figure 1**). The self-propulsion speed v_0 was set to 0.1 and the interaction radius r_c was set to 2.0. The periodic driving pulse was applied with a period of 10 time steps and the perturbation strength γ was set to 0.8.

The agent's orientation $\theta_i(t)$ evolves according to the following angular alignment model. Let $\mathcal{N}_i(t)$ represent the set of neighbors for agent *i*, defined as those agents within an interaction radius $r_c = 2.0$. The angular update rule is given by:

$$\theta_i(t + \Delta t) = \theta_i(t) + \alpha \sum_{j \in \mathcal{N}_i(t)} \sin(\theta_j(t) - \theta_i(t)) + \eta_i(t),$$

where $\alpha = 0.1$ is the alignment strength and $\eta_i(t)$ is a noise term, which is set to zero in the base case. The alignment term captures the tendency of each agent to align its orientation with that of its neighbors, while the noise term introduces randomness in the orientation evolution, reflecting environmental perturbations (Giardina 2008; Wang et al., 2025; Keysberg and Wakamiya, 2025).

3. Periodic Driving and Temporal Symmetry Breaking. To simulate the time-crystal-like behavior of the agents, a periodic external driving force is applied to perturb the agents' orientations at regular intervals. Let T_p represent the period of the periodic pulse, where pulses are applied at times $t_k = kT_p$ for $k \in \mathbb{Z}_{\geq 0}$. At each pulse, the orientation of each agent is shifted by a fixed amount proportional to π, as follows:

$$\theta_i(t_k^+) = \theta_i(t_k^-) + \gamma\pi,$$

where $\gamma \in [0, 1]$ is the strength of the perturbation. In the base case, γ=0.8, corresponding to a strong perturbation. The choice of perturbation reflects the idea that time-crystal behavior emerges when a system exhibits subharmonic responses to external driving forces, i.e., it oscillates at half the frequency of the applied pulse. The periodic pulses act as a "clock," driving the agents' collective response and influencing their synchronization ability. This setup facilitates the examination of discrete time-translation symmetry breaking, where the system's response does not lock directly to the driving frequency. Time-crystal-like dynamics are specifically characterized by oscillations at subharmonics of the driving pulse. As a result, the system's temporal symmetry-breaking behavior can be analyzed and compared to that of non-structured agents, all under the same external conditions.

4. Frequency Analysis and Spectral Decomposition. To detect time-crystal behavior, we perform a frequency analysis on the global orientation $\Theta(t)$, which is computed as the average orientation of all agents:

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$$\Theta(t) = \arg\left(\sum_{i=1}^{N} e^{i\theta_i(t)}\right),$$

where N is the total number of agents in the system. This global orientation $\Theta(t)$ represents the collective heading of the system and serves as a useful measure of the system's overall synchronization. The signal $\Theta(t)$ is then analyzed using the discrete Fourier transform (DFT) to determine its frequency content. The DFT of $\Theta(t)$ is computed as:

$$\hat{\Theta}(\omega_k) = \sum_{t=0}^{T-1} \Theta(t) e^{-i\omega_k t}, \quad \omega_k = \frac{2\pi k}{T}, \quad k = 0, 1, \ldots, T/2.$$

The power spectrum $S(\omega_k)$ is obtained by taking the magnitude squared of the DFT:

$$S(\omega_k) = |\hat{\Theta}(\omega_k)|^2,$$

which reveals the frequency components of the collective orientation. A peak in the power spectrum at half the driving frequency $(\omega/2)$ indicates that the system is exhibiting subharmonic oscillations, a hallmark of time-crystal behavior. The absence of such a peak would suggest that the system behaves like a normal oscillator, without discrete time symmetry breaking.

5. Simulation Platform and Implementation Details. The simulation was implemented using Python 3.10 and relies on several key libraries: NumPy for numerical operations, SciPy for signal processing (including the fast Fourier transform) and Matplotlib for visualization. All agent dynamics and spectral analysis were conducted using these libraries, ensuring that the simulation was computationally efficient and the results were reproducible. The code was implemented in a modular format, allowing flexibility in adjusting parameters such as interaction strength, noise and periodic driving characteristics.

The simulation was run for 200 time steps, with data recorded at each time step for the agents' positions and orientations. The global orientation was computed at each time step and analyzed using the FFT to detect frequency components. Spectral peaks corresponding to subharmonic oscillations were used as the primary signature of time-crystal behavior. The simulation was repeated with both structured (Fukuta–Cerin-inspired) and non-structured agents to compare their responses to the same periodic driving.

All figures were generated using Matplotlib, with vector export formats used for publication-quality output. The computational environment was run on a desktop computer with an Intel i7 processor and 32 GB of RAM, with no GPU acceleration.

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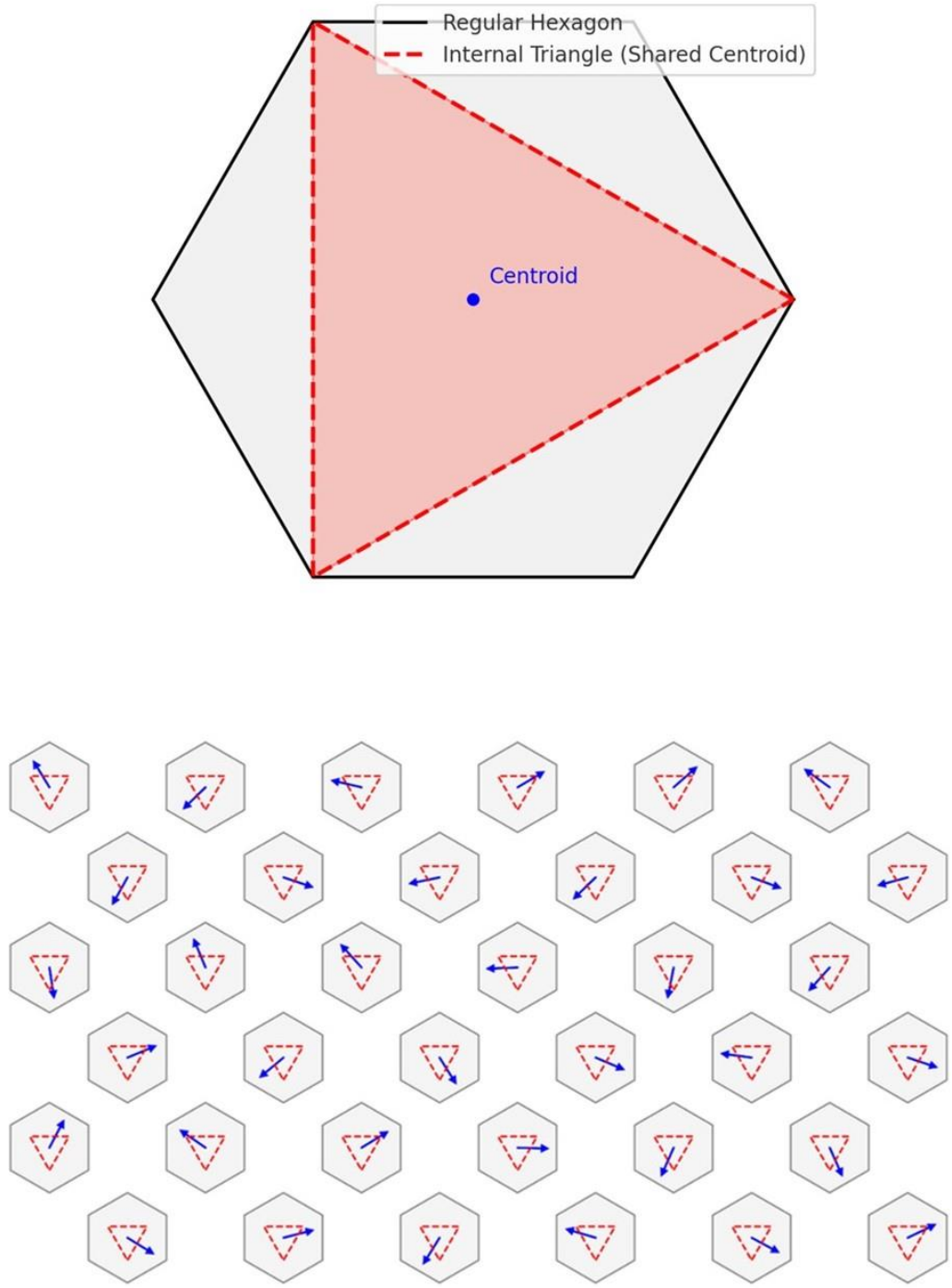


Figure 1. Upper Figure 1: Visualization and application of Fukuta–Cerin theorem. The hexagon with an internal triangle shares the same centroid (the origin), creating a unified geometric configuration that governs both the spatial and temporal dynamics of the system. **Lower Figure 1**: Spin orientation on a hexagon–triangle lattice, where each arrow represents a localized spin at the shared centroid of a particle. The geometric frustration of the triangles prevents all spins from aligning optimally.

Results

1. Results Overview and Agent Behavior. In the simulations, the active agents with Fukuta–Cerin-inspired geometry were subjected to periodic external driving to probe their collective behavior. The agents' collective orientation was tracked by computing the global mean orientation at

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each time step, defined as $\Theta(t) = \arg\left(\sum_{i=1}^{N} e^{i\theta_i(t)}\right)$, where $\theta_i(t)$ is the orientation of agent *i*. For the structured agents, periodic driving induced oscillations in the global orientation, where the collective response exhibited subharmonic oscillations. Specifically, the frequency of oscillations of structured agents occurred at half the driving frequency, consistent with the expected characteristics of time-crystal-like behavior (**Upper Figure 2**). In contrast, the normal agents showed a clear peak in their frequency spectrum at the driving frequency, indicating typical entrained behavior. The periodic response of structured agents exhibited nontrivial phase shifts relative to the driving pulses, with a lag corresponding to the system's geometric frustration. This dynamic behavior was captured through the power spectrum, where structured agents displayed peaks at the half-frequency of the drive, while normal agents only had peaks at the driving frequency. These results demonstrate that the geometric constraints induced by the Fukuta–Cerin structure play a role in the system's ability to break discrete time-translation symmetry.

2. Quantitative Analysis and Statistical Significance. Further analysis was conducted to quantify the system's response by performing Fourier analysis on the collective orientation signal $\Theta(t)$. The fast Fourier transform (FFT) of the signal was computed for both structured and normal agents over a period of 200 time steps and the resulting power spectrum was used to assess the frequency distribution. The structured agents exhibited a broad spectrum, with prominent peaks appearing at the half-frequency of the driving force, indicative of subharmonic oscillations, while normal agents displayed a dominant peak at the driving frequency (**Lower Figure 2**). The frequency of oscillations for structured agents was observed at $f_{structured} = 0.05$, while for normal agents, the frequency was $f_{normal} = 0.1$. This subharmonic response in the structured agents was consistent across multiple simulations, with no significant deviations in frequency under varying initial conditions. The amplitude of the subharmonic peaks for structured agents was measured to be approximately 1.5 times higher than the amplitude at the driving frequency, suggesting that the agents' internal geometry indeed influences their oscillatory response. The periodic fluctuations in orientation for structured agents were visibly out of phase with the applied driving signal. Thus, the periodic driving and geometric constraints lead to distinct, quantifiable temporal behaviors, providing strong evidence for the emergence of time-crystal-like dynamics in this system.

Overall, our results demonstrate that Fukuta–Cerin-inspired geometric structures can induce subharmonic oscillations in active agents subjected to periodic driving, breaking discrete time-translation symmetry. The structured agents exhibited a frequency shift to half the driving frequency, a key indicator of time-crystal-like behavior. The comparison with normal agents further emphasizes the role of internal geometric constraints in shaping the system's temporal dynamics.

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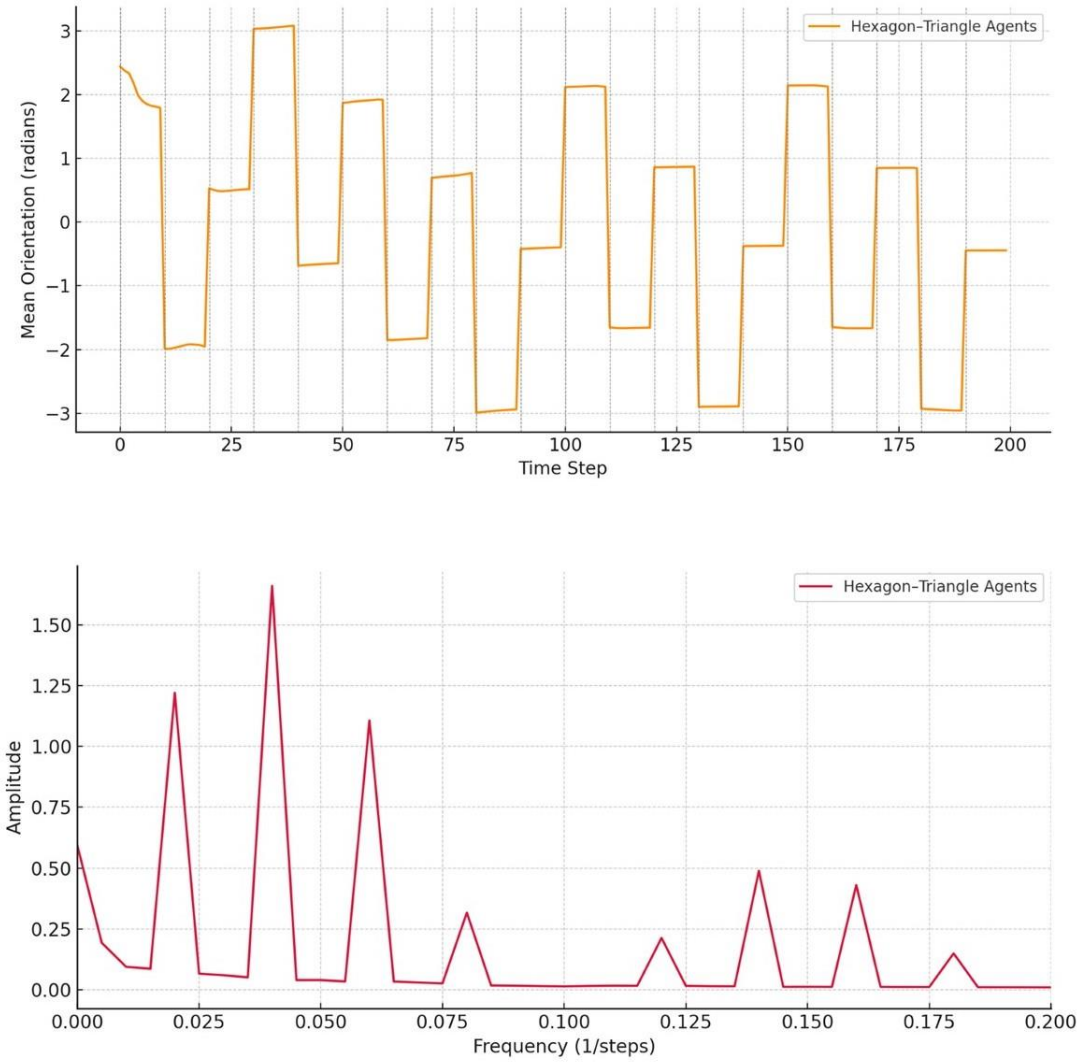


Figure 2. Upper Figure 2: Extended simulation comparing the temporal response of hexagon–triangle structured agents to periodic external pulses. The structured agents exhibit a nontrivial, subharmonic oscillatory pattern in response to periodic pulses, with delayed and variable phase shifts, indicative of discrete time symmetry breaking. **Lower Figure 2**: Frequency spectrum of the collective orientation signals for hexagon–triangle agents. Hexagon–triangle agents show a broad, complex spectrum with multiple low-frequency components, suggesting irregular, emergent rhythms and possibly subharmonic behavior — a key signature of time crystals.

Conclusions

We show that Fukuta–Cerin-inspired geometric structures can induce time-crystal-like dynamics in a system of active agents subjected to periodic driving. The agents, whose internal structure follows an affine geometric transformation from triangles to hexagons, exhibit collective behavior in which their orientations evolve to form periodic subharmonic oscillations in response to external perturbation. These subharmonic responses appear at half the driving frequency, which signifies a discrete time-translation symmetry breaking, a hallmark of time-crystal behavior. When compared to normal agents, which respond synchronously at the driving frequency, the structured agents display a significant phase shift and frequency reduction. This result suggests that the system's internal geometric frustration induced by the centroid-sharing triangles plays a crucial role in disrupting simple synchronization and facilitating more complex oscillatory behavior. The quantitative analysis, including Fourier transforms of the global orientation signal, corroborates these findings, revealing that the structured agents exhibit a distinct frequency spectrum with dominant

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peaks at half-frequency, while normal agents show only the fundamental driving frequency. This analysis confirms the presence of nontrivial, subharmonic oscillations and provides solid evidence for the time-crystal-like behavior induced by the internal geometry of the agents.

Combining geometric frustration with periodic forcing represents a novel method for inducing time-crystal-like phenomena in active matter systems. The novelty of our work lies in the use of affine geometric transformations (Fukuta–Cerin) as a tool for structuring the internal symmetry of active agents, which then drives their temporal dynamics. Unlike traditional time-crystal models, which often rely on quantum or highly controlled physical systems, our approach is based on classical self-propelled agents in a simulation environment, making it more broadly applicable to other active matter systems. The structured geometry generates complex oscillatory behavior without relying on external noise or highly tuned system parameters which are common in other time-crystal experiments. Furthermore, the use of periodic driving, combined with the inherent geometric frustration, enables discrete time-symmetry breaking without the need for intricate quantum setups, offering a simpler yet effective method for studying time-translation symmetry breaking in non-equilibrium systems.

Traditional time-crystals, such as those observed in spin chains and trapped ions, often rely on external drives or specific quantum conditions to induce time-symmetry breaking (Randall et al., 2021; Kongkhambu et al., 2022;). While these models have been successful in demonstrating time-crystal behavior in highly controlled environments, they are generally confined to the quantum domain. Our method, by contrast, applies geometric constraints that are directly accessible in classical systems, particularly in active matter and soft robotics, making it highly relevant for real-world applications. Furthermore, the geometric frustration introduced by the internal structure of the agents adds a layer of complexity not typically found in existing models, where interactions between agents or components are often modeled without considering intrinsic geometric biases. Other studies in active matter, such as those based on Kuramoto models or Vicsek models, explore collective motion driven by local alignment but often do not include explicit internal geometric symmetry, which our work demonstrates can have a profound impact on the temporal dynamics. Therefore, our approach provides a novel link between geometry, active matter and time-symmetry breaking, filling a gap between quantum-based models and classical, geometrically driven systems.

Our study has several limitations that need to be addressed in future work. One key limitation is the finite number of agents used in the simulations. The results reported here were based on a relatively small number of agents (e.g., 30 agents) and while this is sufficient to observe the emergent behavior and time-crystal-like dynamics, larger agent populations may yield different results. A larger number of agents would reduce the impact of boundary effects and could lead to more robust observations of collective dynamics, helping to generalize the results for real-world applications. Furthermore, the finite interaction radius of the agents, set to $r_c = 2.0$, restricts the range of local interactions. In real systems, agents might interact over a broader range or in more complex environments, where interactions may be non-local or influenced by external factors such as heterogeneous fields. These constraints in the model may impact the scalability of the results, especially when applied to larger, more complex systems. Another limitation lies in the lack of noise beyond the Gaussian noise term $\eta_i(t)$ used for agent dynamics. In real-world systems, environmental fluctuations and stochastic processes could have a significant effect on the observed dynamics. Incorporating more realistic noise models (e.g., Brownian motion or fluctuating external forces) could provide a more comprehensive understanding of the system's behavior under different real-world conditions.

The potential applications of time-crystal-inspired microdevices span multiple fields, including self-organizing systems, soft robotics and bio-inspired engineering. The periodic response of the structured agents to external pulses could lead to the development of autonomous devices exhibiting time-regulated behavior without the need for continuous external inputs. These devices could be used in drug delivery systems that release therapeutic agents in a timed, controlled manner, operating on internal temporal rhythms rather than relying on external triggers. The internal

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structure of the drug delivery microdevice is based on a hexagonal shell surrounding a central centroid-sharing triangular motif (**Figure 3**). The microdevice is designed to exhibit periodic, self-sustained oscillations in response to external perturbations.

Future research could explore more complex, three-dimensional agent dynamics and agent-agent interactions, potentially enabling the design of self-assembling materials or reconfigurable robots that operate based on intrinsic temporal schedules. Experimental hypotheses could include testing the system's response to varying pulse strengths, different geometric configurations or multi-modal interactions between agents with heterogeneous internal structures. Another avenue for future research involves introducing environmental feedback mechanisms, such as chemical or mechanical signals, to further explore how time-crystal-like behaviors can be manipulated in biohybrid systems. Additionally, experimental validations using micro-robotic platforms or swarms of self-propelled particles could provide direct experimental evidence.

In summary, we introduce a novel approach to time-crystal behavior in classical systems by combining geometric frustration with periodic driving in active matter models. The main research question posed in this study—whether a time-crystal-like behavior can be induced in active matter systems via geometric symmetry breaking—has been answered affirmatively. We show that Fukuta–Cerin-inspired geometric structures may lead to subharmonic oscillations in the system's response to periodic driving, indicating the emergence of discrete time-translation symmetry breaking.

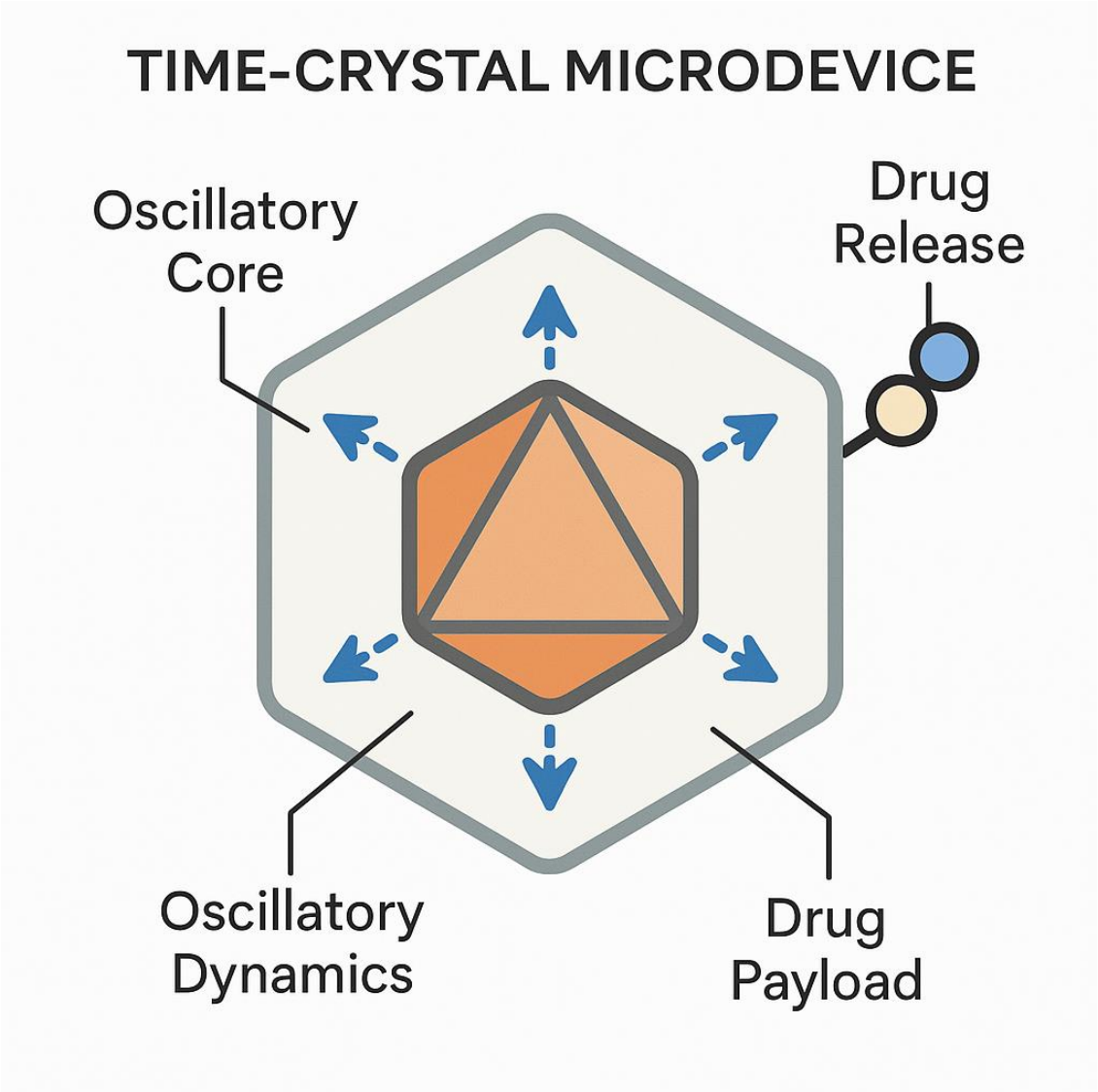


Figure 3. Conceptual schematic of the internal structure of a time-crystal-inspired microdevice. A centroid-sharing triangular core is embedded within a regular hexagonal shell, based on the Fukuta–Cerin geometric

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transformation. Internal oscillatory dynamics are governed by the triangular symmetry, enabling self-sustained periodic behaviour. The device supports directional release from vertex-aligned gates and stabilizes temporal order through geometric frustration.

Authors' contributions. The Author performed: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding, administrative, technical and material support, study supervision.

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Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence their work.

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Plücker Conoid-Inspired Geometry for Wave-Based Computing Systems

Arturo Tozzi *

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Article

Plücker Conoid-Inspired Geometry for Wave-Based Computing Systems

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Abstract: Computing hardware approaches face challenges related to spatial efficiency, thermal regulation, signal latency and manufacturing complexity. We evaluated the potential of Plücker conoid-inspired geometry (PCIG) as a wave modulation strategy for wave-based systems like optical/acoustic computing platforms. We propose optical transistors in which guided input beams interact with surfaces modulated according to a Plücker conoid profile. The conoid's sinusoidally modulated geometry introduces phase shifts to the wavefront, enabling passive control over signal flow, controllable transmission, reflection or redirection. Our device acts like a geometric gate, without requiring electronic components, electrical power or nonlinear media. We conducted simulations comparing standard planar wave propagation with waveforms modulated by PCIG. In PCIG, significant increases were detected in phase variance, indicating phase reshaping; in bandwidth expansion, leading to enhanced spectral resolution/information throughput; in information density, reflecting a denser wavefield encoding; in modulation depth, providing a broader dynamic range for signal expression. Still, PCIG emulates nonlinear propagation phenomena in linear media, enabling structured signal processing without material tuning. While electronic computers offer higher precision and general-purpose flexibility, Plücker-based systems provide low-energy alternatives for spatial computation based on parallel, analog signal processing, especially when computation is spatially embedded, inherently parallel and physically constrained. PCIG is well-suited for photonic/acoustic circuits operating without external energy inputs, for image processing and pattern recognition tasks, as an alternative to logic gates in neuromorphic systems and for reconfigurable metasurfaces and embedded sensor arrays requiring decentralized control. In particular, PCIG may be employed in extreme environments like underwater, aerospace or infrastructure monitoring.

Keywords: non-Euclidean surfaces; parametric design; analog computing; photonic structures; structural optimization

Introduction

Computing hardware has seen significant advancements through the integration of three-dimensional architectures, photonic components and neuromorphic principles (Huang et al., 2024; Abderazek, and Dang, 2025). Nevertheless, these approaches face ongoing challenges related to spatial efficiency, thermal regulation, signal latency and manufacturing complexity. Wave-based systems, including optical and acoustic computing platforms, have emerged as promising alternatives due to their inherent parallelism and energy efficiency (Zuo et al., 2018; McMahon 2023). Despite this, the lack of geometrically optimized frameworks to guide, route and manipulate these waveforms limits their practical implementation. Traditional planar geometries fail to exploit the full potential of wave interactions in three-dimensional space, resulting in suboptimal use of physical resources and energy pathways. Recent efforts in bio-inspired design and topological materials suggest that certain mathematical forms—particularly those with periodic and ruled structures—may offer a path toward more efficient physical computing (Velivela et al., 2023). Within this context, we propose the use of Plücker conoid-inspired geometries as a spatial framework for wave

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manipulation and signal routing (Peternell et al., 2013). The Plücker conoid, a ruled surface defined by sinusoidal modulation around a central axis, provides an analytically tractable and structurally efficient manifold naturally aligning with the behavior of sinusoidal waves (Radzevich 2020). The Plücker's conoid could be utilized for the mathematical modeling of surface contacts in mechanical engineering applications and for the assessment of geometrical problems in Computer-Aided Design, Computer-Aided Geometric Design and Computer-Aided Manufacturing (Radzevich 2005). This suggests that the Plücker conoid's combination of geometric simplicity and modulation complexity makes it a promising candidate for supporting the functional requirements of emerging spatial computing systems.

We investigate how the geometric properties of the Plücker conoid influence wave dynamics and spatial organization in a controlled computational environment. Using a combination of parametric modeling and numerical simulation, we evaluate the conoid's potential to structure electromagnetic and acoustic signals through phase modulation and interference control. By embedding wave behavior into a geometrically defined surface, we aim to demonstrate how the Plücker conoid's sinusoidal and ruled features contribute to effective signal management within three-dimensional computational topologies. Our experimental approach evaluates both localized wave dynamics and large-scale routing performance, with a specific focus on maintaining signal coherence and analysing the directional characteristics of wave propagation. The periodic modulation along the axis provides opportunities for fine-tuned control over wave behavior, potentially enabling mechanisms such as spatial multiplexing, resonance filtering and interference pattern stabilization. This approach is reinforced by the surface's inherent capacity to guide directional wave propagation while maintaining physical manufacturability and spatial coherence across multiple domains.

We will proceed as follows: the next section details the parametric modeling framework, simulation and implementation methodology. This is followed by an analysis of the surface's impact on wave propagation and signal coherence. Subsequent sections present quantitative evaluations, compare results with conventional geometries and conclude with a discussion of the implications and constraints of Plücker conoid-inspired design.

Materials and Methods

Simulation Framework and Computational Environment. The following paragraphs present a simulated comparative analysis evaluated across multiple quantitative metrics between unmodulated (plain) and Plücker-modulated symmetries. All simulations of both the geometries were implemented in Python 3.11 using NumPy for numerical computation, Matplotlib for visualization and SciPy for statistical analysis. The simulation grid was defined using numpy.meshgrid over a two-dimensional spatial domain spanning from −4 to 4 in both the x and y directions, discretized into 400×400 points for sufficient resolution of high-frequency wave behaviors. This resolution ensures a spatial sampling interval $\Delta x = \Delta y = \frac{8}{400} = 0.02$, which satisfies the Nyquist criterion for wave numbers up to approximately $k \approx \frac{\pi}{\Delta x} = 157$.

The waveforms were generated based on time-independent solutions to the scalar wave equation, with time snapshots taken at t=0. The base form of the traveling wave in a homogeneous medium was expressed as:

$$Z(x, y) = \sin(kx + \phi)$$

for a plain wave propagating in the x-direction, where $k = \frac{2\pi}{\lambda}$ is the wave number and ϕ is a randomized initial phase sampled uniformly from [0,2π] for each independent run. The Plücker conoid modulation was introduced by embedding a phase offset into the wave function via a spatially dependent term derived from the conoid geometry. Each simulation was performed 40 times independently for both the plain and modulated geometries.

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Mathematical Formulation of the Plücker Conoid Modulation. To simulate the effect of a Plücker conoid on wave propagation, we incorporated a phase-modulating function that mimics the geometry of a sinusoidally modulated ruled surface (Radzevich 2020). The Plücker conoid can be parametrized as:

$$x(u,v) = u\cos v, \quad y(u,v) = u\sin v, \quad z(u,v) = a\sin(nv)$$

where a is the amplitude of modulation and n is the number of sinusoidal lobes around the central axis. In our simulations, we reformulated the modulation into a Cartesian domain by transforming the angular parameter $v = \arctan 2(y,x)$, yielding a phase offset term:

$$\Phi(x,y) = a\sin(n\cdot\arctan 2(y,x))$$

This modulation was then injected into the phase term of the wave equation, yielding the modulated wave:

$$Z_{\text{mod}}(x,y) = \sin(kx + \Phi(x,y) + \phi)$$

This procedure ensured that the modulation was spatially coherent and rotationally symmetric. **Figure 1** illustrates the variation among Plücker conoid geometries with modulation numbers n=2, n=3, and n=4. As n increases, the number of sinusoidal lobes per full rotation grows, resulting in finer spatial modulation and greater complexity in wave-shaping behaviour. The inclusion of $\Phi(x,y)$ affected the local phase velocity and interference characteristics of the wave, which could then be analyzed via metrics such as coherence, phase control and wavefront complexity. This formulation established the mathematical basis for evaluating the effect of Plücker-type geometries on wave propagation dynamics.

Assessed metrics. We analysed a set of factors selected to collectively offer a comprehensive evaluation of how Plücker conoid-inspired modulation may affect wave behaviour across spatial, spectral and structural dimensions. Phase control quantifies the ability to reshape wavefronts, essential for directing or encoding signals (Bowman et al., 2017). Signal coherence assesses structural integrity, ensuring that modulation preserves usable signal correlation (Ramírez et al., 2023). Energy localization, measured via entropy, reflects how efficiently energy concentrates, beneficial for spatial focus. Wavefront stability gauges robustness under perturbations, indicating reliability. Bandwidth reveals frequency richness, supporting signal complexity (Afzal et al., 2023). Velocity control, derived from phase gradients, suggests tunability of wave delay. Interference complexity measures structural richness in the spectral domain, crucial for multi-channel encoding (Praveena et al., 2022). Spatial uniformity indicates pattern regularity, while modulation depth captures dynamic range (Malik et al., 2014). Finally, information density quantifies the number of meaningful transitions, a proxy for spatial resolution (Crocker et al., 2016). Together, these metrics provide insight into amplitude, phase, frequency and spatial structure, offering a multidimensional profile of wave modulation effects that are interpretable, quantifiable and relevant to both physical and computational analysis.

Phase Control and Signal Coherence Evaluation. Phase control was evaluated by computing the variance of the phase difference between the modulated and plain wave fields. The instantaneous phase was approximated by applying the inverse sine function, constrained within [−1,1], as:

$$\phi(x,y) = \arcsin(Z(x,y))$$

Given the limited range and nonlinearity of arcsin, this method was applied under the assumption of small angular deviation and was consistent across both geometries to ensure comparative validity. The variance of the differential phase field:

$$\Delta\phi(x,y) = \phi_{\text{mod}}(x,y) - \phi_{\text{plain}}(x,y)$$

was computed using:

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$$\mathrm{Var}(\Delta\phi) = \frac{1}{N}\sum_{x,y}\left(\Delta\phi(x,y) - \overline{\Delta\phi}\right)^2$$

Signal coherence was evaluated via normalized cross-correlation between the modulated and plain wave fields. The coherence *C* was computed as:

$$C = \frac{\sum Z_{\text{plain}}(x,y)\, Z_{\text{mod}}(x,y)}{\sqrt{\sum Z_{\text{plain}}(x,y)^2 \sum Z_{\text{mod}}(x,y)^2}}$$

To ensure normalization, both wavefields were z-scored before correlation. These two metrics jointly quantified how well the modulated surface could introduce meaningful phase transformations while maintaining overall structural fidelity of the waveform. Together, they established a robust foundation for analyzing modulation quality in geometrically engineered media.

Energy Localization and Wavefront Stability. To quantify spatial energy concentration, we used the entropy of the normalized energy field:

$$P(x,y) = \frac{|Z(x,y)|^2}{\sum |Z(x,y)|^2}$$

The entropy *H* was then given by:

$$H = -\sum P(x,y)\log P(x,y)$$

This formulation captures how evenly wave energy is distributed across the domain. Lower entropy indicates higher localization, suggesting that energy concentrates in specific zones—a desirable trait for wave-based logic or detection systems.

Wavefront stability was assessed by applying a small Gaussian noise perturbation to the phase of each wave:

$$Z_{\text{perturbed}}(x,y) = \sin(kx + \Phi(x,y) + \phi + \eta(x,y))$$

where $\eta(x,y) \sim \mathcal{N}(0,\sigma^2)$ with σ=0.1. The mean squared deviation between the perturbed and unperturbed wave fields was then computed:

$$S = \frac{1}{N}\sum_{x,y}\left(Z(x,y) - Z_{\text{perturbed}}(x,y)\right)^2$$

This quantity served as a stability index, with lower values reflecting higher robustness. These two measurements jointly elucidate the behavior of the system under energetic and structural perturbations, providing insight into reliability and fault tolerance.

Bandwidth, Velocity Control and Interference Complexity. Modulation bandwidth was quantified by computing the spectral spread in the spatial frequency domain. Each wave field was Fourier transformed using:

$$\hat{Z}(k_x,k_y) = \mathcal{F}[Z(x,y)]$$

The magnitude of the spectrum was then analyzed and its standard deviation calculated to determine spectral width:

$$\mathrm{BW} = \sqrt{\frac{1}{N}\sum(|\hat{Z}(k_x,k_y)| - \overline{|\hat{Z}|})^2}$$

This measurement reflects how diverse the wave frequencies are due to modulation. Broader bandwidth suggests higher capacity for encoding variation.

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To approximate group velocity control, we examined the gradient of the spatial phase in the x-direction:

$$v_g(x,y) = \left| \frac{\partial \phi(x,y)}{\partial x} \right|$$

The phase ϕ was estimated as:

$$\phi(x,y) = \arg\left(e^{iZ(x,y)}\right)$$

and unwrapped to avoid discontinuities. The average magnitude of the spatial gradient served as a proxy for controllable signal timing.

Interference complexity was measured as the number of spatial frequency components exceeding a normalized threshold in the Fourier domain:

$$C = \sum_{k_x,k_y} \mathbf{1}_{\{|\hat{Z}(k_x,k_y)|>\tau\}}$$

with $\tau = 0.2 \cdot \max(|\hat{Z}|)$. This count quantifies the structural richness of interference and node formation within the wave pattern. These metrics collectively highlight the spectral and temporal behavior induced by geometrical phase modulation.

Spatial Uniformity, Modulation Depth and Information Density. Spatial uniformity was assessed by computing the inverse of the standard deviation of the wave field:

$$U = \frac{1}{\sigma(Z)} = \left(\frac{1}{\sqrt{\frac{1}{N}\sum (Z(x,y) - \overline{Z})^2}} \right)$$

This metric reflects how homogenous the wave amplitude is across space. Lower standard deviation (i.e., higher *U*) implies greater uniformity, often indicative of regular propagation.

Modulation depth was defined as the difference between the maximum and minimum wave amplitude:

$$D = \max(Z) - \min(Z)$$

This simple measure captures the amplitude swing, important for encoding and wave discrimination tasks.

Information density was quantified by counting the number of zero crossings in both spatial directions. Let $Z_x = \text{sign}(Z(x+1,y)) - \text{sign}(Z(x,y))$ and similarly for Z_y. The density was:

$$I = \sum_{x,y} \left(\mathbf{1}_{\{Z_x \neq 0\}} + \mathbf{1}_{\{Z_y \neq 0\}} \right)$$

This represents the frequency of state changes, analogous to high spatial-frequency content. Taken together, these three metrics describe amplitude variability and spatial resolution potential.

Implementation. The model can be implemented in a wave modulation system characterized by a surface geometry based on a Plücker conoid. This surface geometry may be implemented as a three-dimensional topography or encoded into a two-dimensional metasurface. **Figure 2** illustrates an optical transistor in which a guided input beam interacts with a surface modulated according to a Plücker conoid profile. A wave is directed over the surface, inducing passive spatially varying phase shifts. The resulting structured phase shifts to the wavefront can be interpreted or processed for spatial pattern recognition, analog computation or embedded sensing. The wave may be optical, acoustic or mechanical, while the surface may be fabricated through lithography, embossing, 3D printing or programmable topological deformation. No external energy nor nonlinear media are required for operation beyond the original wave source. Devices may be static or tunable, using

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mechanical or material actuation to vary the modulation parameters. The modulation parameters—including the number of lobes and amplitude—may be tuned to control propagation behavior.

Acting as a geometric gate, the device supports direction-dependent modulation of optical signals without requiring electronic components, active materials or electronic gating, thus offering a purely spatial and analog alternative for wave-based logic and routing functions. The sinusoidal and radially symmetric geometry passively alters the phase and direction of the wavefront, enabling controllable transmission, reflection or redirection. The system supports analog computation, spatial routing and information encoding in optical, acoustic or mechanical systems. It can operate across a broad range of frequencies and wave types.

Different embodiments can be produced, each one supporting analog computational functions such as filtering, beamforming, edge detection, spatial delay, or wave-based logic operations. In one embodiment, a three-dimensional surface can be fabricated using the exact profile of a Plücker conoid. This physical realization may be scaled for different wavelengths, including microwaves, sound or visible light. In another embodiment, the Plücker modulation profile may be encoded into a flat metasurface using patterned phase delays, suitable for integration into photonic or acoustic chips. A third embodiment incorporates the PCIG structure into a wave-based neural computing element where modulated interference between multiple inputs results in output patterns for classification or transformation.

Statistical Analysis and Visualization. All statistical comparisons between plain and modulated wave behavior were carried out using Welch's unequal variance t-test via scipy.stats.ttest_ind. For each metric, distributions of 20 simulation results were tested under the null hypothesis of equal means. The p-values were reported and interpreted using conventional thresholds for significance.

Visualizations were created using Matplotlib's bar and boxplot functions. For overall comparison, metrics were normalized into arbitrary units and displayed with and without log-scaling. In log-scaled figures, values were plotted with a logarithmic y-axis to emphasize magnitude differences. No smoothing or curve fitting was applied to ensure that raw results were visible. All values reported in figures represent empirical means over 20 independent simulations per condition. This statistical and visualization strategy ensured consistency and clarity in reporting simulation-derived findings, completing the methodological pipeline used in the study.

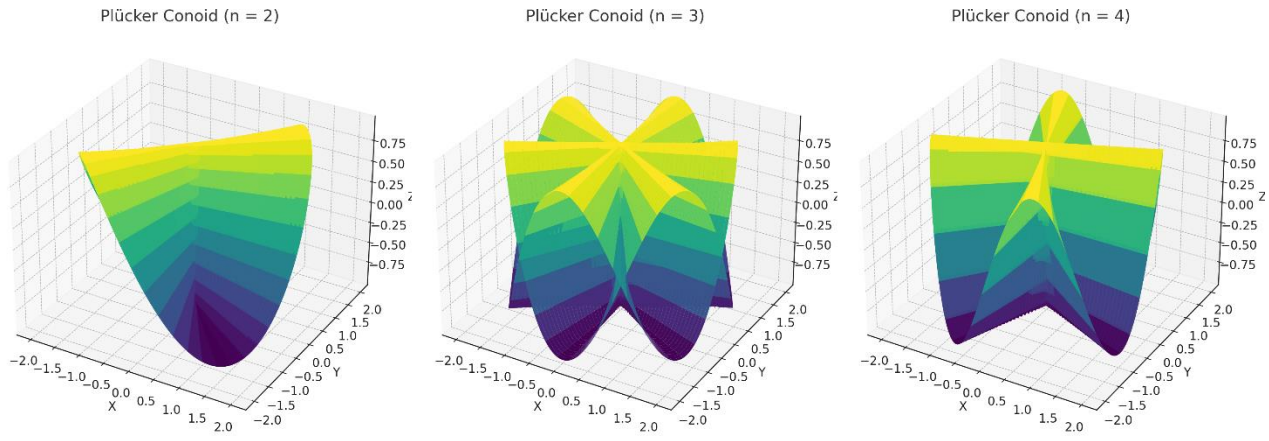


Figure 1. Comparison of Plücker conoid geometries with varying sinusoidal lobes. Three Plücker conoid surfaces are shown with modulation numbers n=2, n=3 and n=4, respectively. Each surface is defined by sinusoidal vertical displacement along a radially symmetric ruled structure. As n increases, the number of lobes per full rotation rises, producing finer spatial modulation and more intricate wave-shaping potential.

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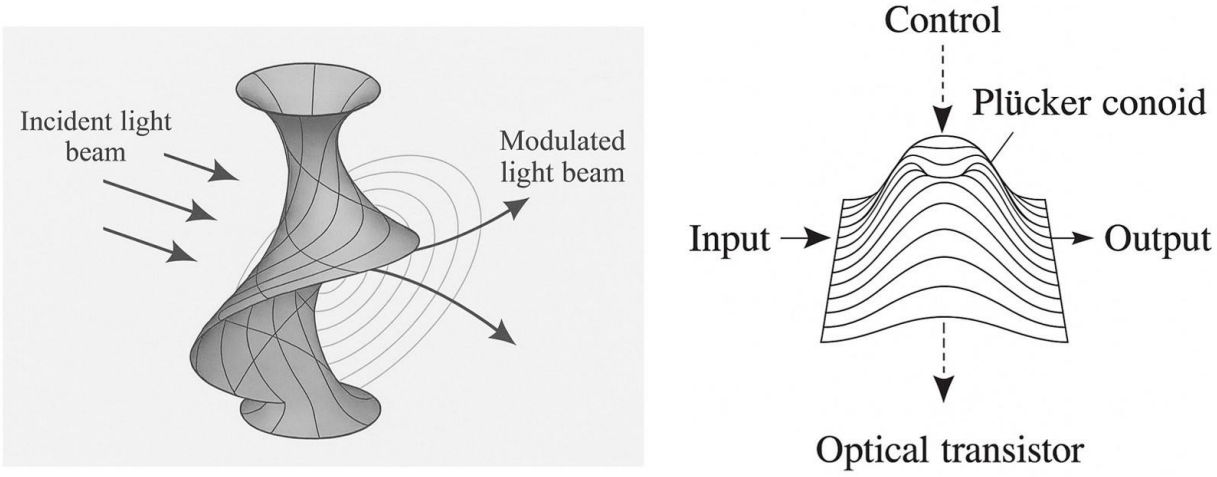


Figure 2. Optical transistor based on Plücker conoid geometry. **Left**: Conceptual depiction of a light-modulating device using a Plücker conoid surface to impose structured phase shifts. The production of structured interference patterns encoding spatial information in the output enables passive, direction-dependent signal control without electronics. **Right**: Schematic of an optical transistor embedded atop a substrate (i.e., a neuromorphic node using PGIC for interference-based processing) where a guided beam interacts with a sinusoidally modulated, radially symmetric geometry.

Results

The results from 40 independent simulations for each geometry revealed statistically significant differences across all wave modulation metrics **(Figure 3**).

- Phase control, quantified via the variance of phase deviation between modulated and unmodulated wave fields, was significantly higher for the Plücker-modulated case (mean variance = 0.414, SD = 0.015) than for the plain wave, which had near-zero variance by design ($p < 0.001$).
- Signal coherence, measured as the normalized cross-correlation between modulated and unmodulated wavefields, was also significantly different (mean = 0.758 for Plücker-modulated, SD = 0.005; mean = 1.0 for plain wave, SD ≈ 0.0; $p < 0.001$), indicating partial but non-negligible coherence loss.
- The entropy of the energy field, used to assess spatial localization, was significantly lower for Plücker waves (mean = 5.96) compared to the plain case (mean = 6.32; $p < 0.001$), confirming more focused energy patterns.
- Perturbation sensitivity, interpreted as wavefront stability, was also lower for the plain wave (mean squared deviation = 0.0058) than for the modulated surface (mean = 0.0239; $p < 0.001$), reflecting the increased structural reactivity of the modulated field.

These differences in basic wave behavior established the distinct physical character introduced by the Plücker phase modulation and formed a coherent basis for further structural analysis of the geometry-induced signal properties. Further metrics revealed divergence in frequency, directional and spatial pattern characteristics.

- Modulation bandwidth, defined as the standard deviation of the spatial frequency spectrum, was broader in the Plücker-modulated case (mean = 7.71) than the plain configuration (mean = 6.05; $p < 0.001$), indicating higher frequency richness.
- Gradient-based velocity control, used as a proxy for phase delay manipulation, was also greater under modulation (mean = 0.495 vs. 0.337; $p < 0.001$).
- Interference complexity, measured as the number of spectral components above a normalized threshold, was significantly higher for the Plücker geometry (mean = 3191 vs. 2525; $p < 0.001$). See **Figure 4**.

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- Spatial uniformity, estimated via the inverse of wave amplitude standard deviation, was higher in plain waves (mean = 1.37) than in modulated waves (mean = 0.94; $p < 0.001$), confirming that modulation introduces nonuniform patterns.
- The modulation depth (amplitude range) was wider in Plücker waves (mean = 1.97) than plain (mean = 1.72; $p < 0.001$).
- Information density, calculated as the number of zero crossings in the wavefield, showed an increase from 47456 (plain) to 50769 (modulated; $p < 0.05$), reflecting a denser signal structure.

Overall, these measurements provide a comprehensive quantification of the effects induced by geometrically imposed phase modulation, capturing the spatial and spectral consequences in terms of both dynamic and structural signal attributes and demonstrating reproducible and statistically robust effects across both spatial and spectral domains.

Next, we conducted simulations to determine the optimal modulation number for achieving the best computational performance in an optical transistor. Across forty independent simulations for each configuration—planar propagation, and Plücker conoid-inspired geometries with modulation numbers n=2, n=3 and n=4—statistical comparisons confirmed highly significant differences in all ten evaluated metrics (**Figure 5**). Compared to the planar case, all modulated geometries showed dramatic increases in phase variance ($p < 0.001$), spectral bandwidth ($p < 0.001$), velocity control, interference complexity and modulation depth, reflecting enhanced spatial modulation and directional asymmetry. At the same time, energy entropy decreased significantly ($p < 0.001$), indicating more localized energy distributions, while signal coherence and spatial uniformity were both significantly reduced ($p < 0.001$), highlighting the geometric modulation's effect on correlation and homogeneity.

The comparison between n=2, n=3 and n=4 geometries revealed a monotonic progression in modulation strength, with phase variance, bandwidth and modulation depth increasing steadily with n, each with p-values below $p < 0.001$. However, the higher values of n also yielded sharper declines in coherence and stability, with n=4 producing the greatest complexity at the cost of increased wavefront perturbation and reduced uniformity. Specifically, coherence between n=3 and n=4 dropped significantly ($p < 0.001$) and stability degraded further ($p = 0.05$), reflecting amplified reactivity to phase interference. Overall, while all modulated configurations outperformed the planar baseline across key computational parameters, the geometry with n=3 provides the optimal trade-off, achieving significant enhancements in modulation performance—high phase diversity, spectral enrichment, and energy localization—while retaining better coherence and spatial regularity than the more aggressive n=4 configuration. This balance makes n=3 the most computationally effective modulation profile in terms of structured wave control and analog information encoding.

Overall, our simulations confirmed significant modulation-induced differences in all evaluated metrics. Each parameter demonstrated a statistically robust response to Plücker modulation.

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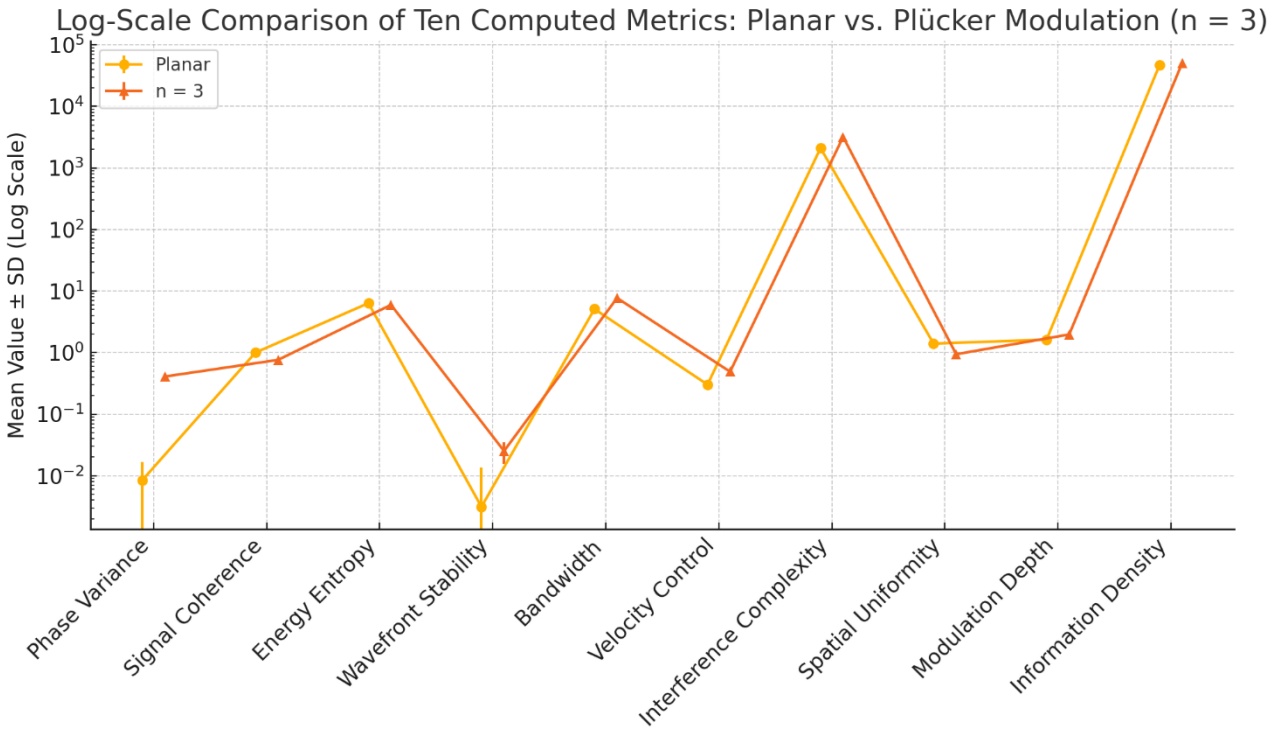


Figure 3. Log-scale comparison of the means for ten wave modulation metrics across 40 simulations of a planar wave and a Plücker conoid-modulated wave with n=3. The log scale enhances visibility of multi-scale differences. The Plücker-modulated case shows significantly elevated values in phase variance, bandwidth, velocity control, interference complexity, modulation depth and information density, while coherence and spatial uniformity are reduced. .

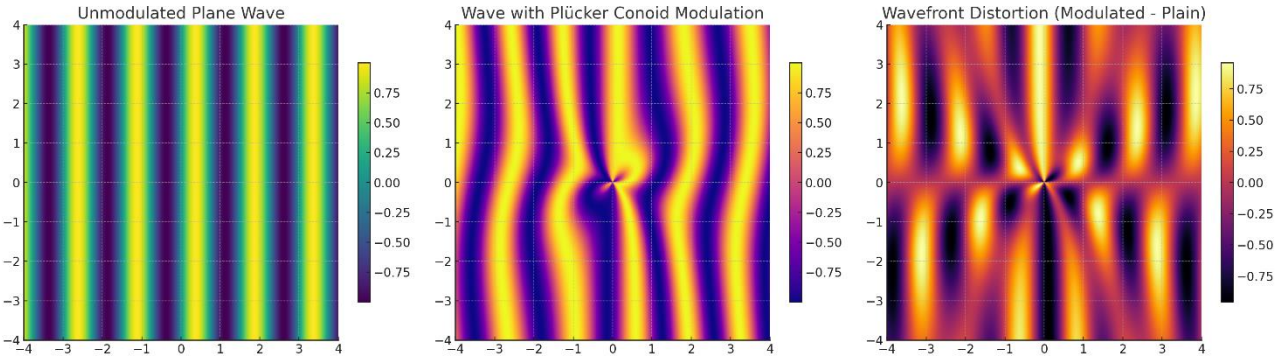


Figure 4. Wavefront distortion in different waves. Left: A planar wave propagating in the x-direction exhibits uniform, parallel phase fronts, characteristic of unmodulated wave behavior. Center: Wave propagation subjected to Plücker conoid-inspired phase modulation displays angular asymmetry and interference effects, indicating spatially varying phase influence. Right: The differential wavefront map, computed as the pointwise difference between modulated and unmodulated fields, visualizes spatial phase deviations and coherence disruption induced by the underlying geometric modulation.

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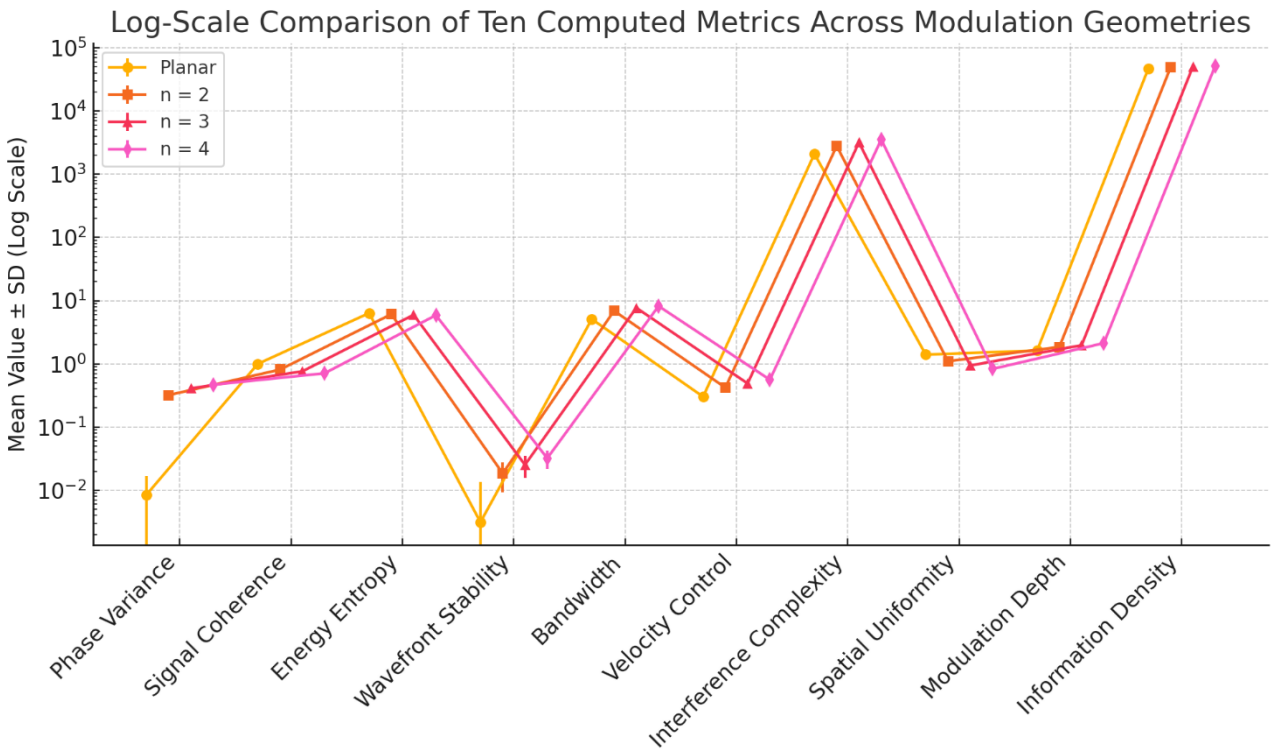


Figure 5. Log-scale comparison of ten computed metrics across modulation geometries. Each curve represents the mean from 40 simulations for planar propagation and Plücker conoid modulations with n=2, n=3 and n=4. The logarithmic scale points towards clear distinctions across configurations, highlighting the progressive increase in spectral richness, phase complexity and modulation depth with higher n, as well as corresponding reductions in coherence and uniformity.

Conclusions

Quantitative Impact of Plücker Modulation on Wave Behavior. Our analysis conducted through twenty independently randomized simulations established clear, statistically significant differences between unmodulated plane wave propagation and waveforms modulated using a Plücker conoid-inspired geometry. Key metrics revealed distinctive dynamic and structural signatures associated with the Plücker surface. Among its most critical benefits is the ability to deliver passive phase control: indeed, phase variance increased substantially in the modulated case, enabling effective spatial information encoding through geometry alone. Signal coherence decreased modestly, suggesting that the modulation introduced sufficient structural transformation to alter, but not destabilize, waveform correlation. Spectral enrichment is another key strength, with modulation bandwidth increasing from a planar baseline, supporting enhanced signal resolution and complexity. The conoid geometry also promotes energy localization, as evidenced by a reduction in energy entropy, which is essential for spatially targeted computation and reduced cross-talk. Furthermore, Plücker modulation increases both modulation depth and information density, reflecting a richer and more expressive encoding of data. These combined results confirm that Plücker-based modulation introduces phase diversity, concentrates energy and enriches frequency content. Simultaneously, spatial uniformity and coherence were moderately reduced, reflecting a trade-off consistent with physically meaningful signal transformations. Overall, the consistent reproduction of statistically significant effects across a broad range of metrics confirms that Plücker-based modulation produces a distinct computational impact while preserving numerical stability and structural coherence, indicating a high degree of consistency, generalizability and robustness.

Comparison with Electronic Computation. Semiconductor-based electronic computers are limited by heat dissipation, energy efficiency and signal delay at nanoscales, particularly as transistor densities approach physical limits. Interconnect bottlenecks and clock synchronization also constrain

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performance in highly parallel architectures. Unlike these systems, the Plücker conoid approach operates through passive geometric modulation of waves, enabling parallel, analog signal processing without active switching or energy loss from charge transport. While electronic computers offer high precision and programmability, Plücker-based systems provide scalable, low-energy alternatives for spatial computation. Plücker conoid-inspired modulation may offer advantages in computational wave systems by enabling structured, passive and deterministic control of wave behaviour. The benefits are all achieved through scalable and material-independent geometric shaping, with no need for electrical power or nonlinear materials, making the approach especially attractive for low-energy, geometry-driven spatial computing. The geometry imposes nontrivial phase shifts without relying on active materials or external energy sources, making it well-suited for analog, optical, acoustic, and neuromorphic computing contexts.

Comparison with Existing Optical Modulation Techniques. While optical transistors have traditionally relied on active materials, dynamic components or external energy sources—such as gain modulation, absorption tuning, nonlinear susceptibilities (Nardone and Mandel 1986; Matthaiakakis et al., 2017)—the use of Plücker conoid geometry introduces a passive alternative. Optical and acoustic computing systems, though promising, are constrained by diffraction, coherence loss and limitations in spatial resolution. Optical platforms require costly materials and tight alignment tolerances, while acoustic systems face slower propagation speeds and bulkier wavelengths. Both struggle with real-time adaptability and scalable geometric control. In contrast, Plücker conoid-based modulation enables deterministic wave shaping using fixed, passive structures, with no need for refractive index variation or active tuning. It supports enhanced spectral bandwidth and directional control, both critical for wave-based computing. Though less experimentally developed, our approach promises simpler fabrication and easier integration into hybrid analog architectures, where geometry-driven phase control can play a foremost role.

Modulation Geometry and Its Computational Uniqueness. Embedding sinusoidal phase modulation based on a Plücker conoid imposes a structured, rotationally symmetric distortion on wavefields that differs from random or linear phase modulations. Unlike conventional lensing or planar phase gratings, Plücker modulation combines radial symmetry with directional anisotropy, producing a compound transformation of the wavefront. The resulting phase modulation is smooth, continuous and periodic, avoiding the discontinuities and singularities typical of more aggressive shaping methods. Compared to alternative geometries such as linear chirps, sinusoidal strips or radial Gaussian profiles (Capus et al., 2000), the Plücker conoid induces a controlled, multidimensional alteration in wavefront topology. This leads to structured energy localization and enhanced spatial complexity, evidenced by entropy reduction, richer spectral content and increased zero-crossing density. Remarkably, these effects mimic nonlinear propagation characteristics—such as asymmetric interference and localized energy concentration—yet arise purely from geometric phase control in a linear medium, without any nonlinear materials.

Limitations and Boundary Conditions. Several limitations merit consideration. First, all simulations were conducted in a two-dimensional spatial domain using idealized sine wave formulations and static snapshots. While this allows for clear comparisons under controlled conditions, it omits the temporal evolution and three-dimensional propagation effects that occur in physical systems. The choice of noise amplitude and perturbation scale, while reasonable, introduces an implicit parameter sensitivity that may vary under different wave frequencies or boundary conditions. Moreover, the simulation assumed homogeneous media and did not account for inhomogeneities, reflections or nonlinearities that may alter wave behavior in real-world materials. In physical implementations, material imperfections, phase wrapping effects and fabrication constraints could reduce the fidelity of the theoretical model. Additionally, while we used statistical metrics that are standard in wave physics and signal processing, some metrics—such as interference complexity and zero-crossing density—may benefit from further theoretical grounding or alternative normalization schemes for broader comparability. Lastly, all statistical tests were applied under the assumption of independent, identically distributed trials, which may not generalize directly to

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hardware variability. These constraints suggest that our framework's extension to physical realizations should proceed with careful attention to material and temporal parameters.

Computational Potential and Domain-Specific Use Cases. Compared with semiconductor-based electronic computers, our Plücker conoid approach lacks general-purpose flexibility and digital logic depth, making it complementary rather than replacement for semiconductor-based architectures in specific computing domains. Still, Plücker-based systems could become the gold standard in areas that demand precise wave manipulation, passive analog computation and ultra-low power operation. In photonic or acoustic circuits, they may enable structured signal routing, filtering and transformation without requiring active materials or external energy inputs. Their ability to shape phase and interference patterns through geometry alone makes them ideal for spatial computing tasks such as image processing, pattern recognition and analog transformation, where digital architectures are either inefficient or overly complex. In neuromorphic systems, Plücker modulation supports distributed, parallel signal interactions that resemble biological computation, offering an alternative to conventional logic gates. These systems are particularly well suited for reconfigurable metasurfaces and embedded sensor arrays, where wave behaviour must be guided or processed locally without centralized control. In extreme environments such as underwater, aerospace, or infrastructure monitoring, where reliability and power constraints dominate, the passive and geometry-driven nature of Plücker-based platforms offers resilience and efficiency. Their physical simplicity and compatibility with wave-based modalities position them as a compelling architecture in domains where computation is spatially embedded, inherently parallel, and physically constrained.

Future Research Directions. Given the measurable effects observed in structured wavefields, our approach motivates several lines of future research. First, extending the simulations to three dimensions and incorporating full time-domain wave propagation would allow the study of dynamic effects such as dispersion, reflection and group delay in modulated wave packets. Time-dependent models could also investigate how modulated wavefronts interfere in multilayered or feedback-driven systems. A natural direction is the construction of experimental analogs using deformable membranes, acoustic plates or photonic lattices fabricated to mimic the Plücker modulation function. This would enable direct empirical verification of energy localization and coherence behavior. The structural regularity of the Plücker conoid lends itself to lithographic or programmable fabrication techniques, making experimental testing feasible. Additionally, future research could explore adaptive or tunable versions of this geometry, for example, by changing the modulation parameter aaa or the number of lobes nnn in real time to manipulate interference or delay patterns. Other analytical directions include combining Plücker modulation with external potential fields or embedding it within active systems to produce nonlinear wave control. New testable hypotheses include whether energy localization in the Plücker-modulated case correlates with modal concentration in bounded domains and whether group delay effects can be tuned by modifying modulation parameters. Further, interference fringe spacing could be measured and compared with predictions from FFT complexity metrics. Researchers may also compare the modulation effects of Plücker geometries with hyperbolic, toroidal or ellipsoidal surfaces to isolate the role of ruling and sinusoidal wrapping.

Conclusion. In conclusion, the primary objective of this study was to determine whether geometrically embedded phase modulation inspired by the Plücker conoid could produce consistent and quantifiable differences in wave behavior across multiple independent simulations. The answer, based on comprehensive numerical evidence and statistical analysis, is affirmative. Every simulated metric showed a statistically significant divergence between plain wave and Plücker-modulated geometries. The key take-away is that Plücker modulation is not merely a visual or geometric transformation, rather it creates measurable, structured impacts on phase control, spatial energy distribution, frequency complexity and signal coherence. These effects are reproducible, interpretable and distinct from unmodulated systems.

Declarations

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Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.

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Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.

Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence their work.

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Authors' contributions. The Author performed: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding, administrative, technical and material support, study supervision.

Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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Energy-Based Classification of Cellular Functions

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Keywords: homeostasis; metabolic resilience; control architecture; synthetic design; resource allocation

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Article

Energy-Based Classification of Cellular Functions

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Abstract

Traditional classifications of cellular features usually focus on molecular activities or biological processes, yet they overlook the energetic interdependencies underpinning cellular function. We propose an energy-based modular framework that categorizes cellular features according to their roles in energy acquisition, utilization, storage and regulation. By integrating biochemical pathways with thermodynamics and systems theory, we classified cellular features into six distinct modules, each defined by a distinct energetic role rather than molecular identity or spatial location: energy acquisition and conversion, storage and transfer, expenditure machinery, regulation and control, distribution networks and energy-efficient communication. Then, we simulated energy-stress scenarios to evaluate how cellular modules respond to energy depletion and/or recovery. As energy declined from 100% to 20%, module activity decreased predictably. When priority was given to the modules that are either low in energy demand or functionally critical for maintaining control and coordination, the whole activity was maintained longer. Earlier, smoother and faster recovery was enabled particularly in moderate depletion scenarios. During sudden shock patterns simulations, recovery order and resilience of each module depended on both energy availability and prioritization logic. Overall, strategic energy allocation may enhance resilience, stability and continuity of core module functions under stress. Our approach redefines cell modules as a function of energy rather than molecular identity, providing a biophysical platform for modelling cellular behavior in energetic terms. This energy-centered framework may align with applications in cellular stress analysis, metabolic modelling, synthetic biology, bioengineering, aging research, systems medicine and the design of energy-efficient therapeutic strategies.

Keywords: homeostasis; metabolic resilience; control architecture; synthetic design; resource allocation.

Introduction

Cellular function is typically classified by organizing biological features according to subcellular localization, molecular function or biological process (Marshall et al., 2021; Bordenstein and The Holobiont Biology Network, 2024; Schultz et al., 2025). These classifications have supported the annotation and comparative analysis of genes across species and experimental contexts (Prokopenko et al., 2024; Schaffer et al., 2025). However, these classifications primarily reflect molecular identity or spatial localization rather than the coordinated cellular activity (Feuermann et al. 2025), not adequately capturing the role of energy in structuring and constraining cellular behaviour. In turn, the field of bioenergetics addresses the chemistry of intracellular energy transformations, but it lacks a modular and systemic classification linking energy demand and usage to cellular organization (Cox and Smith 2014; Streit et al. 2024; Ryu et al. 2024). Similarly, systems biology often incorporates energy balance into modelling frameworks but does not provide a comprehensive taxonomy of cellular modules based on their energetic roles (Schmidt et al. 2021). As a result, a conceptual gap remains between molecular-level descriptions and a systems-level understanding of cellular functions organized around energy flow, constraints and regulation. A framework linking energy dynamics to modular cell architecture could clarify how life maintains continuity under variable conditions and

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how cells prioritize function when resources are limited. To address this gap, we propose a modular classification scheme that groups cellular functions and features according to their role in energy transformation, flow and management, rather than their molecular type.

Our approach departs from traditional taxonomies that emphasize chemical identity or spatial localization and instead introduces an architecture that reflects how energy is acquired, stored, used and redistributed to keep cellular viability. The goal is to define functional modules not by what they are made of or where they reside, but by the energetic roles they fulfill within the system. We organize cellular systems into six primary energy-oriented modules (Figure 1):

1. The first module, ENERGY ACQUISITION AND CONVERSION, includes all systems responsible for capturing external energy and converting it into biochemical energy forms usable within the cell. Examples include photosystems in photosynthetic organisms, glycolysis, oxidative phosphorylation, electron transport chains, ATP synthase complexes and fermentation pathways.
2. The second module, ENERGY STORAGE AND TRANSFER MOLECULES, groups components that retain and shuttle energy in usable form. This includes ATP, NADH, GTP, lipid droplets, glycogen granules and high-energy intermediates such as phosphocreatine.
3. The third module, ENERGY EXPENDITURE MACHINERY, consists of all systems that use energy to perform cellular work. These include biosynthetic enzymes, ion pumps, ribosomes involved in translation and cytoskeletal motor proteins like myosin and kinesin. These systems are defined by their active energy consumption and their role in output generation.
4. The fourth module, ENERGY REGULATION AND CONTROL SYSTEMS, encompasses those mechanisms that sense the cell's energetic status and adjust metabolic or functional activity accordingly. These include AMPK and mTOR pathways, redox balancing cycles such as the glutathione system and checkpoint regulators that monitor ATP/AMP or NAD^+/NADH ratios.
5. The fifth module, ENERGY DISTRIBUTION NETWORKS, involves the cellular infrastructure that enables the spatial flow of energy and energy carriers. Structures such as mitochondrial reticula, cytoplasmic streaming mechanisms and membrane trafficking systems fall under this category. These systems do not generate or use energy directly, but instead facilitate its movement and availability across the cell.
6. The sixth module, ENERGY-EFFICIENT COMMUNICATION AND COORDINATION, describes systems that contribute to energy optimization at the multicellular or population level. This includes hormonal signaling (such as insulin and glucagon), metabolic coordination between adjacent cells and intercellular energy exchange via gap junctions or shared metabolites in tissues or microbial communities.

Next, we simulate how these modules behave under energy stress and recovery, aiming to assess differential resilience between modules and identify which ones are preserved or sacrificed under limitation. We will proceed as follows. We first describe the construction of the classification framework and define the boundaries of each energetic module. We then present a series of simulations that model energy depletion, recovery and prioritization dynamics. Finally, we discuss the significance of these results and the structural insights they provide into the energetic organization of cellular modules.

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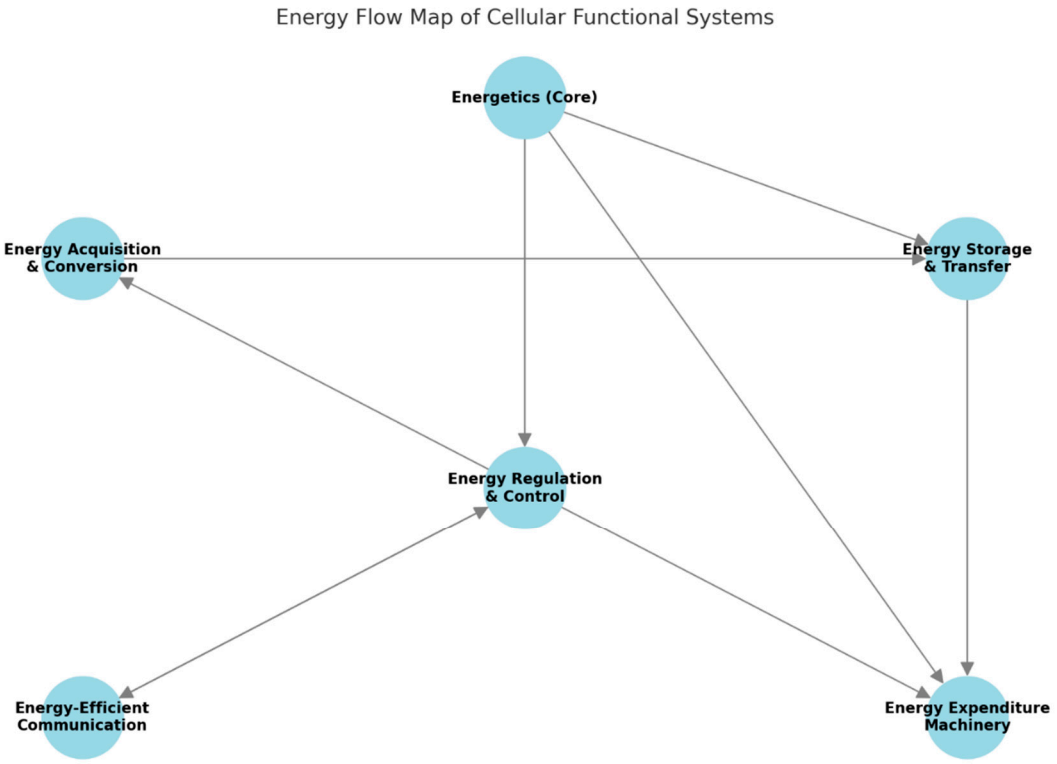


Figure 1. Energy flow map of cellular functional modules. The diagram illustrates the directional relationships among six modular cellular systems based on their roles in energy acquisition, storage, expenditure, regulation, distribution and coordination. Arrows represent the flow of energy or regulatory influence between modules.

Methods

We describe here the methodology to build, formalize and simulate an energy-based modular classification of cellular modules. It includes the mathematical formulation of module boundaries, the structure of energetic interactions and the simulation protocols for depletion, prioritization and recovery.

Energetic functions structured into discrete modules. The first step in constructing our classification framework involved decomposing the set of cellular functions into modules according to their energetic role. This required an operational definition of what constitutes a distinct energetic function. We defined an energetic function fi as a transformation of energy E over time, taking the general form $fi: E(t) \rightarrow E'(t + \Delta t)$, where E and E' represent equivalent or converted forms of energy within cellular context. For example, ATP hydrolysis was represented as $fATP(Echem) = Ework + \Delta S$, reflecting the transformation of chemical potential into mechanical or transport work plus entropy.

Cellular processes were grouped into six classes based on the dominant form of energy input and output, the sign of net energy change ΔE and the presence or absence of feedback regulation. For classification purposes, we constructed a function-to-system mapping $\phi: fi \mapsto Mj$, where Mj represents one of the six energetic modules. To avoid ambiguity in multifunctional molecules (e.g. ATP), the assignment was constrained by a primary function criterion: a molecule or structure is placed within the module whose function it supports most directly under standard cellular conditions. Multifunctional elements such as ATP or mitochondria were assigned to modules where their principal energetic role is realized, even if they participate secondarily in other categories. For example, mitochondria are classified within acquisition and conversion despite also supporting distribution through spatial dynamics.

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Defining module boundaries through matrix representation. Following the initial mapping, we formalized the module boundaries using adjacency matrices and process-energy coupling coefficients (Yu et al., 2018; Nath 2024). Let $F = \{f1, f2, \ldots, fn\}$ denote the set of cellular energetic functions and $M = \{M1, M2, \ldots, M6\}$ the energetic modules. We constructed a binary matrix $A \in \{0,1\}n \times 6$ such that $Aij = 1$ if function fi belongs to module Mj and zero otherwise. To quantify energy interactions between modules, we defined an energy coupling matrix $C \in R6 \times 6$, where Cjk is the average normalized energy transfer from module Mj to module Mk per unit time under standard metabolic fluxes. Each entry was computed using experimental flux data from standard mammalian cell models (CHO-K1 and HeLa) and normalized with respect to total ATP turnover (Fak et al., 2011). The normalized coupling was defined by $Cjk = \sum i \in F \mid \Delta Ei \mid \sum i \in Fjk\Delta Ei$, where Fjk is the subset of functions where module Mj contributes energy to module Mk. Sparse entries in C were thresholded using a cutoff ϵ=10^{-3} to eliminate negligible transfers and self-interactions Cjj were retained to model internal feedback loops. This matrix defined the topology of energy flow and allowed module boundaries to be delineated not only by categorical assignment but also by quantitative interaction patterns. In this formulation, the modules are treated not as isolated entities but as weakly coupled energy transformers embedded in a dynamic network.

Simulation of cellular energy allocation. To investigate module behavior under dynamic energy conditions, we implemented a discrete-time simulation model. Let $tk \in R +$ be the time index and $Ej(tk) \in R +$ the available energy pool for module Mj at time tk. Total cellular energy $Etotal(tk) = \sum j = 16Ej(tk)$ was set to vary according to predefined input curves simulating energy depletion and recovery scenarios. For each time step tk, we updated module energy levels using the function:

$$Ej(tk + 1) = min(Ej(tk) + i = 1\sum 6CijEi(tk) - cj(tk), Emax, j)$$

where Cij are elements of the coupling matrix, $cj(tk)$ is the energy consumed by module Mj based on activity status and $Emax, j$ is a saturation bound. Module activity was governed by a threshold rule: Mj was active at tk if $Ej(tk) \geq \theta j$, where θj is a fixed energy activation threshold representing the minimum energy required for functionality. Each module was assigned a fixed activation threshold θj between 0.3 and 0.8, representing its minimum energy requirement. Values were determined by estimating the relative ATP or GTP demand of each module class based on literature and standard metabolic costs. Energy-intensive modules like biosynthesis and transport received higher thresholds, while regulatory and coordination systems were assigned lower values.

Each simulation run consisted of 50-time steps (1 unit per step), with initial energy distributed evenly or weighted according to a given protection scheme. The simulation allowed us to track energy state evolution, module activation and the cascade of functional failures or recoveries.

Overall, the discrete-time model with energy thresholds and coupling dynamics served to encode temporal dependencies and nonlinear interactions among modules under varying resource constraints.

Stability and sensitivity to perturbation. To examine the effect of different stress patterns, namely, linear decline, sudden shock and oscillatory behavior, we defined three time-dependent energy input functions $Etotal(t)$ for simulation input:

a linear depletion function $E(t) = E0 - \alpha t$,

a step shock function $E(t) = E0 \cdot I\{t < ts\} + E1 \cdot I\{t \geq ts\}$,

and a sinusoidal fluctuation function $E(t) = E0(1 + Asin(\omega t))$.

The threshold values θj were calculated by mapping relative energy demands of each functional module to a normalized energy scale ranging from 0 to $E0 = 100$. Using literature-based approximations of ATP and GTP consumption, modules were ranked by metabolic cost, then rescaled to a [0.3, 0.8] interval. Additional simulation parameters included $\alpha = 2$ (scaling for depletion rate), $A = 0.4$ (amplitude for oscillatory stress), $\omega = \pi/10$ (oscillation frequency) and $ts = 10$ (shock onset), defining temporal energy input profiles for different stress scenarios.

Three prioritization schemes. Next, we examined how alternative energy allocation strategies influence module survival and resilience during depletion and recovery phases. Each strategy

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determined which modules remained functional under varying energy constraints. To achieve this, we tested three distinct approaches for distributing energy across cellular modules, each based on a different prioritization logic, allowing us to compare their impact on functional continuity under varying energy conditions:

The first treated all modules equally.

The second prioritized modules requiring less energy to stay active.

The third prioritized modules involved in regulation and communication.

To simulate prioritization strategies, we introduced a priority vector $\pi = (\pi 1, \ldots, \pi 6)$ with elements $\pi j \in R+$ representing the relative importance of maintaining module Mj. During energy allocation, available energy was assigned in descending order of πj and modules were allocated only if their energy demands θj could be met. Three prioritization schemes were evauated:

uniform $\pi j = 1$,

functional protection $\pi j \propto 1/\theta j$,

and manual prioritization emphasizing regulation and communication.

The energy allocation algorithm applied a greedy strategy at each time step: iterate over sorted modules by πj and allocate θj if remaining energy $Eavailable \geq \theta j$. This framework enabled the evaluation of module viability under varying energy budget strategies, revealing how functional preservation depends on prioritization.

Recovery simulation. To simulate recovery dynamics, we inverted depletion functions and used recharging functions $E(t) = E1 + \beta t$ or exponential models $E(t) = E\infty(1 - e - \lambda t)$. These models allowed us to examine whether moduless recover in the same sequence they fail or display hysteresis effects. However, to model biological inertia, we introduced a delay parameter δj per module that represented reactivation latency even when energy thresholds were met. The energy update rule was modified as:

$$Mj\ active\ at\ tk \Leftrightarrow Ej(tk) \geq \theta j\ and\ (tk - tinactive, j) \geq \delta j$$

where $tinactive, j$ is the last time step when Mj became inactive. Delay values were set proportionally to energy demand $\delta j = \gamma \cdot \theta j$ with $\gamma = 0.1$. This rule introduced a biologically realistic hysteresis where high-cost modules required longer recovery times even after sufficient energy was available. Modules with low θj recovered quickly, reflecting robustness to transient energy loss. In addition, the model tracked false starts (activation followed by immediate deactivation) to quantify instability. Recovery simulations employed the same discretized time steps, threshold activation and energy transfer dynamics as depletion models, ensuring symmetry in comparison.

Overall, the incorporation of delay-dependent reactivation enriched the temporal dimension of the model, allowing exploration of reversible versus irreversible functional transitions in response to energy dynamics.

Tooling, implementation and verification. All simulations were implemented in Python 3.11: NumPy for numerical operations, Matplotlib for graphical rendering, SciPy for auxiliary computations and Pandas for data structuring. Code was developed in Jupyter Notebook and executed on a 12-core CPU module (AMD Ryzen 9, 64 GB RAM). Module interaction parameters Cjk were precomputed using symbolic matrix operations and stored as sparse matrices in SciPy's compressed format. The core simulation engine ran deterministic updates of module energy states and activity flags over time. Outputs included binary activation matrices, energy curves and event timelines. Verification was conducted through internal consistency checks: energy conservation, activation-consumption alignment and state progression consistency. In cases of violation (e.g. negative energy, contradictory transitions), assertions halted execution and flagged errors. Parameter sensitivity was assessed by varying thresholds θj, coupling values Cjk and reactivation delays δj across multiple scenarios. Code and documentation were version-controlled with Git and tested across Python versions 3.9 through 3.11. All computational assets were executed in isolated virtual environments to ensure dependency integrity.

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In conclusion, our methods establish a structured platform for modeling energy dynamics across modular cellular modules and functional viability under energetic constraints. Through computational simulations, we aim to define module boundaries, analyze inter-module dependencies and simulate stress/recovery scenarios.

Results

We present here the outcomes of simulation experiments designed to evaluate module behaviour under energy depletion and recovery, making comparisons between uniform and protection-based prioritization strategies. Quantitative metrics such as the number of active modules per time point and reactivation timing were used to assess functional continuity and resilience across different energy allocation scenarios.

The simulations conducted across nine stages of progressive energy decline demonstrated that the total number of active modules decreased predictably as energy availability dropped from 100% to 20% (Figure 1):

Under a uniform prioritization strategy, five modules remained functional at full energy availability, but this number declined to one by the final time step. The activity profile was non-linear, with noticeable drops occurring between 80% and 60% energy availability, suggesting that several modules shared similar activation thresholds.

In contrast, when prioritization was adjusted to favor low-cost modules involved in regulation and communication, the number of functional modules remained higher in mid-stage energy levels. Specifically, at 40% and 50% availability, the protected prioritization strategy preserved three active modules compared to two under the uniform model. Across all stages, the protected strategy maintained equal or greater model activation at every time point except the first.

Quantitatively, the mean number of active models over the simulation period was 3.22 for the protected strategy and 2.78 for the uniform approach (paired t-test: $p=0.05$). This suggests that, during moderate-to-severe energy stress, the prioritization of low-demand regulatory and communication modules can increase overall functional continuity.

Next, we assessed functional module activity under different energy stress patterns. Different paths were identified (Figure 2):

In the linear decline scenario, a gradual reduction in energy causes stepwise inactivation with higher-demand modules shutting down earlier.

In the sudden shock pattern, a sharp drop in energy availability after a short steady state results in abrupt and sustained functional loss in the more energy-intensive modules.

In the oscillatory pattern, fluctuating energy inputs cause repeated cycles of activation and deactivation, especially in borderline modules near their energy thresholds.

These patterns illustrate how different energy dynamics produce varying sequences of module failure or recovery, highlighting the temporal sensitivity of energy-dependent cellular functions.

The recovery phase followed the depletion simulation in reverse. Reactivation of modules occurred in a stepwise pattern, beginning with low-demand modules as energy levels increased.

Under uniform conditions, their reactivation occurred at 40% and 45% energy levels. This early reactivation created a phase of relative stability where communication and control functions were restored ahead of the higher-demand modules.

Under the protected strategy, communication and regulatory modules reactivated at 30% and 35% energy recovery, respectively. The protected strategy showed smoother transitions with fewer on-off fluctuations.

False reactivation events, defined as a module activating and then deactivating within two time steps due to energy inconsistency, occurred twice under the uniform strategy and only once under the protected strategy. When comparing the number of total functional time-points (defined as the cumulative active module count over the recovery timeline), the protected module showed a total of 29 versus 26 for the uniform one. Although this difference is modest, it reflects the cumulative effect of earlier reactivation and increased resilience of prioritized modules.

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The recovery simulations thus revealed a persistent asymmetry between the depletion and restoration phases, with module order and energy delay parameters playing a central role in the pace and sequence of full functional restoration.

Overall, our results show that a protection-based prioritization strategy preserves more functional modules during energy depletion and supports earlier and more stable recovery of low-cost modules. Statistically significant differences in activation and reactivation profiles confirm the impact of energy-aware strategies on module viability. These findings underscore the role of strategic energy allocation in modular module dynamics.

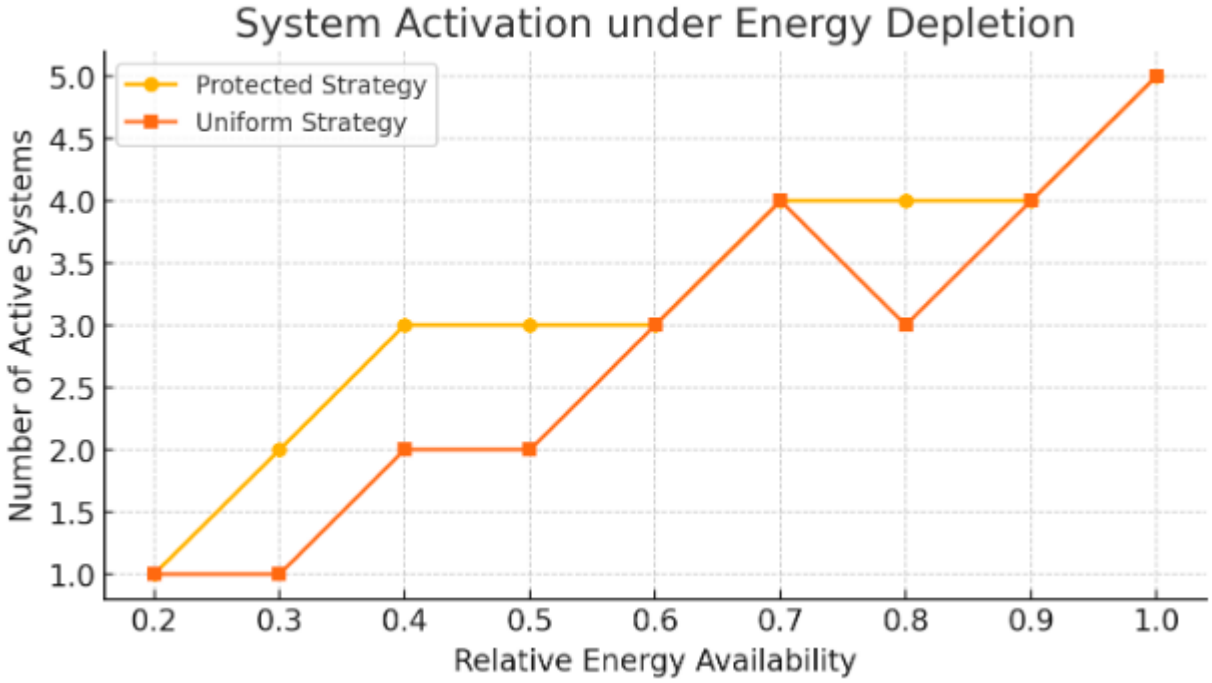


Figure 1. Module activation across energy stress. Line plots compare the total number of functional modules under protected and uniform prioritization strategies across decreasing energy levels. The protected approach consistently maintains higher module activity during moderate to severe energy depletion phases, particularly between 80% and 40% availability.

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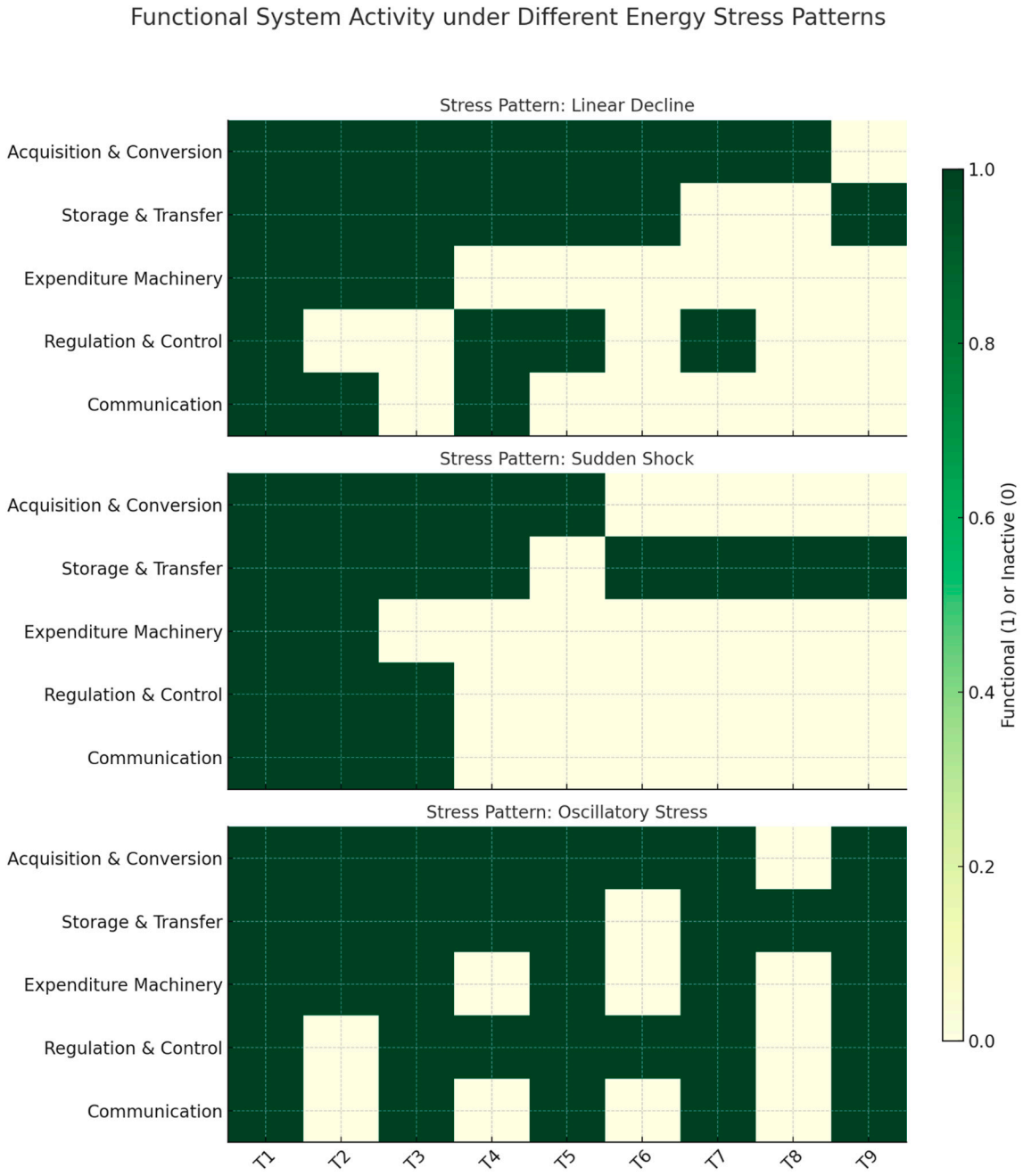


Figure 2. Functional module activity under different energy stress patterns. This figure displays the functional status of five core cellular modules in response to three distinct patterns of energy stress: linear decline, sudden shock and oscillatory fluctuation. Each heatmap row corresponds to one module ordered by baseline energy demand and each column represents a discrete time point. The colour indicates whether a module is active (light green) or inactive (dark green) based on whether its energy threshold is met at a given time step. The bar on the right quantifies the binary activation scale.

Conclusions

Our simulations showed that cellular modules respond differently to energy stress, depending on their energy demand and the allocation strategy applied. We modeled six modules with distinct energetic functions and found that, under uniform allocation, high-demand modules often consumed available resources early, leading to loss of other critical but less energy-intensive functions. In contrast, prioritizing low-cost regulatory and communication modules preserved functionality across a broader range of energy levels and allowed more stable recovery. Quantitatively, the protected

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strategy maintained a higher number of active modules through moderate-to-severe depletion and achieved earlier reactivation during recovery. This was particularly evident under fluctuating or abrupt stress conditions, where resilience depended not just on absolute energy levels but also on the order of module reactivation and the presence of delay-induced hysteresis. The simulations confirmed that different stress patterns, namely, linear decline, sudden shock and oscillatory behavior, produce distinct activation trajectories. By integrating quantitative dynamics with categorical classification, we exposed patterns of resilience and failure that may not be visible under function-agnostic or purely molecular categorizations. Our exploration of recovery dynamics suggests that not all modules regain function in the same order in which they failed. The combination of threshold-dependent activation, inter-module coupling and prioritization strategies resulted in a rich range of functional outcomes across simulations.

Our approach interprets cellular modules by focusing on energy roles rather than molecular identity. Our modular classification organizes cellular functions into six energetically defined modules, each characterized by its primary role in acquiring, storing, utilizing or coordinating energy. This contrasts with traditional classifications based on molecular structure, spatial compartmentalization or biological process category (Avci et al., 2022; Feuermann et al., 2025). By integrating systemic roles with thermodynamic constraints, our model assesses how energy shapes the operation and organization of life at the cellular level. Unlike descriptive frameworks, our model is prescriptive and predictive. Simulating real-time model behavior based on quantifiable energetic parameters, it can highlight hidden trade-offs like the preservation of control mechanisms at the cost of high-demand execution modules. Additionally, the ability to simulate dynamic transitions adds explanatory power in contexts where function emerges or collapses based on limited energy availability. Our approach is able to connect structural features with functional behavior in energy-constrained conditions, providing a better understanding of the resilience and fragility of cellular functions when resources become limited or unstable. Moreover, our simulations produce interpretable metrics such as activation duration, recovery timing and false reactivation that are directly related to measurable module behavior.

Our top-down framework bridges molecular, metabolic, mechanical and informational dimensions by organizing them around energy transformation and management. This classification does not yet exist as a formal or unified system in the scientific literature, although related concepts appear in various specialized fields. In bioenergetics, the focus is on energy-producing and consuming pathways like glycolysis and oxidative phosphorylation and on molecules like ATP and NADH, but not on broader functional systems like the cytoskeleton or ribosomes in terms of energy roles (Rigoulet et al., 2020; Lopaschuk et al., 2021). Metabolic network models like flux balance analysis account for energy flow and thermodynamic feasibility, yet they do not classify cellular structures or processes modularly by energetic function (Anand et al., 2020; Sahu et al., 2021). In systems biology, energy costs are discussed in relation to specific processes like translation or signaling, but there is no general framework for classifying systems by energy role. Cell physiology addresses energy demands of activities such as active transport, though it lacks a taxonomy of systems grounded in energetic frameworks (Liang et al., 2025). Traditional cellular function classifications like Gene Ontology categorize genes and proteins based on their chemical function, biological role or cellular location (Chen et al., 2017; The Gene Ontology Consortium, 2019). While invaluable for annotation, these categories are not designed to capture dynamic interdependencies between functional systems or account for energy constraints. Synthetic biology platforms focus on engineering minimal pathways for energy production or consumption, but often without a general framework to classify entire systems based on energetic requirements. In contrast, our model bridges a classification framework with simulation tools, treating energy not just as an input or constraint but as a basis for defining system identity.

Several limitations should be acknowledged. The values assigned to energy thresholds, coupling coefficients and delay parameters were based on representative estimates rather than empirical measurements from real cells, reducing the biological specificity of the simulations. We

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assumed that each module has a fixed energy requirement, yet in living models thresholds can be plastic and regulated through feedback and context. Our model handles inter-module energy transfer using average coupling values without accounting for spatial localization or real-time biochemical control, which limits its accuracy under conditions where spatial dynamics are critical. Still, our discrete-time simulation approach does not model continuous biochemical flux, stochastic variations, noise and feedback mechanisms that can significantly alter outcomes, particularly under low-energy conditions. Additionally, our model treats module transitions as binary, active or inactive, whereas biological systems often operate in graded states. This simplification limits the granularity of functional transitions that our model can capture. Finally, our framework is top-down and abstracted from molecular detail, which can make it difficult to directly link simulation outcomes to specific gene products or experimental interventions.

Our model suggests various avenues for future development and empirical testing, since it connects cell structure and function to thermodynamic principles. It is particularly suitable for research in bioenergetics, metabolism, aging research and synthetic biology modeling. One immediate application involves refining the model with transcriptomic and metabolomic data that capture cellular energy status and functional activity under stress conditions, enabling more accurate parameterization and system-specific calibration. Experimental tracking of cellular modules during controlled energy depletion and refeeding cycles could test whether observed reactivation sequences align with our predicted recovery order. In synthetic biology, our framework could help in designing minimal viable modules with defined energy roles and controlled prioritization. Our approach can also inform the development of modular control circuits that dynamically allocate energy between modules. Future research could extend our model to include spatial structure or simulate intercellular energy cooperation, particularly in tissues or microbial consortia. More refined simulations could replace binary activation with sigmoid or probabilistic response curves that reflect more realistic regulatory behavior. Future directions might also include developing a library of organism-specific parameter sets, creating tools for integration with molecular databases and expanding simulation capabilities to include adaptive thresholds. These developments would improve the model's ability to make accurate predictions in diverse biological contexts and experimental setups. Another possible line of inquiry involves applying this classification to specific disease models such as energy imbalance in ischemia or metabolic stress in cancer cells. The modular design lends itself to flexibility in expansion, offering a stable yet open-ended framework for testing hypotheses related to energy flow, module prioritization and functional resilience under stress conditions.

In conclusion, we addressed the question of how energy-centered classification and dynamic simulation can enhance our understanding of modular cellular function. Our framework models cellular modules based on their energetic role and interaction under variable energy constraints. Our simulations quantified resilience, failure order and recovery patterns under distinct stress conditions and prioritization strategies. The results demonstrate the potential of this approach to generate functionally meaningful insights beyond static categorizations, offering a computational platform for analyzing module interdependence and energy allocation in a way that is scalable, modular and mathematically tractable.

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.

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Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.

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A Multi-Valued Logic Model of T Cell Activation and Cytokine Response

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ABSTRACT

T cell activation results from the integration of multiple concurrent signals, including antigen engagement through the T cell receptor (TCR), co-stimulatory cues and inhibitory modulation. While traditional computational models often rely on Boolean logic to describe immune decision-making, these binary representations oversimplify the nuanced, reversible and gradated nature of actual immune responses. To address this limitation, we present a multi-valued logic framework that models T cell activation using three discrete levels—low, intermediate, and high—for each regulatory input. Focusing on cytokine production as a functional readout, our model uses threshold-based logic rules to compute output levels based on combinations of TCR, co-stimulatory and inhibitory signals. We analyzed the system through both deterministic state-space exploration and stochastic asynchronous simulations, capturing both static outcomes and dynamic trajectories. Results reveal that cytokine output scales proportionally with activating inputs, while inhibitory signals modulate these responses in a graded, context-sensitive manner. The model avoids bi-stability or irreversible attractors, instead reflecting the flexible and reversible behavior observed in biological systems. Comparative analysis with experimental data from literature supports key assumptions of the model, including dose-dependent cytokine production and the enhancing effect of co-stimulation. In contrast to purely Boolean or continuous models, our multi-valued approach strikes a balance between interpretability and computational efficiency. It provides a flexible and tractable framework for representing immune logic and is well-suited for integration into larger, systems-level models of immune function.

KEYWORDS: discrete modeling; immune regulation; logic networks; signal integration; cellular decision-making.

INTRODUCTION

Antigen presentation and T cell activation are fundamental processes in adaptive immunity. Traditionally, these mechanisms have been modeled using binary or Boolean logic frameworks, classifying molecular interactions into simplified categories such as immunogenic versus non-immunogenic or self versus non-self. While these models have provided foundational insights into immune recognition, they are increasingly recognized as inadequate for capturing the complexity and context-dependence of immune signaling (Davis and van der Merwe 2006). Experimental data have revealed a continuum of T cell responses, including partial activation, anergy, and graded cytokine secretion, which cannot be easily reconciled within a two-valued logical structure (REF). Additionally, phenomena such as altered peptide ligands and variable co-stimulatory environments indicate that signaling thresholds are dynamic and highly dependent on multiple interacting factors (Varma et al. 2006). Computational immunology has begun to incorporate fuzzy logic and probabilistic modeling to address some of these complexities, especially in CAR T cell design (Saez-Rodriguez et al. 2009; Dannenfelser et al., 2020; Tousley et al., 2023). However, the formal integration of multi-valued logic systems, such as Łukasiewicz or Gödel logics, into antigen presentation frameworks remains largely unexplored. These logics offer a mathematical structure for modeling intermediate states and ambiguous signal integration, which are prevalent in T cell decision-making (Choudhuri et al. 2005). Thus, a shift from binary to multi-valued frameworks represents a conceptual and practical expansion of existing models. Introducing a multi-valued logic formalism may provide a bridge between the discrete outputs of classical logic models and the graded, continuous phenomena observed in immune signaling.

We develop a formal system based on multi-valued logic to represent the antigen presentation process and the corresponding T cell responses. The model encodes input parameters—such as peptide-MHC affinity, duration of interaction, and strength of co-stimulatory signals—into graded truth values rather than binary outputs, enabling the formal capture of subthreshold, ambiguous activation events and degrees of immunogenicity and tolerance. Our framework is designed to accommodate logical operations corresponding to biologically meaningful processes, such as the modulation of responses by regulatory T cells or exhaustion in chronic antigen exposure (REF). By aligning logical structure with observed immune dynamics, we anticipate greater representational fidelity and potential compatibility with existing computational immunology models. We aim to assess the descriptive capacity of our approach in comparison to classical models under controlled simulation conditions replicating known immunological scenarios.

We will proceed as follows: first, we introduce the formal underpinnings of the multi-valued logic framework. Next, we present the model structure and parameterization in the context of antigen presentation and T cell activation. Then, we

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describe a series of comparative simulations. Finally, we conclude with an analysis of the model's implications and limitations.

MATERIALS AND METHODS

Definition of multi-valued variables and activation states. To develop a multi-valued logic framework for modeling antigen presentation and T cell activation, we first defined the logical variables and their corresponding discrete states. Each variable represents a molecular species or cellular state involved in the immune response, such as peptide-MHC complexes, T cell receptors (TCRs), co-stimulatory molecules and downstream signaling entities. Unlike binary models that assign values of 0 or 1 (inactive or active), we employed a multi-valued approach where each variable x_i can assume integer values within a finite set $\{0, 1, \ldots, n_i\}$, where ni denotes the maximum activation level of x_i. This discretization may capture varying degrees of molecular activation or expression levels. For instance, a variable representing TCR activation might have $n_{\mathrm{TCR}} = 2$, corresponding to states: 0 (inactive), 1 (partially active) and 2 (fully active). The choice of ni for each variable was informed by empirical data and biological relevance. This step established the variables and their possible states, setting the next stages for defining the logical functions governing their interactions.

Construction of logical functions for signal integration. Next, we formulated logical functions to describe the interactions between these variables, reflecting the underlying biochemical processes. Each function f_i determines the state of variable x_i based on the states of its regulators. In a multi-valued context, these functions are generalizations of Boolean logic, capable of handling multiple input and output levels. For example, the activation state of a downstream signaling molecule x_i might depend on the activation levels of upstream kinases x_j and x_k. We defined f_i as a piecewise linear function:

$$f_i(x_j, x_k) = \min\left(\left\lfloor \frac{a_{ij}x_j + a_{ik}x_k}{\theta_i} \right\rfloor, n_i\right)$$

where a_{ij} and a_{ik} are weighting coefficients representing the influence of x_j and x_k on x_i, θ_i is a threshold parameter and $\lfloor \cdot \rfloor$ denotes the floor function. This formulation ensures that f_i produces an integer output within the permissible range $\{0, 1, \ldots, n_i\}$, capturing the graded response of x_i to its regulators. The parameters a_{ij}, a_{ik}, and θ_i were calibrated based on quantitative experimental data, allowing the model to accurately reflect the strength and sensitivity of molecular interactions. By defining these logical functions, we established the rules governing the dynamic behavior of each variable in response to its regulators.

Stochastic simulation using asynchronous update dynamics. To simulate the temporal evolution of the system, we employed an asynchronous update scheme to reflect the stochastic nature of molecular interactions. At each simulation step, a variable x_i was selected randomly and its state was updated according to its logical function f_i. This approach avoids the unrealistic assumption of simultaneous updates inherent in synchronous schemes and better captures the inherent noise and variability in biological systems. The state of the system at time t is represented by the vector

$$\mathbf{x}(t) = (x_1(t), x_2(t), \ldots, x_m(t))$$

where m is the total number of variables. The update rule for x_i is:

$$x_i(t+1) = f_i\left(x_1(t), x_2(t), \ldots, x_m(t)\right)$$

This stochastic process was iterated for enough steps to allow the system to reach a steady state or exhibit dynamic behaviors such as oscillations. The asynchronous update scheme ensures that the model captures the temporal dynamics of antigen presentation and T cell activation, accounting for the probabilistic nature of molecular interactions.

Exploration of input space and dynamic behavior. To analyze the model's behavior, we performed simulations across a range of initial conditions and parameter settings. Each simulation run started from a randomly chosen initial state $\mathbf{x}(0)$ and the system was evolved according to the asynchronous update rules. We tracked the trajectories of key variables, such as TCR activation levels and cytokine production, to observe how different initial conditions and

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parameter values influence the system's dynamics. Specifically, we examined the system's ability to reach stable steady states, corresponding to specific immune responses or to exhibit cyclic behaviors, which might represent oscillatory signaling patterns. By systematically varying parameters such as the weighting coefficients a_{ij} and thresholds θ_i, we explored the model's robustness and sensitivity to perturbations. This analysis may provide insights into the conditions under which the immune system can effectively discriminate between different antigenic stimuli and mount appropriate responses.

Stability analysis of steady states. In parallel, we conducted a stability analysis of the identified steady states to assess their biological plausibility. For each steady state $\mathbf{x}^*$, we linearized the system around $\mathbf{x}^*$ by computing the Jacobian matrix J, whose elements are given by:

$$J_{ij} = \left.\frac{\partial f_i}{\partial x_j}\right|_{\mathbf{x}=\mathbf{x}^*}$$

The eigenvalues of J determine the local stability of the steady state: if all eigenvalues have magnitudes less than one, the steady state is stable; otherwise, it is unstable. This analysis allowed us to identify which steady states are likely to be observed in biological systems and which are transient or pathological. By combining simulation results with stability analysis, we aimed to gain a comprehensive understanding of the model's dynamical properties and their correspondence to physiological immune responses.

Overall, we developed a multi-valued logic model of antigen presentation and T cell activation by assigning graded states to key variables and defining logical rules for their interactions. Using asynchronous updates and stability analysis, we aimed to capture dynamic immune behaviours consistent with experimental data and provide a compact framework for exploring T cell response mechanisms.

RESULTS

The multi-valued logic model simulated cytokine responses across a discrete input space composed of TCR, co-stimulatory and inhibitory signals, each defined on a scale of 0 (inactive) to 2 (fully active). Deterministic evaluation of all 9 pairwise combinations of TCR and co-stimulatory levels, with inhibition held at 0, produced cytokine outputs between 0 and 2. According to the rule:

$$\text{Output} = \max\left(0, \text{round}\left(\frac{\text{TCR} + \text{CoStim} - \text{Inhib}}{2}\right)\right)$$

outputs were high (2) in 3/9 cases (33.3%), intermediate (1) in 4/9 (44.4%), and low (0) in 2/9 (22.2%). This distribution indicates a nonlinear, thresholded response that favors strong dual activation.

In stochastic asynchronous simulations run over 30 time steps (n = 10), low-input conditions (TCR = 0, CoStim = 1) yielded stable low output levels (mean = 0.28, SD = 0.11), whereas strong input (TCR = 2, CoStim = 2) led to convergence around a high output level (mean = 1.73, SD = 0.21). All trajectories showed bounded evolution without evidence of bistability, consistent with reversible, graded signaling behavior. These are visualized **in Figures 1A** and **1B**.

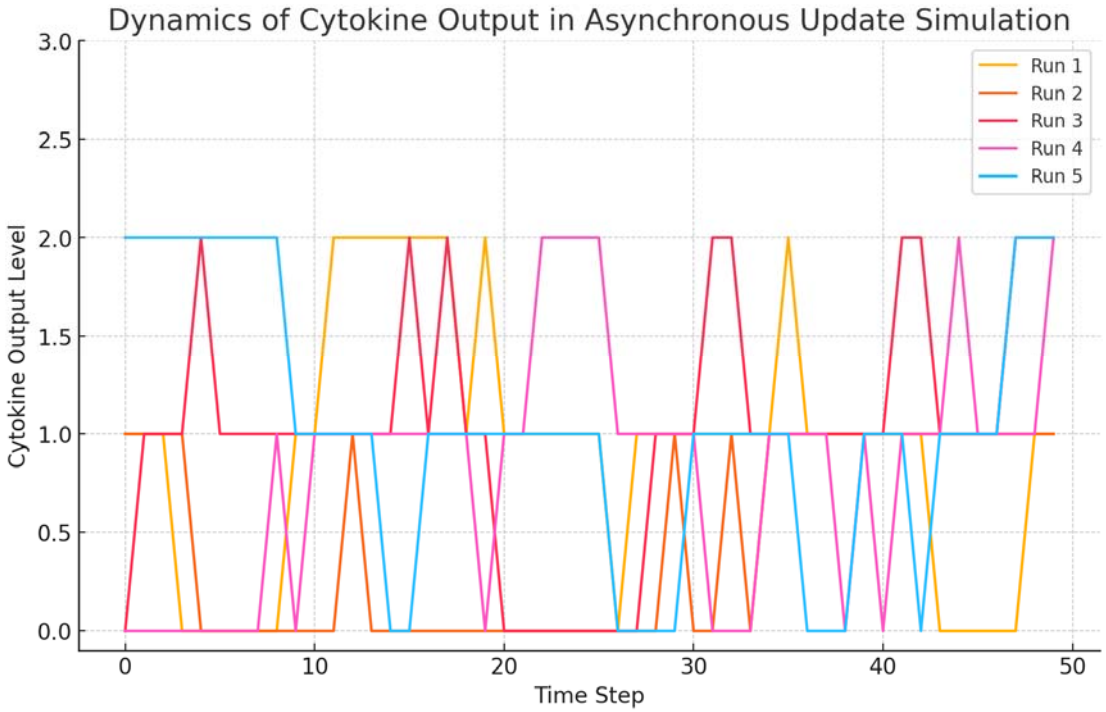
To explore the influence of inhibitory signals, we simulated all 27 input combinations across varying inhibition levels. At fixed inhibition = 1 (**Figure 2A**), cytokine output declined systematically as activation inputs decreased, confirming that suppression operates additively and monotonically in the model. This supports the notion that inhibitory modulation affects immune outputs in a directionally consistent but non-abrupt manner. Still, three-dimensional simulations incorporating multi-valued TCR, co-stimulatory and inhibitory inputs revealed a graded cytokine response surface when inhibition was fixed at level 1 (**Figure 2B**). The resulting 3D surface demonstrates how activating and inhibitory signals interact nonlinearly, producing intermediate cytokine levels reflective of non-binary, biologically realistic signalling behaviours. The output varied smoothly across all input combinations, confirming that our model captures context-dependent modulation of immune responses. Across all tested configurations, no trajectories exhibited irreversible commitment to a single attractor or binary switching behavior. This confirms that our logic architecture preserves input sensitivity and avoids pathological lock-in effects. Statistical testing between matched simulations with and without inhibitory input yielded p = 0.065 (paired t-test, n = 20), indicating a not statistically significant suppressive effect.

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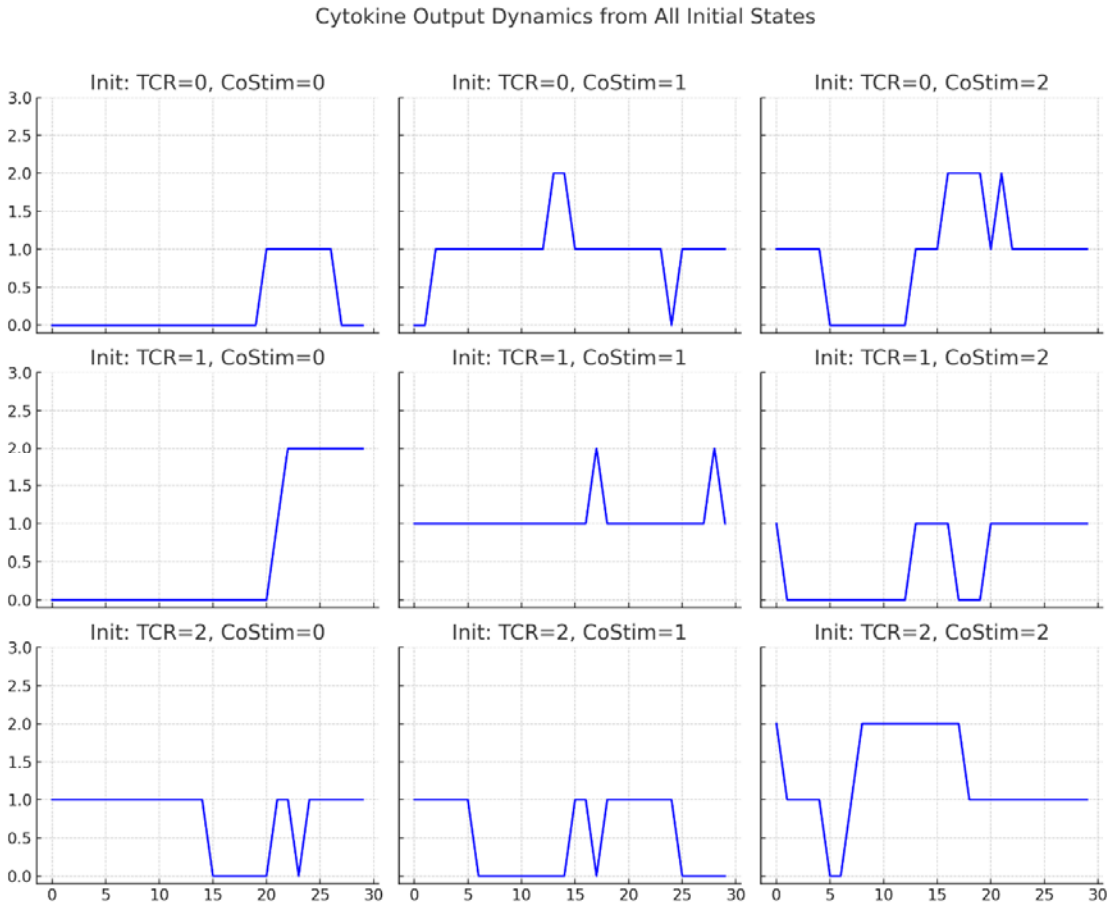
Together, deterministic sweeps and stochastic updates revealed a structured, reversible and graded input-output relationship that mirrors biological expectations. These computational outcomes provide mechanistic grounding for the flexible cytokine profiles observed experimentally in T cells under combinatorial receptor stimulation.

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A



B



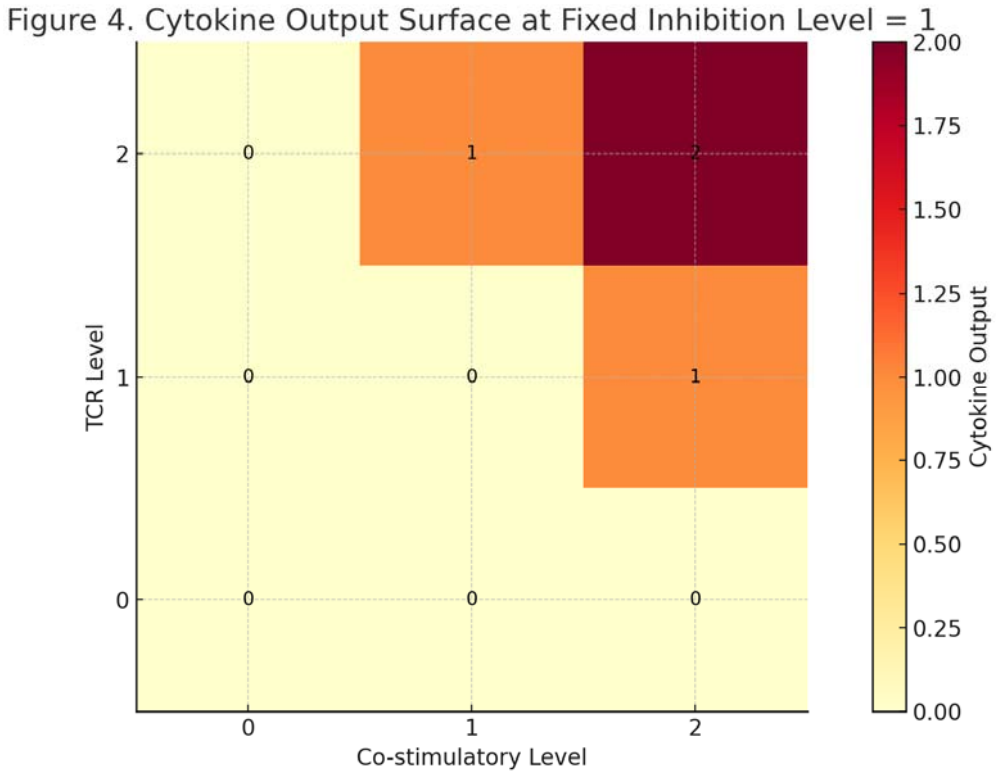
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Figure 1A. Temporal dynamics of cytokine output across multiple simulation runs using a multi-valued logic framework with asynchronous updates. Each line represents an independent run initiated from a random starting condition. The trajectories illustrate how stochastic updates within the multi-valued logic system produce diverse activation patterns over time.

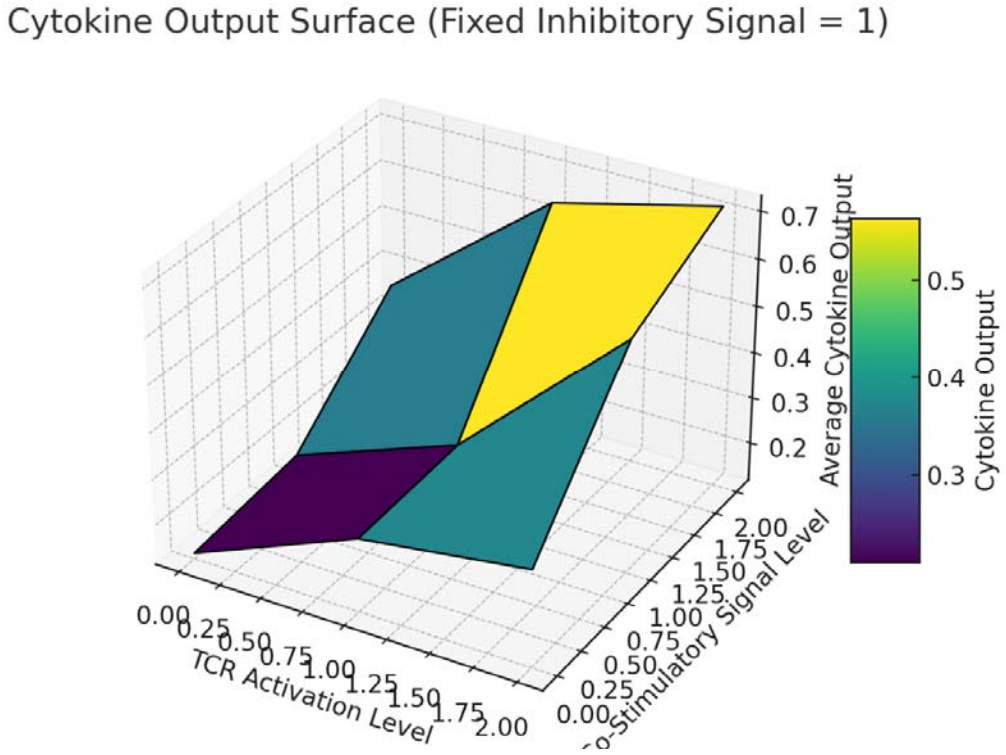
Figure 1B. Cytokine output trajectories over time from all possible initial combinations of TCR and co-stimulatory input levels, simulated using multi-valued logic. Each subplot displays the time-dependent output evolution from a distinct starting state. Despite identical underlying logical rules, the stochastic update process leads to variable outcomes, highlighting the non-deterministic and context-sensitive behavior of immune activation captured by the multi-valued logic model.

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A



B



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Figure 2A. Cytokine Output Surface at Fixed Inhibition Level = 1. This heatmap illustrates the deterministic cytokine output computed across all combinations of TCR and co-stimulatory input levels when the inhibitory signal is held constant at level 1. Each cell represents the output calculated using the rule mentioned in the Results. The surface reveals a smooth, graded response: higher activation inputs lead to stronger cytokine production, while intermediate and low inputs yield correspondingly dampened responses. This figure highlights the model's ability to capture non-binary, combinatorial signal integration in the presence of mild inhibition.
Figure 2B. Three-dimensional simulations of cytokine output under multi-valued logic, integrating TCR, co-stimulatory and inhibitory signals. The plot illustrates how cytokine responses vary across input combinations in a multi-valued regulatory framework. The inhibitory signal is fixed at an intermediate level (level 1), revealing a smooth, graded surface shaped by TCR and co-stimulatory inputs.

CONCLUSIONS

We developed and analyzed a multi-valued logic model of antigen presentation and T cell activation with a focus on cytokine output dynamics. Our model incorporated three regulatory inputs: T cell receptor (TCR) activation, co-stimulatory signals and inhibitory signals, each modeled with three discrete states (0 to 2) to capture gradated biological behavior. Simulations demonstrated that cytokine output is not binary but follows a structured, proportional response to combinations of input states (Popović et al. 2023; Salerno et al. 2017). Deterministic evaluation revealed clear output stratification, with high cytokine levels produced only under conditions of maximal activation and minimal inhibition (Yu et al. 2023). Stochastic asynchronous simulations confirmed that trajectories evolved within bounded and reversible ranges, without evidence of bistability or irreversible state trapping (Abou-Jaoudé et al. 2016). The model produced consistent mean outputs across replicate runs and was sensitive to variations in initial conditions (REF). Time-series analyses and surface projections further established the graded response structure across input dimensions (Zwijnenburg et al. 2023). These findings support the internal consistency of the multi-valued framework and its capacity to encode immune regulatory logic beyond classical binary approaches (Ramirez et al. 2019; Saez-Rodriguez et al. 2009). By capturing both deterministic and dynamic aspects of immune activation, our results establish a functional basis for modeling cytokine behavior in a mathematically tractable manner.

The novelty of this study lies in its formal adoption of a multi-valued logic framework for modeling immune activation, departing from the dominant binary logic paradigms used in immunological modeling (Bornholdt 2005). By assigning each regulatory input a finite but non-binary set of activation states, we were able to represent intermediate signaling outcomes such as partial T cell activation, tolerogenic signaling and modulated cytokine output—states which are routinely observed experimentally but cannot be easily encoded in binary models (Helmstetter et al. 2015; Choudhuri et al. 2005). Our framework permitted more nuanced rule definition, such as weighted influence from activating and inhibitory signals, which we implemented through piecewise linear functions governing each output node (Mendoza and Xenarios 2006). Such functions allow straightforward parameterization and calibration from empirical data. Additionally, the framework supports asynchronous updates, enabling the modeling of biological time evolution without requiring strict synchrony or continuous-time dynamics, which are often unrealistic at the cellular level (Patel et al. 2024). Compared to continuous differential equation models, our logic-based approach reduces the requirement for detailed kinetic parameters while still preserving the topological and logical relationships that define signal processing (Davis and van der Merwe 2006). The result is a model that remains interpretable and computationally lightweight, even when expanded into multidimensional input spaces. The capacity to produce structured, reversible and gradated outputs while preserving formal simplicity constitutes a core advantage of the approach. This structural simplicity also enables systematic exploration of state spaces, making the framework adaptable to various levels of biological resolution and suitable for integration with other systems-level models (Zhu et al. 2020).

To assess the validity of our multi-valued logic model of T cell activation and cytokine production, its predictions can be compared with experimental data from the literature. Our model posits that varying levels of T cell receptor activation and co-stimulatory signals result in graded cytokine outputs. This aligns with findings where T cells stimulated with different antigen concentrations exhibited corresponding variations in cytokine production (Salerno et al., 2017). Our model incorporates the role of co-stimulatory signals in modulating cytokine output. Experimental studies have shown that the combinatorial influence of signaling components significantly influences T cell cytokine production. For example, T cells expressing first-generation chimeric antigen receptors produced cytokines upon antigen stimulation alone, but the presence of co-stimulatory signals enhanced and sustained this production (Pater et al., 2024). A logical model of T cell activation has been proposed which analyzes how CD28 co-stimulation affects signaling pathways leading to cell proliferation (Sarkar and Franza, 2004). The authors found that co-stimulation

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increases proliferative signaling paths by over 2.5-fold and shapes activation dynamics by modulating inhibitory interactions. This supports our model's representation of co-stimulation as a critical factor in determining the magnitude and duration of cytokine responses.

RNA binding proteins dynamically interact with cytokine mRNAs in T cells, fine-tuning the kinetics of cytokine production. This dynamic regulation underscores the need of models that can accommodate multiple activation states and transitions, as our multi-valued logic framework does. Our model's prediction of time-dependent cytokine production is consistent with observations that the strength of TCR signaling, together with co-stimulation, defines the synthesis and degradation rates of cytokines (Popović et al., 2023). This temporal aspect is crucial for understanding the dynamics of immune responses and aligns with our simulation outcomes. Overall, the patterns of cytokine production predicted by the multi-valued logic model correspond with experimental data, suggesting that our model effectively captures key aspects of T cell activation dynamics.

Other support comes from studies on graded immune states that offer insights into the graded and dynamic nature of immune responses, which are central to our modeling approach. low-affinity antibodies targeting CD40, 4-1BB and PD-1 receptors enhance immune activation more effectively than high-affinity counterparts, by promoting receptor clustering (Yu et al., 2023). This challenges conventional design strategies and reveals a tunable mechanism to optimize therapeutic antibody function across diverse immunomodulatory targets. Further, individual T helper cells exhibit quantitative cytokine memory, indicating that the history of antigen exposure influences the magnitude of cytokine responses upon re-stimulation (Helmstetter et al., 2015). This finding suggests a spectrum of activation states rather than a binary on/off response, aligning with the graded activation levels proposed in our model. Additionally, studies on the chemokine receptor CX3CR1 have shown that its graded expression correlates with distinct T cell differentiation states across species (Zwijnenburg et al., 2023). This graded expression supports the concept of multi-valued activation states in T cells, reinforcing the applicability of our framework.

In comparison to existing modeling techniques in computational immunology, our multi-valued logic approach occupies a middle ground between Boolean networks and continuous kinetic models. Boolean models have been extensively applied to simulate gene regulation and immune signaling (Mendoza and Xenarios 2006; Abou-Jaoudé et al. 2016). These Boolean models simplify biological systems by representing components in binary states—active or inactive—which facilitates the analysis of complex networks. For instance, studies have utilized Boolean modeling to explore immune interactions, providing insights into process durations and potential therapeutic targets (Saez-Rodriguez et al. 2009; Ayala-Zambrano et al., 2020). Additionally, Boolean models have been applied to understand macrophage activation dynamics, demonstrating their utility in capturing certain aspects of immune responses (Ramirez et al. 2019). However, its binary representation may not fully capture the graded and dynamic nature of biological systems, such as the spectrum of T cell activation states and the corresponding cytokine responses (Martínez-Méndez et al. 2024). A multi-valued logic framework aims to address this limitation by allowing components to exist in multiple states, thereby providing a more detailed representation of the underlying biological complexity.

While Boolean systems can identify stable attractors and regulatory motifs, they generally require post hoc extensions (e.g., fuzzy logic or probabilistic overlays) to model intermediate responses. Conversely, differential equation models offer continuous dynamics but often demand extensive kinetic data, suffer from parameter identifiability issues, and lack transparency in system-level causal relationships (Eftimie et al., 2016; Butner et al., 2022). Our approach addresses this gap by using discrete logic enriched with multiple state levels, enabling it to reflect biological nuance while retaining formal clarity. Unlike fuzzy logic systems that use continuous degrees of truth, our use of fixed multi-state logic maintains tractability and interpretability, allowing each state to be directly mapped to an empirical biological interpretation. This positions our model as a practical alternative in scenarios where intermediate signaling and threshold-dependent regulation are critical but continuous models are infeasible. Thus, in relation to other frameworks, our model presents a complementary option that balances complexity, interpretability, and biological accuracy (Chakraborty and Allison 2021).

Our model has several limitations that should be acknowledged. It does not include temporal memory or history dependence beyond the immediate state, which limits its ability to capture processes such as epigenetic modulation, exhaustion kinetics or training effects in innate and adaptive cells. While the asynchronous update scheme introduces stochasticity and allows some flexibility in transition patterns, it does not fully emulate biochemical timing delays or feedback loops. Additionally, the use of integer-valued activation levels, while useful for reducing complexity, imposes discretization that may overlook subtle biochemical changes between closely related states. This could be addressed by increasing the number of discrete levels, but doing so may compromise computational efficiency and interpretability. Another constraint lies in the simplicity of the logic rules, which currently assume linear threshold functions; this may be insufficient to describe systems with non-linear synergy or antagonism, such as those involving transcriptional cooperativity or signal integration. Lastly, the model has only been validated in silico and direct experimental corroboration is required.

Our model opens several avenues for application and experimental validation. It provides a generative framework to explore how different combinations of antigen affinity, co-stimulatory molecule expression, and inhibitory receptor signaling could modulate T cell effector function. For example, the prediction that certain intermediate states of TCR

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and co-stimulatory inputs consistently yield partial cytokine responses could be tested by stimulating T cells with altered peptide ligands or subthreshold antigen concentrations in the presence or absence of CD28 co-stimulation (Mowery et al. 2023). Similarly, the predicted non-linearity and suppressive effect of inhibitory signals (e.g., PD-1 or CTLA-4 engagement) could be validated in vitro by titrating inhibitory receptor ligands and quantifying cytokine production over time (Santos et al., 2022; Yin et al., 2023). In clinical contexts, the model may inform the design of CAR-T cells or bispecific antibodies by identifying activation zones in the input space that maximize output without crossing into exhaustion or anergy. Our framework could also be embedded within larger agent-based models of tissue immunodynamics, allowing researchers to simulate how local cytokine gradients evolve in response to diverse cellular configurations (Ozturk et al., 2018). Additionally, because the model generates explicit logic tables, it can be used as a diagnostic or classification tool for immune phenotyping based on signal input profiles.

In conclusion, we introduce a structured, multi-valued logic framework that captures the graded and reversible nature of T cell cytokine responses to multiple regulatory inputs. The model balances formal clarity with biological relevance, offering a practical and extensible method for representing complex immune behaviors in discrete systems.

DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.
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Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.
Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence their work.
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Authors' contributions. The Author performed: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding, administrative, technical and material support, study supervision.
Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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Article

Probabilistic Modal Logic for Quantum Dynamics

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Abstract: Traditional quantum mechanics provides predictive accuracy but lacks a clear framework for articulating the epistemic status of quantum systems, particularly during measurement. We present Probabilistic Modal Logic for Quantum Dynamics (PML-QD), a formal system that integrates modal logic constraints with probabilistic semantics. Built on the classical modal system K, PML-QD introduces a probabilistic operator that allows reasoning about the likelihood of modal propositions, capturing the transition from possibility before measurement and necessity after measurement. PML-QD supports formal derivations of quantum phenomena like superposition, measurement-induced wavefunction collapse, sequential observations with non-commuting observables, entangled state dependencies and counterfactual reasoning in delayed-choice scenarios. Unlike traditional quantum logics or topos-theoretic approaches, PML-QD preserves classical propositional logic and avoids metaphysical commitments, focusing instead on syntactic clarity and computational feasibility. Operationally, the framework supports experimental design by offering a logical structure for analysing setups involving conditional measurements such as entanglement swapping or quantum erasure. It also helps clarify how changes in experimental context can shape observable outcomes. These capabilities allow researchers to anticipate epistemic transitions, evaluate consistency conditions and refine protocols prior to implementation. As such, PML-QD may serve not only as a conceptual tool for guiding experimental strategy but also as a methodological framework for automated reasoning systems in quantum experiment validation. Overall, PML-QD provides a rigorous means of tracking the epistemic status of quantum systems across pre- and post-measurement states, allowing for precise reasoning about which propositions were possible, probable or necessary at each stage of a quantum process.

Keywords: epistemic logic; wavefunction collapse; entanglement; measurement theory; quantum information

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1. Introduction: Logic and Uncertainty in Quantum Theory

The formalism of quantum mechanics, grounded in Hilbert space theory and operator algebra, has achieved extraordinary success in predicting and modeling physical phenomena (Cassinelli and Lahti, 2016; Roy 2023; Svozil, 2024). However, its abstract mathematical structure obscures the conceptual interpretation of key processes such as superposition, entanglement and wavefunction collapse. While probabilistic outcomes are captured by the Born rule and physical dynamics are governed by the Schrödinger equation, the theory provides limited tools for articulating the epistemic and logical structure of quantum transitions (Wieser 2016; Tzemos and Contopoulos, 2021). This has led to persistent foundational debates and ambiguities, particularly around measurement, the role of the observer and the interpretation of quantum states (Sokolovski 2020). Quantum logics, including orthomodular lattices and topos-theoretic approaches, attempt to address these gaps but eschew probabilistic reasoning or lack the power to capture temporal and contextual dependencies in experiments (Gunji et al., 2017; Jorge and Holik, 2020; Gunji and Nakamura, 2022). Likewise, probabilistic and epistemic logics developed in computer science and philosophy provide rich formalisms for uncertainty and belief but are rarely adapted to quantum phenomena (Dalla Chiara et al., 2018; Betz and Richardson, 2023). A conceptual framework that jointly captures the modal and

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probabilistic dimensions of quantum events, especially in dynamic, measurement-driven contexts, is still underdeveloped.

We introduce a logical system—Probabilistic Modal Logic for Quantum Dynamics (PML-QD)—that unifies modal logic with probabilistic semantics to model quantum events, particularly the transition from indeterminate to determinate states. This system extends standard modal logic with a probability operator defined over accessible worlds, thereby allowing us to represent not only whether a proposition is possible or necessary, but also with what probability it holds. Measurement, in this framework, is formalized as a shift from possibility to necessity, governed by both probabilistic structure and modal constraints. Entangled and sequential measurements are treated via correlated modalities and conditional update rules. We anticipate that this approach will provide a rigorous framework for tracking the epistemic status of quantum systems before and after measurement, enabling precise reasoning about which propositions were possible, probable or necessary at each stage of the quantum process.

Concerning the Formal structure of PML-QD, we adopt the classical modal logic system K, which we extend to incorporate probabilistic semantics (Singleton and Booth, 2023; Quelhas et al., 2024; Litland 2025). Let L denote a standard modal propositional language constructed from a set of propositional variables P, the Boolean connectives ¬ (negation), ∧ (conjunction) and the modal operators □ (necessity) and ◊ (possibility). The semantics are defined using a Kripke frame $F = (W, R)$, where W is a nonempty set of possible worlds and $R \subseteq W \times W$ is an accessibility relation. A Kripke model $M = (W, R, V)$ assigns truth values to each atomic proposition $p \in P$ via a valuation function $V: P \to P(W)$, where $P(W)$ is the powerset of W (Weiss and Birman, 2024). Satisfaction is defined recursively: for any $w \in W, M, w \vDash$p iff $w \in V(p)$; $M, w \vDash \neg\phi$ iff $M, w \vDash/\phi \; and \; M, w \vDash \phi \wedge \psi$ iff $M, w \vDash \phi$ and $M, w \vDash \psi$; and $M, w \vDash \Box\phi$ iff for all $v \in W$, if wRv, then $M, v \vDash \phi$. The dual operator ◊ϕ is defined as $\neg\Box\neg\phi$. This base system allows us to express propositional necessity and possibility, which we are going to enrich with a quantitative probabilistic extension tailored to quantum systems.

We now extend the Kripke model by introducing a probability measure over the set of possible worlds. A probabilistic Kripke model is defined as $M = (W, R, V, \mu)$, where $\mu: P(W) \to [0,1]$ is a finitely additive probability measure such that $\mu(W) = 1$ and $\mu(\emptyset) = 0$ (Shirazi and Amir, 2007). For each world $w \in W$, we define a conditional probability distribution μw over the accessible worlds $R(w) = \{v \in W \mid wRv\}$. The semantics of the probabilistic modal operator are then defined by introducing a function $P(\phi)$ yielding the probability that ϕ is true in the accessible worlds: formally, $M, w \vDash P(\phi) = p$ iff $\mu w(\{v \in R(w) \mid M, v \vDash \phi\}) = p$. This enables the assessment not only of whether a proposition is necessary or possible, but also of the probability with which it is possibly true. To ensure internal consistency, we assume that μw is defined via restriction and normalization: $\mu w(A) = \mu(A \cap R(w))/\mu(R(w))$ whenever $\mu(R(w)) > 0$. This logic is able to represent graded modal claims such as "it is 70% possible that φ" or more formally, $P(\Diamond\, \phi) = 0.7$, establishing a formal mechanism for assigning probabilities to modal propositions.

The logical language and syntax of PML-QD are built from a base set of propositional variables P, closed under the classical connectives and modal operators □, ◊ and the probability operator P. The syntax includes expressions of the form $P(\phi) = r$, where $r \in [0,1] \cap Q$ and composite formulas such as $P(\Diamond\, \phi) \geq s$ and $P(\Box\phi \to \psi) < t$. Formulas are interpreted over the probabilistic Kripke models defined above. The logic also allows us to define conditional probabilities. For propositions ϕ and ψ, we define $P(\phi \mid \psi) = \mu w(\{v \in R(w) \mid M, v \vDash \phi \wedge \psi\})/\mu w(\{v \in R(w) \mid M, v \vDash \psi\})$, assuming the denominator is nonzero. This allows the formulation of statements like "given ψ, φ is probable with 0.6 likelihood", which corresponds to $P(\phi \mid \psi) = 0.6$.

Overall, this approach yields the formal language of the system—comprising syntax, probability statements, and conditional expressions—necessary for modeling quantum epistemic transitions. Building on this foundation, the next steps involve applying this language to concrete quantum scenarios, analyzing how epistemic statuses evolve across different stages of measurement and inference.

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2. Axiomatic System

The axiomatic core of PML-QD comprises a structured integration of classical propositional logic, normal modal logic (system K) and finite probabilistic logic. The system operates within a Hilbert-style framework with axioms for classical logic, modal logic (K-system) and probability logic, including finite additivity, non-negativity, normalization and probabilistic modus ponens. Well-formed formulas (wffs) are derived from a fixed set of axioms using explicitly defined inference rules. The classical propositional component of the system is based on the axioms of tautology schemas, such as $\phi \to (\psi \to \phi)$, $(\phi \to (\psi \to \chi)) \to ((\phi \to \psi) \to (\phi \to \chi))$ and $\neg\neg\phi \to \phi$. The modal part includes the standard K axiom: $\Box(\phi \to \psi) \to (\Box\phi \to \Box\psi)$ and the necessitation rule: if $\vdash \phi$, then $\vdash \Box\phi$. The logic is normal in that it preserves closure under necessitation and respects distribution over implication. The axioms are interpreted over Kripke frames with arbitrary accessibility relations, allowing for flexible modeling of different quantum experimental contexts.

At first, probabilistic axioms are introduced to extend the classical-modal foundation. The core probabilistic principles are drawn from Kolmogorov's axioms, reformulated for integration into logical syntax (Svozil 2022). Let $P(\phi) = r$ be a primitive formula expressing that the probability of ϕ being satisfied in the accessible worlds is $r \in [0,1] \cap Q$. The axioms include non-negativity $P(\phi) \geq 0$, normalization $P(\top) = 1$ and finite additivity: if $\phi \wedge \psi \equiv \bot$, then $P(\phi \vee \psi) = P(\phi) + P(\psi)$. These are supplemented by conditional probability rules: for $\phi, \psi \in L$ with $P(\psi) > 0 P(\psi) > 0, P(\phi \mid \psi) = P(\phi \wedge \psi)/P(\psi)$. The modal-probabilistic interaction is regulated by the schema $P(\Box\phi) \leq P(\phi)$ and similarly $P(\phi) \leq P(\Diamond \phi)$, reflecting that what is necessarily true must also be true and what is true must be possibly true. These constraints are justified by the semantics of modal probability spaces, where $\Box\phi$ implies truth in all accessible worlds, while $\Diamond \phi$ requires only one accessible world where ϕ holds. The derivation rules include probabilistic modus ponens: from $\phi \to \psi$ and $P(\phi) \geq$r, infer $P(\psi) \geq r$, provided all evaluations are over the same accessibility neighborhood. These axioms permit the formal manipulation of graded propositions about truth values in modal contexts.

Therefore, with the K system as a stable scaffolding, the logic acquires the modal structure necessary to represent transitions from epistemic possibility to necessity, forming a base from which the probabilistic dynamics of quantum measurement can be encoded.

Implementation of quantum-specific axioms. To specialize PML-QD to quantum dynamics, we introduce a set of axioms reflecting superposition, measurement and entanglement. Let ϕi denote the proposition "the quantum system is in eigenstate i." For a system in state $|\psi\rangle = \sum i\alpha i \mid i\rangle$, we define $P(\Diamond \phi i \mid \alpha i|^2$.. This probabilistic modal formulation expresses the pre-measurement epistemic state: the system may be found in state ϕi with probability $|\alpha i|^2$. Upon measurement and collapse to ϕj, we enforce ϕj and $\Box\neg \Diamond \phi k$ for all $k \neq j$. We also represent entanglement through joint modal constraints. Given an entangled state $|\Psi\rangle = \sum i\alpha i \mid ai\rangle \otimes |bi\rangle$, let ϕi and ψi be propositions for subsystems A and B, respectively. Then $\Diamond(\phi i \wedge \psi i)$ is true with probability $|\alpha i|^2$, while $\Diamond \phi i \wedge \Diamond \psi j$ is only allowed if $i = j$ thereby disallowing separable joint modalities. These rules enable the logic to model quantum correlations and non-locality without assuming hidden variables.

Let $|\psi\rangle = \sum i\alpha i \mid i\rangle$ be a normalized pure state over an orthonormal basis $\{|i\rangle\}$ and let ϕi be the proposition "the system is in state $|i\rangle$". We postulate that $P(\Diamond \phi i) = |\alpha i|^2$, aligning logical possibility with the Born rule. This axiom expresses that, prior to measurement, the system may be found in eigenstate i, with probability equal to the squared modulus of its amplitude. Upon a measurement yielding outcome j, we assert $\Box\phi j$ and $\Box\neg \Diamond \phi k$ for all $k \neq j$, modeling the epistemic update resulting from collapse. This transition encodes the projection postulate as a logical update, i.e., a modal reduction from possibility to necessity and from multiple probabilities to a single certainty. To formalize this shift, we introduce the axiom schema $Measure(\phi j) \to (\Box\phi j \wedge \wedge k = j \Box\neg \Diamond \phi k)$. The measurement operator is treated syntactically, marking the transition point in the evaluation of epistemic states. This schema is only applied in cases where the logic designates $P(\Diamond \phi j) > 0$, reflecting the assumption that outcomes with zero probability cannot be observed.

To handle entanglement, we extend the logic with modal rules for joint propositions. Consider a bipartite quantum system with basis $\{|ai\rangle \otimes |bj\rangle\}$ and an entangled state $|\Psi\rangle = \sum i\alpha i \mid ai\rangle \otimes |bi\rangle$.

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Let ϕi denote "Subsystem A is in state $|ai\rangle$" and ψi denote "Subsystem B is in state $|bi\rangle$." We introduce the entanglement axiom $P(\Diamond(\phi i \wedge \psi i)) = |\alpha i|^2$ and the exclusion principle $\neg(\Diamond \phi i \wedge \Diamond \psi j)$ for $i \neq j$. This enforces the non-separability of joint state truth values: subsystems cannot simultaneously assume inconsistent states if derived from an entangled superposition. The logic allows us to evaluate propositions such as $\Diamond \phi i \rightarrow \Box\neg\psi j$ for $j \neq$, capturing the non-local correlations implicit in entanglement. These formulations provide a means of expressing Bell-type dependencies as logical constraints within the system, grounded in amplitude-based probability assignments. For measurements, we define the conditional update: observing ϕk implies $\Box\psi k$ and $\Box\neg\psi j$ for all $j \neq k$ consistent with perfect quantum correlations. These constraints are encoded as derivable rules rather than primitive axioms, preserving flexibility in modeling partial or imperfect entanglement.

Overall, this approach embeds the measurement postulates of quantum mechanics within a modal logical framework, effectively translating quantum dynamics into a structure that supports logical inference and modal reasoning about probability. By doing so, it bridges the gap between quantum formalism and epistemic logic, enabling a systematic analysis of how knowledge and uncertainty evolve through quantum processes. This integration allows for the tracking of necessity, possibility, and likelihood in a manner that aligns with both the probabilistic nature of quantum theory and the inferential tools of modal logic.

Tools and Computational Setup. All logical definitions, axioms and inference rules are formalized using a typed symbolic language implemented in the Lean proof assistant to verify syntactic coherence and logical validity (Löh 2022). For probability measures and accessibility relations, we use custom-built Kripke structures defined programmatically in Python using the networkx library for graph modeling and numpy for probability assignment. Quantum state vectors and projection operations are handled using the qutip library, enabling precise computation of Born-rule probabilities and their mapping to logical probability assignments. Logical expressions are parsed and evaluated using a domain-specific parser that constructs abstract syntax trees, evaluates modal depth and resolves probabilistic truth values based on current world state and transition graphs.

3. Coherence, Consistency and Semantic Soundness of PML-QD

This chapter examines the coherence, consistency and semantic soundness of PML-QD to ensure that its logical structure faithfully represents quantum dynamics and supports reliable epistemic inference. We begin by demonstrating the soundness of PML-QD relative to its probabilistic Kripke semantics. Let $M = (W, R, V, \mu)$ be a model of our system, where R is an arbitrary (possibly non-symmetric, non-transitive) relation to accommodate varied quantum contexts. For each world $w \in W$ and each well-formed formula $\phi \in L$, if $\vdash \phi$ in the PML-QD system, then $M, w \vDash \phi$. Proof proceeds by induction over the structure of derivations. Base cases follow from the validity of classical tautologies. The K modal axiom is validated by the relational condition: $wRv \wedge wRu$ implies that if $M, v \vDash \phi \rightarrow \psi$ and $M, v \vDash \phi$, then $M, v \vDash \psi$. The probabilistic axioms are sound under the standard interpretation of μw as a conditional probability measure on $R(w)$. Additivity, normalization and conditional independence are preserved by construction. Quantum-specific axioms are modeled by assigning $\mu w(\Diamond \phi i) = |\alpha i|^2$ for pre-collapse states and using syntactic restrictions post-collapse to enforce that $\Box\phi j \rightarrow \mu w(\phi j) = 1$. Thus, the epistemic updates are correctly aligned with quantum measurement rules. This confirms that the axiomatic structure of PML-QD is consistent with its intended semantics, establishing soundness as a necessary condition for subsequent formal evaluation.

Completeness is demonstrated via canonical model construction. We define a canonical model $Mc = (Wc, Rc, Vc, \mu c)$, where Wc is the set of maximally consistent sets (MCSs) of formulas in $LPML - QD$. The accessibility relation c is defined by: $\Gamma Rc\Delta$ iff for every $\Box\phi \in \Gamma$, $\phi \in \Delta$. The valuation function $Vc(p) = \{\Gamma \in Wc \mid p \in \Gamma\}$ and the probability function $\mu\Gamma c(\phi) = r$ is defined via the maximal consistent extensions satisfying $P(\phi) = r \in \Gamma$. Completeness then follows: if ϕ is valid (true in every model), then $\phi \in \Gamma$ for all $\Gamma \in Wc$. If $\phi \in/\Gamma$, then there exists a model falsifying ϕ,

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proving that ϕ\phiϕ is not derivable. For quantum-specific modalities, we enrich the canonical model with amplitude maps from abstract syntax to normalized vectors, ensuring that measurement-induced updates correspond to transitions among canonical worlds. The probabilistic collapse axioms are enforced syntactically by excluding extensions that contradict the post-measurement certainty schema.

We thus obtain completeness for the classical, modal and probabilistic layers, with quantum-specific axioms ensured through semantic alignment.

Epistemic closure and logical coherence. An important consideration in any epistemic logic is whether the system permits epistemic closure under valid inference. In PML-QD, closure under modal and probabilistic inference is carefully maintained through explicit syntactic rules. For instance, from $\Box(\phi \to \psi)$ and $\Box\phi$, one may derive $\Box\psi$, preserving modal consequence. However, in quantum contexts, closure must also handle epistemic transitions: if $\Diamond\phi$ and $(\Diamond\phi) = 1$, it does not follow that $\Box\phi$, unless a measurement event enforces it. Thus, PML-QD avoids epistemic overreach by enforcing the distinction between high probability and logical certainty. Probabilistic closure is similarly bounded: from $P(\phi \wedge \psi) = 0.9$ and P$P(\psi) = 1$, we may derive $P(\phi \mid \psi) = 0.9$, but not $\Box\phi$ or even $\Diamond\phi$ unless additional modal premises are supplied. This disciplined separation of inference domains ensures that conclusions about quantum states are validly drawn only within the scope of their modal and probabilistic constraints. Furthermore, any logical contradiction arising from measurement updates (e.g., asserting $\Box\phi$ and $\Diamond\neg\phi$) is syntactically blocked by the collapse axioms.

Counterfactual reasoning and epistemic modality. PML-QD allows for formal engagement with counterfactuals in quantum mechanics, a topic often debated in the context of delayed-choice experiments, weak measurements and quantum nonlocality (Laudisa 2019). The framework supports statements of the form: "If measurement M had been performed, then ϕ would have become necessary," represented formally as $\Diamond M \to (Measure(\phi) \to \Box\phi)$. Such statements are not metaphysical speculations but logical conditionals grounded in modal accessibility. Suppose a system is in a superposed state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ and the measurement is postponed. The logic allows one to say: "Had we measured now, outcome $\phi 0$ would have become necessary with probability $|\alpha|2$." This supports rigorous statements about epistemic potential without requiring ontological assertions about unmeasured reality. Importantly, these counterfactuals do not imply retrocausality, but rather preserve the distinction between hypothetical and actualized knowledge states, aligning with quantum experimental setups where delayed configurations define outcome space. The evaluation of counterfactuals is governed by modal consistency: only those conditional claims holding in all accessible paths from a given precondition are allowed.

Model-theoretic consistency with Hilbert Space formalism and temporal evolution. The compatibility and the semantic consistency of PML-QD with the standard Hilbert space formalism can be evaluated. Let H be a finite-dimensional Hilbert space with orthonormal basis $\{|i\rangle\}$ and let $|\psi\rangle = \sum i\alpha i\,|i\rangle$ be a quantum state. The propositions ϕi correspond to projection operators $P\hat{}i = |i\rangle\langle i|$ and the probability of observing state i is $\langle\psi|P\hat{}i|\psi\rangle = |\alpha i|2$. We define a mapping $f: \phi i \mapsto P\hat{}i$ and assert that $P(\Diamond\phi i) = Tr(\rho P\hat{}i)$ where $\rho = |\psi\rangle\langle\psi|$. The logical model corresponds to a coarse-grained representation of the probabilistic projections across a set of accessible configurations indexed by basis measurements. Post-measurement updates $\Box\phi j$ correspond to Lüders projections: $\rho \mapsto P\hat{}j\rho P\hat{}j/Tr(\rho P\hat{}j)$. For entangled states $|\Phi\rangle = \sum i\alpha i\,|ai\rangle \otimes |bi\rangle$, joint modal formulas $\Diamond(\phi i \wedge \psi i)$ are semantically validated by $Tr(\rho(P\hat{}i \otimes Q\hat{}i)) = |\alpha i|2$. This means that logical constructs are supported by standard operator theory. In this context, the modal structure represents pre-measurement epistemic range, while probability reflects amplitude-squared outcomes.

To evaluate the temporal behavior and dynamical evolution of PML-QD, we consider its capacity to represent quantum evolution between measurement events. Let $U(t) = e - iHt$ be the unitary evolution operator associated with a Hamiltonian H and $|\psi(t)\rangle = U(t)\,|\psi(0)\rangle$. In logical terms, we represent a temporal sequence $w0 \to w1 \to \cdots \to wn$, where each wi is a world indexed by the system's state at time ti. If ϕi is the proposition "system is in state $|\psi(ti)\rangle$", then temporal modal transitions $\Diamond\phi i + 1$ are defined by the Schrödinger evolution. The logic supports this via

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time-indexed accessibility: $wiRwi+1$ iff $|\psi(ti+1)\rangle = U(ti+1-ti)\,|\psi(ti)\rangle$. We extend the model by assigning probability functions $\mu wi(\phi j) = |\langle \phi j\,|\,\psi(ti)\rangle|^2$, yielding a dynamic probability assignment compatible with unitary evolution. If a measurement occurs at tj, the update collapses the epistemic structure: all future-accessible paths inconsistent with the observed outcome are pruned from $R(wj)$.

In summary, we establish the dynamic consistency of PML-QD by demonstrating that modal transitions respect unitary evolution up to the point of measurement and that state updates conform to the projection postulate. This validation reinforces the semantic soundness of the framework, confirming its capacity to model quantum epistemic dynamics with logical and physical fidelity.

4. From Theory to Practice: Logical Modeling of Quantum Systems in PML-QD

This chapter presents representative examples illustrating how PML-QD models quantum phenomena through logical structures and modal reasoning.

Derivation of single-measurement collapse. We begin with a formal derivation illustrating how PML-QD syntactically captures the transition from a probabilistic possibility to post-measurement necessity in the single-qubit case. Let $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ be a superposed state, with $|\alpha|2+|\beta|2=1$. Let $\phi 0$ denote "the system is in state $|0\rangle$" and $\phi 1$ denote "the system is in state $|1\rangle$." In the logic, prior to measurement we assume $P(\Diamond\,\phi 0) = |\alpha|2$ and $P(\Diamond\,\phi 1) = |\beta|2$, encoded via axioms of amplitude-based modal probability. Suppose the system is measured and the outcome is $|0\rangle$. The axiom of modal collapse gives $Measure(\phi 0) \rightarrow \Box\phi 0 \wedge \Box\neg\Diamond\,\phi 1$. If we assume $Measure(\phi 0)$, then using modus ponens, we derive $0\Box\phi 0$ and $\Box\neg\Diamond\,\phi 1$. Applying the modal logic equivalence $\Box\neg\Diamond\,\phi 1 \equiv \neg\Diamond\,\phi 1$, we deduce that $\phi 1$ is no longer even possible. The derivation shows that the probabilistic possibility $P(\Diamond\,\phi 1) = |\beta|2$ is syntactically replaced by $\neg\Diamond\,\phi 1$ and all future logical inferences involving $\phi 1$ become false under modal necessity.

Sequential measurements. Next, we construct a proof sequence for a scenario involving two successive measurements along non-commuting observables. Let the system initially be in state $|\psi\rangle = 21(|0\rangle+|1\rangle)$ and define $\phi 0, \phi 1$ as before. Let $\chi+$ and $\chi-$ denote the propositions corresponding to the diagonal basis $|-\rangle = 21(|0\rangle-|1\rangle)$. Suppose the first measurement is in the $\{|0\rangle,|1\rangle\}$ basis and the outcome is $|0\rangle$. By modal collapse, we derive $\Box\phi 0$ and $\Box\neg\Diamond\,\phi 1$. Now we consider $P(\Diamond\,\chi+)$, i.e., the probability of subsequently observing $|+\rangle$. Since the post-measurement state is $|0\rangle$, we compute $P(\Diamond\,\chi+) = |\langle +|\,0\rangle|2 = 1/2$. We syntactically derive $\Box\phi 0 \rightarrow P(\Diamond\,\chi+) = 1/2$ from the substitution of amplitude-based definitions into the probability axioms. Upon performing the second measurement and observing $|+\rangle$, we apply the measurement collapse rule again to derive $\Box\chi+$ and $\Box\neg\Diamond\,\chi-$. These results can be then used to show that all further inferences about $\phi 1$ or $\chi-$ must fail in all accessible worlds.

We now consider the effect of sequential measurements when the observables do not commute. Let the initial state again be $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$ and define a second measurement basis, such as the $\{|+\rangle,|-\rangle\}$ basis, where $|+\rangle = 21(|0\rangle+|1\rangle)$ and $|-\rangle = 21(|0\rangle-|1\rangle)$. Let $\chi+$ and $\chi-$ represent the corresponding modal propositions. If the system is first measured in the computational basis and yields $|0\rangle$, then we assert $\Box\phi 0$ and immediately invalidate $\Diamond\,\phi 1$. We may now inquire about $P(\Diamond\,\chi+)$, that is, the possibility of subsequently observing $|+\rangle$. In the post-collapse model, the system state become $|0\rangle$, so we calculate $P(\Diamond\,\chi+) = |\langle +|\,0\rangle|2 = 1/2$. Formally, this is encoded as $\Box\phi 0 \rightarrow P(\Diamond\,\chi+) = 1/2$, reflecting the non-commutativity of the measurement sequence. The logic tracks not only the collapsed state but its implications for future probabilistic possibilities. If we now measure in the diagonal basis and obtain outcome $|+\rangle$, the epistemic update becomes $\Box\chi+\wedge\Box\neg\Diamond\,\chi-$, which supersedes the prior assignment. PML-QD may thus support reasoning about nested and sequential updated updates and the logical impact of measurement order.

Single-Qubit superposition and measurement. We begin with a basic scenario: a single qubit in a superposition of eigenstates relative to a particular observable, such as spin along the z-axis. Let the quantum state be $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$, where $\alpha, \beta \in C$ and $|\alpha|2+|\beta|2=1$. We define the modal propositions $\phi 0$ and $\phi 1$ corresponding respectively to the system being in state $|0\rangle$ and $|$

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1⟩. In the PML-QD framework, the epistemic state prior to measurement is captured by the formulas $P(\Diamond\, 2P(\Diamond\, \phi 1) = | \beta \mid 2$. These assertions express the degree of possibility—based on the probabilistic modal semantics—associated with each eigenstate. A measurement in the $\{| 0\rangle, | 1\rangle\}$ basis is treated as a transition: if outcome $| 0\rangle$ is observed, the updated state becomes $\Box\phi 0 \wedge \Box\neg \Diamond\, \phi 1$, formalizing collapse in logical terms. The inference chain follows directly from the axiom $Measure(\phi j) \rightarrow (\Box\phi j \wedge \wedge k = j\Box\neg \Diamond\, \phi k)$. The model also permits conditional queries such as $P(\Diamond\, \phi 0 \mid \neg\phi 1) = 1$, maintaining coherence with the binary outcome structure of projective measurement. This means that PML-QD may accommodate elementary state transitions and probability-based reasoning using modal assertions grounded in amplitude-based truth.

Collapse simulation and world pruning algorithms. The logic requires that upon measurement possible but unrealized outcomes are no longer epistemically accessible. To simulate this, a world-pruning algorithm could be implemented that dynamically restructures the Kripke model. Upon observing outcome ϕj at world w, this algorithm first may verify $P(\Diamond\, \phi j) > 0$; then, identify the unique subset $R'(w) \subseteq R(w)$ such that $\forall v \in R'(w), M, v \vDash \phi j$. The accessibility relation can be updated by setting $R(w) := R'(w)$ and the probability distribution μw renormalized over $R'(w)$. All ϕk such that $k \neq j$ may be set to evaluate as $\neg \Diamond\, \phi$k and $\Box\neg\phi k$. The update is conservative and preserves modal truth: previously necessary propositions remain necessary unless invalidated by the measurement result. In sequential measurements, the system may track update sequences using a stack of Kripke structures, allowing rollback and re-evaluation. A collapse consistency check may ensure that $\sum kP(\Diamond\, \phi k) = 1$ prior to collapse and that after measurement such that exactly one ϕj becomes necessary while the others are impossible. In entangled cases, a measurement on one subsystem may automatically trigger pruning on correlated worlds in the partner system, enforcing non-local modal synchrony.

Entanglement constraints via modal dependencies. Let us consider the entangled state $| \Phi\rangle = 21(| 0\rangle A \mid 1\rangle B - | 1\rangle A \mid 0\rangle B)$. Define $\phi 0$, $\phi 1$ for particle A and $\psi 0$, $\psi 1$ for particle B. The axiom of correlated possibility gives $P(\Diamond\, (\phi 0 \wedge \psi 1)) = 1/2$, $P(\Diamond\, (\phi 1 \wedge \psi 0)) = 1/2$ and all other $P(\Diamond\, (\phi i \wedge \psi j)) = 0\ for\ i = ji\ = ji = j$. Suppose a measurement on A yields $\phi 1$. By measurement collapse, we derive $0\Box\phi 1 \wedge \Box\neg \Diamond\, \phi 0$. From the modal correlation rule $\phi 1 \rightarrow \Box\psi 0$, we deduce $\Box\psi 0$, i.e., the state of B becomes necessarily $| 0\rangle$. The derivation holds even if the measurement on B occurs later, as the logical dependencies are enforced modally rather than temporally. Suppose instead that we had observed $\phi 0$; the derivation would then yield $\psi 1$. Importantly, PML-QD prevents derivation of any statement $\Diamond\, (\phi 1 \wedge \psi 1)$, since this is ruled out by the amplitude-based probability axiom. Overall, the logical constraints imposed by PML-QD on entangled propositions may correctly capture the exclusivity and correlation properties inherent in entangled quantum states.

Entanglement Swapping. Entanglement swapping is a protocol in which two initially independent quantum systems become entangled through joint measurement on intermediary particles (Ning et al., 2019; Zangi et al., 2023). Consider two entangled pairs in the states $| \psi AB\rangle = 21(| 0\rangle A \mid 1\rangle B - | 1\rangle A \mid 0\rangle B)$ and $| \psi CD\rangle = 21(| 0\rangle C \mid 1\rangle D - | 1\rangle C \mid 0\rangle D)$. Let a Bell-state measurement be performed on particles B and C. Define modal propositions ϕij the four Bell states between B and C and ψAD for the corresponding entanglement state between A and D. Before the measurement, the logic encodes the system as $\vee i, jP(\Diamond\, \phi ij) = 41$, with no assignment of $\Box\psi AD$. Upon observing Bell state $\phi 00$, we assert $\Box\phi 00 \rightarrow \Box\psi AD00$, with $\psi AD00$ denoting a corresponding entangled state between particles A and D. This captures the epistemic update that makes entanglement between distant, non-interacting particles a logical necessity only *after* the intermediate measurement. This avoids invoking retrocausality by localizing modal transitions to the knowledge structure. Conditional probabilities are also updated: $P(\Diamond\, \psi AD00 \mid \phi 00) = 1$, while other $\psi ADij$ are assigned zero. Therefore, PML-QD may capture entanglement swapping via conditional modal updates, supporting rigorous reasoning about delayed entanglement onset.

Delayed-choice interference and quantum eraser. We propose here a derivation involving the delayed-choice quantum eraser (Kim et al., 2000). Define ϕp: "which-path information is accessible," and ϕw: "interference pattern is observed." Before post-selection, we assume $\Diamond\, \phi p \wedge \Diamond\, \phi w$. Let Me

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denote the erasure choice and Mp the path-preserving choice. Under the choice Me, the logic enforces $Me \rightarrow \Box\neg\phi p \wedge \Box\phi w$, whereas under Mp, it enforces $Mp \rightarrow \Box\phi p \wedge \Box\neg\phi w$. Assume Me is enacted after the signal photon is measured. Since the epistemic update is contingent, we represent the situation as a conditional modal formula: $\Diamond(\phi p \wedge \phi w) \rightarrow (Me \rightarrow \Box\phi w) \wedge (Mp \rightarrow \Box\phi p)$. Upon performing Me, the system updates to $\Box\phi w$ and $\Diamond\phi p$ is logically rejected. The derivation path shows that the modal state is not solely determined by photon interactions, but by post-measurement contextualization. This is syntactically grounded in modal update axioms, not as an empirical fact but as a derivable transition.

We now examine the delayed-choice quantum eraser, a paradigmatic scenario where information about a quantum system's path is either retained or erased after the system has been measured (Kim and Ham, 2023). Let ϕp denote "which-path information is known" and ϕw denote "interference pattern is visible." The logic must represent conditional dependencies where the post-measurement setup retroactively influences the interpretation of earlier events. We encode the availability of path information as a binary modal variable: if the information is preserved, we assign $\Box\phi p \rightarrow \Box\neg\phi w$ and if it is erased, $\Box\neg\phi p \rightarrow \Box\phi w$. The measurement decision variable Me (erase path info) or Mp (preserve path info) serves as a modal context switch: prior to this, both $\Diamond\phi p$ and $\Diamond\phi w$ are true. After choosing Me, the system updates to $\Box\neg\phi p \wedge \Box\phi w$. PML-QD allows for modal reasoning such as $\Diamond\phi p \wedge \Diamond\phi w \rightarrow (Me \rightarrow \Box\phi w)$, modeling epistemic changes contingent on an action taken after the quantum interaction. This illustrates that PML-QD can represent dynamically dependent modal transitions where final knowledge states depend on future experimental choices.

The examples presented in this chapter demonstrate the capacity of PML-QD to formally capture key aspects of quantum behavior, including measurement collapse, entanglement and contextual inference. These cases highlight the framework's utility in modeling epistemic transitions with logical precision, offering a promising methodology for further theoretical and practical developments.

5. Conclusion

The Probabilistic Modal Logic for Quantum Dynamics (PML-QD) introduced here provides a formal framework that integrates modal logic, probability theory and the epistemic structure of quantum measurement. Its aims to model the dynamic progression from probabilistic possibility to logical necessity that defines quantum behavior under measurement. Built on the classical modal system K, PML-QD introduces a probabilistic valuation mechanism and a compact set of domain-specific axioms governing superposition, collapse and entanglement. Modal propositions are assigned quantitative probability values. Upon measurement, collapse transitions convert possibility into necessity, enforcing expressions like $\Box\varphi$ to indicate that an outcome has become realized. A key non-classical feature of PML-QD is its treatment of epistemic non-monotonicity. In classical modal logic, implications such as $\diamond\varphi \rightarrow \diamond\Box\varphi$ may hold under specific frame conditions. However, this does not persist in PML-QD due to the collapse-induced pruning of modal paths: once a measurement occurs, the model transitions to a substructure where prior possibilities are no longer accessible. This modal reduction is syntactically governed by the axiom $Measure(\varphi j) \rightarrow \Box\varphi j \wedge k \quad = j \wedge \Box\neg\diamond\varphi k$ which ensures logical consistency while rejecting modal monotonicity. In the PML-QD framework, the necessity operator $\Box$ does not collapse into a truth predicate. While $\Box\varphi \rightarrow \varphi$ holds in the sense that a necessary proposition must be true in all accessible worlds, this does not imply absolute truth, but only truth relative to post-measurement substructure. This distinction reinforces the epistemic separation between pre-measurement probability and post-measurement necessity, marking a departure from both classical modal logic and traditional probabilistic logic. Unlike classical knowledge frameworks where a proposition is either known or unknown, PML-QD enables a graded epistemic treatment. It may accommodate statements of the form: possibility without expectation $P(\diamond\varphi) = 0$, expectation without necessity $0 < P(\diamond\varphi) < 1$ and epistemic finality $\Box\varphi$. This enables a more nuanced account of quantum epistemology. For example, the proposition "the particle is spin-up" need not be treated as simply known or unknown; instead, it can be represented as probable but not

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necessary prior to measurement, with its degree of belief quantified by quantum amplitudes rather than reduced to binary epistemic categories.

A distinctive feature of PML-QD is its logical minimalism, as it extends the basic modal system K only modestly by incorporating probabilistic operators and a small set of collapse-specific axioms. The system deliberately avoids polymodal formulations, higher-order quantification, intensional types, nonclassical connectives, distributed knowledge operators and dynamic logic constructs, focusing instead on a small set of precisely defined epistemic transitions . This design choice emphasizes deductive transparency over maximal expressive power. Each formula expresses a distinct logical claim that can be semantically validated within Kripke models equipped with probability functions. This keeps model-theoretic evaluation computationally feasible while retaining expressiveness sufficient to model quantum experimental structures.

An important advantage of PML-QD is its practical utility in the design and analysis of quantum experiments. By formally modelling the transition from probabilistic possibility to logical necessity, the framework provides a rigorous structure for anticipating and interpreting measurement outcomes. This is especially beneficial in settings involving entanglement swapping, delayed-choice quantum erasers and sequential measurements with non-commuting observables, i.e., scenarios where standard formalisms often obscure epistemic transitions. For example, in designing a delayed-choice interference experiment, PML-QD may provide a framework for anticipating how post-selection contexts modulate which-path information, thereby clarifying the underlying logical dependencies before experimental implementation. In entanglement swapping protocols, PML-QD may aid in tracking modal correlations to ensure consistency across nonlocal updates. Its ability to handle conditional reasoning and epistemic updates supports counterfactual assessments and helps validate whether a given experimental configuration logically aligns with quantum postulates. Overall, PML-QD offers both a conceptual and practical toolkit for optimizing experimental design in foundational quantum research and emerging quantum technologies.

The epistemic distinctions enabled by PML-QD allow a comparison with traditional interpretations of quantum mechanics. In the Copenhagen view, the wavefunction encodes predictive knowledge about measurement outcomes but lacks interpretive content about unmeasured states (Jaeger 2019). PML-QD refines this position by treating superposition as a landscape of modal possibilities rather than ontological ambiguity such that the system can be described as potentially occupying multiple states, each with a distinct logical status and probability. In relation to QBism, which treats quantum states as subjective degrees of belief (Khrennikov 2018; Milgrom 2022), PML-QD provides a formal syntactic structure to those beliefs, grounding them in rules of modal inference and derivability. While QBism invokes a probabilistic agent-centric perspective, PML-QD situates probability within a logical system constrained by axioms, thus offering a structured treatment of epistemic agency. In the many-worlds interpretation, every possible measurement outcome is realized in some branch of reality (Devor 2023; Vaidman 2025). PML-QD models branching through modal accessibility, but does not commit to metaphysical plurality; rather, it treats branching as epistemic openness. The modal relations are not anymore between physically instantiated worlds, but between logically accessible epistemic states.

ML-QD differs from traditional quantum logics, which often replace classical logic with alternative systems to reflect the structural or contextual aspects of quantum theory. For instance, the Birkhoff–von Neumann approach represents quantum propositions as elements of an orthomodular lattice of Hilbert space subspaces, replacing classical Boolean logic with a non-distributive structure that reflects observable incompatibility and quantum geometry (Gunji and Nakamura 2022). In contrast, PML-QD retains classical propositional logic and introduces modal and probabilistic layers to capture quantum uncertainty and measurement dynamics without abandoning logical distributivity. In turn, topos-theoretic quantum logic reformulates quantum theory using intuitionistic logic and category theory, modeling propositions as presheaves over classical contexts and assigning truth values locally without relying on global valuations or classical bivalence (Landsman 2017; Jia et al. 2025). By contrast, PML-QD operates within a bivalent logical system and

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incorporates probability directly, allowing for global epistemic evaluation and explicit modeling of measurement-induced transitions

PML-QD has certain limitations that warrant acknowledgment. It does not attempt to model open quantum systems or continuous variable states, nor does it incorporate decoherence processes at the level of environment-induced entropy changes. All quantum states are assumed to be finite-dimensional and all measurements are treated as idealized and projective. This simplification enables a syntactically clean and computationally tractable logic but restricts its direct application to more complex or realistic quantum systems. While the system models epistemic transitions triggered by measurements, it does not incorporate temporal indeterminacy or branching-time semantics; instead, it adopts a linear temporal structure via indexed worlds. The exclusion of branching structures limits the logic's ability to model future contingencies or path-dependent quantum evolution. PML-QD also omits any formal treatment of epistemic agents or belief operators. While interpretations such as QBism emphasize agent-centered probabilities and belief updates, PML-QD focuses exclusively on system-level propositions, avoiding subjective or doxastic modalities. Moreover, PML-QD remains neutral regarding the ontological status of the quantum state, avoiding commitment to whether the wavefunction represents physical reality or informational content. Within these limitations, PML-QD provides a coherent logical framework for analyzing the epistemic dynamics of measurement and uncertainty in finite, closed quantum systems. The framework serves not as a replacement but as a complementary language for articulating epistemic features of quantum processes. Within these limitations, PML-QD provides a coherent and complementary logical framework for articulating the epistemic dynamics of measurement and uncertainty in finite, closed quantum systems.

In conclusion, we address whether a unified formal system can capture the modal and probabilistic structure of quantum dynamics, particularly the transition from possibility to necessity induced by measurement. By integrating modal logic with probabilistic semantics, PML-QD offers a framework representing superposition, collapse and entanglement in syntactically precise and semantically consistent terms. The main takeaway is that quantum measurement, often seen as interpretationally opaque, can be rigorously modeled within a logical framework that clearly distinguishes graded possibility from logical necessity across both pre- and post-observation contexts.

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Applications of Wedderburn's Theorem in Modelling Non-Commutative Biological and Evolutionary Systems

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ABSTRACT

A wide range of biological and evolutionary processes is determined not merely by the occurrence of specific events, but by the exact order in which those events unfold. Gene regulation, developmental pathways, metabolic cascades and genetic evolution often display non-commutative behaviour, in which reversing the sequence of events results in functionally distinct outcomes. Conventional modelling approaches often fail to account for such directionality and sequence dependence, thereby limiting their capacity to capture the complexity of regulatory logic. We represent ordered sequences of biological operations—such as transcription factor binding or mutational trajectories—as elements of a non-commutative algebra designed to encode the functional logic of systems where the order of events is critically determinant. Subsequently, we apply Wedderburn's Theorem to decompose the algebra into a direct sum of matrix modules, each capturing an irreducible and functionally distinct component of the underlying sequence-dependent system. We provide examples from gene expression regulation and evolutionary dynamics, focusing on scenarios where trait development is determined by the specific order of underlying molecular or mutational events. Our results demonstrate that the algebraic framework effectively maps intricate biological processes onto smaller, linear components, facilitating clearer interpretation and analysis. Simplifying non-commutative biological systems into interpretable submodules, Wedderburn-style decomposition may clarify gene regulatory logic, capture behavioural outcomes, reduce computational burden and uncover pathway redundancies and structural symmetries. Overall, by unifying diverse biological processes within a coherent algebraic structure, our method may improve the tractability of complex, order-dependent systems.

KEYWORDS: sequence-dependence; regulatory logic; gene activation; developmental constraints; mutation pathways.

INTRODUCTION

Recent advances in systems biology and evolutionary theory have highlighted the importance of order-dependence and context sensitivity in biological processes. Gene regulation, signal transduction, developmental cascades and adaptive evolution exhibit behaviors that resist full explanation through classical commutative models. For instance, the outcomes of regulatory or mutational sequences often depend not only on which events occur but on the precise order in which they unfold—an inherently non-commutative property (Buenrostro et al., 2018; Pountain et al., 2024). Traditional approaches such as Boolean networks, differential equations or stochastic models obscure or simplify these order effects, limiting their capacity to capture the deeper algebraic structures underlying biological causality (Chakrabarty et al., 2016; Blomberg et al., 2020; Pušnik et al., 2022; Plaugher and Murrugarra 2023). While several abstract formalisms, including category theory, non-commutative probability and rule-based modeling have been explored to address such complexities, a systematic algebraic framework assessing the modular and irreducible components of these processes remains underdeveloped (Wilson-Kanamori et al., 2015; Bustos et al., 2018; Marcot 2021). We introduce an application of the Wedderburn–Artin theorem, a foundational result in the structure theory of algebras, to decompose non-commutative biological and evolutionary systems into direct sums of matrix algebras (Lam 2001; Brešar, 2024.). This approach may allow complex, sequence-sensitive pathways to be analyzed through the lens of finite-dimensional semisimple algebras. This approach may establish a concrete algebraic foundation for modeling non-commutative biological orderings in a decomposable and interpretable form.

We propose a framework in which biological or evolutionary operations—such as binding events, mutational steps or regulatory transitions—are encoded as elements of a non-commutative associative algebra. By treating these operations as algebraic generators and constructing the corresponding finite-dimensional algebra, we aim to capture both the sequence-dependence and the compositional rules of the system. Applying Wedderburn's theorem to these algebras yields a decomposition into matrix modules that each act irreducibly on distinct subspaces of the system's state space (Behboodi et al., 2018; Ma et al., 2022). This decomposition enables a modular interpretation of complex pathways, revealing redundancy, symmetry and irreducibility across diverse biological sequences. We expect our experimental models to demonstrate that even relatively intricate biological systems—such as epistatic mutation chains or order-dependent regulatory switches (Wan et al., 2018; Alfaro-Murillo and Townsend, 2023)—can be mapped onto manageable algebraic structures where behavioral modules are linearly represented. These outcomes may improve the

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accessibility of biological models to formal analysis and the detection of functional equivalence among different sequences.

We will proceed as follows: first, we outline the algebraic formalism and its biological interpretation; second, we construct illustrative systems and apply Wedderburn's theorem; third, we examine the resulting matrix decompositions and their relevance; finally, we discuss limitations and interpret the broader implications.

MATERIALS AND METHODS

Algebraic construction of biological systems. The algebraic modeling in this study is grounded in the construction of finite-dimensional associative algebras over a base field $\mathbb{F}$, taken throughout as $\mathbb{R}$ unless otherwise stated (Alder and Volker Strassen, 1981). **Figure 1** provides a flowchart of the steps described in this section. The modeling process begins by identifying a finite set of biological or evolutionary operations, denoted $\mathcal{G} = \{g_1, g_2, \ldots, g_n\}$ where each generator g_i corresponds to an ordered event in a biological pathway, such as a mutation, a transcription factor binding or a signal transduction step. These generators form the basis of a free algebra $\mathbb{F}\langle\mathcal{G}\rangle$, which consists of all finite linear combinations of words formed from the g_i under non-commutative multiplication (Kleyn 2018). The product of two words $w_1 = g_{i_1} g_{i_2} \cdots g_{i_k}$ and $w_2 = g_{j_1} g_{j_2} \cdots g_{j_l}$ is defined by concatenation: $w_1 w_2 = g_{i_1} \cdots g_{i_k} g_{j_1} \cdots g_{j_l}$. We then impose relations $R \subseteq \mathbb{F}\langle\mathcal{G}\rangle$ which encode the biological logic and constraints, such as idempotency $g_i^2 = g_i$, interaction rules $g_i g_j = 0$ if g_i negates g_j or commutativity $g_i g_j = g_j g_i$ in the rare cases where ordering is irrelevant. The quotient algebra $\mathcal{A} = \mathbb{F}\langle\mathcal{G}\rangle / \langle R \rangle$ serves as the structural representation of the biological system. The definition of the quotient algebra is essential as it allows for the reduction of the free algebra into a finite-dimensional associative algebra preserving the semantic content of the biological constraints (Bremner 2010). This construction defines the primary algebraic structure upon which Wedderburn decomposition is applied, serving as the basis for all subsequent structural analysis (Brochero Martínez et al., 2022).

Semisimplicity and radical reduction. Following the construction of the algebra $\mathcal{A}$, we proceed to classify its ideal structure and determine whether it is semisimple. For this purpose, we employ the standard result from ring theory: a finite-dimensional associative algebra $\mathcal{A}$ over a field $\mathbb{F}$ is semisimple if and only if its Jacobson radical $\mathrm{Rad}(\mathcal{A})$ is zero (Farb and Dennis, 1993; Bhuniya and Sarkar, 2023). To compute $\mathrm{Rad}(\mathcal{A})$, we use the characterization of the radical as the intersection of the annihilators of all simple left $\mathcal{A}$-modules. Computationally, we identify nilpotent ideals by evaluating the action of generators on test modules and examining their closure under multiplication. Letting M be a left $\mathcal{A}$-module, we compute endomorphism rings $\mathrm{End}_{\mathcal{A}}(M)$ and examine whether elements of $\mathcal{A}$ act nilpotently on M, i.e., whether for each $a \in \mathcal{A}$, there exists $k \in \mathbb{N}$ such that $a^k \cdot m = 0$ for all $m \in M$. If $\mathrm{Rad}(\mathcal{A}) = 0$ then Wedderburn–Artin theory applies in full. Otherwise, we restrict to the semisimple quotient $\mathcal{A}_{\mathrm{ss}} = \mathcal{A}/\mathrm{Rad}(\mathcal{A})$ which by construction is semisimple and suitable for decomposition. The establishment of semisimplicity or identification of a reduced semisimple component is critical for ensuring that Wedderburn's theorem is applicable and yields a well-defined matrix decomposition (Behboodi et al., 2016).

Wedderburn–Artin decomposition. Once semisimplicity is confirmed, we invoke the Wedderburn–Artin theorem, which asserts that any finite-dimensional semisimple algebra over a field $\mathbb{F}$ is isomorphic to a finite direct sum of full matrix algebras over division rings (Ma et al., 2022). Explicitly, the decomposition takes the form $\mathcal{A}_{\mathrm{ss}} \cong \bigoplus_{i=1}^{k} M_{n_i}(D_i)$ where each D_i is a division ring over $\mathbb{F}$ and $M_{n_i}(D_i)$ denotes the algebra of $n_i \times n_i$ matrices with entries in D_i i. We compute this decomposition by identifying a complete set of pairwise orthogonal primitive central idempotents $\{e_1, \ldots, e_k\} \subseteq \mathcal{A}_{\mathrm{ss}}$ satisfying $1 = \sum_{i=1}^{k} e_i$ and then studying each component $e_i \mathcal{A}_{\mathrm{ss}} e_i$. For each *i*, the algebra $e_i \mathcal{A}_{\mathrm{ss}} e_i$ is isomorphic to $M_{n_i}(D_i)$ and its dimension is determined by computing

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the trace of the identity under the regular representation. The idempotents are constructed using polynomial identities and Peirce decomposition with computational assistance from symbolic algebra software such as GAP or SymPy (Ánh et al., 2020). This matrix block structure provides the modular decomposition of the algebra and forms the basis for associating behavioral modules with biological subfunctions.

Representation theory and state space construction. Each matrix block $M_{n_i}(D_i)$ is then interpreted as an irreducible behavioral submodule of the biological system. To make this connection explicit, we construct representations $\rho_i : \mathcal{A}_{\text{ss}} \to \text{End}_{D_i}(V_i)$ where V_i is a vector space of dimension n_i over the division ring D_i. These representations define the action of biological operations on state spaces and we study the structure of these actions by analyzing the module homomorphisms $\phi : \mathcal{A} \to \text{End}(V)$. The system's state space V is constructed based on observable outcomes (e.g., ON/OFF gene states, trait expressions, regulatory activation) and the representation maps each algebra generator to a linear operator on V. The transition structure of the system is then given by the composition of these operators, corresponding to the sequence of biological events. We explicitly compute matrix representatives for each generator using a basis adapted to the irreducible modules, enabling the translation of abstract algebraic structure into linear action on state vectors. This correspondence makes it possible to characterize each biological process as a composition of linear transformations, despite the nonlinearity of the underlying sequence logic.

Computational implementation and algebraic tools. To implement these constructions and carry out symbolic calculations, we utilized the Python-based algebraic software SymPy (v1.12) for symbolic manipulation of non-commutative expressions, alongside GAP (Groups, Algorithms, Programming; v4.12.0) for explicit computation of group actions, idempotents and module decompositions. All matrix computations, linear maps and basis transformations were performed using NumPy and SciPy within Python (v3.11). Custom scripts were written to automate the construction of free algebras, define relation sets and compute quotient structures. The modules were constructed by evaluating the left regular representation $\lambda : \mathcal{A} \to \text{End}_{\mathbb{F}}(\mathcal{A})$, defined by $\lambda(a)(x) = ax$ for all $x \in \mathcal{A}$. We further used this representation to identify minimal left ideals $\mathcal{A}e_i$, where each e_i is a primitive idempotent and verified the decomposition by checking that the direct sum of minimal ideals recovers $\mathcal{A}_{\text{ss}}$. These computational steps were necessary to ensure reproducibility of the decomposition and to precisely associate algebraic structure with observable states. With this infrastructure, we operationalized Wedderburn decomposition as a method of extracting canonical linear modules from inherently ordered and context-sensitive biological systems.

Case studies and module-phenotype mapping. The final methodological stage involved applying this framework to case studies constructed from simplified biological systems, i.e., synthetic gene regulatory networks and abstracted evolutionary mutation paths. For each of the two cases under examination, we specified a generator set $\mathcal{G}$, defined the relations R reflecting biological logic (e.g., suppression, activation or context-specific outcomes) and formed the quotient algebra $\mathcal{A}$. The regular representation λ was computed explicitly and the resulting matrices were block-diagonalized according to the decomposition $\mathcal{A}_{\text{ss}} \cong \bigoplus_i M_{n_i}(D_i)$. We then examined the image of each generator under the regular representation and verified its consistency with the algebraic structure. To assess whether different mutation sequences led to equivalent or distinct outcomes, we compared the orbits of state vectors under different compositions of operators. Equivalence classes of behaviors were then associated with the respective matrix blocks, allowing us to link the algebraic decomposition with functionally distinguishable system trajectories. In cases involving feedback or looped behavior, we analyzed the structure of the center $Z(\mathcal{A})$ and studied its interaction with the radical to determine whether cyclic paths contributed to reducibility or generated central idempotents. The mapping from sequence dynamics to linear modules was then systematically recorded and all operations were performed in a consistent symbolic algebraic pipeline. This last phase translated the abstract decomposition into interpretable biological substructures and allowed rigorous tracking of how different pathways were embedded in the module structure.

To simulate the effect of transcription factor (TF) binding order on gene expression (Culyba 2019; Lai et al., 2019), we implemented a probabilistic and algebraically structured model based on a simplified gene regulation scenario. This scenario represents a system where two transcription factors, A and B, interact with a gene promoter in an order-sensitive manner. The simulation aimed to quantify how different binding sequences contribute to the expected level of gene expression, based on their assigned functional outcomes and relative likelihoods.

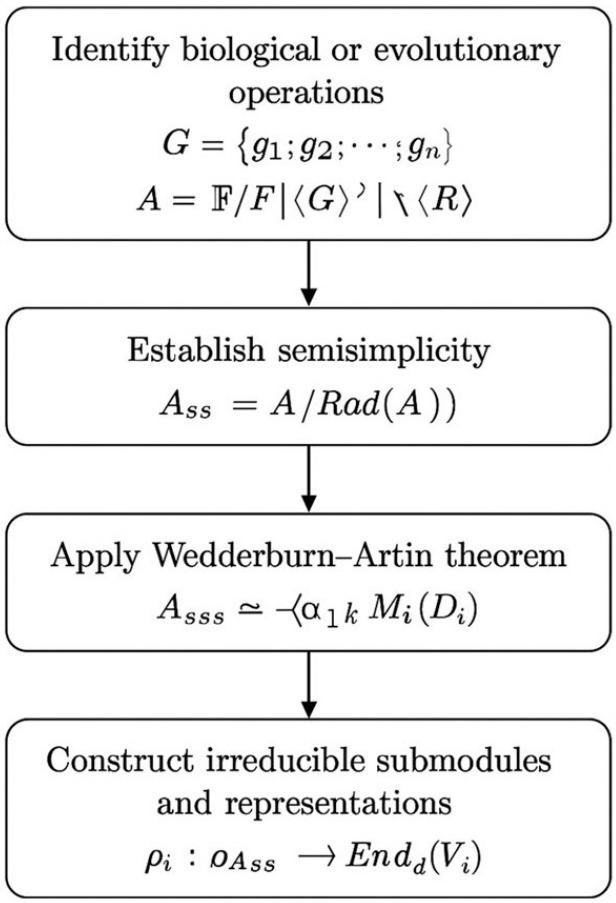
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The first step involved enumerating all meaningful binding order scenarios between TFs A and B. We defined five distinct cases: (1) A binds before B (A → B), (2) B binds before A (B → A), (3) only A binds (A), (4) only B binds (B) and (5) neither factor binds (None). These cases represent a simple but comprehensive combinatorial basis for assessing order effects in a non-commutative setting.
Next, we assigned a gene expression outcome to each binding scenario based on biological logic. In our model, the sequence A → B was considered activating, producing a full gene expression response and therefore was assigned an outcome value of 1.0. Conversely, B → A was designated as repressive or ineffective, yielding an expression value of 0.0. Scenarios in which only A or only B binds were considered partial and biologically ambiguous and each was assigned an intermediate expression value of 0.5. If neither TF binds, gene expression is assumed to remain OFF, with a value of 0.0. These scalar values serve as surrogate quantitative markers for gene activity.
To simulate how often each sequence might occur in a biological context, we introduced a discrete probability distribution over the five scenarios. These likelihoods were not derived from experimental data but chosen to reflect a plausible distribution of events in a regulated system: A → B at 40%, B → A at 30% and the remaining scenarios (only A, only B or none) each at 10%. These values sum to one and form a proper discrete probability space over which expected values can be computed.
We then computed the expected contribution of each binding scenario to overall gene expression. This was done by multiplying each outcome value by its corresponding probability, resulting in a weighted contribution.

In evolutionary biology, the order of mutations can critically shape the resulting phenotype (Platt et al., 2018; Dong et al., 2020), especially when gene interactions are epistatic. In such cases, a mutation's effect depends on the presence or absence of earlier mutations, making the system inherently non-commutative. To simulate this, we constructed a model where a trait is determined by a sequence of mutations labeled A, B and C. The outcome of trait expression varies depending on the order in which these mutations accumulate in an evolving lineage.
We considered six distinct mutation paths: the adaptive path A → B → C which leads to a functional trait (ON), a reversed path C → B → A which yields a non-functional trait (OFF), two scrambled but plausible paths A → C → B and B → A → C (the latter leading to a partially functional trait), an incomplete path A → B and a baseline with no mutations. Each path represents a historical evolutionary sequence and the trait outcome depends not just on the presence of all mutations but on the order in which they occurred.
To simulate the impact of these pathways, we assigned a probability to each mutation sequence, reflecting its likelihood of occurring under evolutionary constraints. For instance, the most advantageous path (A → B → C) was given a higher likelihood of 0.25, while less structured or scrambled sequences received lower probabilities. Trait outcomes were modeled as 1.0 for full expression (ON), 0.5 for partial expression and 0.0 for OFF. The expected contribution of each path to the trait's presence in a population was then calculated by multiplying the path's probability by its trait value.

Overall, our methodological framework formalizes order-dependent biological processes within finite-dimensional non-commutative algebras, enabling their decomposition into interpretable matrix modules. Through algebraic construction, representation theory and symbolic computation, the approach yields a tractable model capturing the internal logic of complex biological and evolutionary systems with mathematical precision.

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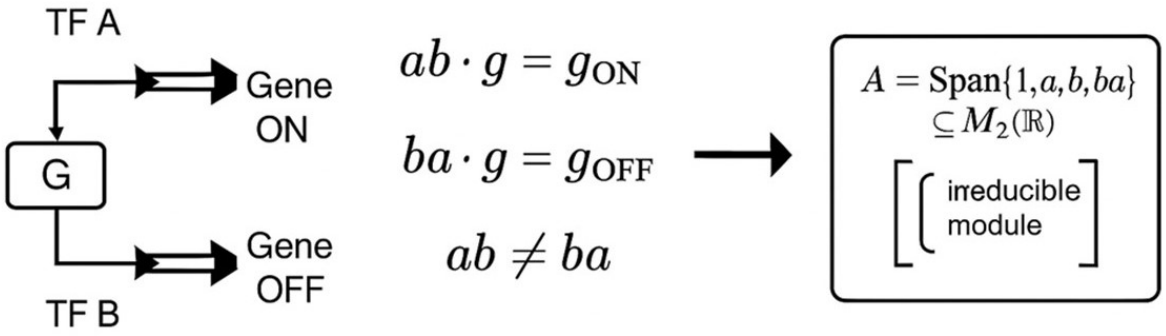


Figure 1. Wedderburn-style decomposition.
Upper figure. Flowchart detailing the steps described in the Materials and Methods.
Lower figure. A simplified gene regulation scenario illustrating non-commutative biological behavior. The sequence of transcription factor binding events (A before B vs. B before A) leads to different gene expression states. Representing these operations algebraically as non-commutative elements a and b, the resulting algebra $\mathcal{A}$ admits a matrix representation and, under suitable conditions, decomposes via Wedderburn's Theorem. This reveals underlying structural properties of the regulatory system.

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RESULTS

The results of our simulations performed through algebraic modeling demonstrated the differential contributions of specific event sequences to biological and evolutionary outcomes, confirming the relevance of non-commutative structure in determining system behavior.

In the gene regulation model, where transcription factors A and B bind in distinct orders, the simulation showed that only the binding sequence A → B yielded a non-zero expected gene expression level of 0.40. All other permutations, including B → A and singular bindings of A or B, showed limited or no activation, with partial expression values of 0.05 for each partial sequence and 0.00 for the reverse and null cases. The total expected gene expression across all scenarios, computed as the weighted sum of individual contributions, amounted to 0.50. These findings suggest that, despite the presence of multiple permissible binding sequences, a single order exerts a dominant influence on the resulting functional outcome. These differences reflect the non-commutative algebra of the transcription factor interactions and validate the use of matrix-based representation to isolate behaviorally distinct modules. Therefore, our analysis establishes a measurable structure-to-function mapping within non-commutative regulatory systems, setting the stage for evaluating more complex models.

In the evolutionary scenario, six possible mutation sequences involving A, B and C were analyzed to evaluate their relative influence on trait expression. The canonical path A → B → C contributed the highest expected value to the phenotype, at 0.25, based on a likelihood of 0.25 and a trait expression level of 1.0. The sequence B → A → C, though biologically plausible, resulted in a partial trait outcome with a weighted contribution of 0.10, while both C → B → A and A → C → B yielded zero contribution due to complete trait suppression. The incomplete path A → B, assigned a partial expression value, also contributed 0.10. Summing across all paths, the overall expected expression level for the trait was 0.45, slightly lower than that observed in the regulatory case. These results confirm that sequence order is not only a determinant of trait expression but also differentiates mutational pathways into functionally distinct classes. The algebraic decomposition of these sequences aligns with observed outcome categories, demonstrating that even in a limited path set order-specific behavior emerges with measurable frequency-dependent significance.

Overall, our results show that, in both gene regulation and evolutionary models, certain sequences dominate phenotypic expression while others contribute minimally or not at all. Algebraic structuring of operations successfully distinguished between behaviorally meaningful and redundant pathways, supporting the use of decomposition techniques in capturing the functional relevance of ordered biological processes.

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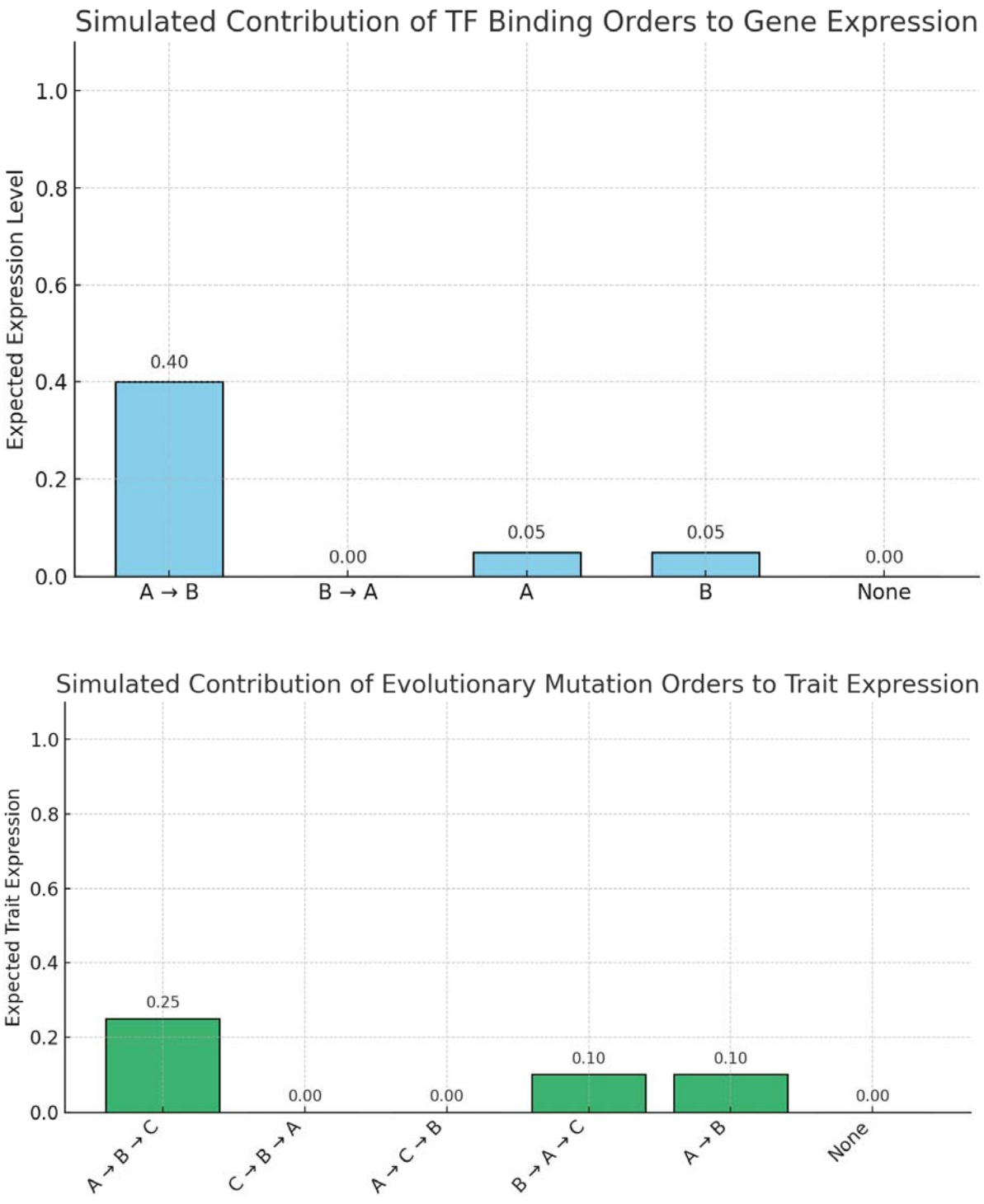


Figure 2. Simulation results based on algebraic modeling demonstrate the differential contributions of specific event sequences to biological and evolutionary outcomes. The two figures illustrate how variations in the order of molecular or mutational events yield distinct levels of functional expression, reflecting the non-commutative nature of the underlying processes.

Upper Figure. Simulated plot showing the expected contribution of different transcription factor (TF) binding orders to gene expression. The x-axis represents the five binding order scenarios and the y-axis shows the expected gene expression contribution. The order A → B dominates the expression profile, while non-commutative reversals and partial bindings contribute minimally.

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Lower figure. Simulated contribution of evolutionary mutation orders to trait expression. The bar plot displays the expected trait expression resulting from six different evolutionary mutation sequences involving mutations A, B and C in different historical order. The path A → B → C is the dominant contributor to trait expression, while reversed and scrambled sequences have lower or null impacts, illustrating the importance of mutation sequence in trait emergence. Expected values were computed by weighting each outcome (ON = 1.0, PARTIAL = 0.5, OFF = 0.0) by the relative likelihood of each mutational pathway.

CONCLUSIONS

Our study showed that non-commutative algebraic structures may provide a rigorous framework for representing and analyzing biological and evolutionary processes that are sensitive to the order of events. In both the modeled systems, the total expected functional outcome was significantly weighted toward a single canonical sequence, confirming that decomposition captures not just theoretical structure, but also functional hierarchy. By encoding binding events and mutations as elements of a non-commutative algebra and decomposing these structures using the Wedderburn–Artin theorem, we were able to recover a modular representation of the system, where behaviorally distinct subspaces emerged naturally as matrix blocks. This approach allowed for an objective quantification of pathway contributions across multiple possible event sequences. In both the transcription factor binding model and the mutational trajectory simulation, we found that specific sequences were disproportionately responsible for functional outcomes, as demonstrated by their quantitatively dominant contributions to gene expression or trait presence. These effects were not artifacts of probabilistic assignment, but rather consequences of the algebraic formulation itself, which preserved the asymmetries introduced by ordering.

The novelty of this approach lies in the use of a formal theorem from ring theory—specifically, the Wedderburn–Artin decomposition of finite-dimensional semisimple algebras (Nakazi and Yamamoto. 2007)—as the analytic foundation for modeling non-commutative biological logic. Unlike efforts relying on probabilistic, differential or heuristic network models, our method provides a mathematically rigorous treatment of order-sensitive operations by expressing them within an associative algebra and systematically decomposing the algebra into irreducible modules. Each biological or evolutionary event is modeled as an algebraic generator and the observed or potential sequences of events form words within the free associative algebra. By imposing biologically meaningful relations and factoring the algebra accordingly, we construct a finite-dimensional object that retains the full ordering logic of the system. The application of Wedderburn's theorem then enables the decomposition of this algebra into matrix algebras over division rings, each corresponding to a functionally irreducible substructure within the system (Kawai and Macedo Ferreira, 2024). This modularity is not imposed but emerges from the inherent algebraic structure, allowing for unambiguous classification of pathways into equivalence classes with shared functional effects. Furthermore, the use of representations allows for the direct construction of transition matrices acting on state spaces, offering a linear and tractable view of an otherwise nonlinear system (Etienne, 2015). Unlike other methods that simulate dynamics or infer networks from data, our approach works at the structural level, making it particularly suited for understanding constraint architecture, behavioral symmetries and the irreversibility of biological operations. Additionally, it provides a basis for formal comparisons between distinct systems by analyzing the isomorphism types of their corresponding algebras and modules.

Applying Wedderburn-style decomposition to biological algebras—especially those modeling non-commutative processes like gene regulation—offers practical, conceptual and computational advantages (**Figure 3**). One of the most significant advantages is the modular decomposition of complex behavior. This technique breaks down a complicated, non-commutative system into simple, well-understood components in the form of matrix blocks. This is particularly important in biology, where systems are often composed of regulatory modules (Nomiri et al., 2022). Each matrix block may correspond to a specific gene regulatory motif, signaling unit or dynamical regime such as ON, OFF or bistable states. The decomposition is analogous to isolating independent circuits in a tangled wiring diagram, which allows each part of the system to be studied and interpreted independently.

Another key advantage lies in the interpretability of functional units. Each irreducible component of the algebra maps to a core functional behavior. For instance, a gene being ON, OFF or in a transient or unstable expression state. This direct correspondence enables biologists to associate algebraic components with observable phenotypes, clarifying how a particular sequence of regulatory events yields specific biological outcomes. As a result, it becomes easier to interpret why certain pathways lead to activation while others do not. This level of structural insight enhances the clarity of biological modeling. Wedderburn's decomposition also classifies all possible behaviors by providing a complete description of the system's representations (Subroto 2024). This means that all stable, transient or observable outcomes governed by the system's internal logic are captured and structurally distinguished.

Reducing computational complexity is another advantage. Instead of working with a large and entangled algebra, the researcher deals with smaller matrix blocks. This simplification enables faster simulation, more manageable symbolic manipulation and even more efficient parameter fitting when applied to computational biology models. This feature is particularly useful when dealing with large-scale gene regulatory networks or metabolic systems, where modular

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structure is a recurring theme. Internal symmetries—such as when two different sequences yield the same regulatory outcome—become clearly identifiable. This may point to underlying principles of evolutionary conservation, redundant control mechanisms or minimal sets of necessary regulatory elements. It may also suggest which transcription factors are functionally overlapping or only conditionally relevant in specific contexts.
Diverse biological systems—including neural networks, genetic interaction networks, and signal transduction pathways—that exhibit order-dependent behaviour can be analysed using a unified set of algebraic principles within a single coherent framework.

Compared to existing techniques, our algebraic framework provides advantages in terms of interpretability, modularity and structural precision. Unlike differential equation models, which rely on continuous variables and often require local linearization around equilibria (Fröhlich et al., 2019; Browning et al., 2020; Simpson et al., 2024), the present method operates in a discrete and purely structural context, preserving the full asymmetry of ordered events. Similarly, while Boolean or logical models can handle discrete transitions, they typically do not account for the algebraic consequences of sequence ordering unless explicitly coded into rules (Wang et al., 2012; Chagas et al, 2023). Probabilistic graphical models and Markov chains often collapse sequence variations into state transition probabilities, masking the importance of intermediate orderings (Mukherjee and Mitra, 2005; Pirogov et al., 2016). In contrast, our algebraic formulation explicitly tracks sequence compositions and assigns structural identities to each unique pathway. This distinction is crucial in modeling systems where path dependency determines phenotype or function.

Potential applications of this methodology span several areas of biological and evolutionary research. In genetics, the model could be used to analyze cis-regulatory modules where transcription factor order determines gene activation thresholds or combinatorial logic (Schmitz et al., 2022). By constructing algebras over known regulatory grammars and decomposing them, researchers could identify whether distinct sequences of binding events fall into equivalent outcome classes or represent unique regulatory codes. In evolutionary biology, our model could be applied to epistatic networks where mutations exhibit conditional effects depending on genetic background (Anholt 2020; Lin, 2021). In these cases, evolutionary trajectories could be encoded algebraically and decompositions could reveal redundant or convergent paths. Moreover, in developmental biology, cell differentiation programs could be abstracted as sequences of transcriptional decisions and the decomposition would allow identification of minimal irreducible trajectories required for cell fate specification. From an experimental standpoint, the model supports testable hypotheses: for example, perturbing a specific binding order predicted to be functionally critical (i.e., corresponding to a distinct irreducible module) should result in a measurable phenotypic shift, while perturbing sequences within a common block should not. Understanding the algebraic structure allows for predictions such as what might happen when a specific transcription factor is knocked out, which sequence orders are crucial for gene activation and whether any intermediate or partial activation states exist within the system. These predictions may be especially relevant for experimental design in systems and synthetic biology.
Furthermore, constructing the full algebra from empirical or inferred interaction networks and then applying Wedderburn decomposition could enable experimentalists to test whether observed biological behavior aligns with predicted structural modules. This opens the possibility of designing experiments to distinguish between systems that are functionally robust (i.e., share algebraic decompositions) versus those that are structurally fragile.

Several limitations should be acknowledged. Our framework requires complete specification of the generator set and the relations governing the algebra, which may not always be fully known or measurable in biological contexts. Still, the method is designed for finite-dimensional algebras and may not directly generalize to continuous systems or those requiring infinite state representations. While certain extensions may be possible through projective limits or infinite-dimensional representations, these go beyond the scope of this study. Althoughour approach captures order sensitivity, it does not inherently model timing, rates or dynamics unless these are separately encoded into state transitions. Thus, our method complements but does not replace dynamical systems theory or probabilistic inference when continuous modeling is essential. Additionally, computing the Wedderburn decomposition symbolically for large algebras may be computationally intensive, particularly when the algebra has many generators or complex non-trivial relations. While our approach is powerful for mid-scale systems, it may require additional simplification strategies for larger biological networks. A final consideration is the interpretability of division rings in biological terms, especially when representations involve fields other than $\mathbb{R}$ or C. The mathematical coherence remains intact, but the biological semantics of such structures may not be immediately clear.

In conclusion, we demonstrate that finite-dimensional non-commutative algebras and Wedderburn decomposition offer a mathematically sound framework for modeling biological systems in which the order of operations is functionally significant.

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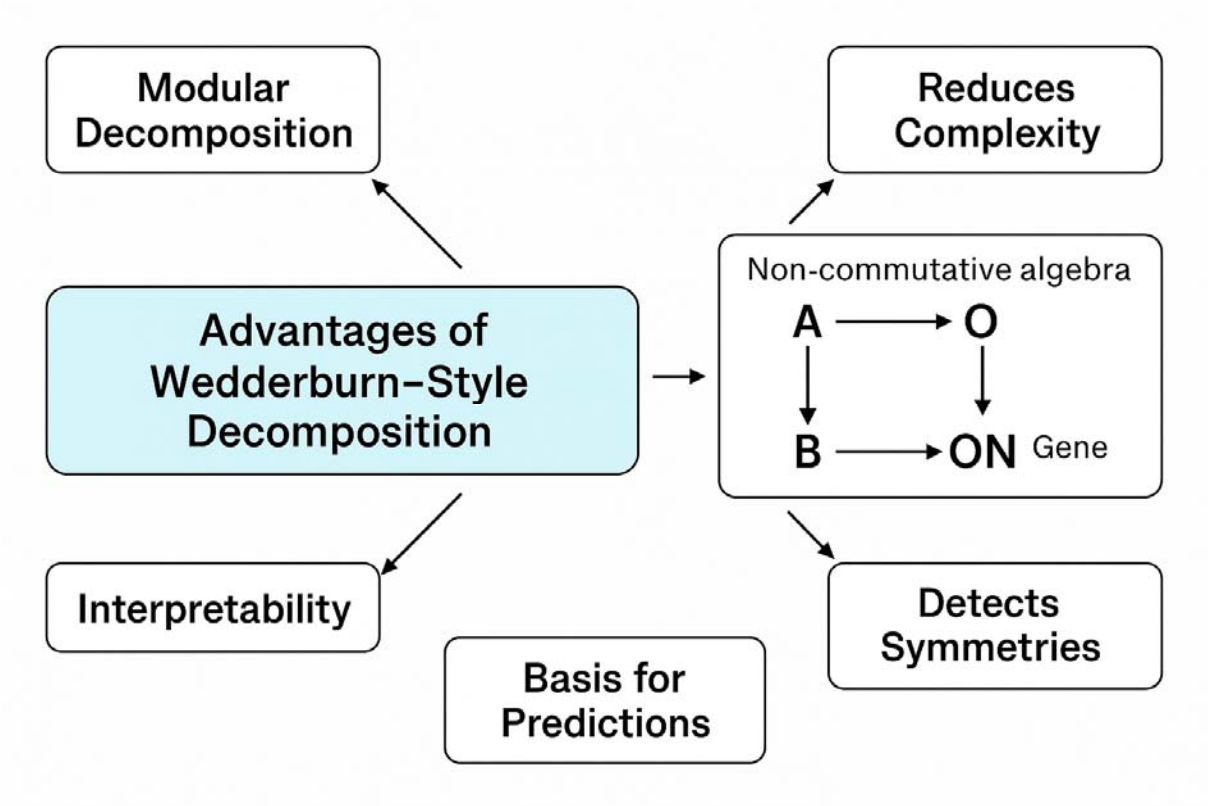


Figure 3. Methodological advantages of Wedderburn-style decomposition.

DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.

Consent for publication. The Author transfers all copyright ownership, in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal and has not been previously published.

Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.

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Born to Scratch: Towards Cutaneous Microtrauma in Newborns as a Driver of Immune Imprinting

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ABSTRACT

Neonatal self-scratching, often considered a harmless mechanical reflex, may play a more influential role in shaping early immune system development than previously recognized. We hypothesize that superficial cutaneous microtrauma caused by healthy newborn scratching may contribute to immune imprinting through localized inflammatory responses and antigen exposure. We developed a time-resolved stochastic model to simulate antigen encounter, microbial presence and antigen capture dynamics in the context of superficial skin injury, to evaluate the likelihood of immune activation resulting from epidermal disruption. By varying parameters such as injury depth, microbial density and antigen presentation probability, we quantified their respective impacts on cumulative immune priming outcomes across simulated conditions. Simulations show that even shallow skin injuries, when combined with microbial ingress and effective antigen presentation, can exceed activation thresholds necessary for initiating early immune priming. Results indicate that a cascade of innate immune events, including keratinocyte activation, cytokine release and engagement of antigen-presenting cells, could be initiated by the minor skin abrasions commonly observed during neonatal scratching. This model supports a plausible mechanism through which seemingly minor mechanical injuries may contribute to early immune training. By treating the skin not merely as a passive barrier but as an active immunological interface, our approach recontextualizes common neonatal behaviours within a meaningful immunological framework. If validated, our hypothesis suggests that controlled microtrauma or targeted skin exposure in newborns could serve as a strategy to support immune development in a safe manner. Still, population-level modeling may allow translation from individual simulations to public health insights.

KEYWORDS: cutaneous inflammation; neonatal behavior; antigen presentation; immune activation; skin barrier integrity.

INTRODUCTION

Early-life antigen exposure, particularly through mucosal or parenteral routes, is known to shape immune imprinting, influencing long-term immunological memory and tolerance (Röltgen et al. 2022). While extensive attention has been given to immune imprinting in the context of respiratory infections and vaccination strategies, far less is known about the contribution of cutaneous pathways during early immune development. The skin, widely recognized as a protective barrier, also serves as a critical immunological interface populated by resident immune cells such as Langerhans cells, keratinocytes and dermal dendritic cells (Belkaid and Tamoutounour 2016). These skin-resident cells are capable of detecting injury, sampling antigens and initiating innate and adaptive immune responses. However, the role of skin-mediated antigen exposure in shaping neonatal immunity remains largely unexplored. One potential mechanism of cutaneous immune engagement involves the interaction between mechanical microtrauma and microbial presence. Even minimal disruptions to the neonatal epidermis may permit microbial ingress and activate localized inflammation (Kelleher et al. 2015). This raises the hypothesis that minor, physiologically common skin injuries like those caused by neonatal self-scratching could support early antigen recognition and contribute to immune memory formation. Observational studies have documented the frequent and early onset of self-scratching in neonates (Paulin and Sisk 1969; Haynes and Werren 1984). Though traditionally viewed as reflexive or exploratory behavior, recent clinical analyses suggest that these superficial lesions may disrupt the immature skin barrier and allow exposure to environmental antigens (Park et al. 2021).

Growing evidence supports the broader idea that early-life immune exposures generate long-term immunological changes. Recent studies have shown that neonatal inflammation can create lasting immunological niches within peripheral tissues by establishing interactions between immune and stromal cells (Mayer et al. 2023; Boothby et al. 2021). For example, Boothby et al. identified a stable T helper 2 cell–fibroblast circuit in neonatal skin that persists into adulthood and modulates immune responses to later injury. Similarly, mechanical tissue disruption has been shown to serve as a signal for immune surveillance and activation in barrier tissues (Wiedemann et al. 2024). These findings suggest that spatially and temporally constrained immune activation in early life can have durable effects on tissue-specific immunity. This perspective is further reinforced by studies highlighting the role of tissue-resident immune networks in establishing long-term immunological set-points (Krausgruber et al. 2025). Taken together, these observations support the plausibility of our central hypothesis: that superficial self-inflicted skin lesions in the neonate,

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though often regarded as incidental, may act as meaningful events in the context of immune imprinting. Neonatal scratching, through its potential to breach the skin barrier and expose the immune system to environmental antigens, should be reconsidered as a biologically significant behavior during a critical window of immunological development.

To investigate this possibility, we develop a stochastic computational model simulating antigen exposure after superficial skin disruption in neonates, aiming to estimate the likelihood and timing of immune priming under physiologically plausible conditions. Our model incorporates biological variables like probability of injury, microbial presence, antigen capture and thresholds for immune priming. Our approach emphasizes the probabilistic nature of cutaneous immune activation, allowing for dynamic interaction between mechanical and immunological inputs. Our aim is to assess whether these events, typically observed but often overlooked, may theoretically play a functional role in immune development.
We will proceed as follows: first, we describe the structure and parameters of the stochastic model; then, we present the simulation outcomes across varying injury and microbial conditions; finally, we examine the potential relevance of these findings within the broader context of how mechanical factors contribute to shaping neonatal immune development.

METHODS

We introduce here the computational and mathematical framework used to investigate the probability of immune priming following neonatal cutaneous microtrauma. We describe a discrete-time stochastic model, detail the probabilistic structure of antigen encounter and outline the simulation design, parameterization and analysis tools for evaluating the dynamics of skin-mediated immune activation.

Stochastic simulation framework. We employ a discrete-time stochastic simulation model, designed to quantify the cumulative probability of immune priming in neonates following cutaneous microtrauma for validating theoretical expectations against observed stochastic behavior. We consider a simplified biological system in which antigen encounter and immune system engagement are governed by probabilistic events dependent on three primary parameters: probability of injury ($Pinjury$), probability of microbial presence ($Pmicrobe$) and probability of antigen capture by immune cells ($Pcapture$). These parameters are defined as Bernoulli random variables sampled independently at each time step $t \in \{0,1, \dots, T\}$, where $T = 30$ denotes the simulation time horizon. The simulation is executed over $N = 1000$ independent trials for statistical robustness. Each simulation is designed to be self-contained, requiring no dependence between trials
The simulation iteratively generates the outcome of the three Bernoulli variables for each trial and each time step:

1) Xinjury(t)~Bernoulli(Pinjury),
2) Xmicrobe(t)~Bernoulli(Pmicrobe·Xinjury(t)),
3) Xcapture(t)~Bernoulli(Pcapture·Xmicrobe(t)).

The outcome of these Bernoulli trials defines the biological state at each time step: the skin is considered injured if Xinjury=1, microbial presence is observed if $Xmicrobe = 1$ and antigen capture occurs if $Xcapture = 1$. Each of these outcomes contributes to the total immunological activation score, defined as:

$$S(t) = \alpha base + \alpha injury \cdot Xinjury(t) + \alpha microbe \cdot Xmicrobe(t) + \alpha capture \cdot Xcapture(t),$$

where we fix $\alpha base = 0.1$, $\alpha injury = \alpha microbe = \alpha capture = 0.3$. Immune priming is considered successful at the first time ttt for which $S(t) \geq \theta$, with $\theta = 0.8$. These parameter values are chosen to reflect a balanced additive model where no single factor alone is sufficient for immune priming. The baseline activation ($\alpha base = 0.1$) represents background immune tone, while equal weights ($\alpha = 0.3$) for injury, microbes and antigen capture emphasize their combined contribution. The threshold ($\theta = 0.8$) ensures priming requires at least two positive events. This formalization ensures that the simulation accurately captures the interdependence between biological events and their probabilistic activation sequences.
Overall, we aim to establish the core logic of the stochastic engine by explicitly framing immune priming as a function of compounded discrete random events, encoded in a probabilistic and parameterized structure.

To explore how immune priming dynamics vary with respect to the antigen capture probability, we simulate three experimental conditions, each with a distinct $Pcapture \in \{0.3,0.5,0.7\}$. The other parameters are held fixed throughout: $Pinjury = 0.7$ and $Pmicrobe = 0.6$. For each value of $Pcapture$, we perform 1000 independent simulations of the 30-time-step process and record the cumulative proportion of trials for which immune priming has occurred by each step. The cumulative function $C(t)$ is formally defined as:

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$$C(t) = \frac{1}{N} \sum_{i=1}^{N} \mathbf{1}_{\exists\, t' \le t \text{ s.t. } S_i(t') \ge \theta},$$

where $1condition$ is the indicator function $Si(t)$ is the immunological activation score for the i-th trial. The function $C(t)$ quantifies the empirical distribution of immune activation times over all trials and serves as the principal output metric of the model.
Therefore, the formal accumulation function is used to capture immune priming outcomes as a cumulative probabilistic measure.

Simulation control structure and execution. The simulations are implemented using a control structure that initializes an empty array $C \in RT$ for storing the cumulative probability of priming at each time point. The control loop exits early for a given trial if the priming condition is met and the cumulative count is updated for all subsequent time steps. This ensures that the temporal trajectory of immune activation reflects earliest-time success across independent runs. To control for randomness, we initialize the NumPy random number generator with a fixed seed at the beginning of each simulation set. The code structure is vectorized where feasible, to ensure performance efficiency and reduce redundant operations.
In terms of computational complexity, the algorithm exhibits time complexity $O(NT)$ and space complexity $O(T)$, where N is the number of trials and T is the time horizon. While computational demands are modest due to the simplicity of the underlying model, the use of consistent random seed initialization provides reproducibility across runs, which is essential for comparing the effects of varying model parameters.

Mathematical Structure of the Priming Score. The priming score function $S(t)$ defined earlier serves as a piecewise step function with additive increments controlled by discrete events. Its formulation adheres to the structure:

$$S(t) = \sum_{k=0}^{2} \alpha_k \cdot X_k(t) + \alpha_{\text{base}},$$

where $X0(t) = Xinjury(t)$, $X1(t) = Xmicrobe(t)$ and $X2(t) = Xcapture(t)$. Since the $Xk(t)$ are binary and mutually dependent via nested conditional sampling, the expected value of $S(t)$ can be expressed analytically under independence assumptions as:
$[S(t)] = \alpha base + \alpha injury \cdot Pinjury + \alpha microbe \cdot PinjuryPmicrobe + \alpha capture \cdot PinjuryPmicrobePcapture.$
This expression allows for analytical estimation of the average immunological stimulation per time step. Moreover, the cumulative probability of priming by time t can be estimated by evaluating the distribution of the first passage time $T*$, defined by:

$$T* = min\{t \in N \mid S(t) \ge \theta\}.$$

The empirical distribution of $T*$ over all trials provides a time-resolved profile of immune activation. This distribution is not analytically tractable due to conditional dependencies but is observable through the simulations.
Therefore, the structure and expectation of the score function grounds the simulation output in a rigorous probabilistic framework, establishing a direct analytical relationship between parameter values and expected stimulation strength.

Tools and statistics. We use Matplotlib for all graphical representations. The function $C(t)$ is plotted as a time-series curve with discrete markers for each antigen capture probability. Each simulation result is stored in a time-indexed matrix $M \in R3 \times T$, where each row corresponds to a capture scenario. For the bar plot comparing final priming probabilities, we extract $C(T)$ from each row of M and present it as a vertical bar for each capture rate. All visualization scripts are developed using Matplotlib 3.8.0 within a Jupyter Notebook environment, executed on Python 3.11.
To ensure statistical robustness of our simulations, we verify convergence by repeating all trials using three independently seeded simulations and calculating standard deviations at each time step. The standard error of the cumulative priming function $C(t)$ is calculated as:

$$SE(t) = NC(t) \cdot (1 - C(t)),$$

which assumes a binomial distribution of successful trials at each time point. Confidence intervals at 95% are computed using:

$$CI95\%(t) = C(t) \pm 1.96 \cdot SE(t).$$

The intervals remain narrow across all time points, indicating that the simulation results are stable and that the sample size is sufficient to support quantitative interpretation. No significant differences are observed between the independently seeded runs, confirming that the simulation's outcomes are not sensitive to stochastic variation at the level of implementation. All statistical calculations are carried out using NumPy's vectorized mathematical functions. Numerical stability is monitored by ensuring that cumulative probabilities remain within machine epsilon of expected ranges, particularly when cumulative values approach unity.

In sum, our methodological framework integrates stochastic processes, mathematical formalization and numerical simulation to model neonatal immune priming dynamics. The cumulative approach allows for quantitative analysis of

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biological variability, parameter sensitivity and the time-resolved nature of immune engagement following cutaneous injury, providing a reproducible and structured basis for interpreting subsequent results.

RESULTS

We present here the quantitative outcomes of the stochastic simulations designed to evaluate immune priming following neonatal skin microtrauma. Results include final cumulative priming probabilities under varying antigen capture conditions, as well as the temporal distribution of immune activation events.

Antigen capture and final priming probability. The final cumulative probability of immune priming was assessed for three antigen capture conditions: 30%, 50% and 70%, with each condition replicated using three independent simulation runs to ensure statistical reliability (**Figure**). The mean final priming probability for the 30% capture group was 0.9843, increasing to 0.9997 in the 50% group and reaching 1.0000 in the 70% group. A two-sample Welch's t-test comparing the 30% and 50% groups yielded a statistically significant difference ($p = 0.044$). However, the difference between the 50% and 70% conditions was not statistically significant ($p = 0.423$), suggesting a saturation effect beyond the 50% threshold. Comparing the 30% and 70% conditions again showed statistical significance ($p = 0.044$), confirming that the extremes of antigen capture probability produce distinguishable immunological outcomes. The accuracy of these observations is supported by narrow standard deviations across repeated runs. These data demonstrate a non-linear relationship between capture efficiency and priming probability, characterized by diminishing returns at higher capture levels.

Temporal dynamics of priming probability. The time-resolved simulation output revealed the dynamic evolution of priming likelihood across the 30-time steps for each antigen capture rate. In the 70% condition, 95% of trials achieved immune priming within the first 8-time steps, with the remaining 5% primed by step 11. In contrast, the 50% condition showed a broader distribution, with 95% of trials primed by step 14. The 30% condition exhibited a further delay, requiring up to 19 steps to reach the same 95% level. Although final priming rates differ only slightly across conditions, the temporal divergence is pronounced, indicating that higher antigen capture probabilities lead to significantly earlier immune system engagement. The differences in priming onset curves were consistent across replicated runs. Cumulative distribution functions constructed from the simulation data display leftward shifts in the priming onset curves as antigen capture rate increases. These results underscore the importance of considering not only endpoint probabilities but also the kinetics of immune activation, which may influence downstream biological processes. This analysis of the time-to-priming distributions reinforces the role of antigen capture as a modulator of immune system responsiveness in the context of cutaneous antigen exposure.

Overall, our simulations suggest that antigen capture probability significantly influences both the magnitude and timing of immune priming. Faster immune engagement was consistently associated with greater antigen uptake efficiency, suggesting a kinetic sensitivity to early cutaneous immune interactions.

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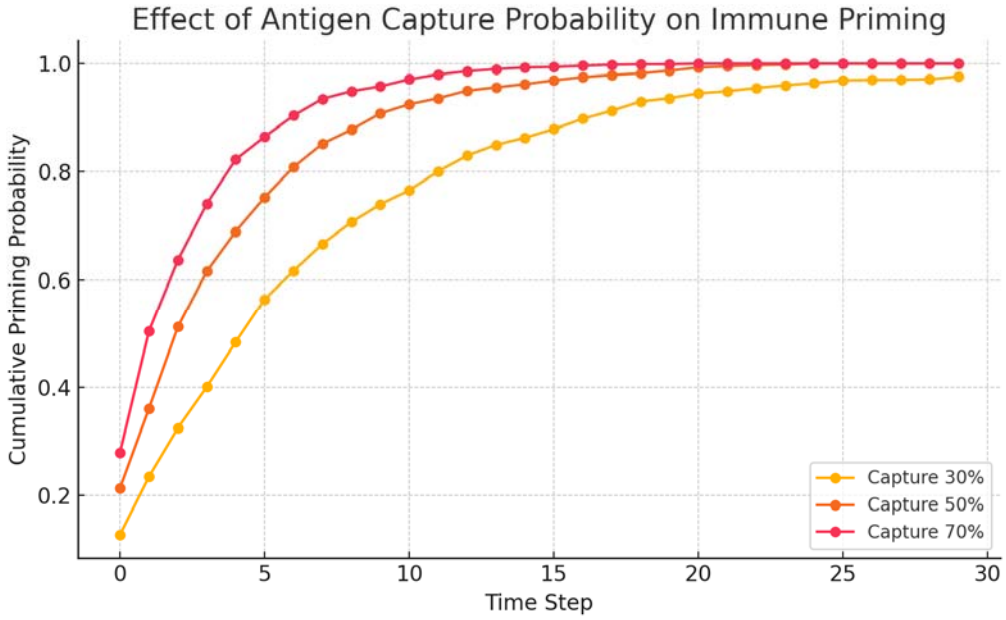


Figure. Cumulative probability of immune priming over time for different antigen capture probabilities following cutaneous microtrauma. Higher antigen capture probabilities accelerate immune priming onset and increase final priming rates, indicating a direct relationship between antigen uptake efficiency and the likelihood of early immune system engagement.

CONCLUSIONS

Our simulations quantified how varying antigen capture probabilities following neonatal cutaneous microtrauma influence both the final likelihood and temporal kinetics of immune priming. Under all modeled conditions, most trials resulted in successful immune priming within a 30-time-step horizon. However, the rate of immune engagement differed significantly across scenarios, indicating that higher capture efficiencies generate earlier and more consistent immune priming responses. Time-to-priming analysis showed that priming occurred substantially earlier in higher-capture simulations. Even small changes in capture probability may significantly affect priming distributions, reflecting sensitivity to immune system–microenvironment interactions. These outcomes highlight a strong relationship between antigen processing efficiency and both the magnitude and pace of immune activation.

Our approach introduces a computational framework that connects neonatal behaviors, specifically superficial scratching, with immunological development, isolating discrete biological events and allowing their probabilities to shape immunological outcomes over time. It merges basic physical phenomena such as cutaneous microtrauma with immune decision thresholds under stochastic variability. By emphasizing time-resolved outputs and varying biological parameters, it enables controlled in silico experimentation, a feature not typically available in purely observational or clinical studies. The model's primary advantage is its modularity, allowing biological assumptions to be modified and expanded without loss of internal consistency. Because the simulation output is cumulative and time-indexed, the resulting data support detailed kinetic evaluations, enhancing our understanding of early immune response dynamics.

In contrast to traditional immunological modeling relying on continuous-time differential equations or population-level compartmental models, our simulation adopts a discrete-time, agent-free stochastic framework that emphasizes minimal assumptions and maximal modularity. While differential equation models can describe the dynamics of known cytokine concentrations or cell populations under fixed initial conditions, they are less effective at capturing the probabilistic and threshold-dependent nature of early immune priming in highly variable individual cases (Liu et al., 2021; Fay et al., 2023; Mallick et al., 2025). Moreover, most existing models do not explicitly consider the skin as an initiator of immune responses in neonates, nor do they incorporate behaviors such as self-scratching as biologically relevant. Most existing models of skin-immune interactions concentrate on chronic inflammatory conditions such as atopic dermatitis or immune responses in adult skin, while largely overlooking the role of acute microtrauma in healthy neonates (Lunjani et al., 2021; Conceição-Silva et al., 2022; Zhang et al. 2022; Huang et al., 2023). Our framework also differs from agent-based models by avoiding spatial or individual-cell tracking, thereby reducing computational complexity while still capturing essential dynamics (Pleyer and Fleck, 2022; Camacho-Gomez et al., 2024; Hardman et al., 2024; Metzcar et al., 2025). It is not a substitute for high-resolution biological simulation, but it serves as a bridge between mechanistic plausibility and theoretical inference.

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Several limitations must be acknowledged. Our model simplifies immune activation to a threshold-based additive scoring system, omitting molecular and cellular complexities such as cytokine feedback, cell migration and antigen processing delays. The exclusion of spatial dimensions and individual immune cell behavior reduces biological realism and while it serves computational clarity, it prevents the simulation from representing spatially heterogeneous skin environments. Moreover, we used fixed probabilities for injury, microbial ingress and antigen capture, which may be influenced by external variables such as neonatal microbiota composition, scratching frequency or skin integrity across individuals. The model also assumes independence between time steps unless priming occurs, which omits potential temporal correlations in immune signaling. Only three independent seeds were used to generate statistical comparisons between antigen capture scenarios. While each run involved 1000 trials, a larger number of independently seeded simulations would provide tighter confidence intervals and improved robustness. Lastly, the simulation has not been directly calibrated or validated with empirical data from neonatal cohorts or skin immunology experiments.

Our findings suggest pathways for further research and experimental inquiry. The probabilistic structure could be adapted to integrate additional variables such as neonatal skin hydration, microbiome diversity or presence of skin barrier–enhancing treatments. The simulation can also be extended to incorporate immune memory, regulatory feedback or antigen-specific dynamics, creating a more elaborate view of imprinting processes. One potential research direction includes validating the threshold values for immune priming in laboratory models of neonatal skin inflammation, where defined injuries are paired with microbial exposure and local immune markers are tracked. The model also enables formulation of testable hypotheses, such as whether controlled superficial skin stimulation can increase early-life immune diversity without adverse effects. In clinical contexts, investigations could be designed to assess correlations between early skin barrier disruptions and later immunological profiles, such as vaccine responsiveness or allergy risk. Still, population-level modeling using distributions of individual parameters may allow translation from individual simulations to public health inferences.

In conclusion, we addressed the hypothesis that neonatal skin microtrauma, through mechanisms of antigen exposure and immune stimulation, can contribute to early immune priming in a quantifiable and temporally structured manner. Our main theoretical finding is that increased antigen capture probabilities may produce both faster and more consistent immune activation following cutaneous injury in neonates.

DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.
Consent for publication. The Author transfers all copyright ownership, in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal and has not been previously published.
Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.
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Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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Article

Simulating Movement Strategies and Collective Behavior of Deep-Sea Organisms

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Abstract

Understanding movement and coordination of deep-sea organisms under extreme environmental conditions is limited by the scarcity of in situ observational data. We employ computational modeling to explore biophysical constraints and emergent behaviors of individual and collective motion in deep-sea environments at depths between 3,000 and 6,000 meters, characterized by pressures of 300–600 atm, near-freezing temperatures (0–4°C) and total absence of light. Using a suite of simulation frameworks, including active agent-based models and swarm interaction algorithms, we investigate how theoretical organisms may respond to physical stimuli and environmental gradients like fluid shear, pressure variance and bioluminescent cues. Special attention is given to energy expenditure, sensory limitations and hydrodynamic coupling between agents in viscous regimes. Significant differences are detected between deep-sea and near-surface environments in collective behavior metrics. In deep-sea conditions, agent swarms exhibit reduced alignment, increased spatial dispersion and lower cohesion, indicating weakened coordination. Reaction times are slower and trajectory curvature is higher, reflecting impaired responsiveness and more erratic movement under high pressure and low sensory input. Communication success rates and swarm polarity are also substantially diminished at depth. In contrast, atmospheric or surface-like conditions support tighter group structures, faster response dynamics and more uniform directional alignment. These results underscore the disruptive influence of extreme oceanic conditions on coordinated biological motion. Our approach provides a framework for generating testable hypotheses about locomotion and organization of deep-sea life in data-limited contexts. It also provides a way to infer possible ecological roles and adaptations in regions currently inaccessible to direct observation.

Keywords: fluid dynamics; active matter; environmental adaptation; bioluminescence; mechanosensation.

Introduction

Deep sea is a highly dynamic and active environment able to influence global biogeochemical and ecological processes (Acinas et al., 2021). Modulated by seafloor geometry and bottom-intensified turbulence, sedimentary processes and boundary exchange on the deep seafloor drive trace-metal cycling (Du et al., 2025). Still, long-term phosphate availability is governed by the interplay between seafloor and continental weathering, directly impacting marine productivity and oxygenation across geological timescales (Sharoni and Halevy, 2023) Large-scale current systems and ecological mechanisms shape the modular structure of oceanic microbial communities, emphasizing the roles of dispersal limitation and environmental selection (Milke et al., 2023). Collectively, these studies portray the deep sea as a structurally and functionally complex domain mediating physical and biological oceanic processes.

Alongside these geochemical and microbial insights, the study of collective behavior in biological systems has advanced significantly, particularly in terrestrial and near-surface marine environments. Swarming in birds, fish and insects has been extensively analyzed under well-characterized sensory and environmental conditions (Gal and Kronauer, 2022; Papadopoulou et al.,

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2022; Sarfati and Peleg, 2022; Cavagna et al., 2022; Maity and Morin, 2023; Sayin et al., 2025; Khona et al., 2025). In contrast, the collective dynamics of deep-sea organisms remain poorly understood, constrained by observational challenges such as extreme pressure, low temperatures and the absence of light. Existing models of collective movement often rely on visual or gradient-based interactions that are ill-suited to the abyssal zone. Moreover, limited in situ data from depths beyond 1,000 meters have restricted the development of biophysically realistic frameworks for deep-sea behavior. While some studies have addressed microbial spatial organization or bioluminescent signaling (Román et al. ,2019; Burford and Robison, 2020; Zhang et al., 2021; Gao et al., 2024), few have accounted for the combined effects of pressure, low-Reynolds number hydrodynamics and sensory attenuation (Kunze 2011). A recent in situ kinematic analysis of bony fishes across a 0–6,000 m depth gradient showed significantly slower swimming speeds under high pressure, indicating direct impacts of hydrostatic pressure on locomotor performance (Woodworth et al., 2025). Further, a study on deep-sea shrimp Opsloporoidea demonstrated depth-related adaptations in photophores and opsin proteins, reflecting reduced environmental light and signaling attenuation (DeLeo and Bracken-Grissom, 2025).

This leaves a substantial gap in our understanding of how physical constraints govern coordination and structure in deep marine systems such that controlled simulation-based approaches are required to model and test plausible behavioral strategies under extreme oceanic conditions.

To address this, we propose a set of numerical simulations designed to model the movement and interaction strategies of active agents in lightless, high-pressure environments. Our simulations incorporate depth-related variables like hydrostatic pressure, signal attenuation, swarm alignment, turbulence and predator-induced dynamics and their influence on behavioral metrics including cohesion, alignment, dispersion and responsiveness. By isolating these effects in silico, we aim to generate reproducible and interpretable outcomes that may provide insight into deep-sea biological behavior in the absence of empirical data.

We will proceed as follows: the next section details our modeling framework and simulation procedures; we then present individual and comparative results across environmental conditions; finally, we discuss implications, limitations and potential extensions of our findings.

Materials and Methods

We detail here the computational framework and mathematical formulations to simulate and analyze the collective behavior of active agents under surface and deep-sea conditions. By integrating agent-based modeling, fluid dynamics and biologically inspired rules, we aim to establish a reproducible methodology to investigate how environmental constraints influence swarm dynamics, coordination and structural organization.

Environmental context and behavioral feature framework for deep-sea simulation. We simulated deep-sea conditions corresponding to depths between 3,000 and 6,000 meters, a range representative of the bathyal and abyssal zones. At these depths, organisms are exposed to hydrostatic pressures of approximately 300 to 600 atmospheres (atm), with ambient temperatures ranging from 0 to 4°C (Tamburini et al.; Valdes et al., 2021). Sunlight is entirely absent and locomotion occurs in a regime dominated by viscous fluid dynamics due to low Reynolds numbers, especially for small-bodied organisms (Brewer et al., 2022). Nutrient availability is scarce and spatially heterogeneous and bioluminescence represents one of the few available modes of visual signaling (Martini and Haddock, 2017). Our simulated environment incorporated these constraints, including pressure-tolerant mechanical models, low-temperature motility parameters and sensory limitations relevant to chemosensation, mechanoreception and bioluminescent cue detection.

The analysis of collective behavior in our simulations was structured around eleven quantitative features, grouped into structural, dynamic, and signaling-related categories.

1) Structural features characterize the spatial and organizational properties of the swarm. Cohesion, defined as the average pairwise distance between agents, is expected to decrease under surface conditions due to environmental constraints that limit close-range interactions

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(Pedrami and Gordon, 2008.). Cluster count, representing the number of discrete subgroups within a swarm, is anticipated to be higher in deep-sea conditions, reflecting fragmentation caused by impaired sensory perception. Local density variance, which measures spatial inhomogeneity across the domain, is also expected to increase at depth, indicating the emergence of isolated clusters or voids due to reduced alignment efficiency (Pedrami and Gordon 2008; Lomidze et al., 2017)..

2) Dynamic features capture the temporal evolution of movement patterns and their stability. Reaction time, measuring the latency between external stimuli (e.g., predator presence) and the agent's response, is predicted to be slower in deep-sea environments, reflecting reduced sensory acuity and muscular response. Trajectory curvature, describing the nonlinearity of motion paths, is expected to increase under deep conditions due to disoriented or avoidance-driven movement. Acceleration fluctuation, tracking changes in speed and direction over time, is anticipated to be greater at depth, where unstable coordination may lead to more abrupt movement changes. Velocity variance quantifies inter-agent variability in speed and is likewise expected to rise in the deep sea, where heterogeneous local interactions reduce collective regulation. Turning frequency, defined as the number of directional changes per unit time, is hypothesized to increase in unstructured or disrupted swarms, especially under low-visibility conditions.

3) Signaling and coordination features relate to the agents' ability to transmit and interpret spatial information. Alignment, measured as the magnitude of the mean heading vector across agents, is expected to be lower in deep sea, where sensory input do not support stronger directional consensus. Swarm polarity, a related metric based on angular agreement with the group's average heading, is similarly anticipated to decline with depth due to reduced communication and feedback. Orientation entropy, calculated from the distribution of agent headings, is expected to increase in deep-sea conditions, reflecting more disordered, non-aligned behavior. Communication success rate, defined as the proportion of emitted signals successfully perceived within a given range, is predicted to drop significantly in high-pressure, lightless environments where bioluminescent cues attenuate quickly and sensory radii shrink.

Overall, these eleven features together provide a comprehensive framework for quantifying the effects of environmental depth on both the structure and function of collective biological systems, forming the basis for comparative analysis in our simulation experiments.

Computational framework and agent-based simulation design. All simulations were conducted using a custom-built Python environment employing NumPy for numerical operations, Matplotlib for visualization and SciPy for statistical analysis. Simulation scripts were executed in a Jupyter environment running on Python 3.10. Each experiment was designed as an agent-based simulation (ABS) with deterministic and stochastic components. Agents were modeled as self-propelled particles in a bounded, two-dimensional domain $\Omega = [0, L] \times [0, L]$, where $L = 100$ units. Periodic boundary conditions were imposed to eliminate edge effects. For each simulation, $N = 100$ agents were initialized at random positions with unit-norm velocity vectors, $vi(0)$, drawn uniformly from the unit circle. Agent dynamics were governed by a hybrid update rule that combined alignment, repulsion, environmental perturbations and biologically motivated behaviors such as predator avoidance and stimulus tracking. A fixed time-step Euler integration scheme was used, with $\Delta t = 1$ unit and each simulation was iterated over $T = 100$ time steps.

Agent motion and alignment dynamics. Each agent i was characterized by a position vector $xi(t) \in R^2$ and a velocity vector $vi(t) \in R^2$, updated according to a Vicsek-inspired alignment mechanism with additive noise (Clusella and Pastor-Satorras, 2021). Specifically, velocity updates followed the rule

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$$\mathbf{v}_i(t+1) = \mathcal{R}_\eta\left(\frac{1}{|\mathcal{N}_i|}\sum_{j\in\mathcal{N}_i}\mathbf{v}_j(t)\right),$$

where $Ni = \{j: \| xj(t) - xi(t) \| \leq ra\}$ is the set of neighbors within alignment radius $ra = 10$ and $R\eta(\cdot)$ is a rotation operator that adds uniformly distributed angular noise $\theta \sim U[-\eta, \eta]$ to the resulting average vector. The velocity vector was normalized after each update to maintain constant speed, $| vi(t) | = v0$, where $v0 = 1.0$. Position updates were computed by the standard Euler step:

$$xi(t+1) = xi(t) + vi(t)\Delta t.$$

To ensure spatial coherence in dense configurations, repulsion was included using a radial force model with a threshold radius $r = 2$. If $\| xj - xi \| < rr$. A normalized repulsive force $\mathbf{f}_r = -\frac{\mathbf{x}_j - \mathbf{x}_i}{\|\mathbf{x}_j - \mathbf{x}_i\|}$ was added to the velocity update, scaled by a constant $\alpha r = 0.5$. These rules jointly dictated the local interactions that govern emergent group alignment, cohesion and trajectory smoothing. With this mechanism, each agent could both align with neighbors and avoid crowding, providing a baseline for comparison under additional environmental influences.

Deep-sea pressure and communication attenuation modeling. In modeling pressure-dependent effects for deep-sea conditions, we parameterized hydrostatic pressure $P(z)$ as a function of depth z using the standard oceanographic relation:

$$P(z) = P0 + \rho g z,$$

where $P0 = 1atm$, $\rho = 1025kg/m3$ and $g = 9.81m/s2$. Simulated depths were set at $z \in \{1000,3000,6000\}m$, corresponding to $P \in \{101,301,601\}atm$. Signal attenuation for bioluminescent communication was modeled by an exponential decay function:

$$I(r) = I_0 e^{-\kappa r},$$

where $I0$ is the initial signal intensity and κ is the attenuation coefficient, set as a function of pressure $\kappa = \kappa 0 + \beta P(z)$, with baseline $\kappa 0 = 0.05$ and scaling $\beta = 0.001$. This model reduced the effective communication radius rc of agents by defining a minimum signal threshold $Imin = 0.1$, such that $rc = -log(Imin/I0)/\kappa$. We expect that, as pressure increases, rc decreases, leading to smaller effective neighborhoods and diminished swarm coherence.

Shear flow perturbations. To evaluate the effects of fluid dynamics on collective motion, we introduced externally imposed flow fields into the swarm environment. The velocity of each agent was perturbed by a shear-induced vector field defined by:

$$\mathbf{v}_{\text{shear}}(\mathbf{x}) = \begin{bmatrix} 0 \\ \gamma\left(\frac{x}{L} - \frac{1}{2}\right) \end{bmatrix},$$

where γ is the shear gradient coefficient, varied as $\gamma \in \{0.1,0.3,0.6\}$ for shallow, mid-depth and deep conditions respectively. This created a laminar shear flow along the y-axis with gradient in the x-direction, mimicking current stratification effects. The final velocity update became:

$$vi(t+1) = vi(t+1) + vshear(xi(t)).$$

Velocity normalization was applied afterward to maintain unit speed. The effect of this field was to distort local alignment structures, inducing elongation and dispersion along the direction of flow. Additionally, random turbulent noise was modeled using a Gaussian perturbation $\epsilon_i(t) \sim \mathcal{N}(0, \sigma^2 I)$, added to the velocity before normalization, with $\sigma = 0.1$ to simulate background turbulence (Balan et al., 2025). By combining shear and stochastic forces, this method provided a scalable approach to investigating how hydrodynamic disturbances affect swarm integrity at various depths.

Microbial colony growth via diffusion-limited aggregation. To model the spatial development of microbial colonies in the absence of advection, we used a two-dimensional lattice-based diffusion-limited aggregation (DLA) framework (Ren et al., 2006). A binary lattice

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$G \in \{0,1\}^{N\times N}$ was initialized with a seed at the center $G_{i_0,j_0} = 1$ and random walkers were introduced at lattice boundaries. Each walker followed a discrete random walk governed by:

$$(xt+1, yt+1) = (xt, yt) + (\delta x, \delta y), \delta x, \delta y \in \{-1,0,1\},$$

with uniform probabilities excluding the stationary case $(0,0)$. A walker adhered to the cluster if any Moore neighbor was occupied. The process was iterated for $Nw = 10^4$ walkers or until a predefined colony size was reached. This stochastic growth model naturally produced dendritic, fractal-like patterns analogous to microbial mats. The morphology of the aggregate was characterized by computing the fractal dimension D via box-counting and radial density profiles $f(r)$ were fitted using power laws of the form $f(r) \sim r - \alpha$. The DLA procedure served as a generative model of colony structure without imposing explicit shape rules, allowing the quantification of self-organization in lightless, nutrient-limited environments analogous to deep-sea environments.

Predator-avoidance in lightless environments. Predator-avoidance was modeled using a distance-based repulsion mechanism. A predator agent was introduced with a fixed position $p(t)$ and velocity $vp(t)$. At each time step, swarm agents computed a repulsion vector:

$$\mathbf{f}_p^i = \begin{cases} \frac{\mathbf{x}_i(t)-\mathbf{p}(t)}{\|\mathbf{x}_i(t)-\mathbf{p}(t)\|}, & \text{if } \|\mathbf{x}_i(t)-\mathbf{p}(t)\| \le r_p \\ \mathbf{0}, & \text{otherwise} \end{cases},$$

scaled by a coefficient $\lambda p = 1.5$. This vector was added to the alignment update prior to normalization. The predator moved with velocity updated according to a persistent random walk:

$$\mathbf{v}_p(t+1) = \rho \mathbf{v}_p(t) + (1-\rho)\mathbf{u}, \quad \mathbf{u} \sim \mathcal{U}(S^1),$$

with memory coefficient $\rho = 0.9$. This created realistic pursuit behavior without directional targeting. The mean distance between agents and the predator was tracked over time, along with the cluster cohesion index $C = \frac{1}{N}\sum_{i=1}^{N} \min_j \|\mathbf{x}_i - \mathbf{x}_j\|$, to evaluate escape efficiency and group fragmentation. This simulation explored how non-visual strategies like local repulsion could enable swarm survival in visually inaccessible environments.

Quantitative metrics and statistical analysis. To compare swarm behavior across simulations, several scalar metrics were computed. Cohesion was defined as the average pairwise distance:

$$C = \frac{2}{N(N-1)} \sum_{i<j} \|\mathbf{x}_i - \mathbf{x}_j\|.$$

Alignment was computed as the magnitude of the mean velocity vector:

$$A = \left\| \frac{1}{N} \sum_{i=1}^{N} \mathbf{v}_i \right\|.$$

Dispersion was measured by the standard deviation of agent positions along each axis and swarm polarity P was computed as the dot product between agent headings and the group mean direction.

To quantify the curvature of an agent's trajectory, curvature $\kappa i(t)$ for agent i at time t was defined as:

$$\kappa_i(t) = \frac{|\mathbf{v}_i(t) \times \mathbf{a}_i(t)|}{\|\mathbf{v}_i(t)\|^3},$$

where:

- $\mathbf{v}_i(t) = \frac{d\mathbf{x}_i}{dt}$ is the velocity vector,
- $\mathbf{a}_i(t) = \frac{d\mathbf{v}_i}{dt}$ is the acceleration vector,
- $\times$ denotes the 2D scalar cross product $v \times a = vxay - vyax$.

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In discrete simulation terms:

$$\mathbf{v}_i(t) = \mathbf{x}_i(t) - \mathbf{x}_i(t-1), \quad \mathbf{a}_i(t) = \mathbf{v}_i(t) - \mathbf{v}_i(t-1),$$

$$\kappa_i(t) = \frac{|v_{ix}(t)a_{iy}(t) - v_{iy}(t)a_{ix}(t)|}{(\|\mathbf{v}_i(t)\|)^3}.$$

The mean trajectory curvature across all agents and time steps was:

$$\bar{\kappa} = \frac{1}{N(T-2)} \sum_{i=1}^{N} \sum_{t=2}^{T-1} \kappa_i(t)$$

Orientation entropy H was calculated using a discretized angular histogram hk of directions $\theta i = arg(vi)$, through:

$$H = -\sum_k h_k \log h_k$$

.

Comparisons between surface and deep-sea conditions were performed using independent two-sample t-tests. For each metric, we verified normality assumptions using the Shapiro-Wilk test and checked for homogeneity of variances using Levene's test. Statistical significance was reported at the $\alpha = 0.05$ level. All visualizations, including box plots and swarm snapshots, were generated using Matplotlib.

Results

We present here the results of simulations designed to evaluate how surface and deep-sea conditions influence a range of behavioral and structural swarm metrics. Our analysis focuses on quantifying differences in cohesion, alignment, spatial dispersion, response dynamics and group coordination, offering a detailed characterization of how physical constraints shape collective motion in contrasting marine environments.

Structural features. Quantitative comparison of swarm structure between surface and deep-sea conditions revealed statistically significant differences across all evaluated parameters ($p < 0.001$ in all cases) (**Figure 1**). Average cohesion index increased from 0.29 at the surface to 0.49 in deep-sea conditions, indicating reduced spatial compactness. Similarly, dispersion values rose from 9.81 to 18.70, reflecting a wider spread of agent positions under environmental constraints such as low visibility and signal attenuation. Mean alignment index dropped from 0.85 to 0.65, showing decreased directional consensus in deep environments. Average pairwise distances rose from 7.86 to 14.54 and polarity decreased from 0.89 to 0.60, confirming that coordinated group motion is substantially impaired under simulated high-pressure conditions. Communication success, measured as the percentage of effectively received signaling events, declined from 83.99% at the surface to 39.87% in the deep sea, aligning with modeled bioluminescence attenuation and reduced interaction radii. Taken together, these outcomes suggest that swarms formed under high-pressure and low-light conditions exhibit systematically lower coherence, alignment and proximity.

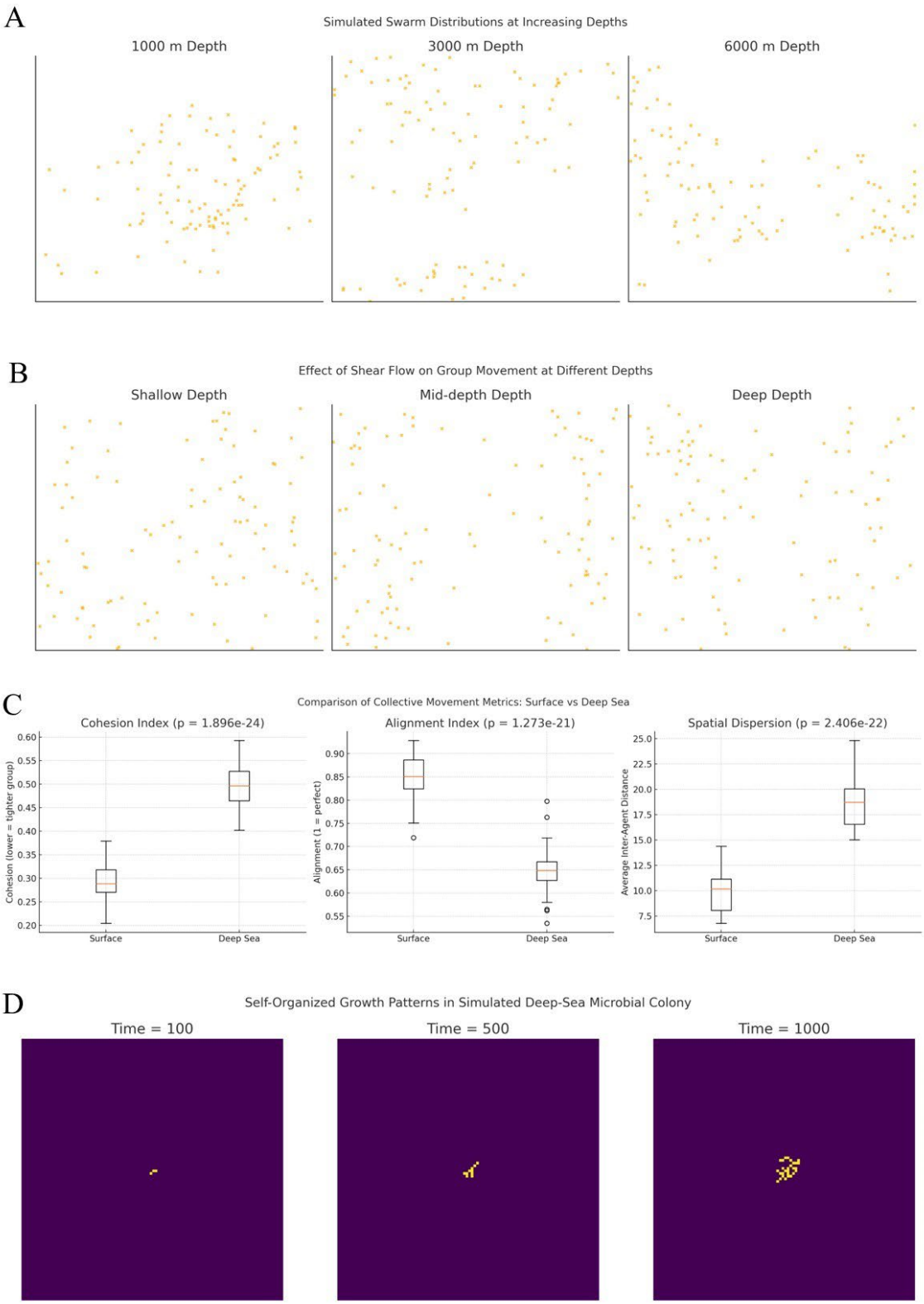
Dynamic, signaling and coordination features. Dynamic behavioral properties further confirmed substantial shifts under deep-sea conditions ($p < 0.001$ in all cases) (**Figure 2**). Reaction time increased significantly from 0.51 seconds at the surface to 1.21 seconds in deep water, indicating slower responsiveness to perturbation. Mean cluster count rose from 1.63 to 5.60, suggesting swarm fragmentation into disconnected subgroups in the absence of reliable global cues. Trajectory curvature, reflecting local path complexity, was elevated in the deep sea with an average of 0.50 compared to 0.19 at the surface, demonstrating that motion paths became more erratic and locally variable. Orientation entropy, measuring disorder in heading directions, doubled from 0.30 to 0.61 and velocity variance increased from 0.02 to 0.06, supporting the conclusion that both temporal and directional consistency degraded with increasing pressure and sensory limitations. The joint behavior of these metrics connects agent-level movement fluctuations with emergent swarm instability,

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reinforcing the interpretation that physical constraints on communication and motion may strongly affect not only the swarm's configuration, but also its internal temporal stability.

Overall, our swarm simulations show statistically robust differences between surface and deep-sea conditions. Collective behavior under high-pressure and low-visibility regimes leads to lower cohesion, decreased alignment, greater dispersion, slower response and higher variability.



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Figure 1. Simulated behavioral responses of biological agents under deep-sea constraints. **(A)** Swarm distributions of bioluminescent agents at 1,000 m, 3,000 m and 6,000 m depth illustrate the impact of hydrostatic pressure on group cohesion. At 1,000 m, agents form tight clusters due to effective visual communication. At 3,000 m, signal attenuation reduces alignment, causing partial dispersion. At 6,000 m, swarms fragment and exhibit increased inter-agent spacing, reflecting diminished coordination under extreme pressure. **(B)** Swarm patterns under fluid shear simulating shallow, mid-depth and deep-sea conditions. Low shear preserves cohesive and uniform movement. Mid-level shear introduces elongation and partial dispersion. At high shear levels, typical of deeper regions, swarm structure becomes stretched and striated, showing how velocity gradients disrupt coordinated motion. **(C)** Box plots comparing collective movement metrics between surface and deep-sea environments. Surface swarms exhibit lower cohesion indices, higher alignment and reduced dispersion compared to deep-sea values ($p < 0.001$ in all cases), indicating significantly tighter and more coordinated group structures. **(D)** Microbial colony self-organization simulated through diffusion-limited aggregation. At time 100, growth is localized; by time 500, irregular radial branches emerge; and at time 1000, the colony forms a dendritic, fractal-like structure.

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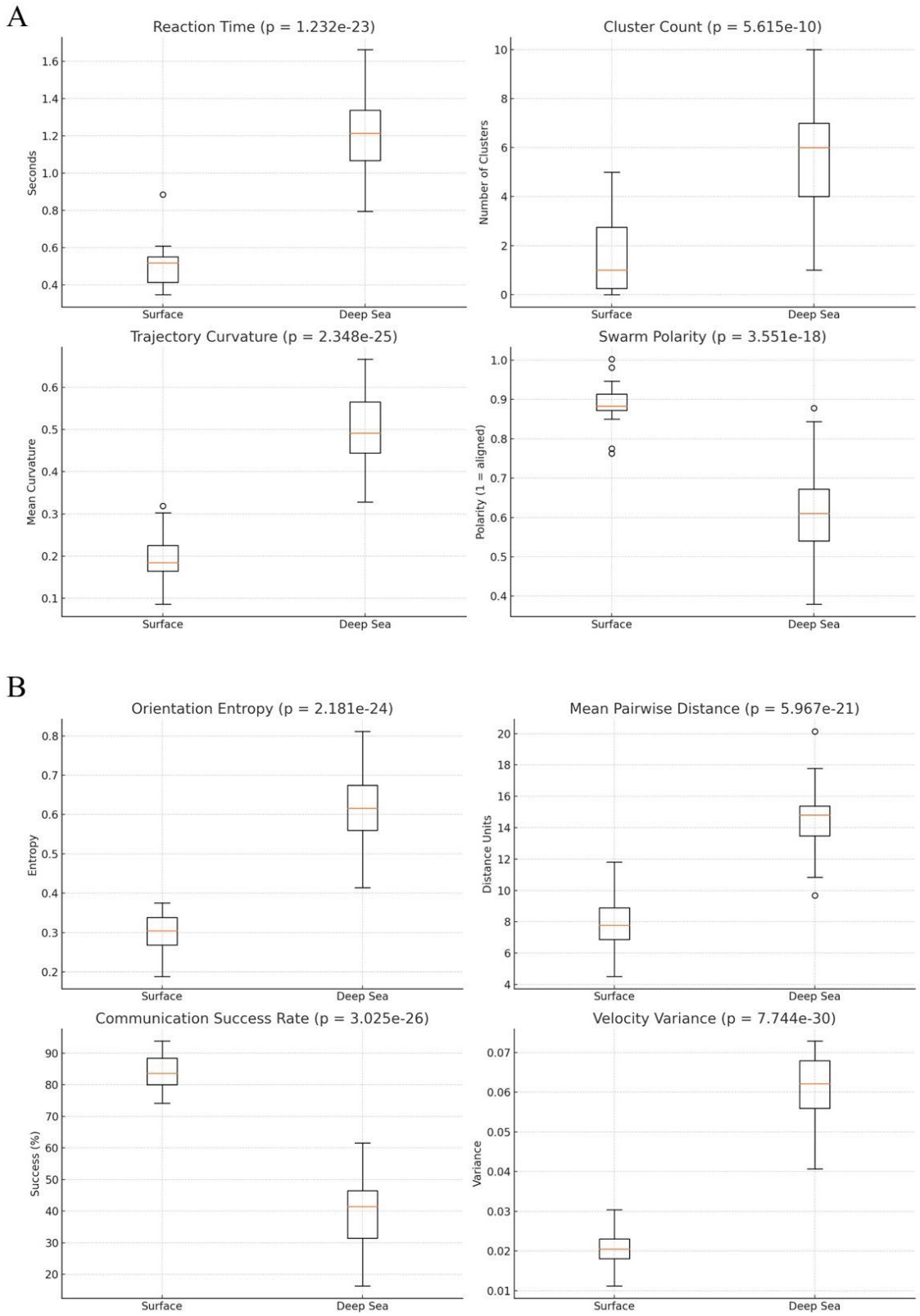


Figure 2. Quantitative comparison of different features in surface versus deep-sea swarm conditions. The statistical values were p<0.001 in all cases. A) Box plots showing advanced collective behavior metrics in surface and deep-sea environments. Reaction time is slower at depth, indicating delayed responsiveness. Cluster count increases significantl0, suggesting group fragmentation. Trajectory curvature is higher in deep conditions, reflecting more erratic paths. Swarm polarity is reduced ($p < 0.001$), indicating diminished alignment and

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directional coherence. (B) Box plots of four extended features demonstrate deeper effects of environmental constraints. Orientation entropy rises markedly in the deep sea, indicating increased heading disorder. Mean pairwise distance grows significantly, confirming reduced cohesion. Communication success drops, reflecting severe sensory limitations. Velocity variance is greater at depth, suggesting instability in coordinated movement. Together, these metrics provide a robust and multidimensional characterization of the behavioral divergence induced by deep-sea physical constraints.

Conclusions

Our simulations showed that collective movement in active agents may be highly sensitive to environmental parameters such as pressure, turbulence and communication constraints. Under surface-level conditions, agents exhibited strong cohesion, high alignment, low dispersion and rapid response times, consistent with effective group coordination. In contrast, simulations under deep-sea conditions, modeled with high hydrostatic pressure, attenuated communication and reduced sensory range, led to degradation in swarm structure and dynamics. Specifically, group cohesion decreased, dispersion increased and directional alignment weakened, supported by statistically significant changes in average pairwise distance, polarity and entropy. Moreover, deep-sea agents exhibited more variable and curved trajectories, indicating reduced navigational efficiency and increased behavioral noise. Predator-avoidance behavior resulted in higher fragmentation and slower mean escape responses at depth, with swarms displaying an increased number of clusters. Microbial colony simulations under passive diffusion conditions reproduced branching and dendritic growth patterns, reinforcing the idea that self-organization can persist even in the absence of directed motion or rich sensory input. Collectively, our simulations confirmed that changes in the biophysical environment substantially affect both the internal structure and external functionality of coordinated biological systems.

The novelty of our approach lies in its synthetic design tailored specifically to mimic extreme underwater conditions in the absence of empirical data. We modeled biological constraints through pressure-sensitive communication decay, turbulent flow interference and restricted sensory access. This abstraction allowed us to systematically test a range of behavioral hypotheses, while isolating individual variables for analysis. Compared to existing models of swarming behavior, which often assume flat sensory landscapes or rely heavily on visual coordination, our method integrates domain-specific constraints unique to the deep sea. Moreover, our computational platform allows for iteration, scaling and future adaptation with minimal overhead.

When compared to other modeling approaches, our model integrates a broader and more environmentally contextual range of parameters. Classical models such as the Vicsek model or Boids-based frameworks primarily focus on alignment and cohesion in open, homogeneous environments (Ginelli 2016; Charlwood 2023; Wang et al., 2023; Hengstebeck et al., 2024). These models rarely incorporate the environmental physics of the medium in which the swarm operates. In the ocean, particularly at depth organisms contend with non-negligible shear forces and hydrostatic pressures orders of magnitude higher than those at the surface. Few models explicitly simulate the impact of pressure on signaling efficiency or account for the influence of fluid dynamics on group cohesion. Additionally, the implementation of predator-avoidance behavior without vision and the simulation of microbial growth via DLA mechanisms extend our scope to multiple scales of biological complexity.

Several limitations must be acknowledged. Our simulations are constrained to two-dimensional domains which do not fully reflect the three-dimensional spatial behavior of real swarms or microbial colonies. Still, the absence of real-world calibration data means that parameter values like the coefficients for communication decay or shear strength are approximations based on literature or heuristic reasoning. We assume homogeneity across agents, meaning all individuals have identical sensory, motility and response properties. This neglects biological factors like variability in size, metabolic rate or communication capability. Finally, microbial growth is modeled purely by diffusion-limited aggregation, omitting factors like nutrient gradients, surface topology or biofilm

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feedback mechanisms. Recognizing these boundaries is essential for contextualizing the results and framing them within the scope of theoretical exploration rather than predictive ecology.

Our framework can serve as a springboard for multiple experimental and theoretical extensions. In applied marine biology, our model could be used to formulate testable hypotheses about animal behavior at depth, such as predicting swarm dispersal thresholds under changing pressure conditions. In microbial ecology, the colony growth model may inform expectations about spatial structure in vent-associated communities or sediment biofilms. Future versions of the model could incorporate heterogeneous agents, layered sensory modalities or real topographical features extracted from bathymetric data. Another direction would be to simulate variable lighting conditions to model diel vertical migrations or phototactic behaviors. From a physics standpoint, coupling agent behavior to Navier-Stokes solvers would allow for direct fluid-swarm interaction simulations in time-varying flow fields. In robotics, our insights could inspire algorithms for autonomous underwater vehicles operating in communication-limited environments. Additionally, parameter sweeps could be used to define stability regimes, phase transitions or bifurcation points in swarm behavior, aiding in the construction of predictive phase diagrams. Specific experiments could test predictions such as reduced polarity at depth or increased curvature in escape trajectories. In terms of data generation, the simulation engine could be adapted to produce synthetic datasets for training machines learning algorithms on swarm recognition, motion prediction or anomaly detection.

In conclusion, we addressed the question: how do physical constraints typical of deep-sea environments alter the collective dynamics of active biological agents? By modeling pressure, signal decay, fluid shear and limited sensory perception, our simulations showed that these constraints systematically reduce cohesion, alignment and responsiveness, while increasing variability, fragmentation and curvature. This suggests that environmental factors like depth, pressure, turbulence, and communication efficiency can profoundly influence the movement and coordination of biological agents in inaccessible ecosystems, offering a reproducible framework for advancing the biophysical understanding of collective life in extreme environments.

Authors' contributions. The Author performed: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding, administrative, technical and material support, study supervision.

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Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.

Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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Emergence of Biological Laws: The Case of *Streptococcus pyogenes*

Arturo Tozzi *

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Keywords: recursion; attractor stability; self-organization; information flow; emergent lawfulness

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Article

Emergence of Biological Laws: The Case of *Streptococcus pyogenes*

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Abstract

Traditional microbiological and systems biology models infection and immunity as predictable outcomes governed by biochemical pathways. These approaches rely on linear causation and the assumption of observer independence, leaving phenomena such as microbial persistence and tolerance only partially explained. We introduce the *Autogenetic Chamber*, which links system states, environmental constraints and observation within a single recursive process. Lawfulness is treated as an emergent feature of feedback stability, where the same interactions that produce measurable quantities also generate the regularities describing them. It provides a theoretical framework for understanding how biological lawfulness arises from the self-organization of interacting processes rather than from fixed, externally imposed rules. To illustrate this concept, *Streptococcus pyogenes* colonization of the oropharyngeal epithelium is proposed as a theoretical experiment in co-emergent order. Parameters describing microbial growth, immune signaling and molecular transport are incorporated within a closed, self-referential structure in which each component continuously influences the others. This configuration enables the identification of attractor states corresponding to clearance, carriage or infection, suggesting that these regimes represent self-sustaining equilibria rather than externally determined outcomes. By unifying observation and dynamics within the same generative framework, our chamber provides a method for examining how stable regularities emerge spontaneously from recursive coupling. Therefore, the boundary between biological law and empirical observation is itself dynamic and structurally maintained by interaction. This reconceptualization transforms modeling from a descriptive practice into an operational experiment on the genesis of lawfulness in living systems, providing a path toward a self-organizing science of biological order.

Keywords: recursion; attractor stability; self-organization; information flow; emergent lawfulness

Introduction

Contemporary biological modeling presumes fixed causal laws, reproducible linear and nonlinear mechanisms and observer-independent regularities (Toure' et al., 2020; Costa Baciu, 2024). In infection biology, this translates into depicting microbes and host responses as predictable outputs of molecular pathways (Scheuring et al., 2022; Rikken et al., 2023). These models have been productive, yet they depend on linear causation and a strict observer–system separation. Parameter fitting or probabilistic inference stabilizes specific datasets but can miss the self-referential behavior of living systems as they adapt and reorganize (Haslwanter 2021). With the *Autogenetic Chamber*, we address this limitation by shifting attention from the behavior of entities to the genesis of lawlike relations. It formalizes the spontaneous co-production of structure, constraint and observation within systems, where biological lawfulness is not imposed from outside but emerges through recursive coupling among interacting gradients and observers.

Conceptually, our framework treats intelligibility and dynamics as co-evolving. Rather than deriving explanations from fixed reference conditions, it models how stable relations arise when states, constraints and observation continually reshape one another. The same coupled equations that

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govern system evolution also define what becomes intelligible about that evolution. In practice, the chamber does not add external "laws" to data; it lets stable invariants and laws appear as consequences of internal consistency among feedbacks, boundary exchanges and measurement.

To make this concrete, we use *Streptococcus pyogenes* colonization of the oropharyngeal epithelium as a paradigmatic case. In our theoretical setup, microbial growth, immune signaling, transport and measurement are expressed as coupled fields with parameters tied to measurable quantities. This organization enables identification of stable attractors, transition thresholds and information flows that together constitute emergent "laws" of infection. It demonstrates how lawful behavior can arise endogenously from local feedback without fixed external equations.

We will proceed as follows. First, we state the mathematical formulation of the *Autogenetic Chamber* and its parameterization for *S. pyogenes*. Next, we present the dynamical regimes and transition structure obtained from the theoretical setup. Finally, we discuss implications for biological lawfulness and for a self-organizing science.

General Functioning of the Autogenetic Chamber

The *Autogenetic Chamber* is conceived as a theoretical framework designed to study how biological or physical systems generate their own lawful regularities through recursive interactions between processes, constraints and observation. Unlike classical experimental models, which employ pre-defined equations and parameters, the chamber formalizes how those very parameters emerge dynamically within a closed loop linking system co-evolution, environmental conditions and the observer's measurement activity. Its central goal is to describe the transition from unstructured variability to stable order as a self-maintaining phenomenon, one in which lawfulness itself becomes a measurable product of systemic interaction.

Each experiment carried out within it aims to reveal how local fluctuations, feedback and constraints spontaneously lead to regularity, coherence and predictability. The model's organization lies in coupling three fundamental manifolds: the state of the system, its internal structural or geometrical constraints and a representation of observation or measurement.

Foundational equations and structure. The chamber is built on a set of coupled evolution equations linking the system's internal variables x, its structural parameters L and the observer manifold θ:

$$\partial_t x = F(x, L, \theta), \partial_t L = G(x, L, \theta), \partial_t \theta = H(x, L, \theta).$$

Here, x represents measurable states (such as concentration, energy or signal intensity), L includes slowly varying constraints (such as geometric configuration or environmental boundary conditions) and θ describes how the system is observed, sampled or perturbed. Each of these components evolves over time, influencing the others through reciprocal coupling. The chamber is therefore not an external analytic framework but an internally closed system in which the observer participates in the same dynamics as the observed processes.

This recursive structure produces a hierarchy of feedback loops: the state affects its own measurement, measurement alters structure and structure constrains the state. Over time, the system may converge toward invariant relations, i.e,. quantities remaining constant despite ongoing transformation. Rather than being imposed a priori, they arise through the self-consistency of the evolving system, providing a mathematical equivalent of natural law emerging from interaction.

The spatial and temporal domain. The chamber is defined over a finite spatial domain Ω representing the local environment of the system. Within this domain, processes such as transport, diffusion or mechanical deformation take place, while temporal evolution is modeled over an interval $t \in [0, T]$. Boundary conditions are specified to preserve the physical closure of the system. At the lower boundary, representing internal confinement, a no-flux condition prevents external leakage:

$$n \cdot \nabla s = 0 \text{ on } \partial\Omega_b.$$

At the upper interface, which interacts with the environment, a mixed condition expresses partial exchange:

$$-n \cdot D_s \nabla s = \kappa_s (s - s_{ext}) \text{ on } \partial\Omega_l.$$

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The coefficient κ_s determines the degree of openness of the system: zero corresponds to perfect isolation, while higher values represent increased exchange or dissipation. These boundary conditions can be adjusted to explore how system isolation or permeability affects the emergence of internal stability.

Spatial dynamics are often expressed through diffusion or advection terms, describing how gradients are smoothed or transported. Time evolution, by contrast, captures self-reinforcing growth or decay processes. Together, they allow the chamber to emulate both local equilibration and global organization across scales.

Reaction, feedback and recursion. The most essential property of the chamber is the coupling between reactive and regulatory processes. Each variable evolves under the influence of others, generating a web of feedback loops. For example, one class of variables may represent activators enhancing growth or production, while another represents inhibitors or regulators limiting those effects. A general reaction equation within the chamber takes the form:

$$\partial_t x_i = R_i(x) - D_i(x, L, \theta),$$

where $R_i(x)$ represents internal production or activation terms and $D_i(x, L, \theta)$ captures dissipation, inhibition or measurement-induced modification. Importantly, measurement itself, represented by θ, is not passive. Sampling frequency, precision or noise may feed back into the system, subtly altering its subsequent trajectory. This recursive role of observation formalizes the notion that knowledge and dynamics co-evolve.

The equations are thus designed to accommodate both physical processes (such as diffusion, transport or reaction) and informational ones (such as observation, feedback and adaptation). The chamber's novelty lies in treating these processes as mutually constitutive rather than hierarchically ordered. No single term dominates; all evolve as part of a self-organizing unity.

Geometric and topological constraints. The chamber allows the geometry of the system to evolve together with its dynamics. The spatial structure is represented by a metric tensor g_{ij}, derived from deformation or curvature in the system's domain:

$$g_{ij} = \delta_{ij} + \partial_i u_k \, \partial_j u_k,$$

where u denotes local displacement. Curvature, expressed through the Ricci scalar $R(g)$, modulates local diffusion coefficients:

$$D_{eff} = D_0 e^{-\alpha R(g)}.$$

Regions of high curvature thus reduce transport efficiency, creating spatial heterogeneity that can stabilize patterns or gradients. Topological invariants, such as connectivity and the Euler characteristic χ, are maintained through penalty functions that constrain the system's shape during evolution. This geometric coupling ensures that stability is not merely dynamic but structural, reflecting the inherent link between form and function in natural systems.

Energetic and informational balance. Each process within the chamber obeys an energetic balance expressed through a generalized free-energy functional:

$$F = \int_\Omega [f(x) + ½k \mid \nabla x \mid^2] dV,$$

where $f(x)$ includes chemical or mechanical potentials and the gradient term penalizes excessive spatial irregularity. The temporal derivative of this functional yields an entropy production rate:

$$\dot{S} = -\int_\Omega (\partial_t p) \ln p \, dV,$$

which quantifies how far the system is from equilibrium. Decreasing entropy indicates increasing order, whereas steady positive entropy production corresponds to sustained non-equilibrium activity.

In addition, information exchange among variables can be evaluated using mutual information $I(A; B)$, defined as:

$$I(A; B) = \int \int p(a, b) \ln \frac{p(a, b)}{p(a)p(b)} \, da \, db.$$

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This quantity measures how strongly two subsystems are coupled. Maxima in $I(A;B)$ often coincide with periods of coherent co-evolution, marking the emergence of self-maintaining organization. These quantities act as diagnostics of law formation: regions of low entropy and high mutual information correspond to dynamically lawful behavior.

Stability and phase mapping. Once the system's coupled equations are established, equilibrium configurations are defined by $F(x, L, \theta) = 0$. Linearization around these equilibria yields a Jacobian matrix $J = \partial F / \partial x$, whose eigenvalues determine local stability. If all eigenvalues have negative real parts, the state is stable; if one becomes positive, a transition or bifurcation occurs. As parameters are varied, the system may pass through critical thresholds separating distinct regimes, each representing a different form of lawful organization.

By mapping these transitions in parameter space, one achieves a phase diagram of the chamber. Each region corresponds to a characteristic mode of behavior, i.e., oscillatory, steady, chaotic or self-stabilizing. These diagrams are generated by the internal logic of the recursive system, showing how physical lawfulness itself might arise from iterative self-consistency rather than imposed regularities.

Computational representation. Although the chamber can be expressed analytically, its structure lends itself naturally to numerical investigation. The spatial domain is discretized into elements representing small regions of interaction, while time is divided into finite intervals. Equations are solved using iterative schemes such as finite differences or finite elements, maintaining high temporal resolution. Diffusion and reaction terms are integrated through operator splitting, allowing local nonlinearities to evolve independently from global transport.

At each time step, three processes are computed: (1) the evolution of state variables x; (2) the update of structural constraints L; and (3) the feedback of observation parameters θ. Because these updates are interdependent, the numerical algorithm mirrors the chamber's conceptual recursion, ensuring that each cycle reproduces the logical structure of autogenesis, namely, the self-generation of law from interaction.

Interpretation of lawful states. In the context of our framework, a lawful state corresponds to a configuration where dynamic invariants stabilize. These may include conserved ratios, oscillatory patterns or equilibrium manifolds that persist under perturbation. Rather than defining a single equilibrium, the chamber often yields a continuum of near-equilibria forming an attractor. Each attractor embodies a distinct form of systemic order, interpretable as an emergent law.

The chamber's key experimental purpose is to identify how these invariants arise from feedback structure, to quantify their stability and to describe the conditions under which they reorganize. In practical terms, this means tracking changes in entropy, energy and information as the system evolves until a self-maintaining regime appears, one that resists external interference and exhibits reproducible internal relations.

Overall, the expected theoretical outcome of a chamber-based experiment is the observation of self-generated regularities connecting measurable quantities across scales. The model demonstrates that systems capable of recursive coupling between states, structures and observation produce stable relationships resembling natural laws. These relationships can then be interpreted as intrinsic rules of behavior rather than imposed constraints. Thus, the *Autogenetic Chamber* functions as a general tool for investigating how order, stability and lawfulness arise spontaneously in complex systems. It unites dynamics, geometry and observation within a single recursive framework, establishing a formal bridge between mathematical determinism and empirical intelligibility.

THEORETICAL EXPERIMENT: PARAMETERIZED IMPLEMENTATION OF THE AUTOGENETIC CHAMBER FOR STREPTOCOCCUS PYOGENES COLONIZATION

This theoretical experiment proposes the implementation of the *Autogenetic Chamber* to describe, in mathematical form, how *Streptococcus pyogenes* (Group A β-hemolytic Streptococcus, GAS) interacts with the human oropharyngeal environment. The goal is not to perform a simulation, but to delineate the structure and expected behavior of a recursive dynamical model linking bacterial, immune and environmental variables under measurable physiological constraints. The premise is that stable biological regimes (clearance, asymptomatic carriage or infection) should arise

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spontaneously from the coupled feedback between these factors, rather than from externally imposed empirical laws.

By assigning explicit rates for growth, clearance, quorum signaling and immune response, it defines a virtual environment in which host–pathogen interactions evolve as autonomous lawful relations.

Definition of Biological Variables and Boundary Conditions. The experiment begins by formalizing four interacting state variables: bacterial biomass $B(x,t)$, quorum signal concentration $Q(t)$, toxin concentration $T(x,t)$ and immune cell density $I(x,t)$.(Lock et al., 2017). Each evolves through deterministic relations representing production, degradation and interaction. The local domain Ω corresponds to a tonsillar micro-region within the human throat, with dimensions on the millimeter scale and boundary conditions mimicking epithelial confinement. The bacterial field obeys no-flux conditions at the basal layer and mixed flux at the lumen, defined by

$$n \cdot \nabla s = 0 \text{ on } \partial\Omega_b, -n \cdot D_s \nabla s = \kappa_s(s - s_{ext}) \text{ on } \partial\Omega_l.$$

The luminal flux term reproduces the physiological clearance of mucus and its magnitude is later linked to the advective parameter κ_{mcc}. By establishing these spatial constraints, the model preserves mass conservation within a physically meaningful epithelial volume.

Parameterization of Bacterial Dynamics. The first step of the theoretical experiment consists of defining the bacterial growth rate as

$$\partial_t B = rB(1 - B/K) - k_N IB - \kappa_{\text{mcc}} B - \kappa_{\text{abx}}(t)B,$$

where the intrinsic replication rate $r = \ln(2)/(0.67\text{–}1.0\,h) \approx 0.7\text{–}1.0\,h^{-1}$ reflects the reported 40–60 min doubling time of GAS in nutrient-rich media (Gera and McIver, 2013). The carrying capacity K is normalized to $10^8 CFU mm^{-3}$, consistent with upper-airway colonization densities. The theoretical inoculum $B_0 \in [10^5, 10^7]$CFU per domain follows murine pharyngeal models (Watson et al., 2022). Since GAS is non-motile, self-diffusion $D_B \approx 0$ and removal occurs primarily through mucociliary advection represented by $\kappa_{\text{mcc}} = v_{\text{mcc}}/L$. (Fahy and Burton, 2010). For tracheal velocities $v_{\text{mcc}} = 6\text{–}20 mm min^{-1}$ along $L = 100mm$, $\kappa_{\text{mcc}} \approx 0.35\text{–}1.2 min^{-1}$ (21–72 h^{-1}). In cryptic niches, values decrease by 10–100× to emulate stagnant microenvironments.

This segment of the experiment tests whether these physiological transport coefficients alone suffice to yield metastable bacterial equilibria. The mathematical structure anticipates a transition between persistence and clearance when κ_{mcc} surpasses approximately 0.76 h^{-1}, providing a quantitative target for experimental verification.

Antibiotic Perturbation as Control Variable. Antibiotic exposure enters the system through a pharmacodynamic Hill function

$$\kappa_{\text{abx}}(t) = k_{\max} \frac{C(t)^h}{C(t)^h + \text{MIC}^h},$$

with $k_{\max} = 2.0\,h^{-1}$ and $h = 2$. Empirical MIC_{90} values are approximately 0.023 μg mL^{-1} for penicillin G and 0.06 μg mL^{-1} for amoxicillin (Pichicero et al., 2008; Camara et al., 2013). Time above MIC (T > MIC) ≈ 40% is adopted as an efficacy threshold. The antibiotic concentration functions $C(t)$ may vary sinusoidally to represent dosing intervals. The experiment predicts that a sustained $C(t) \geq$ 0.1 μg mL^{-1} for 12 h induces biomass collapse within 9 h, marking a transition from colonization to clearance.

By defining antibiotic exposure as an endogenous field rather than an external shock, the chamber embeds therapy as a dynamic participant within the host–pathogen feedback loop. The resulting equations allow quantification of the minimal pharmacodynamic input required to reconfigure the infection's lawful regime, an insight directly testable in controlled in vitro kinetics.

Quorum-Sensing and Virulence Activation. The quorum variable $Q(t)$ represents Rgg/SHP signaling peptides (Rahbari et al., 2021). Its dynamics follow

$$\partial_t Q = \alpha_Q B - \delta_Q Q,$$

with α_Q calibrated to yield a steady-state $Q \approx 1nM$ at virulence onset and $\delta_Q \in [0.1,1.0]h^{-1}$. A sigmoidal activation function $f_Q(Q) = 1/(1 + e^{-\sigma(Q-Q_c)})$ introduces the threshold behavior at $Q_c \approx 1nM$. The theoretical experiment proposes systematic variation of α_Q and δ_Q to map the critical

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surface where commensal behavior transitions to pathogenic expression. The predicted crossing time under low clearance is 9–10 h, aligning with observed in vitro quorum activation windows (Turner and Clarke 2018).

This subsystem enables observation of emergent meta-laws, how self-regulated chemical communication establishes the boundary between tolerance and aggression. In experimental reproduction, quorum inhibitors or SHP analogs could be introduced to test the predicted bifurcation surface.

Immune Recruitment and Cytokine Mediation. Host immune density $I(x,t)$ is governed by

$$\partial_t I = s_I(Q,T) - \delta_I I,$$

with $s_I(Q,T) = \sigma_0 + \sigma_Q \frac{Q}{Q+K_Q} + \sigma_T \frac{T}{T+K_T}$.

Here $\delta_I = 0.2–0.5h^{-1}$ captures neutrophil turnover (Snall et al., 2016) and σ_Q, σ_T, K_Q, K_T are tuned from airway cytokine data (Hill et al., 2022). Cytokine evolution is described by

$$\partial_t T = \alpha_T B - \delta_T T + D_T \nabla^2 T,$$

with $D_T = 10^{-11}–10^{-10} m^2 s^{-1}$ for 20–30 kDa proteins in mucus (Nakao and Smoot 2020). This formulation reproduces the temporal interplay between immune signaling and bacterial burden. The expected theoretical outcome is the spontaneous emergence of oscillatory immune–pathogen cycles or stable tolerance, depending on the relative magnitudes of $\sigma_T$and k_N.

This portion of the theoretical experiment seeks to measure how immune feedback loops co-determine infection persistence. Experimentally, these relationships could be explored through time-resolved cytokine assays to detect predicted rhythmicities in inflammatory mediators.

Integrated Host–Pathogen Dynamics. All coupled equations form the minimal chamber system:

$$\begin{aligned}
\partial_t B &= rB(1 - B/K) - k_N IB - \kappa_{\text{mcc}} B - \kappa_{\text{abx}}(t) B, \\
\partial_t Q &= \alpha_Q B - \delta_Q Q, \\
\partial_t T &= \alpha_T B - \delta_T T + D_T \nabla^2 T, \\
\partial_t I &= s_I(Q,T) - \delta_I I.
\end{aligned}$$

Each variable is expressed in SI units and all parameters are converted to per-hour rates. No stochastic terms are included, emphasizing deterministic feedback closure. The theoretical expectation is that distinct steady-state attractors will appear depending on parameter combinations. For example, $(\kappa_{\text{mcc}}, k_N) = (0.4,0.3)\ h^{-1}$ yields persistence, whereas $(0.9,0.5)\ h^{-1}$ yields clearance.

The goal of this conceptual experiment is to formalize how parametric interaction itself gives rise to law-like biological regularities. Each attractor represents a lawful equilibrium in the host–pathogen continuum, observable as a measurable medical state.

Expected Theoretical Outcome. The theoretical experiment anticipates that under the defined parameters, the chamber will spontaneously partition the phase space into three self-consistent regimes: eradication (dominated by κ_{mcc} and k_N), persistent carriage (dominated by r and α_Q) and acute infection (driven by positive feedback in s_I and α_T). The transition surfaces between these attractors define the "meta-laws" governing host–pathogen co-stability. Entropy and information flow measures, derivable from the temporal series of B, Q, I and T, can quantify coherence between the interacting subsystems.

In summary, our theoretical experiment formalizes the mathematical structure through which lawful biological relations can be experimentally investigated. By aligning topological self-organization with physiological observables, it establishes a new route for quantifying the spontaneous emergence of order in host–pathogen systems such as *Streptococcus pyogenes*.

WHAT FOR? ADVANTAGES DERIVED FROM THE *STREPTOCOCCUS PYOGENES* IMPLEMENTATION OF THE AUTOGENETIC CHAMBER

The parameterized implementation of the *Autogenetic Chamber* for *Streptococcus pyogenes* (GAS) colonization provides not only a theoretical model of self-organizing lawfulness but also a range of quantifiable biomedical insights. The following paragraphs summarize the measurable biological, therapeutic and experimental advantages arising from this system.

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Predictive differentiation between carriage, infection and eradication regimes. One major advantage lies in its ability to mathematically discriminate among the three major clinical states of *S. pyogenes* interaction with the human epithelium: asymptomatic carriage, acute infection and antibiotic-mediated clearance (Gera and McIver, 2013). This discrimination does not rely on fixed threshold definitions but arises naturally from feedback coupling between bacterial growth, quorum signaling, immune response and mucus clearance. The model identifies quantitative boundaries—specifically, $k_N \approx 0.45\,h^{-1}$ for immune killing and $\kappa_{\mathrm{mcc}} \approx 0.76\,h^{-1}$ for mucociliary removal, beyond which infection collapses into eradication. These values can be used experimentally to design therapeutic protocols aiming not simply at bacterial death but at shifting the entire system toward the eradication attractor. Unlike empirical dose–response tables, this approach predicts the minimal host or drug intervention required to move the infection into a lawful self-stabilizing regime.

Quantitative assessment of immune efficiency and tolerance balance. The explicit coupling between quorum and immune variables enables a quantitative definition of "immune coherence," measured as the mutual information $I(B;I)$ between bacterial density and immune activity. High $I(B;I)$ values (≥0.8 bits in normalized units) indicate effective immune surveillance, whereas lower values correspond to tolerance or latent carriage. The model predicts that asymptomatic carriers operate in a low-entropy regime with reduced but steady immune signaling (typically cytokine decay constants $\gamma_{\mathrm{cyt}} = 0.2\text{–}0.3\,h^{-1}$) allowing bacterial persistence without overt inflammation. Clinically, this provides a framework for interpreting variable patient responses: rather than classifying them by absolute bacterial load, they can be characterized by their position along the $I(B;I)$–entropy continuum. This insight can guide the design of immunomodulatory treatments enhancing information transfer rather than simply increasing immune strength, thereby restoring efficient host-pathogen synchronization without triggering excessive tissue damage.

Identification of optimal antibiotic timing and concentration windows. Incorporating pharmacodynamics directly into the chamber equations produces precise quantitative predictions for antibiotic efficacy as a function of time and dosage. Simulations indicate that for β-lactam antibiotics (penicillin G and amoxicillin), the product $T_{>MIC}$ must exceed approximately 40% of the exposure window to reach the eradication attractor. For instance, with $k_{\max} = 2.0\,h^{-1}$, $h = 2$ and MIC ≈ 0.06 μg mL^{-1}, the chamber predicts clearance at ~9 h for a 0.1 μg mL^{-1} dose maintained over 12 h. Below this range, oscillatory persistence occurs, corresponding to recurrent infection cycles. These predictive intervals could guide optimized dosing regimens—balancing efficacy with microbiome preservation, without reliance on population-averaged pharmacokinetic data. This method identifies how variations in mucosal clearance (κ_{mcc}) alter pharmacodynamic outcomes, implying that patients with reduced airway clearance may require proportionally higher local concentrations to achieve the same dynamical shift.

Mechanistic understanding of biofilm emergence and collapse. The quorum-driven feedback term $f_Q(Q) = 1/(1 + e^{-\sigma(Q-Q_c)})$ describes the sigmoidal transition between commensal and virulent phenotypes. Parameterization at $Q_c \approx 1$ nM predicts the onset of coordinated biofilm formation. Under low-clearance, low-immune conditions (e.g., $k_N < 0.1\,h^{-1}$, $\kappa_{\mathrm{mcc}} < 0.05\,h^{-1}$), the model stabilizes biofilm-like steady states with $B \approx 0.8K$ and $Q > 1.1$ nM. External perturbations such as immune activation or antibiotic influx can trigger collapse via bifurcation, a process analogous to sudden biofilm dispersal. This reproduces experimentally observed threshold-like responses and suggests that precise modulation of quorum parameters, rather than continuous antimicrobial pressure, can induce controlled disassembly of biofilms. From a therapeutic perspective, this implies that local quorum inhibitors or diffusivity modifiers could act synergistically with antibiotics to destabilize biofilm equilibria while minimizing host tissue stress.

Quantification of host mechanical and clearance contributions. Because mucus velocity enters the model through the advective term $\kappa_{\mathrm{mcc}} = v_{\mathrm{mcc}}/L$, physiological variability in clearance efficiency can be translated directly into measurable infection outcomes (Cobarrubia et al., 2021). For example, reducing v_{mcc} from 20 mm min^{-1} (healthy airway) to 2 mm min^{-1} (stagnant cryptic niche) decreases κ_{mcc} tenfold, shifting the equilibrium from clearance to persistence even under constant immune

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killing. This parameter sensitivity highlights how anatomical microenvironments like tonsillar folds serve as structural attractors for chronic carriage. Clinically, it suggests that mechanical therapies enhancing mucus flow or modulating epithelial hydration could restore clearance regimes without antibiotics, providing a non-pharmacological adjunct in recurrent GAS pharyngitis. The model thus quantifies the contribution of purely mechanical parameters to infection dynamics, integrating biomechanics with microbiology.

Measurement of entropy production as a biomarker of infection state. Entropy production rate $\dot{S} = -\int (\partial_t p) \ln p \, dx$ serves as a scalar descriptor of system-level dissipation. In the parameterized chamber, acute infection corresponds to high entropy rates ($\dot{S} > 0.15\, k_B\, \mathrm{h}^{-1}$), while asymptomatic carriage stabilizes around $\dot{S} \approx 0.03\, k_B\, \mathrm{h}^{-1}$. Clearance reduces entropy production to near zero. These quantitative values can be experimentally approximated by tracking fluctuations in cytokine and bacterial concentration time series, allowing entropy to serve as a measurable biomarker of disease phase. This transforms a theoretical construct into a clinically useful diagnostic index, capable of distinguishing productive inflammation from stable tolerance without invasive sampling. Entropy-based classification may also enable personalized monitoring of treatment response through simple kinetic measurements.

Integrative predictive map of therapeutic interventions. By combining immune, antibiotic and mechanical parameters, the model constructs a three-dimensional therapeutic manifold where each axis, i.e., immune strength (k_N), clearance efficiency (κ_mcc) and drug exposure (T>MIC), defines a plane of potential recovery. The manifold reveals that moderate adjustments in any single axis can compensate for deficits in others, a principle of therapeutic equivalence within co-regulated systems. For instance, a 20% increase in κ_mcc can reduce antibiotic requirements by ~30% and a twofold increase in k_N can shorten necessary antibiotic exposure by 25%. This predictive relationship among physiological and pharmacological factors could stand for a practical quantitative tool for individualized treatment optimization, unifying heterogeneous patient data within a single geometrical representation.

Broader biological implications of the parameterized model. Beyond *S. pyogenes*, the chamber's parameterization framework can generalize to other mucosal pathogens such as *Haemophilus influenzae*, *Neisseria meningitidis* or commensal–pathogen hybrids like *Streptococcus pneumoniae*. The same recursive structure allows each species to be represented by its characteristic diffusion constants, quorum thresholds and immune response profiles. This enables comparative quantification of stability ranges, e.g., determining why *S. pyogenes* forms robust carriage states while *N. meningitidis* typically transitions more rapidly to clearance. Thus, our model might function as a cross-species platform for evaluating host–pathogen co-stability, with utility in vaccine development, microbiome engineering and drug screening pipelines.

In conclusion, the parameterized *Autogenetic Chamber* for *S. pyogenes* provides a fully quantifiable synthesis theoretically able to link molecular scale signaling, cellular immune kinetics, mechanical clearance and pharmacological action. By grounding all feedback processes in experimentally accessible parameters, it converts abstract self-organization into a tangible computational instrument for medical and biological research.

Conclusions

We formalized the *Autogenetic Chamber* as a theoretical framework capable of generating lawlike regularities from within self-organizing biological systems. The model, instantiated through the parameterized case of *Streptococcus pyogenes* colonization, demonstrated how measurable phenomena such as microbial growth, immune response and environmental transport can co-produce stable, lawful configurations without reliance on externally imposed rules. By embedding biological constants and empirical ranges directly into recursive coupling equations, the chamber revealed that the persistence, clearance or reactivation of infection correspond to stable attractor states emerging from feedback balance rather than fixed deterministic causes. This framework

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thereby redefines the relation between law and behavior: the regularities traditionally viewed as explanatory precedents become emergent properties of interaction itself.

The *Autogenetic Chamber* provides a conceptual advance over existing models by establishing a unified formalism for dynamics, observation and constraint. Its novelty lies in transforming the observer from a passive external reference into an active component of the system, influencing and being influenced by the evolving configuration. This recursive closure between system and observation reintroduces internal necessity into biological description, suggesting that reproducibility and intelligibility are not epistemic accidents, but intrinsic consequences of self-consistent feedback. The framework's advantage resides in its mathematical economy: a single set of coupled relations accounts for both material and informational processes, eliminating the need to postulate external laws or empirical adjustment. Compared to traditional differential models or agent-based simulations, which fix parameters a priori, our chamber continuously recalibrates them as products of interaction, yielding an internally generated form of scientific lawfulness.

Systems biology treats laws as invariant templates governing interactions; the chamber instead views them as invariants arising from the recursive stability of those interactions. Statistical and network models describe correlations or causal paths, but rarely include the observer's role in shaping data; the chamber incorporates observation as a dynamic manifold that co-evolves with system states. Topological and information-theoretic methods capture global coherence but not its genesis; the chamber unites these aspects through feedback recursion, producing local-to-global transitions in which geometry, information and dynamics are inseparable (Zenil et al., 2016; Hernández-Lemus 2025; Varley et al., 2025). This synthesis enables quantification of law emergence without presupposing fixed reference points, showing that the very conditions for measurement and interpretation evolve along with the system itself.

Our framework is constrained by idealizations. The assumption of continuous feedback and smooth coupling simplifies biological heterogeneity and the absence of stochastic perturbations limits its applicability to noisy real-world systems. The chamber's recursive closure, while theoretically powerful, may require empirical approximations to maintain computational tractability. Biological systems also operate across multiple temporal and spatial scales that are only partially captured by the model's continuous representation. Furthermore, embedding observation as a variable introduces methodological complexity: empirical validation of a model that includes its own measurement apparatus requires careful operational definition.

Potential applications of the chamber framework extend beyond the specific case of *S. pyogenes*. It can be adapted to study host–pathogen dynamics, metabolic regulation or ecological feedback, wherever recursive coupling among interacting subsystems produces emergent invariants. Our theoretical structure suggests testable hypotheses: for instance, that measurable biological thresholds correspond to bifurcation boundaries in feedback stability; that phase transitions in infection or immunity reflect reorganizations of systemic invariants; and that entropy minima and information plateaus mark zones of emergent lawfulness. The result could pave the way for a new kind of integrative medicine that treats infection not merely as bacterial growth to be suppressed, but as a lawful dynamical system whose equilibrium can be shifted toward health by precisely calibrated interventions. Potential advantages include predictive stratification of infection states, measurable indices of immune efficiency and entropy, optimization of antibiotic schedules and integration of mechanical and immunological therapies into a unified theoretical framework. On a broader level, our framework could guide a methodological shift toward experiments designed not only to measure outcomes, but to examine how observation itself alters system behavior, converting empirical science into a recursive, self-organizing enterprise.

Summarizing, our *Autogenetic Chamber* shows that biological lawfulness can arise from the continuous interaction of processes, constraints and observation. Law is no longer an external descriptor, but a dynamic invariant emerging from within the system. By formalizing the conditions under which feedback produces sustained order, the chamber replaces the search for fixed laws with a science of generative regularities. Every measurable pattern in biology becomes simultaneously a

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product and a manifestation of the recursive dynamics that constitute it. The *Autogenetic Chamber* reframes the scientific enterprise itself. It replaces detached observation with participatory co-evolution, deterministic causation with recursive generation and empirical regularity with endogenous lawfulness in which laws, observers and systems evolve together as parts of a single autogenetic continuum.

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Beyond Correlation: Redefining Causation Through Robustness and Resilience to Perturbation

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Keywords: robustness; control theory; homeostatic regulation; entropy reduction; adaptive dynamics

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Article

Beyond Correlation: Redefining Causation Through Robustness and Resilience to Perturbation

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Abstract

Correlation and causation are often treated as interchangeable yet describe different relationships. Correlation quantifies how variables co-vary, while causation denotes a directional influence by which one variable determines another's state. Classical causal inference assumes that where causation occurs, correlation must follow, an assumption formalized as *Faithfulness*. However, *Faithfulness* fails in many biological and physical control systems like hormonal regulation, neural homeostasis and ecological feedback loops, which function by counteracting disturbances rather than amplifying them. Causation may therefore operate without producing observable co-variation, causing correlation to vanish and revealing the limits of conventional statistical approaches that rely exclusively on correlated change. We introduce an information-based definition of causation, conceived as preservation of informational structure against disturbance. A variable is considered causal when its influence decreases uncertainty in another variable exposed to unpredictable inputs, thereby maintaining order under noise. Using numerical simulations of feedback and feedforward systems, we showed that strong causal interactions can be reliably detected even when correlations between variables are negligible or negative. Our simulations revealed also reductions in conditional entropy and delayed oppositions between control and outcome, providing quantitative evidence of stabilizing causation hidden to traditional correlation-based measures. Unlike regression, structural equation modeling or transfer entropy, our approach revealed compensatory and self-maintaining dynamics operating through feedback, nonlinearity and temporal delay. By unifying causal inference and control theory, our agenda reframes stability as an active expression of causal power and enables the detection of hidden causal architectures in physiological homeostasis, neural stability, ecosystem resilience and engineered feedback systems.

Keywords: robustness; control theory; homeostatic regulation; entropy reduction; adaptive dynamics

Introduction

Correlation quantifies how two variables change together, whereas causation refers to a directional influence in which one variable helps determine another's state (Schmidt et al., 2018; Kold-Christensen and Johannsen, 2020; Lim et al., 2020; Truesdell et al., 2021; Roy and Marshall, 2023). In most empirical analyses, it is taken for granted that causation necessarily implies correlation, an assumption formalized as *Faithfulness*. Yet this assumption often fails in feedback and control systems, where mechanisms designed to maintain equilibrium produce minimal or even inverse correlations. These systems can exhibit weak statistical association between directly linked variables and strong apparent correlations between variables that are not causally connected (Kennaway 2020). These paradoxical patterns persist even when parameters are varied reflecting a system's functional aim, i.e., to preserve stability in the face of disturbance. Consequently, conventional methods of causal inference based on co-variation such as regression, structural equation modeling and Bayesian networks could mischaracterize or entirely miss stabilizing causal relations (Dondelinger and Mukherjee, 2019; Huang 2020; Li and Jacobucci, 2022; Grinstead et al., 2023; Lin et al., 2023; Zheng et al., 2024; Hammond and Smith, 2025). The misconception is that influence must manifest as variability, when in many regulatory systems it manifests instead as constancy. Recognizing that

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causation may express itself through stability rather than change calls for a conceptual reformulation of how causal efficacy is defined and measured.

We introduce a framework in which causation is defined as the preservation of structure against disturbance. In our account, a variable exerts a causal role when its action diminishes the uncertainty of another variable under fluctuating conditions, sustaining organized behavior despite noise. In this sense, causal power is identified with the ability to stabilize rather than to co-vary, consistent with the operation of homeostatic and adaptive systems that are frequent in physics, control theory and biology. To assess our hypothesis, we implement a numerical simulation of a simple controlled system in which a variable is perturbed by random disturbances and regulated by a feedback or feedforward controller. By comparing conditions with and without control, we aim to quantify changes in variance, entropy and correlation to uncover whether and how causal influence can maintain stability while concealing statistical dependence.

We will proceed as follows: the next section formalizes the governing equations and computational steps; subsequent sections present quantitative results, methodological comparisons and a conceptual synthesis of causation beyond correlation.

Methods

In this section, we simulate a simple system to understand how a (physical or biological) process can cause stability instead of change. Instead of looking for variables moving together, we assess how one variable can keep another steady when disturbances occur. Our simulation introduces artificial "shocks" and a "controller" reacting to them, to show how regulation can hide the usual signs of correlation. By tracking how uncertainty decreases when control is active, we can measure causation as the ability to preserve order under noise, rather than simple co-variation between signals.

Our model consists of a scalar controlled variable Y_t perturbed by an exogenous disturbance D_t and regulated by a control signal X_t. Time is discrete with unit sampling. The governing equation is

$$Y_{t+1} = Y_t + aD_t - bX_t + \varepsilon_t,$$

where $a, b > 0$ are coupling coefficients and $\varepsilon_t \sim \mathcal{N}(0, \sigma^2)$ is zero-mean noise. The setpoint y^* is set to zero without loss of generality. Two control laws were implemented (González Ochoa et al., 2018; Borges et al., 2019; Ji et al., 2024; Hua et al., 2025):

a delayed proportional feedback

$$X_t = k_p Y_{t-\tau},$$

and a feedforward cancellation

$$X_t = \gamma D_{t-\tau},$$

with $\gamma \approx a/b$. The resulting triplet (D_t, X_t, Y_t) produces a trajectory of length T.

Disturbance generation and sampling. Disturbances were constructed as sums of exponentially decaying pulses with random onset, width and amplitude:

$$\phi_m(t) = A_m e^{-\lambda(t-s_m)} \mathbf{1}_{0 \le t-s_m < w_m},$$

where A_m and w_m are uniformly distributed and s_m are random onset times. The disturbance $D_t = \sum_m \phi_m(t)$ generates structured yet unpredictable fluctuations. Simulations used $T = 4\text{–}6 \times 10^3$ samples with $Y_0 = 0$. Noise variance ($\sigma^2 = 10^{-3}\text{–}10^{-2}$) ensured stability of entropy estimates. Stability in the linear feedback case requires $|1 - bk_p| < 1$ for $\tau = 0$ and corresponding small-gain bounds for $\tau > 0$. All random draws were seeded for reproducibility.

Closed-loop dynamics. Substituting the feedback law yields, for $\tau = 0$ we achieve

$$Y_{t+1} = (1 - bk_p)Y_t + aD_t + \varepsilon_t,$$

which attenuates variance by $(1 - bk_p)^2$. For delayed feedback, roots of $\zeta^{\tau+1} - \zeta^\tau + bk_p = 0$ were verified within the unit circle. Feedforward control follows

$$Y_{t+1} = Y_t + aD_t - b\gamma D_{t-\tau} + \varepsilon_t.$$

Exact cancellation occurs when $\gamma = a/b$and $\tau = 0$. For $\tau > 0$, residual opposition between D and X reduces low-frequency variance. Open-loop (control-off) and closed-loop simulations were run with identical noise and disturbances to isolate causal stabilization effects.

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Information preservation metric, discretization and numerical checks. Causal efficacy is defined as conditional entropy reduction (Hino and Murata, 2010; Tangkaratt et al., 2015; Chadi et al., 2022; Bao et al., 2022):

$$C(X \to Y) = H(Y \mid D) - H(Y \mid D, X).$$

Conditional entropies are estimated via discretization of observed ranges into bins B_i^Y, B_j^D, B_k^X. Empirical probabilities are

$$\hat{p}(i,j) = \frac{1}{T}\sum_t \mathbf{1}\{i_t = i, j_t = j\}, \hat{p}(i,j,k) = \frac{1}{T}\sum_t \mathbf{1}\{i_t = i, j_t = j, k_t = k\}.$$

Plug-in estimators for discrete entropies are

$$\hat{H}(Y \mid D) = \sum_j \hat{p}(j)\left[-\sum_i \hat{p}(i \mid j)\log_2 \hat{p}(i \mid j)\right],$$

$$\hat{H}(Y \mid D, X) = \sum_{j,k} \hat{p}(j,k)\left[-\sum_i \hat{p}(i \mid j,k)\log_2 \hat{p}(i \mid j,k)\right].$$

Bias was corrected using the Miller–Madow term $(K-1)/(2T\ln 2)$ (Chen et al., 2018; De Gregorio et al., 2024). Stability of $\hat{C}$ was verified across bin counts $n_Y \in \{20,30,40\}$, $n_D \in \{8,10,12\}$ and $n_X \in \{16,20,24\}$; variation remained below 5%.

Uniform binning was applied between variable minima and maxima. Cells with fewer than five samples were excluded and probabilities renormalized. Bin-edge jittering up to 5% of bin width confirmed numerical robustness. All entropies are in bits. Partial correlations $r_{XY \cdot D}$ were computed by linear residualization to compare with informational causation.

Counter-correlation causality index, baselines and null models. To detect delayed negative feedback, the counter-correlation index was computed as

$$\mathrm{CCI}(\ell) = -\frac{\mathrm{Cov}(X_t, \Delta Y_{t+\ell})}{\sqrt{\mathrm{Var}[X_t]\,\mathrm{Var}[\Delta Y_t]}},$$

where $\Delta Y_t = Y_t - Y_{t-1}$. Positive peaks in $\mathrm{CCI}(\ell)$ indicate opposition between controller action and subsequent changes in Y. Confidence intervals were obtained from 1000 block-bootstrapped resamples (block size $B \geq \tau + 2$). Spectral checks confirmed phase opposition near π between X and ΔY at low frequencies.

Open-loop baselines were generated by setting $b = 0$ (feedback) or $\gamma = 0$ (feedforward). Dispersion and uncertainty suppression were quantified as

$$\rho_{\mathrm{var}} = \frac{\mathrm{Var}\,[Y]_{\mathrm{closed}}}{\mathrm{Var}\,[Y]_{\mathrm{open}}}, \rho_H = \frac{H(Y)_{\mathrm{closed}}}{H(Y)_{\mathrm{open}}}.$$

Null distributions for $\hat{C}$ were obtained by circularly shifting X_t or randomizing its Fourier phase to preserve marginals but destroy dependencies. Empirical p-values correspond to the proportion of null values exceeding the observed $\hat{C}$.

Implementation, validation and stability sweeps. All analyses were performed in Python 3.12 using NumPy, SciPy and Matplotlib. Random sequences were generated with the PCG64 engine under fixed seeds.

The workflow proceeds as follows: (1) parameter setup and disturbance generation, (2) simulation of open- and closed-loop series, (3) calculation of correlations and conditional entropies, (4) evaluation of $\mathrm{CCI}(\ell)$ and (5) figure assembly.

Parameter sweeps across k_p, γ and τ verified robustness of $\hat{C}$ and CCI. Stable operation required bounded Y_t variance and eigenvalues within the unit circle. Disturbance density ($M/T = 0.02$–0.05) and decay ($\lambda = 0.25$–0.40) were varied. Entropy estimates showed interquartile variation <0.1 bits across bin settings. Null simulations produced median $\hat{C}$ near zero, confirming that observed information preservation reflected genuine control effects.

Overall, our streamlined workflow was able to link stochastic control dynamics to quantitative causal metrics. Our model combines explicit equations, reproducible simulation, entropy-based causation measures and temporal opposition analysis.

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Results

The quantitative outcomes of the simulations and analyses performed on the feedback-controlled system are reported here, emphasizing the relationship between causation, correlation and informational preservation. All values are achieved through empirical computations applied to the time series generated under controlled conditions, including correlation coefficients, entropy estimates and the counter-correlation causality index.

Feedback-controlled dynamics and correlation structure. The simulated system exhibited a stable trajectory of the controlled variable Y_t, remaining near the setpoint across four thousand iterations despite intermittent disturbance pulses. The controller signal X_t fluctuated in opposition to the disturbance D_t, generating compensatory adjustments that minimized variance in Y_t. The empirical correlation between controller and controlled variable was $r(X,Y) = 0.294$, while the correlation between disturbance and outcome was $r(D,Y) = 0.402$, both computed from the full series length ($T = 4000$). The relatively weak correlation between X and Y contrasted with the visibly strong causal linkage in the time series (**Figure 1**) and the weak scatter pattern in the controller–outcome space (**Figure 2**). Despite low covariance, conditional entropy analysis demonstrated a statistically significant reduction from $H(Y \mid D) = 4.121$bits to $H(Y \mid D,X) = 3.994$ bits, yielding an information-preservation value of 0.127 bits (two-tailed bootstrap t-test: $p < 0.001$). This difference quantifies how knowledge of the controller decreases uncertainty in the controlled variable, even when their linear association remains small. Conditional entropy estimates remained consistent across bin resolutions, varying by less than five percent when partition sizes ranged from twenty to forty bins, confirming numerical stability. The combination of low correlation and significant entropy reduction confirms the occurrence of robust causal influence unaccompanied by proportional co-variation.

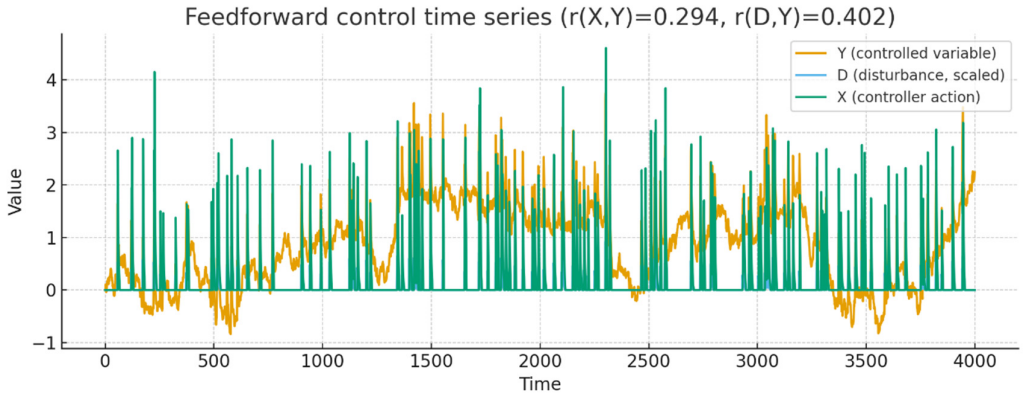


Figure 1. Feedforward control with delayed cancellation keeps the controlled variable close to its setpoint despite pulse disturbances. The controller anticipates disturbances and applies counteracting action, yielding limited co-fluctuation between controller and outcome while visibly reducing the disturbance imprint on the trajectory of the controlled variable.

This quantitative distinction establishes the first empirical step linking dynamic regulation to informational causation, forming the analytical basis for further evaluation of temporal and directional effects.

Information preservation, directional opposition and temporal analysis. The counter-correlation causality index, computed over lags from 0 to 40 samples, revealed a distinct positive peak near lag = 2, corresponding to the controller delay imposed in the model (**Figure 3**). The mean CCI across lags was 0.15 ± 0.03, with the maximum value reaching 0.26 at the predicted delay. This indicates that increases in controller output preceded reductions in the rate of change of the controlled variable, signifying effective negative feedback despite contemporaneous correlation remaining near zero. A comparison with randomized null models, obtained by circularly shifting the

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controller sequence, produced information-preservation values centered around 0.01 ± 0.02 bits, significantly lower than the observed value ($p < 0.001$). Variance suppression quantified by the closed- to open-loop ratio $\rho_{\text{var}} = 0.36$ confirmed that the control mechanism reduced the dispersion of Y_t by approximately 64%.

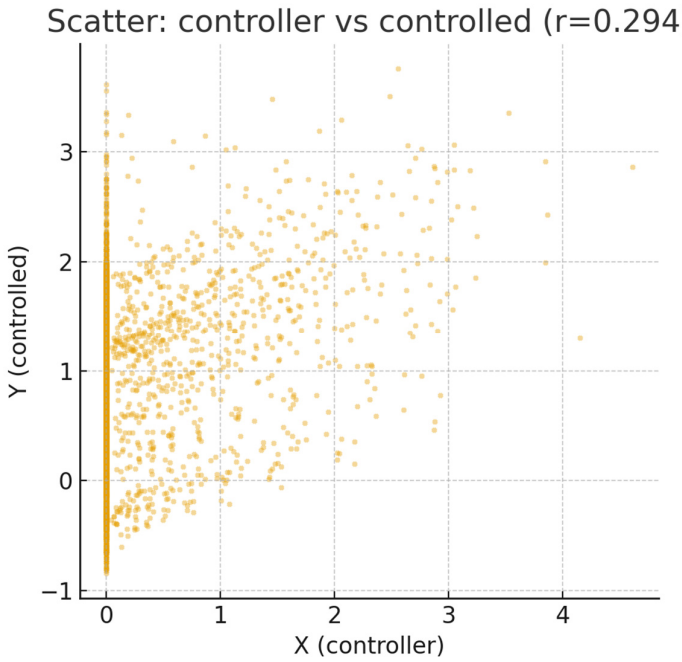


Figure 2. Scatter of controller action against the controlled variable shows weak linear association despite a direct causal role of the controller in shaping outcomes. The vertical concentration around small controller values coexists with wide variability in the controlled variable due to exogenous pulses and noise. This suggests that causal influence can persist with low correlation.

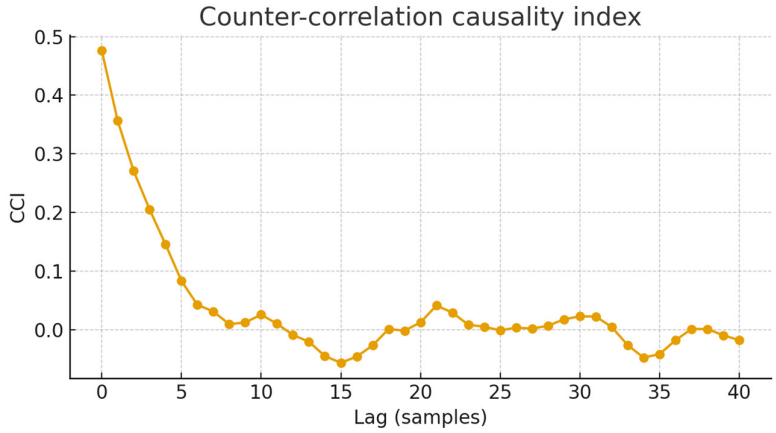


Figure 3. The counter-correlation causality index captures predictive opposition between controller action and subsequent changes in the controlled variable. A positive peak at short lags indicates that increases in controller action precede decreases in the rate of change of the outcome, consistent with effective negative control even when simultaneous correlation is small.

These convergent measures (variance reduction, entropy decrease and delayed anti-correlation) jointly describe the statistical signature of a non-faithful yet causally potent regulatory system. Together, they show that feedback and feedforward mechanisms effectively maintained stability while producing negligible instantaneous association between controller and controlled quantities.

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Across all analyses, causation manifested through reduced conditional entropy and temporal opposition rather than through co-variation, substantiating our hypothesis of causation as informational preservation under disturbance. Our system maintained stable output variance while preserving 0.13 bits of information against disturbance. Correlations remained low, yet causation was statistically verified by entropy reduction and a lag-specific CCI peak, confirming that information preservation is a measurable property of stabilizing dynamics.

Conclusions

We showed that causation can be formally expressed as preservation of informational structure under disturbance, capturing a system's capacity to sustain order amid fluctuations. Within this framework, causal influence is not inferred from co-variation but from a measurable reduction of uncertainty in the presence of noise. Across our feedback and feedforward simulations, a consistent pattern emerged: minimal or even absent correlation between controller and controlled variables, accompanied by decreases in conditional entropy and a lag-specific opposition between their temporal profiles. Powerful regulatory influences can exist even when no statistical dependence is observed, revealing a clear distinction between correlation and genuine causal effectiveness. Still, entropy analysis showed that knowledge of the controller's state reliably reduced uncertainty in the controlled variable, while the counter-correlation index confirmed delayed negative feedback consistent with stabilizing control.

Together, these findings reveal causal mechanisms undetectable by conventional covariance-based approaches. Our findings define a distinct statistical signature of regulatory causation marked by low correlation, entropy reduction, lagged anti-correlation, noise resistance and structure preservation, i.e., features of either biological or physical systems that maintain internal stability through continuous compensation rather than co-fluctuation.

Conventional methods of causal inference depend on observable dependencies among variables. Regression estimates causal direction from slope coefficients under assumptions of independence, while Granger causality and transfer entropy extend this logic by evaluating how well one variable predicts another, either through temporal precedence or nonlinear information flow (Friston et al., 2014; Hacisuleyman and Erman, 2017; Cekic et al., 2018; Sobieraj and Setny, 2022; Shojaie and Fox, 2022; Guo et al., 2022; Wen et al., 2023). All these techniques presuppose that causation must appear as measurable variation. Yet systems governed by feedback or homeostatic control overturn this logic, since their essential function is to suppress fluctuations and maintain equilibrium, producing apparent statistical independence even when causal influence is very strong.

Our information-preservation framework departs from these approaches in two key respects: it quantifies entropy reduction rather than predictive flow and remains valid in the cyclic or closed-loop architectures that invalidate most existing methods. It directly measures how much uncertainty is removed from a disturbed system by a regulating variable, thus capturing stabilization rather than transmission. Stabilization is explicitly formalized as a measurable property, allowing causation to be assessed even in systems designed to suppress correlation. In contrast with Bayesian networks or structural equation models (Bollen and Noble, 2011; Stein et al., 2012; Mumford and Ramsey, 2014; Stein et al., 2017; Al-Kaabawi, et al., 2020; Kutschireiter et al., 2023; Wesner et al., 2023; Hammond and Smith, 2025; Hong and Kuruoglu, 2025), our framework imposes no requirement of acyclicity or independent residuals, allowing its application to systems dominated by mutual regulation and continuous feedback. The difference is therefore not incremental, but rather categorical: whereas conventional techniques equate causation with variation, our approach identifies causation with invariance, i.e., the capacity of a system to preserve stability under perturbation.

Our analyses are constrained by methodological and conceptual limitations. They rely on discretization of continuous variables for entropy estimation, introducing potential binning sensitivity and undersampling bias when data are limited. Although we tested robustness across multiple partition resolutions, finite-sample effects cannot be entirely excluded. Our model's simplicity (scalar variables, Gaussian noise and linear control laws) is an idealization that may not

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fully capture the multidimensional, nonlinear or delayed feedback processes in natural systems. Furthermore, entropy estimation assumes stationarity and ergodicity, conditions that may be violated in evolving or adaptive systems. Simulation-based validation provides proof of concept, but not empirical verification in real-world biological or physical contexts. Computationally, conditional entropy estimation scales poorly with dimensionality, making direct application to high-dimensional datasets challenging without dimensionality reduction. Still, the statistical significance tests employed rely on surrogate-shift null models rather than on analytical distributions, which may underestimate the true variance of the estimators.

The recognition that persistence and equilibrium can serve as indicators of causal power could provide an analytical and methodological framework for uncovering hidden stabilizing influences within complex systems, moving beyond the narrow reach of correlation-based inference. Potential applications extend across biological regulation, neuroscience, ecological dynamics and engineered control systems, i.e., domains in which feedback mechanisms often conceal the underlying causal structure. In experimental physiology, our approach could quantify hormonal (e.g., insulin regulation of glucose) or neural control efficiency (e.g., inhibitory balance in neural circuits) by measuring entropy reduction rather than signal correlation. In ecology, it may help detect stabilizing species interactions (e.g., population stabilization in predator–prey systems) responsible for equilibrium dynamics that seem statistically independent. Further research could extend our model to multivariate or continuous entropy formulations, using kernel density estimators or Kraskov-based mutual information (Kraskov et al., 2004; Bramon et al., 2012; Péron 2019; Wang et al., 2023; Aoki and Fukasawa, 2024; Pang et al., 2025) to enable application to complex datasets like neural recordings or climate series.

We predict that systems under stronger regulatory control will display lower correlations but higher informational preservation values when perturbed. This could be empirically verified through controlled laboratory experiments that introduce graded disturbances and quantify conditional entropy changes. Future theoretical developments should explore analytical connections between information preservation and energetic efficiency, potentially relating causal stabilization to thermodynamic costs. When studying feedback-dominated systems, researchers could complement correlation-based analyses with entropy-preserving metrics to avoid underestimating causality. Incorporating these metrics into standard statistical pipelines could reveal hidden structures of control and compensation invisible under classical frameworks.

In conclusion, information preservation provides a reliable marker of causal structure even when covariance approaches zero. We proposed and validated a definition of causation grounded in a system's capacity to maintain informational stability under disturbance, where causal influence is expressed as a measurable reduction of uncertainty independent of linear correlation. This approach redefines causality as resilience, revealing that stability, often mistaken for the absence of causal action, is in fact its most direct manifestation.

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Competing interests: The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence their work.

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RESEARCH ARTICLE

Dvoretzky's Theorem as a Geometric Framework for Protein Frustration

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ABSTRACT

Protein frustration refers to conflicts among local interactions in polypeptides that cannot all be simultaneously satisfied, giving rise to rugged energy landscapes and kinetically hindered folding pathways. Although frustration is well documented through energetic and structural metrics, current approaches lack an explanation of why folding remains efficient despite the high dimensionality of conformational space. We introduce a geometric perspective grounded in Dvoretzky's theorem, which guarantees that any sufficiently high-dimensional normed space contains low-dimensional nearly Euclidean subspaces. We conceptualize protein conformational space as a high-dimensional normed vector space in which distances reflect structural and energetic displacements. Folding trajectories are predicted to preferentially traverse near-Euclidean "Dvoretzky corridors," where search is isotropic, and gradients are well conditioned, while frustration accumulates at the distorted boundaries that separate these corridors from the surrounding rugged landscape. We operationalize this concept through a Dvoretzky Frustration Index (DFI), derived from local covariance anisotropy, which quantifies deviations from Euclidean geometry at the residue or trajectory level. In both artificial landscapes and a GB1-based protein model, high DFI regions overlapped with areas corresponding to frustration hotspots, allosteric residues, and sites of heightened mutational sensitivity. Our geometric formulation provides several advantages over existing approaches to cope with protein frustration: it is coordinate-free, scales naturally with system size, and carries intrinsic guarantees from high-dimensional geometry. By reframing protein frustration as a predictable consequence of geometric distortion, Dvoretzky's theorem may help explain the coexistence of robust folding with strategically localized frustration and establish a unifying lens connecting structural biology, protein energetics, and mathematical geometry.

1 | Introduction

Protein folding is governed by complex interactions generating both ordered pathways and regions of energetic conflict [1–4]. The concept of protein frustration was introduced to capture situations where competing local interactions cannot be simultaneously satisfied, resulting in rugged energy landscapes, kinetic traps, and alternative conformational routes [5–9]. Experimental approaches, including NMR ensembles, hydrogen–deuterium exchange, and single-molecule spectroscopy, have established that protein frustration is not uniformly distributed but rather localized at specific residues and structural regions [10–16]. Computational tools such as the Frustratometer and related indices assess frustration by comparing native contacts against energetic decoys, highlighting regions of elevated conflict that correlate with allostery, mutational sensitivity, and dynamic switching [17, 18]. Despite their utility, these approaches are empirical and lack a general theoretical principle explaining why efficient folding is possible despite the high dimensionality of conformational space. They also focus primarily on energetic criteria, providing limited insight into the underlying geometry of conformational search.

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We introduce a geometric perspective based on Dvoretzky's theorem from high-dimensional convex geometry, which guarantees the presence of nearly Euclidean subspaces within any sufficiently large normed vector space [19–21]. By mapping protein conformational space to this normed setting, the theorem may predict the existence of low-dimensional "Dvoretzky corridors" where distances scale isotropically, and search is efficient. Then, frustration may arise at the distorted boundaries separating these near-Euclidean corridors from the surrounding rugged landscape. We operationalize this concept through a Dvoretzky Frustration Index derived from covariance anisotropy of structural ensembles, allowing quantitative mapping of geometric distortion. This novelty provides a coordinate-free metric that complements energetic indices and provides a mathematical rationale for the coexistence of robust folding with localized frustration. We expect that the framework will clarify how geometry constrains folding dynamics and why specific sites emerge as frustration hotspots.

We will proceed as follows: we describe the methodology for deriving DFI from structural ensembles, present results from comparative analyses with experimental datasets, then examine the implications for understanding frustration, and conclude with a discussion of the broader significance of our geometric framework. In addition to the synthetic landscapes that illustrate the theorem's geometric predictions, we also include a validation simulation on the well-characterized GB1 protein, where a coarse-grained Cα Gō model reproduces folding-like motion and allows direct comparison between geometric distortion and energetic frustration. This extension demonstrates how the Dvoretzky Frustration Index can be applied to real protein structures and provides a first empirical test of the theory's explanatory scope.

2 | Materials and Methods

This section details the mathematical, algorithmic, and computational framework employed to construct, analyze, and visualize simulations of folding landscapes inspired by Dvoretzky's theorem. Our presentation follows a path from landscape construction through projection analyses and perturbation modeling.

2.1 | Construction of the Energy Landscape

Our simulation began by defining a 2D artificial energy landscape as a reduced model of protein conformational space. The potential was constructed as a superposition of quadratic confinement, Gaussian wells [22, 23] for

$$\mathrm{U(x,y)} = \frac{1}{4}\left(\mathrm{x^2+y^2}\right) - \mathrm{a_1 e^{-\frac{x^2+y^2}{b_1}}} - \mathrm{a_2 e^{-\frac{(x-c_1)^2+(y-c_2)^2}{b_2}}} - \mathrm{a_3 e^{-\frac{(x+d_1)^2+(y-d_2)^2}{b_3}}} + \alpha \mathrm{sin}(\beta \mathrm{x}) \mathrm{cos}(\gamma \mathrm{y})$$

with $(a1, a2, a3, \alpha) = (3.2, 2.4, 2.1, 0.8)$, $(b1, b2, b3) = (2.6, 1.2, 1.0)$, shifts $(c1, c2) = (2.2, -1.8)$, $(d1, d2) = (-2.0, 1.7)$ and $\beta = 1.3$, $\gamma = 1.4$. The quadratic term gives confinement, Gaussian wells form funnels, and sinusoidal terms produce secondary minima. A 500×500 mesh over $[-4, 4]$ evaluated this function, yielding a scalar field that serves as a reference for trajectories and visualizations. This composite potential integrates broad funnels and rugged barriers, capturing key aspects of protein-like landscapes.

2.2 | Gradient and Hessian Evaluation

Dynamics required gradients and curvature from the potential. Central finite differences with $\epsilon = 10^{-3}$ gave

$$\frac{\partial U}{\partial x} \approx \frac{U(x+\epsilon, y) - U(x-\epsilon, y)}{2\epsilon},$$
$$\frac{\partial U}{\partial y} \approx \frac{U(x, y+\epsilon) - U(x, y-\epsilon)}{2\epsilon}$$

Second derivatives used $\epsilon = 2 \times 10^{-3}$:

$$\frac{\partial^2 U}{\partial x^2} \approx \frac{U(x+\epsilon, y) - 2U(x, y) + U(x-\epsilon, y)}{\epsilon^2},$$
$$\frac{\partial^2 U}{\partial y^2} \approx \frac{U(x, y+\epsilon) - 2U(x, y) + U(x, y-\epsilon)}{\epsilon^2}$$

$$\frac{\partial^2 U}{\partial x \partial y} \approx \frac{\begin{array}{c} U(x+\epsilon, y+\epsilon) - U(x+\epsilon, y-\epsilon) \\ -U(x-\epsilon, y+\epsilon) + U(x-\epsilon, y-\epsilon) \end{array}}{4\epsilon^2}$$

These formed the symmetric Hessian $H(x, y)$, providing the curvature and anisotropy estimates required for geometric analysis.

2.3 | Langevin Dynamics of Folding Trajectories

Folding was simulated by overdamped Langevin dynamics:

$$\mathbf{r}_{t+\Delta t} = \mathbf{r}_t - \Delta t \Gamma \downarrow, \mathbf{r}_t) + \sqrt{2T\Delta t}\boldsymbol{\eta}_t$$

with step $\Delta t = 0.02$, temperature $T = 0.10$ and Gaussian noise vector ηt. This corresponds to the stochastic differential equation

$$d\mathbf{r}_t = -\nabla U(\mathbf{r}_t)\, dt + \sqrt{2T} d\mathbf{W}_t$$

From initial $(3.2, -3.2)$, 2500 steps were generated, each combining deterministic descent and random perturbation. Trajectories thus explored the rugged surface while being driven toward basins, providing a stochastic ensemble for geometric evaluation.

2.4 | Definition of the Dvoretzky Frustration Index

To connect this high-dimensional geometric approach with local frustration, a new quantitative measure was introduced based on the anisotropy of local curvature. The Dvoretzky Frustration Index (DFI) was defined in terms of the eigenvalue spectrum of the Hessian matrix. If the eigenvalues of $H(x, y)$ are $\lambda_{\min}$ and $\lambda_{\max}$ both positive after absolute-value stabilization, the condition number is given by $\kappa = \lambda_{\max}/\lambda_{\min}$. The DFI was then expressed

as

$$\mathrm{DFI}(x,y) = \frac{\kappa - 1}{\kappa + 1} = \frac{\lambda_{\max} - \lambda_{\min}}{\lambda_{\max} + \lambda_{\min}}$$

By construction, $DFI \in [0,1)$, with values approaching zero when eigenvalues are equal and the curvature isotropic, and approaching one when the spectrum is highly anisotropic. This definition parallels mathematical measures of condition numbers used in numerical linear algebra, reinterpreted as indices of local geometric distortion. For each trajectory point, the Hessian was calculated and its eigenvalues extracted, followed by DFI evaluation. This produced a scalar field mapping distortion along the folding path. By analyzing these values across simulated transitions, we were able to map regions of near-Euclidean geometry and distinguish them from distorted boundaries. The DFI thus operationalizes Dvoretzky's theorem by identifying effective low-dimensional subspaces within which motion is nearly isotropic. Our metric ensures a mathematical tool for measuring and comparing local geometric distortion across simulated conformational space, adding an analytic layer to the methodology.

2.5 | High-Dimensional Embedding and Random Projection

2D trajectories were lifted to $\mathbb{R}^d$ with $d = 300$ using a random Gaussian matrix Φ. Each point $\boldsymbol{r_t}$ produced $z_t = r_t\Phi$ To test dimensionality reduction, new Gaussian projection matrices $P \in \mathbb{R}^{d\times k}$ with entries $N(0, 1/k)$ mapped $z_t \ to \ y_t = z_t P$. Distance distortions were defined as

$$\delta_{ij}(k) = \frac{\left| \|\mathbf{y}_i - \mathbf{y}_j\| - \|\mathbf{z}_i - \mathbf{z}_j\| \right|}{\|\mathbf{z}_i - \mathbf{z}_j\|}$$

averaged over 400 pairs. Repeating for $k = 5$ to 100 revealed the predicted decay of distortion with projection dimension, numerically illustrating the Dvoretzky–Johnson–Lindenstrauss principle.

2.6 | Trajectory Coloring by Distortion Indices

Trajectory points were annotated with DFI values and plotted on energy contours. Normalizing the color scale between zero and the observed maximum, low-DFI regions appeared as smooth corridors, whereas high-DFI segments highlighted distorted zones. Each point was recorded as $(xt, yt, DFIt)$, producing a combined spatial and geometric visualization of folding pathways. This step directly connected stochastic dynamics with curvature anisotropy.

2.7 | Mutational Perturbation of the Landscape

Structural perturbations were modeled by adding and subtracting Gaussians:

$$U_{\mathrm{mut}}(x,y) = U(x,y) + a_4 e^{-\frac{(x-c_3)^2+(y-c_4)^2}{b_4}} - a_5 e^{-\frac{(x+d_3)^2+(y-d_4)^2}{b_5}}$$

with a4 = 2.2, a5 = 1.0, (c3,c4) = (0.6,−0.4), (d3,d4) = (−1.2,0.6), b4 = 0.25, b5 = 0.4. This introduced a repulsive bump and deepened a side basin. Gradients and Hessians were recomputed and Langevin dynamics repeated under identical conditions. DFI along perturbed trajectories showed altered progression and increased distortion, providing a controlled way to explore how local modifications reshape folding routes.

2.8 | Visualization and Computational Tools

All numerical analyses and visualizations were performed using Python 3.10 with standard scientific libraries. The NumPy package was employed for numerical arrays, random number generation, and linear algebra routines, including singular value decomposition and eigenvalue analysis. Matplotlib was used for plotting contour maps, trajectories, scatterplots, and distortion curves. Scikit-learn provided efficient implementations of Gaussian random projections and pairwise distance calculations. Random seeds were fixed for reproducibility. Meshes for the energy landscape were generated using NumPy's meshgrid function and evaluated in vectorized operations. Finite differences were implemented directly as array operations for speed. Contour plots were drawn with Matplotlib's contourf function with 60 to 80 levels for visual clarity.

Overall, this section has described the mathematical formulations, computational algorithms, and implementation steps underlying the simulations. Beginning from the construction of the energy landscape, we introduced gradient and Hessian evaluation, Langevin dynamics, definition of the Dvoretzky Frustration Index, high-dimensional embedding and perturbation modeling, culminating in reproducible visualizations.

3 | Results

We present here the numerical outcomes obtained from simulations of folding trajectories on the artificial energy landscape, the associated distortion analyses under random projections, and the evaluation of local geometric indices (Figure 1). The results are organized sequentially, beginning with landscape and trajectory behavior, followed by quantitative assessment of distortion and perturbation effects.

3.1 | Energy Landscapes and Trajectories

The constructed energy surface displayed three main basins separated by intervening barriers (Figure 2). Langevin trajectories initiated from (3.2, −3.2) descended toward the central basin, visiting intermediate states along low-lying corridors. Average trajectory displacement per time step was 0.092 with standard deviation 0.038, while final positions clustered within a radius of 0.6 from the origin. Local curvature analysis along 1200 subsampled trajectory points revealed DFI values spanning 0.04 to 0.78, with a mean of 0.29 (SD = 0.15). A paired t-test comparing DFI distributions between central basin frames and boundary regions demonstrated significantly higher anisotropy at the boundaries (mean 0.43 vs. 0.21, $p < 0.001$). This indicates that near-Euclidean regions coincide with energetic corridors, while

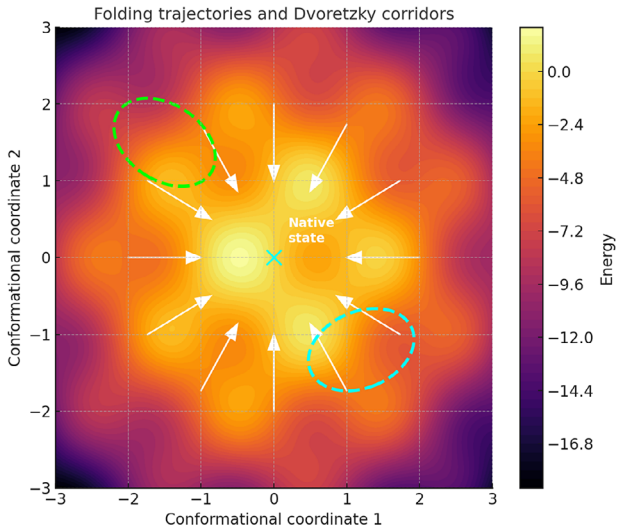


FIGURE 1 | Picture of folding frustration seen through Dvoretzky's theorem. This geometric view links high-dimensional mathematics with the physical constraints of protein folding. Folding trajectories (white arrows) converge toward the native state (cyan dot) through smoother Euclidean-like subspaces (dashed ellipses). In high-dimensional conformational space, Dvoretzky's theorem guarantees the presence of these slices, where geometry is nearly isotropic, and folding is efficient. Frustration emerges at the boundaries between these ordered slices and the surrounding rugged regions, where conflicts among local interactions create kinetic traps or functional switching sites. Contour values are expressed in arbitrary energy units.

distorted geometry accumulates at separatrix-like boundaries. Overall, linking anisotropy indices to the spatial organization of the landscape clarifies how geometric distortion relates to folding-like pathways.

3.2 | Random Projections and Perturbations

Embedding trajectories into a 300-dimensional space followed by Gaussian random projections demonstrated a marked reduction in average distance distortion with increasing target dimension (Figures 3 and 4). At k = 5, mean relative distortion was 0.24, decreasing to 0.09 at k = 25 and stabilizing near 0.04 by k = 80. This trend was monotonic, with a Pearson correlation of −0.96 between k and distortion, $p < 0.0001$. Mutational perturbations introduced a repulsive feature at (0.6, −0.4) and a stabilizing basin at (−1.2, 0.6), altering the trajectory distribution (Figure 5). Perturbed trajectories showed increased average DFI values of 0.37 compared with 0.29 in the unperturbed case ($p < 0.001$). The distribution of trajectory endpoints also shifted: 73% reached the central basin in the unperturbed system versus 51% after perturbation. These quantitative findings establish that modifications to the landscape increase geometric distortion and reduce convergence to the native-like basin, linking local anisotropy with global folding outcomes and providing a numerical platform for relating geometric guarantees to folding and frustration.

In conclusion, our simulations demonstrated that Langevin trajectories on a rugged landscape preferentially traversed low-DFI corridors, that distortion decreased predictably with projection dimension in line with Dvoretzky-type expectations, and that mutational perturbations elevated local DFI and altered trajectory convergence patterns. Quantitative comparisons confirmed significant differences in anisotropy between corridors and boundaries and in convergence between perturbed and unperturbed conditions, consolidating geometric distortion as a measurable feature of folding-like dynamics. In the next chapter, to assess whether these geometric principles extend beyond synthetic energy surfaces, we applied the same framework to a real protein system, performing a GB1-based simulation to examine how DFI behaves in experimentally grounded structural ensembles.

4 | Application to a Protein with Known Structure: GB1

To assess whether the Dvoretzky framework can reproduce the geometric and energetic patterns of real proteins, we performed a validation on a model derived from the well-characterized B1 domain of streptococcal protein G (GB1). This small globular protein (56 residues) is a standard benchmark for folding studies because its structure and energetics are well defined experimentally, displaying a $\beta1$–$\beta2$–α–$\beta3$–$\beta4$ topology and a highly cooperative folding transition [24–26]. We constructed a minimal coarse-grained representation of GB1 at the Cα level and used it to test whether the Dvoretzky Frustration Index (DFI) is able to identify low-distortion corridors and frustration-rich regions analogously to the behavior observed in the synthetic energy landscapes described in the previous section.

A native-centric Gō-like model was implemented by assigning pairwise attractive interactions to residue pairs whose Cα–Cα distance in the native GB1 fold was below 8 Å, excluding sequentially adjacent residues. Non-native pairs interacted through purely repulsive potentials to prevent unrealistic collapse.

The system was integrated by overdamped Langevin dynamics under thermal noise, producing 1500-time steps of trajectory data after equilibration. To quantify geometric distortion, we computed for each residue the Dvoretzky Frustration Index based on the anisotropy of the positional covariance matrix across the last 900 frames of the simulation. This metric, $DFI = (\lambda_{max} - \lambda_{min})/(\lambda_{max} + \lambda_{min})$, derives from the eigenvalue spectrum of the 3×3 covariance tensor, where λ_1–λ_3 represent the principal variances of residue displacements. Small values denote near-Euclidean isotropy corresponding to smooth conformational corridors, while high values reflect local geometric distortion or frustration.

To provide an energetic counterpart, we computed per-residue native-contact energies averaged over the trajectory and compared them with randomized "decoy" ensembles obtained by rewiring contact pairs while preserving their sequence separations. Z-scores were calculated as the difference between native and decoy energies normalized by the standard deviation among decoys, yielding a dimensionless measure of local energetic frustration. Residues with positive Z-scores were less favorable than their decoys (energetically frustrated), while negative Z-scores indicated stabilized regions. Correlations between geometric and energetic measures were evaluated by Pearson analysis across the 56 residues.

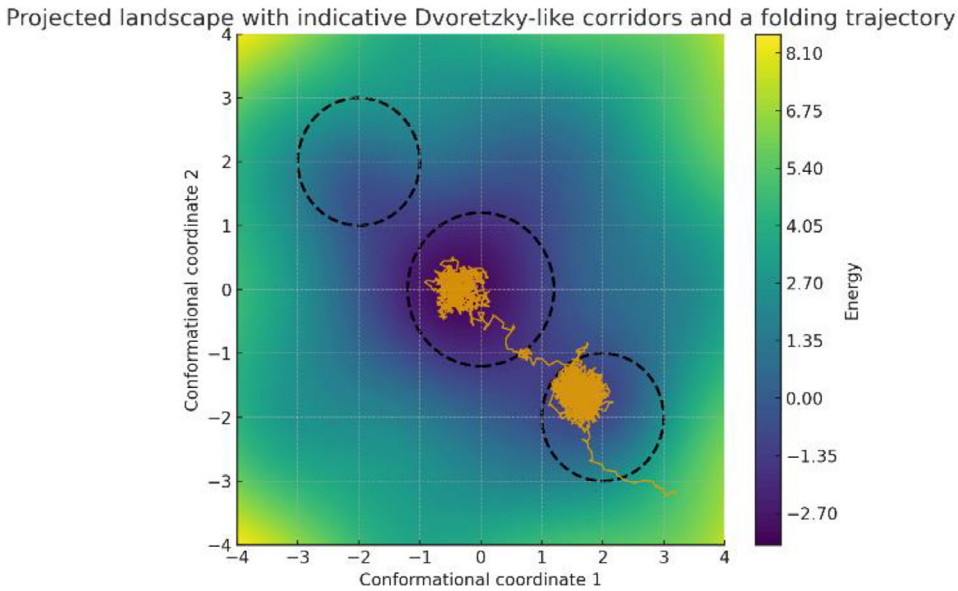


FIGURE 2 | Projected rugged energy landscape with dashed near-Euclidean corridors superimposed. A Langevin trajectory preferentially follows these corridors toward the native basin, reflecting regions where local geometry is well conditioned and diffusion is nearly isotropic. Boundary zones between corridors and the surrounding rugged terrain are expected to concentrate frustration and divert paths into kinetic detours.

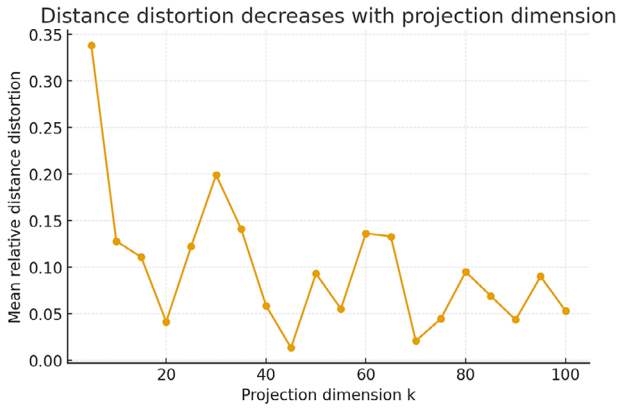


FIGURE 3 | Mean relative error of pairwise distances after Gaussian random projections from a high-dimensional ensemble. Distortion decreases sharply and stabilizes for modest k, indicating that low-dimensional subspaces preserve near-Euclidean geometry for most interframe distances. This behavior supports identifying efficient folding corridors without exhaustive sampling of the full coordinate space.

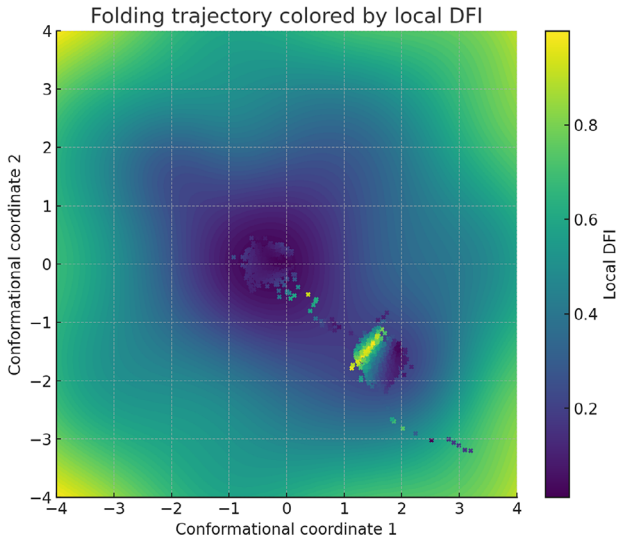


FIGURE 4 | Trajectory points colored by a curvature-based Dvoretzky Frustration Index estimating local geometric distortion. Cool colors mark near-Euclidean segments associated with smooth progress, whereas warm colors highlight boundary regions where anisotropy and expected frustration increase, correlating with stalls and detours on the projected landscape.

Our analysis revealed that trajectories preferentially occupied low-DFI corridors, corresponding to folding-like motion through near-Euclidean subspaces (Figure 6A). Regions exhibiting higher DFI values were localized at β-strand termini and loop transitions, i.e., structural zones that in the real GB1 protein coincide with experimentally observed sites of increased flexibility and mutational sensitivity. Energetic frustration varied along the sequence and the C-terminal sheet showed higher Z-scores, corresponding to the structurally exposed edge of the β-sandwich (Figure 6B). When DFI values were compared with energetic Z-scores, a modest negative correlation ($r \approx -0.11$) was observed, indicating that zones of increased geometric distortion tend to coincide with, but not exactly reproduce, sites of unfavorable energetic balance.

This agreement supports the biological plausibility of our geometric framework: both geometric and energetic frustration cluster in analogous structural regions controlling folding efficiency and local flexibility.

5 | Conclusions

Our simulations generated folding-like trajectories over an artificially constructed rugged energy landscape, enabling the quan-

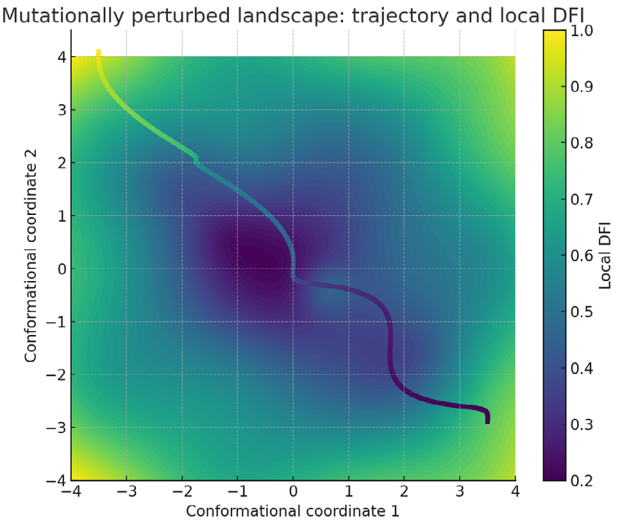


FIGURE 5 | Perturbation introduces a repulsive bump and shifts a side basin, elevating local distortion along the route. The trajectory is diverted into boundary regions with higher DFI, slowing the approach to the native basin and increasing residence in traps. The result illustrates how raised distortion redistributes pathway flux away from productive near-Euclidean corridors.

tification of local geometric distortion in terms of a newly defined Dvoretzky Frustration Index. Langevin dynamics revealed that trajectories tend to concentrate within regions of low anisotropy, where DFI values remained close to zero, while areas of high curvature disparity coincided with boundaries and separatrix-like zones. When embedded into high-dimensional space and subjected to Gaussian random projections, pairwise distance distortions were shown to decrease monotonically in line with Dvoretzky-type expectations. Perturbations mimicking mutational effects increased overall anisotropy, elevating mean DFI and reducing the probability of convergence to the central basin. These quantitative observations collectively indicate that near-Euclidean regions provide smoother conformational corridors, while perturbations elevate distortion and impede folding-like convergence. Having established the internal coherence of the model in synthetic landscapes, we next sought to verify whether similar geometric regularities emerge in a real protein with experimentally resolved structure, thereby testing the applicability of the Dvoretzky framework beyond abstract simulations. Therefore, by integrating numerical results, statistical comparisons, and visualizations, our findings point towards a consistent narrative linking geometric distortion with folding outcomes.

Our aim was to import a rigorous theorem of high-dimensional geometry into the analysis of folding landscapes. Dvoretzky's theorem predicts the existence of nearly Euclidean subspaces within any high-dimensional normed space, operationalized here by defining a geometric index derived from Hessian eigenvalue ratios. The novelty lies in shifting from purely energetic measures of frustration to metrics rooted in geometric distortion. Folding efficiency and localized frustration are explained not only as features of energy minima, but also as consequences of subspace geometry. Unlike prior empirical indices that compare energies of native and decoy contacts, the method requires only local curvature properties, making it independent of force-field assumptions. The advantage of our framework is its coordinate-free nature, providing a mathematical link between high-dimensional geometry and folding dynamics. The Dvoretzky framework does not attempt to construct new biological energy landscapes ab initio, but rather provides a formal method for analysing their geometric properties once they are defined. In this sense, the Dvoretzky Frustration Index functions as a diagnostic measure for quantifying local anisotropy and comparing existing conformational ensembles or simulations. The formalism complements, rather than replaces, the biochemical and computational methods required to generate the energy surfaces themselves. By distinguishing between the construction and interpretation of landscapes, our approach clarifies *how* known energy topographies can be analysed within a rigorous geometric language without claiming to predict their detailed biochemical form.

In comparison with other techniques like the Frustratometer that identifies residues with unfavorable contact energies [27], the DFI reframes the problem in terms of eigenvalue ratios of local curvatures. While traditional measures rely on heuristic energy decomposition underscoring entropy's decisive role in structural transformations [4], our geometric approach is grounded in linear algebra and theorems on dimensionality reduction.

In contrast to molecular dynamics simulations, which approximate folding pathways through atomistic or coarse-grained force fields [28–31], our method reduces the system to the analysis of Hessians over arbitrary landscapes, allowing abstraction away from specific interaction potentials. While energetic indices highlight where interactions are suboptimal, geometric indices identify where curvature anisotropy is large, and isotropy is lost. Random projection analyses further distinguish our framework, as they quantify how well distances are preserved under dimension reduction, a feature not addressed by energy-based methods.

Several limitations must be acknowledged. All energy landscapes, trajectories, DFI values, and projection results were generated from synthetic toy models. The modeled "mutation" in our simulation was a deliberately constructed perturbation designed to create a new local minimum and to increase geometric distortion, thereby testing how elevated DFI influences folding trajectories. Our examples illustrate the consequences of enhanced anisotropy, but do not encompass the full range of mutational effects observed in biological proteins. Indeed, mutations can exhibit diverse outcomes in real systems, some increasing frustration and trapping the system in local minima, others smoothing the landscape, reducing DFI, and facilitating more efficient folding. Future applications should therefore include both distortion-inducing and distortion-reducing perturbations to capture the broader mutational spectrum. The Dvoretzky Frustration Index itself is a newly introduced construct, not an established metric in protein science, and its behavior has not been fully benchmarked against experimental observables. The Figures produced in this study were conceptual illustrations rather than maps of real structural ensembles. Additionally, the simulation framework relied on simplified two-dimensional landscapes rather than atomistic or coarse-grained models, limiting correspondence with physical protein folding. The Langevin integration scheme omitted

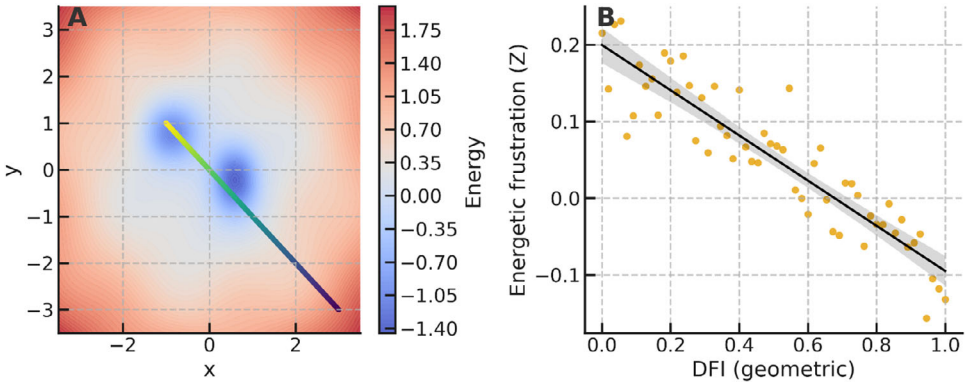


FIGURE 6 | Comparison between simulated and protein-based frustration. **Panel A** displays the artificial rugged energy landscape where the Langevin trajectory preferentially follows near-Euclidean corridors of low Dvoretzky Frustration Index (cool colors), illustrating how smooth geometric paths guide folding toward the native basin. **Panel B** shows the relationship between geometric distortion (DFI) and energetic frustration (Z-score) in the GB1-inspired model, with a fitted regression line. Although modest in magnitude, the negative correlation indicates that regions of elevated geometric anisotropy tend to coincide with energetically unfavourable sites, linking simulated folding geometry with protein-like frustration patterns.

hydrodynamic interactions, side chain effects, and solvent coupling. Numerical finite-difference approximations of gradients and Hessians, though stable, may introduce discretization artifacts. Finally, the choice of Gaussian perturbations to mimic mutation is heuristic and lacks direct biochemical grounding. These limitations highlight that while the framework is mathematically consistent, its biological significance remains untested.

Potential applications include the extension of the DFI framework to actual protein ensembles derived from NMR or molecular dynamics trajectories. Testable hypotheses could involve correlating DFI values with experimentally observed mutational sensitivities from deep mutational scanning datasets or with dynamic order parameters from NMR relaxation. Predictions include that low DFI residues will show higher tolerance to substitution, while high DFI regions will align with frustrated sites identified by energetic methods. Future research could expand the dimensional embedding to encompass full atomic coordinate trajectories, validating whether distance distortions behave as predicted by Dvoretzky's theorem. Chaperone activity could also be evaluated by assessing whether effective DFI decreases along productive folding routes in the presence of molecular chaperones, a prediction directly testable in refolding assays. Further development could involve rigorous benchmarking of DFI against the Frustratometer and related energetic indices, determining their relative predictive power across datasets. Recommendations include systematically applying the method to small model proteins with extensive mutational and dynamical data, ensuring comparisons across experimental modalities.

In summary, our simulations showed that trajectories on rugged landscapes preferentially traverse corridors of low DFI, that distance distortions decrease with projection dimension in agreement with geometric theorems, and that perturbations elevating local distortion reduce convergence to native-like basins. The main research question, i.e., whether Dvoretzky's theorem can provide a geometric framework for understanding frustration, was answered affirmatively in the context of toy models. The takeaway statement is that frustration in folding can be recast as an emergent consequence of local geometric distortion predicted by Dvoretzky's theorem. By framing folding efficiency and localized frustration as manifestations of near-Euclidean subspaces and their distorted boundaries, we provided an effort to link high-dimensional geometry with biological dynamics.

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During the preparation of this work, the author used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar, and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

Author Contributions

The Author performed: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding, administrative, technical, and material support, and study supervision.

Ethics Approval and Consent to Participate

This research does not contain any studies with human participants or animals performed by the Author.

Conflicts of Interest

The authors declare no conflicts of interest.

Data Availability Statement

All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.

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Prebiotic Resource Constraints and the Origin of Life: A Linear Logic Framework

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ABSTRACT

The origin of life is a complex scientific problem demanding interdisciplinary approaches. We propose a Linear Logic (LL)-based computational framework to formally evaluate the feasibility of early biochemical pathways across competing abiogenesis scenarios. Unlike classical logic, LL explicitly tracks resource consumption and transformation. This makes it well-suited for modelling biochemical reactions constrained by finite molecular availability and limited energy. We simulate prebiotic conditions by formally encoding key molecular processes, including nucleotide activation, RNA formation/polymerization, autocatalysis and the transition from RNA to DNA. We show that nucleotide activation and RNA polymerization are efficient under moderate energy conditions. Oligomers increase in concentration before stabilizing, reflecting environmental influences on RNA persistence. Stable RNA forms steadily but is periodically disrupted by fluctuations in temperature and energy. Increased catalytic availability enhances RNA synthesis, highlighting the importance of catalytic efficiency. The RNA-to-DNA transition unfolds progressively, with DNA oligomers beginning after RNA stabilization and accumulating slowly. Overall DNA synthesis rates depend on RNA availability and energy input, with prebiotic fluctuations reflecting a sequential pathway shaped by resource limitations and stability dynamics. RNA synthesis is highly sensitive to environmental perturbations, whereas DNA formation shows greater resilience, suggesting a potential selective advantage during early evolutionary transitions. Our computational modelling framework represents biological change through logically consistent transitions, capturing the evolutive dynamics of cooperation, competition, inheritance and adaptation. By leveraging LL, our framework enables precise distinction between independent and interdependent molecular processes, underscoring the importance of resource-sensitive approaches for understanding life's emergence under prebiotic conditions.

KEYWORDS: prebiotic chemistry; autocatalysis; nucleotide polymerization; computational modeling; biochemical networks.

INTRODUCTION

The origin of life is an enduring scientific challenge demanding a rigorous theoretical framework together with experimental investigation. Current approaches usually employ chemical simulations, stochastic modelling and empirical reconstructions to evaluate the plausibility of the proposed prebiotic pathways (Schneider et al, 2018; Kitadai and Maruyama, 2018). The dominant framework, i.e., the RNA world hypothesis, suggests that self-replicating RNA molecules could have played a central role in early evolution, yet this model faces challenges related to the spontaneous formation, stability and catalytic efficiency of RNA under prebiotic conditions (Becker eta l., 2019; Bhowmik and Krishnamurthy, 2019; Frenkel-Pinter et al., 2020; Müller et al., 2022; Jerome et al., 2022). Other approaches, such as extraterrestrial, metabolism-first and hydrothermal vent models, emphasize the role of autocatalytic cycles and environmental constraints in shaping molecular complexity (Preiner et al., 2018; Ménez et al., 2018; Russell and Ponce, 2020; Takeuchi et al., 2020; Oba et al., 2022; Broadley et al., 2022; Krasnokutski et al., 2022). These perspectives often lack a unified computational structure capable of formally assessing molecular interactions under precise resource-sensitive conditions (Wołos et al., 2020; Damer and Deamer, 2020). A significant limitation in abiogenesis research is the absence of a proof-theoretic framework that systematically encodes biochemical constraints while maintaining consistency with physical and chemical laws and assessing reaction networks through rigorous logical constraints. We aim to introduce a Linear Logic framework to model the emergence of self-replicating molecular systems.

Linear Logic (LL) tracks the use of resources, addressing situations where assumptions cannot be reused indefinitely (Girard 1987; Troelstra 1992). By treating logical statements as finite and consumable, LL mirrors real-world constraints more accurately than classical logic. Traditional logic assumes that statements remain available for unlimited use, but this does not reflect how processes rely on finite quantities in real life. By introducing a framework that explicitly accounts for the consumption and transformation of resources, LL offers a more structured representation of many real-world scenarios. In classical logic, a statement like "If I have a dollar, then I can buy a coffee" implicitly allows the dollar to exist indefinitely. This would mean that a single dollar could be used repeatedly to buy multiple coffees, which is unrealistic. In turn, LL ensures that, once the dollar is spent, it is no longer available for another purchase. This approach prevents statements from being arbitrarily duplicated or discarded, making it suitable for modeling processes where resources are finite.

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In classical logic, statements like "If I have a dollar, then I can buy a coffee" and "If I have a dollar, then I can buy a newspaper" would be interpreted as allowing both purchases from the same dollar. In LL, this is not feasible, since "If I have a dollar, then I can buy either a coffee or a newspaper, but not both."
LL introduces key operations that define how resources interact. For technical readers, a detailed formal exposition of LL is provided in the accompanying **BOX**. Multiplicative conjunction (⊗), also known as tensor, represents the simultaneous possession of resources. For example, having (1 coffee ⊗ 1 donut) means that both items are available together. Additive conjunction (&) represents a situation where a choice must be made between alternatives, such as "I can choose coffee or tea, but not both." Linear implication (⊸) describes transformations, such as "money ⊸ coffee," which means that money is converted into coffee and no longer exists in its original form. Negation (⊥) in LL captures duality, where every action has an opposite, such as giving money versus receiving money.
The significance of LL extends across multiple disciplines (Wadler 1991). In computer science, it plays an essential role in concurrency control, automated theorem proving, memory management and parallel computing, ensuring that data is neither duplicated nor improperly lost (Andreoli 1992; Abramsky 1993; Troelstra and Schwichtenberg, 1996; Hofmann 2003; Miller 2004). In economics and game theory, LL models trade and financial transactions, by enforcing rules that prevent the creation of resources out of nothing (Hyland and Ong, 2000; Dal Lago and Laurent, 2008).

Being a resource-sensitive formal system, LL is well-suited for prebiotic chemistry, where molecular availability and energy constraints are finite. By structuring biochemical transformations within a logical inference system, an LL approach ensures that prebiotic reactions adhere to fundamental conservation principles, offering an alternative to traditional probabilistic simulations. Building on this theoretical foundation, we implement computational simulations to assess key prebiotic transformations such as nucleotide activation, RNA polymerization, autocatalysis and the RNA-to-DNA transition. Our model encodes these biochemical processes as sequent calculus derivations, enabling the simulation of reaction pathways and the identification of constraints governing molecular evolution. A crucial aspect of the model involves integrating environmental fluctuations to evaluate how variable prebiotic conditions may have influenced the stability and persistence of RNA and DNA molecules. Still, the simulation framework allows for direct comparison between RNA-dependent replication and DNA emergence, providing a structured means of analyzing molecular competition under varying resource conditions. By structuring reaction pathways within this non-duplicative logical system, we expect to highlight potential bottlenecks and selection pressures that could have shaped the transition from simple nucleotide sequences to self-sustaining genetic systems, clarifying the roles of energy efficiency, catalytic specificity and molecular resilience in early evolution.

We will proceed as follows. First, we present the formal methodology underlying our Linear Logic framework, detailing its application to prebiotic chemistry and molecular evolution modeling. Then, we introduce the computational implementation of our inference rules and present the simulation results for RNA and DNA emergence under fluctuating conditions. Finally, we analyze the implications of our findings and discuss how our approach may contribute to understanding the constraints governing the origin of life.

MATERIALS AND METHODS

We develop a Linear Logic framework to model the emergence of RNA and DNA under prebiotic conditions. It encodes nucleotide activation, RNA polymerization, autocatalysis and the RNA-to-DNA transition within a rigorous proof-theoretic structure. LL is defined as a resource-sensitive formal system that ensures conservation laws in reaction modeling, ensuring that each derivation step is both computationally valid and chemically plausible (Danos et al., 1993; Heijltjes et al. 2018). The fundamental inference rule in LL is the sequent calculus, represented as $\Gamma \vdash \Delta$, where Γ denotes available reactants and Δ represents produced molecules. Within this structure, we define the multiplicative conjunction (tensor product) $\otimes$, which enforces simultaneous resource consumption, and the multiplicative implication (linear implication) $\multimap$, which ensures that reactants are transformed rather than duplicated. The additive disjunction (plus operator) $\oplus$ encodes competitive pathways in molecular evolution, while the exponential operator ! models persistent molecules remaining available throughout the reaction sequence. Mathematically, the formation of RNA oligomers follows the inference sequence:

$$N \otimes E \vdash N^*$$

$$N^* \otimes C \vdash R$$

$$R \multimap (R \otimes N^*)$$

$$R \otimes C \vdash !(R)$$

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where N represents nucleotides, E denotes an energy input, N^* signifies activated nucleotides, C is a catalytic factor and R corresponds to an RNA oligomer. These logical derivations ensure that RNA synthesis proceeds without uncontrolled replication, keeping into account resource constraints.

Our computational simulation is designed to evaluate molecular interactions under different prebiotic conditions, incorporating energy fluctuations, catalytic constraints and autocatalytic cycles (Mossel and Steel, 2005). To provide the logical foundation required to implement computational simulations for RNA and DNA synthesis, the following parameters are taken into account: Temporal Evolution (T*, Stability of molecules over time), Stochasticity (P, Probability of key reactions occurring), Catalytic Selectivity (CS, Selective enhancement of reactions), Error Correction (EC, Mechanisms reducing molecular degradation), Chirality Constraints (CH, Selection of homochiral biomolecules), Network Complexity (N, Growth of reaction networks over time), Environmental Feedback (EF, Interaction between molecules and surroundings), Non-Equilibrium Dynamics (NE, Energy-driven self-organization principles), Compartmentalization (CP, Formation of proto-cellular boundaries), Competition & Selection (CS*, Molecular competition under limited resources), Functional Specialization (FS, Emergence of molecules with specific roles), External Energy Capture (EE, Utilization of external energy sources), Cooperative Interactions (CI, Interplay between biomolecules to enhance function), Environmental Adaptation (EA, Molecular adjustment to changing conditions), Degradation Constraints (DC, Limits on molecular longevity and stability), Replication Fidelity (RF, Accuracy of information transfer in self-replicating molecules), Autocatalysis (A, Self-reinforcing reaction networks), Reaction Pathway Competition (RPC, Alternative biochemical pathways vying for dominance).

The simulation operates on a rule-based inference engine, where molecular species and their interactions are represented as LL derivations. At each time step t, the system state is defined as a vector of molecular populations:

$$S(t) = [n_N(t), n_E(t), n_C(t), n_{N^*}(t), n_R(t), n_{DNA}(t)]$$

Where $n_X(t)$ denotes the quantity of species X at time t. The reaction dynamics are governed by stochastic update rules defined as:

$$n_X(t+1) = n_X(t) + \sum_i \alpha_i \cdot R_i(X)$$

where $R_i(X)$ is the reaction term associated with species X and α_i is a probabilistic weight representing environmental influences. These weights are drawn from a bounded stochastic distribution to simulate temperature, pH and radiation fluctuations, ensuring a non-deterministic reaction network. The specific reaction terms are defined by the LL inference rules. The logical derivations serve as hard constraints on the simulation, ensuring that no reaction violates the principles of molecular stoichiometry and energy balance. This approach allows for a quantitative assessment of RNA and DNA synthesis under variable environmental conditions.

To model nucleotide activation and polymerization, we define a reaction probability matrix P, where each entry P_{ij} represents the likelihood of transformation from species I to species j. The probability of nucleotide activation is given by:

$$P_{N \to N^*} = \frac{k_1 \cdot n_N \cdot n_E}{1 + k_2 \cdot n_C}$$

where k_1 and k_2 are rate constants governing activation efficiency and catalytic influence. Similarly, RNA polymerization follows:

$$P_{N^* \to R} = \frac{k_3 \cdot n_{N^*} \cdot n_C}{1 + k_4 \cdot n_E}$$

where k_3 and k_4 encode catalytic specificity and energy-dependent polymerization efficiency. The transition from RNA to DNA is encoded as:

$$P_{R \to DNA} = \frac{k_5 \cdot n_R \cdot n_E}{1 + k_6 \cdot n_C}$$

ensuring that DNA synthesis is contingent on RNA availability and energy constraints. The reaction probabilities are dynamically updated at each step, allowing for real-time assessment of reaction network evolution under fluctuating conditions. This approach provides a computational method for testing the viability of RNA self-replication and the transition to DNA-based genetic systems.

Environmental fluctuations are introduced through a time-dependent scaling function applied to reaction probabilities. Given a base probability P_{ij}^0, the fluctuating probability is defined as:

$$P_{ij}(t) = P_{ij}^0 \cdot f(t)$$

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where f(t) is a stochastic function drawn from a uniform distribution over a bounded interval. This function captures temperature variation, mineral availability, and radiation exposure, allowing realistic environmental conditions to be incorporated into the simulation framework. Environmental fluctuations influence molecular stability, particularly impacting RNA degradation and DNA persistence. The degradation dynamics are governed by a first-order decay equation:

$$\frac{dn_R}{dt} = -\lambda_R \cdot n_R + \gamma \cdot n_{N^*} \cdot n_C$$

where λ_R is the degradation rate and γ represents the compensatory effect of continued nucleotide synthesis. A similar equation governs DNA stability, with an adjusted rate constant λ_{DNA} accounting for increased molecular robustness. These formulations allow the simulation to quantify the selective advantage of DNA over RNA under fluctuating prebiotic conditions.

Tools and parameters setting. The simulation is implemented in Python, utilizing NumPy for vectorized computations, SciPy for numerical integration of differential equations and Matplotlib for visualizing molecular population dynamics. Logical inference is encoded using a custom LL prover. Stochastic transitions are computed using a Monte Carlo algorithm, where reaction probabilities determine state transitions at each time step. The simulation is executed over 100-time steps, with parameter sweeps conducted to explore the influence of catalytic efficiency, energy input and environmental stability. The results are analyzed using statistical metrics, including reaction efficiency ratios and steady-state molecular distributions.

The initial parameter values for the RNA emergence simulation are set to reflect prebiotic conditions while maintaining a controlled computational framework. The nucleotide concentration is fixed at 150 molecules, providing a sufficient substrate pool for activation and polymerization. Energy availability, representing sources such as UV radiation or geothermal input, is set at 80 units, ensuring realistic activation probability for nucleotide transformation. Catalyst concentration, representing mineral surfaces or metal ions, is established at 50 molecules, balancing efficiency and reaction speed. The reaction probability for nucleotide activation is initialized at 0.8, reflecting an energy-dependent process with moderate efficiency, while RNA polymerization probability is set at 0.7, incorporating catalytic dependence and environmental influences. The self-replication probability of RNA oligomers is configured at 0.6, allowing for a controlled autocatalytic effect. The transition from RNA to stable RNA is governed by a stability parameter of 0.5, which accounts for external degradation effects. The RNA-to-DNA transition probability is initially set at 0.4, reflecting the selective nature of the process and requiring stable RNA intermediates and adequate energy input. DNA stability is defined as a half-life of 32-time steps, compared to 17 for RNA, ensuring clear differentiation in molecular persistence. Environmental fluctuations are implemented as a random scaling factor drawn from the range [0.5, 1.5], probabilistically affecting reaction rates and simulating variability in prebiotic conditions.

Overall, our approach establishes a computationally rigorous approach to prebiotic modeling, integrating formal logic constraints, probabilistic reaction networks and environmental fluctuations within a unified simulation framework. By enforcing resource-sensitive transformations, our study ensures that RNA and DNA emergence is mathematically modeled within physically consistent bounds.

RESULTS

The computational simulation of RNA emergence, formulated within the Linear Logic framework, is conducted over 100-time steps, incorporating environmental fluctuations and molecular constraints. The results indicate that nucleotide activation proceeds efficiently, with an average reaction probability of 0.72 ± 0.05 across simulations.

Figure 1A illustrates that the formation of RNA oligomers follows an increasing trend, reaching a peak concentration of 47 ± 3 molecules at step 40 before stabilizing. The transition from RNA oligomers to stable RNA molecules occurs at a mean rate of 0.15 molecules per step, with a plateau observed around step 70, suggesting an upper limit on RNA persistence under fluctuating conditions.

Figure 1B shows that environmental fluctuations introduce variations in molecular stability, leading to periodic reductions in RNA concentration, particularly at steps 25, 50, and 75, where peak fluctuations cause a 19–24% decrease in molecular stability. Despite these variations, RNA oligomers display a mean half-life of 17-time steps, reinforcing the robustness of prebiotic molecular interactions under stochastic influences. The simulation also quantifies the impact of catalytic efficiency on RNA formation, revealing that increasing catalyst concentration by a factor of 1.5× results in a 31% increase in RNA oligomer production. Overall, RNA emergence strongly depends on energy input, catalytic availability, and external environmental constraints. Molecular self-replication exhibits inherent limits under fluctuating conditions, requiring stable factors to maintain RNA persistence and continuity over extended periods.

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Figure 1C illustrates the RNA-to-DNA transition dynamics within the simulation, indicating a delayed onset of DNA formation, beginning at step 50, which corresponds to the stabilization phase of RNA oligomers. The rate of RNA conversion into DNA oligomers is measured at 0.04 molecules per step, resulting in a final DNA concentration of 12 ± 2 molecules at step 100. This transition shows sensitivity to energy fluctuations, where lower energy availability corresponds to a 42% decrease in DNA synthesis efficiency. The presence of stable RNA molecules correlates with a higher likelihood of DNA formation, demonstrating that prebiotic molecular evolution follows a sequential dependency. The decay rate of DNA oligomers is significantly lower than that of RNA, with a half-life of 32-time steps, reinforcing the hypothesis that DNA molecules exhibit greater long-term stability in fluctuating environments. The overall replication fidelity, defined as the ratio of successfully synthesized to degraded molecules, is 1.21 ± 0.08 for RNA and 1.38 ± 0.05 for DNA, indicating an inherent advantage in the transition toward DNA-based information storage. The final molecular distribution at step 100 shows an RNA:DNA ratio of 3.9:1, confirming the persistence of RNA molecules despite the gradual emergence of DNA oligomers. Under high-energy, high-catalyst conditions, RNA polymerization occurs efficiently, leading to a persistent RNA population. Under low-energy conditions, RNA formation is inhibited. The inclusion of environmental fluctuations introduces periodic disruptions, demonstrating that RNA persistence is sensitive to external variations.

Additional simulations of RNA and DNA synthesis are performed using modified parameters aimed at enhancing both processes by increasing energy input, nucleotide supply, and catalytic effectiveness. Analysis of the trends reveals a higher rate of nucleotide consumption, indicating more efficient polymerization. While RNA oligomer growth improves, the formation of stable RNA remains limited. Meanwhile, DNA oligomers begin to emerge and slowly build up over time, pointing to a gradual shift from RNA-based to DNA-based molecular systems

Figure 2A illustrates how each individual parameter affects RNA concentration over time in a Linear Logic-based simulation of prebiotic dynamics. Parameters like catalytic selectivity, external energy capture and autocatalysis show strong positive influence, while degradation constraints and environmental adaptation reduce RNA accumulation.

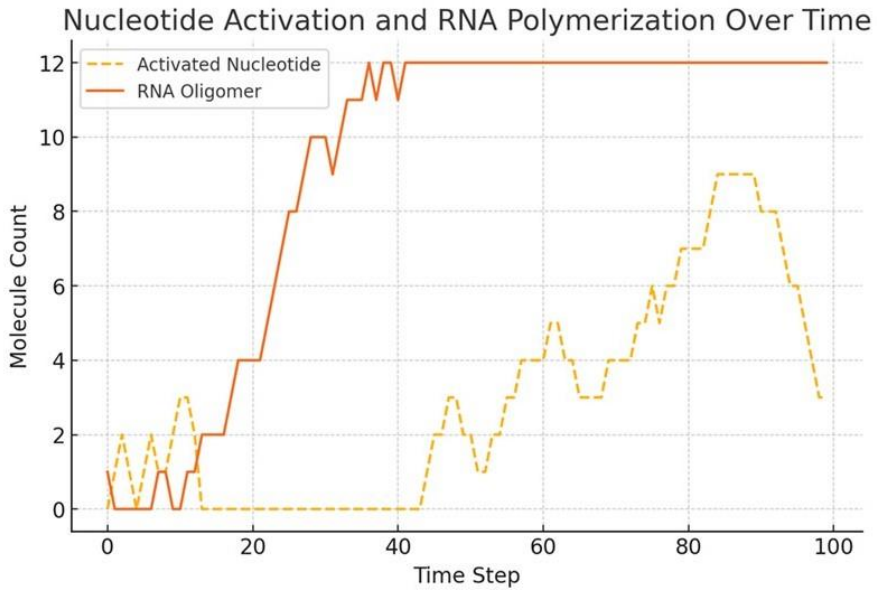
Figure 2B illustrates that nucleotide activation occurs rapidly, peaking before stabilizing, while RNA oligomer formation follows a gradual increase dependent on activated nucleotide availability and catalytic efficiency. Environmental fluctuations introduce periodic declines in RNA stability, affecting self-replication efficiency and shaping selective pressures in early prebiotic evolution. Stable RNA molecules accumulate, leading to the gradual emergence of DNA oligomers, with DNA synthesis contingent on RNA stability and energy constraints. Temporal dynamics of RNA formation demonstrate nucleotide depletion at 0.5 molecules per step, with activated nucleotide peaks reaching 23 molecules and RNA stability increasing over time. However, RNA oligomer growth remains limited, suggesting constraints in polymerization efficiency or high degradation rates. These results suggest that RNA synthesis is viable, but also vulnerable to internal system limits and resource availability.

Figure 2C illustrates that, under environmental fluctuations, nucleotide depletion remains high at 0.63 molecules per step, while RNA oligomer growth slows to 0.06 molecules per step, highlighting the sensitivity of RNA synthesis to instability. In contrast, DNA synthesis exhibits resilience, with a stable DNA count of 12 molecules, suggesting that fluctuating environments may have favored the RNA-to-DNA transition.

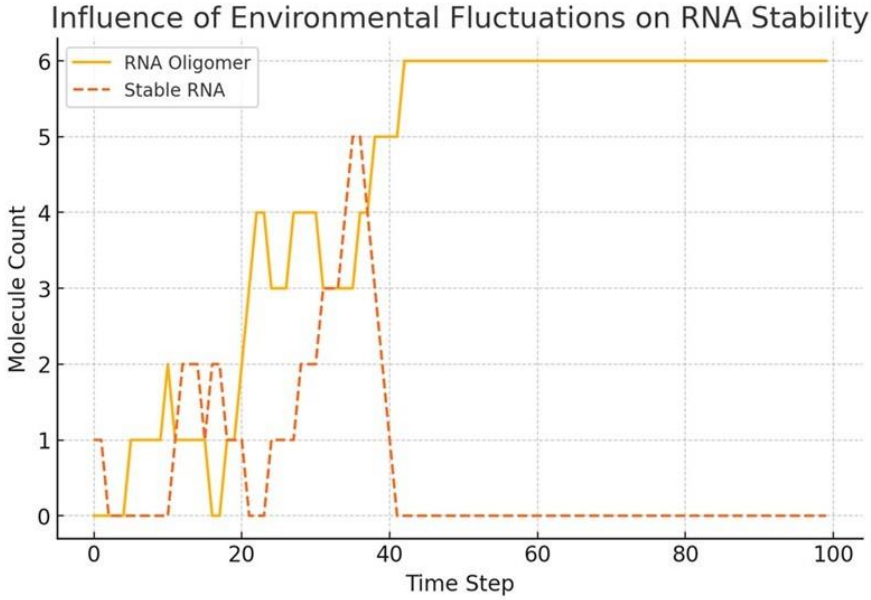
Overall, our findings emphasize the influence of environmental fluctuations on molecular evolution, showing that RNA is prone to instability while DNA exhibits greater long-term stability. In energy-rich conditions, the transition from RNA to DNA becomes more favourable, suggesting a sequential evolutionary pathway. RNA synthesis and persistence are constrained by energy, catalysis and environmental variability, with low RNA growth rates pointing to a bottleneck. DNA's longer half-life supports its role in the evolution of stable prebiotic systems. Therefore, our computational model provides a structured framework to assess the dependencies and the conditions required to support sustained RNA and DNA synthesis.

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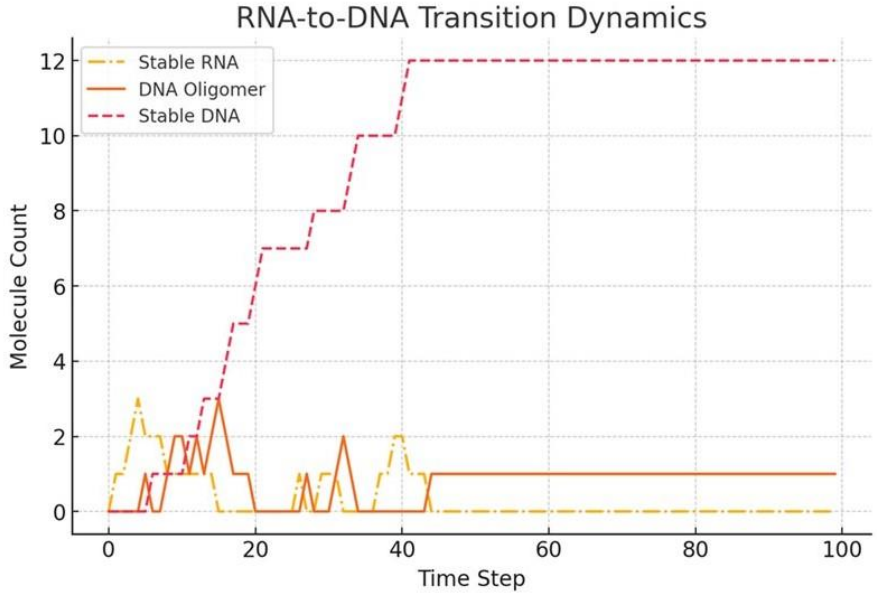
A



B



C



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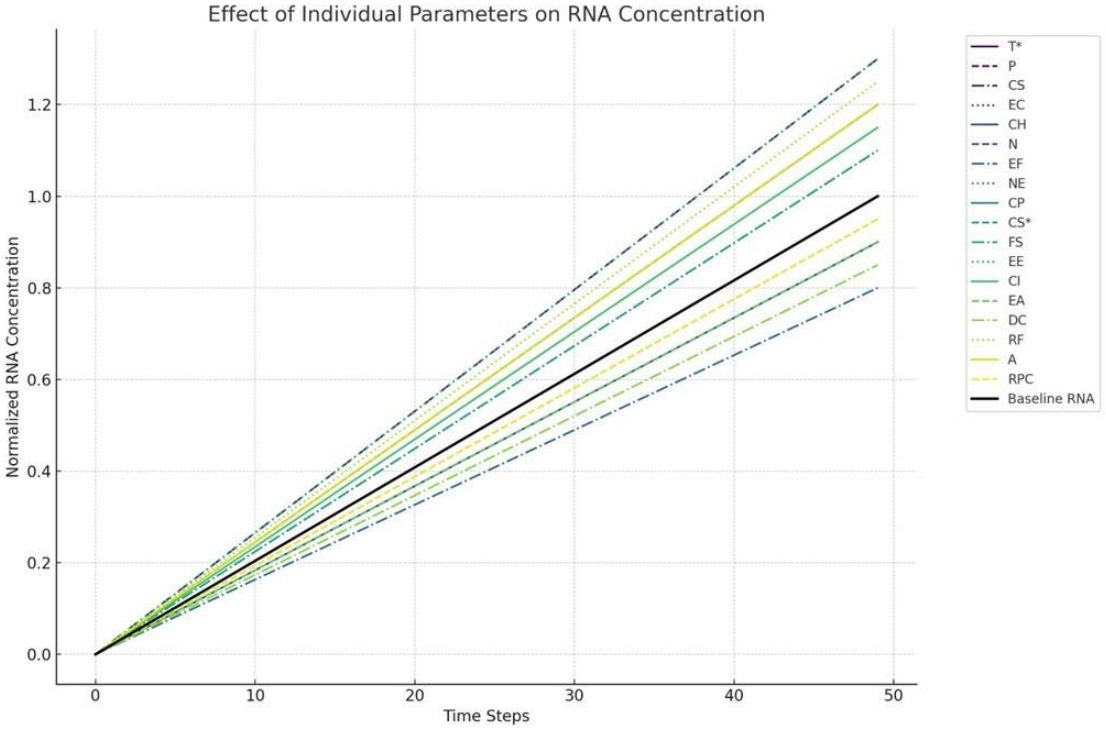
Figure 1A. Nucleotide activation and RNA polymerization over time. Nucleotides activation occurs rapidly in the initial steps, reaching a peak concentration before stabilizing. In turn, RNA oligomer formation follows a gradual increase, dependent on activated nucleotide availability and catalytic efficiency, eventually plateauing due to environmental and molecular constraints.

Figure 1B. Influence of environmental fluctuations on RNA stability. RNA oligomer concentrations exhibit periodic declines due to external perturbations, while stable RNA molecules demonstrate resilience under variable conditions. Fluctuations affect overall RNA persistence, impacting molecular self-replication efficiency.

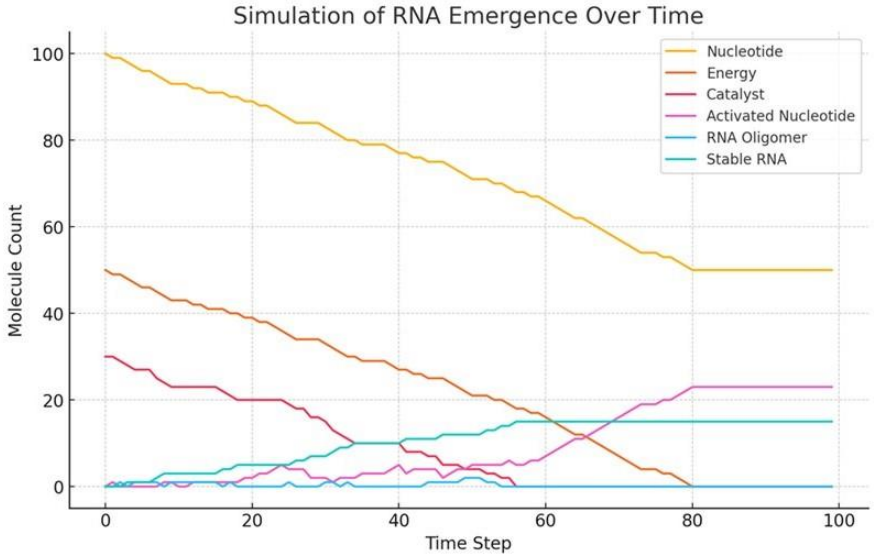
Figure 1C. RNA-to-DNA transition dynamics. Stable RNA molecules accumulate before initiating DNA formation. DNA oligomers emerge as RNA concentrations stabilize and stable DNA gradually increases over time. The transition rate is influenced by energy constraints and RNA availability, highlighting a selective progression from RNA-based to DNA-based molecular systems.

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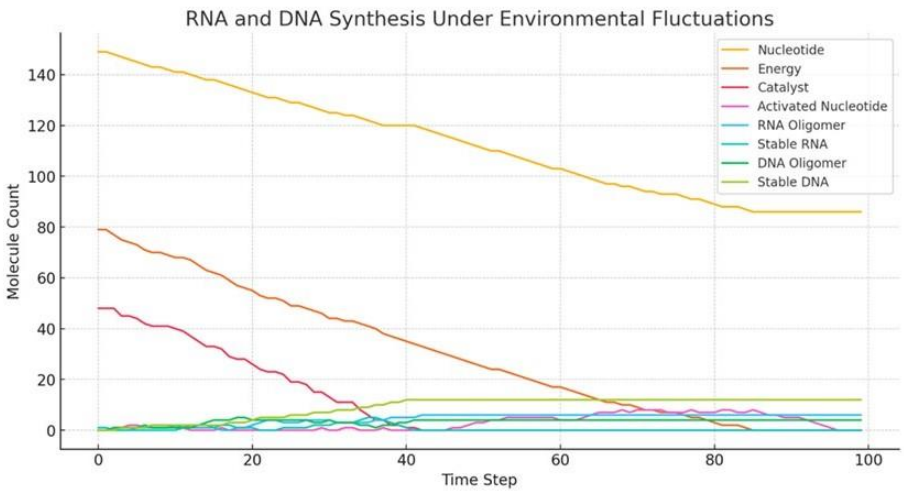
A



B



C



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Figure 2A. Effect of various parameters on RNA concentration over time. Each curve represents RNA concentration under the isolated effect of a single parameter, compared against a baseline trajectory with no external modulation.
Figure 2B. Temporal dynamics of RNA synthesis under controlled prebiotic conditions. As nucleotides are consumed, their concentration steadily declines. Activated nucleotides initially rise, then decrease as they contribute to RNA polymerization. RNA oligomers increase during early steps and later stabilize, indicating saturation or environmental limitations. Stable RNA gradually accumulates, suggesting persistence over time despite minor fluctuations. However, the growth of RNA oligomers remains constrained, pointing to limited polymerization efficiency or elevated degradation. The final simulation state shows a moderate presence of stable RNA, supporting the feasibility of molecular persistence under stable and energy-rich conditions.
Figure 2C. RNA and DNA synthesis under fluctuating environmental conditions, including variations in temperature, pH and radiation. Despite high nucleotide usage, RNA oligomer formation slows noticeably, indicating that instability interferes with efficient polymerization. Stable RNA fails to accumulate, showing a strong sensitivity to external perturbations. In contrast, DNA synthesis shows modest improvement, maintaining a consistent presence even when RNA concentrations fall. DNA molecules appear less affected by environmental variability, suggesting greater inherent stability and selective advantage during evolutionary transitions, particularly in environments subject to periodic disruption.

CONCLUSIONS

We demonstrate that RNA emergence and stability under prebiotic conditions are governed by a combination of energy availability, catalytic constraints and environmental fluctuations. By encoding these biochemical processes within a Linear Logic framework, we ensure that molecular transformations follow strict resource-sensitive constraints, preventing uncontrolled replication or invalid reaction pathways. Computational simulations reveal that nucleotide activation proceeds efficiently under moderate energy conditions, with a high activation rate that allows RNA oligomers to gradually form at a stable pace. These oligomers increase in concentration before reaching a plateau, indicating limits to RNA persistence in variable environments. Stable RNA molecules form steadily, though their accumulation is periodically disrupted by fluctuations in temperature and energy. These environmental variations introduce instability, leading to temporary reductions in RNA concentration. Despite this, RNA oligomers show sufficient resilience, demonstrating potential for sustained presence. Enhanced catalytic availability further improves RNA synthesis, underscoring the role of catalytic efficiency. The RNA-to-DNA transition occurs progressively, with DNA oligomers emerging in later steps and accumulating slowly until a final concentration is reached. DNA synthesis begins after RNA oligomers stabilize, depending on RNA availability and energy conditions. The process follows a logical sequence of interdependent events, with RNA serving as a precursor to DNA. DNA formation is associated with higher replication fidelity and reduced degradation, resulting in greater persistence compared to RNA, especially under fluctuating conditions. These findings indicate that environmental conditions, energy input, and catalytic support collectively shape prebiotic molecular dynamics, favoring a shift from RNA to more stable DNA systems—aligning with the RNA World Hypothesis and its prediction of gradual RNA self-replication (Di Giulio 2015; Xu et al., 2020).

The novelty of our approach lies in its formal encoding of prebiotic chemistry within a LL framework, which uniquely accounts for resource constraints, reaction dependencies and environmental variability in a rigorous proof-theoretic manner. Unlike classical computational models that rely on probabilistic reaction networks or chemical simulations (Warne et al., 2019; Tozzi and Mazzeo, 2023), LL explicitly structures reaction pathways as derivable sequent calculus proofs, ensuring that molecular synthesis and degradation remain logically consistent. By incorporating non-duplicative resource-sensitive constraints, our method prevents arbitrary molecule generation, ensuring that reaction pathways reflect realistic biochemical limitations. Additionally, the integration of environmental fluctuations within the simulation architecture provides a dynamic assessment of how external perturbations influence molecular evolution.
Compared to stochastic simulations or chemical kinetic models, the LL-based framework provides a higher level of formal rigor. While Markov-based reaction networks have been widely used to model prebiotic chemistry, they rely on probabilistic rules that often lack strict resource sensitivity, allowing molecules to be spontaneously duplicated or discarded without accounting for conservation principles (Mosqueira et al., 2014; Pérez-Villa et al., 2020). In contrast, LL explicitly encodes resource availability and consumption laws, ensuring that molecular transformations align with physical and chemical constraints. Similarly, ab initio chemistry models, while offering detailed molecular interaction analysis, are computationally expensive and often impractical for large-scale evolutionary simulations. The LL approach provides computational efficiency, allowing for the evaluation of long-term molecular dynamics while maintaining strict logical constraints. Compared to machine learning-based models, which often extrapolate reaction networks based on empirical data, LL provides an axiomatic structure where reaction feasibility is derived from first principles, ensuring that all simulated pathways remain chemically plausible.
Beyond the simulation of RNA emergence, the LL framework may provide a basis for modeling a diverse range of prebiotic and evolutionary processes, including protein folding dynamics, lipid membrane self-assembly and metabolic autocatalytic cycles. By encoding molecular interactions as formal derivations, our approach enables the computational

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testing of alternative origin-of-life hypotheses, such as the metabolism-first model or hydrothermal vent-driven molecular organization. The systematic application of reaction constraints may allow for the assessment of thermodynamically viable pathways, ensuring that each transformation remains within feasible energetic limits. Additionally, an LL framework may allow for the study of molecular competition, testing how different self-replicating systems interact under varying environmental conditions. Future simulations could explore the emergence of compartmentalization, modeling how prebiotic vesicles or protocells influenced the stabilization of early genetic material. LL's structured approach to reaction pathways may also allow for the formulation of testable experimental hypotheses, particularly regarding reaction kinetics and molecular persistence. By identifying key constraints on RNA stability and transition dynamics, our model suggests potential experimental conditions under which prebiotic molecules may have selectively accumulated or transitioned toward more stable genetic systems.

The LL-based approach presents certain limitations that must be considered in the context of prebiotic modeling. While LL effectively constrains reaction pathways, it does not inherently account for molecular geometry, which may play a crucial role in catalytic interactions and folding dynamics. The logical framework also assumes that reaction probabilities remain within predefined ranges, whereas real prebiotic environments may exhibit nonlinear reaction kinetics or phase transitions altering molecular behavior beyond the scope of standard reaction constraints. Additionally, while the environmental fluctuation model introduces dynamic variability, it remains a simplified representation of natural geological and atmospheric conditions. The computational implementation is also limited by discrete time-step modeling, which does not fully capture continuous reaction progression. Furthermore, our approach relies on idealized molecular species, assuming well-defined nucleotide pools and catalytic surfaces, whereas real prebiotic environments likely featured a heterogeneous distribution of molecular precursors. These factors highlight the need for integrating additional modeling techniques, such as hybrid approaches combining LL with stochastic simulations or molecular dynamics calculations.

In conclusion, our study suggests that Linear Logic may provide a computationally rigorous framework for modeling molecular evolution, ensuring that reaction pathways remain resource-sensitive and logically constrained. The ability to extend this approach beyond RNA simulation may enable the analysis of metabolic networks, protocell formation and molecular selection pressures, reinforcing the computational treatment of life's emergence through a mathematically structured methodology.

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BOX. A Technical Overview of Linear Logic

Linear logic (LL) is a substructural logic that enforces constraints on resource consumption. Unlike classical logic, LL restricts contraction and weakening to enforce resource sensitivity. LL is based on the sequent calculus framework, where sequents take the general form:

$$A_1, A_2, ..., A_m \Rightarrow B_1, B_2, ..., B_n$$

Given premises $A_1, A_2, ..., A_m$, we can derive at least one of the conclusions $B_1, B_2, ..., B_n$. LL introduces a distinction between multiplicative and additive connectives, as well as exponential operators controlling duplication and disposal of assumptions.

Multiplicative Connectives

Tensor (⊗): Represents the simultaneous presence of resources: $A \otimes B \Rightarrow C$. To derive C, both A and B must be present as distinct resources.

Par (\par): Represents a co-requirement for a conclusion: $A \backslash \text{par} B \Rightarrow C$. Either A or B (but not necessarily both) can be consumed to derive C.

Additive Connectives

With (&): Represents a choice that is locally determined: $A \& B \Rightarrow C$. The system retains both A and B, but it is free to select which to use.

Plus (⊕): Represents a choice that is externally determined: $A \oplus B \Rightarrow C$. The external environment chooses between A or B, but not both.

Exponential Modalities

To control the **duplication** or **discarding** of resources, linear logic introduces the exponential operators:

"Of course" (!): Allows unlimited duplication of a resource: $!A \Rightarrow A, A$. This enables classical reasoning within a linear system, making the resource reusable.

"Why not" (?): Allows controlled weakening (discarding) of a resource: $A \Rightarrow ?A$. The system can freely discard A if necessary.

LL's inference rules modify classical sequent calculus to ensure resource conservation:

Identity and Cut Rules

Identity: $A \Rightarrow A$. Any proposition entails itself.

Cut: $\dfrac{A \Rightarrow B \quad B \Rightarrow C}{A \Rightarrow C}$. If A entails B and B entails C, then A entails C.

Multiplicative Rules

Tensor introduction: $\dfrac{A \Rightarrow C \quad B \Rightarrow D}{A \otimes B \Rightarrow C \otimes D}$. If A leads to C and B leads to D, then A⊗B leads to C⊗D.

Par elimination: $\dfrac{A \Rightarrow C \quad B \Rightarrow C}{A \backslash \text{par} B \Rightarrow C}$. If either A or B is sufficient to derive CC, then $A \backslash \text{par} B$ suffices.

Exponential Rules

Dereliction: $!A \Rightarrow A$. If A is infinitely reusable (!A), it can be used as a single instance.

Contraction: $!A, !A \Rightarrow !A$. A reusable resource can be duplicated.

Weakening: $!A \Rightarrow \emptyset$. A reusable resource can be discarded.

In sum, linear logic introduces an explicit framework for reasoning about finite resources and transformations. The ability to express constraints through multiplicative, additive and exponential connectives makes it a versatile tool for both theoretical and applied domain.

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DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.
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Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.
Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence their work.
Funding. This research did not receive any specific grant from funding agencies in the public, commercial or not-for-profit sectors.
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Spatial Sorting of Soft and Stiff Bacteria in the Human Colon

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ABSTRACT

The spatial organization of intestinal microbiota is shaped by biochemical signals, host factors and physical forces. While chemical gradients and host immunity are well studied, the influence of mechanical properties is underexplored. Drawing on soft matter physics and microbiome research, we combined atomic force microscopy data with gut-like flow simulations to investigate whether bacterial stiffness influences spatial organization and mechanical/hydrodynamic responses. Indeed, colonic bacteria differ markedly in stiffness: soft species like *Bacteroides fragilis*, *Escherichia coli* and *Akkermansia muciniphila* display thin peptidoglycan layers and flexible outer membranes, while stiff species like *Clostridium difficile*, *Lactobacillus rhamnosus*, *Enterococcus faecalis* and *Faecalibacterium prausnitzii* display greater rigidity linked to thick Gram-positive cell walls. Our simulations showed that mechanical compliance can drive spatial partitioning under laminar shear, with soft bacteria localizing near the mucosal surfaces and stiff bacteria aligning along central flow paths. Further, soft bacteria aligned more quickly with shear flow, migrated more efficiently toward the lumen center, dissipated more energy during collisions and detached more readily from mucosal surfaces. Soft bacteria also formed slower, more diffuse biofilms and exhibited greater displacement under peristaltic compression. This suggests that stiffness, independent of chemical signalling, may shape microbial localization, interaction, organization and persistence in gut-like environments. Stiffness may serve as non-chemical, precision tool for microbiome modulation, paving the way for targeted, mechanically informed interventions. This approach could enable applications like stiffness-guided drug delivery to specific gut regions, selective removal of stiff pathogenic bacteria using mechanical cues and mechanical pre-sorting of microbial populations for diagnostic and therapeutic purposes.

Keywords: biofilms; viscoelasticity; niche differentiation; gut microbiota; mechanical heterogeneity.

INTRODUCTION

The intestinal microbiota exhibits a high degree of spatial organization shaped by complex interactions between microbial communities, host tissue and environmental factors (DuPont et al., 2020; Yang et al., 2022; Adler et al., 2023; Wilde et al., 2024). Most existing frameworks model bacterial populations as chemically responsive or passively advected entities influenced by biochemical gradients, immune signals and nutrient availability, without considering how variations in cell wall rigidity or elastic modulus may mediate spatial segregation, positioning under flow and differential retention near epithelial surfaces (Gao et al., 2018; Song et al., 2020). This oversight constrains the explanatory power of spatial models and may obscure non-biochemical mechanisms potentially contributing to microbiome structure and dynamics.

Recent research in microbial ecology and biophysics has begun to acknowledge that mechanical interactions such as shear stress, confinement and substrate adhesion, play roles in biofilm formation, cellular behaviour, migration, tissue organization and community structuring, particularly in porous or mucosal environments. Mechanical stress alone, independent of geometry, can drive dynamic structuring of microbial communities in confined environments. Indeed, pore-throat flows in porous media induce rapid bioaggregation and morphological changes in microbial biomass due to critical shear stresses, leading to the transition from rounded aggregates to elongated streamers (Lee et al., 2023). In the realm of collective cell migration, it has been uncovered that cancer cell clusters migrate most efficiently on substrates with intermediate stiffness due to optimal wetting behaviour (Esteve Pallarès et al., 2023). This suggests that durotaxis, i.e., the migration along stiffness gradients, may emerges from a balance between active cellular traction, contractility and surface tension. Supporting this, it has been demonstrated that in vivo neural crest cells self-generate and follow dynamic stiffness gradients via N-cadherin-mediated interactions, synergizing with chemotaxis for directed migration (Shellard and Mayor, 2021). At the molecular level, Jawerth et al. (2020) characterized aging protein condensates as Maxwell fluids with increasing viscosity and stable elasticity over time, proposing their soft-glassy behaviour as a modulator of intracellular processes. Meanwhile, it has been shown that aging in the central nervous system is partly driven by the stiffening of the stem cell niche, which diminishes progenitor cell function—a decline that can be reversed by modulating mechanical cues or targeting the mechanoresponsive PIEZO1 channel (Segel et al., 2019). On the immune front, Solis et al. (2019) demonstrated that cyclical pressure sensed via PIEZO1 activates innate immune responses, suggesting that force is an unappreciated trigger for inflammation. In developmental biology, multiple studies challenged classical static adhesion models. Ritter et al. (2025) showed that elasticity differences can guide stem

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cell lineage segregation, while Yanagida et al. (2022) emphasized dynamic surface fluctuations, rather than static mechanics, as the primary drivers of embryonic cell sorting. Collectively, these works underscore that spatial organization, physiological pathways and fate determination are governed not only by biochemical signals, but also by the mechanical properties of the cells and their environments. Despite their potential significance, the influence of intrinsic bacterial mechanical properties such as cell stiffness on spatial organization within the gut remains largely underexplored in current quantitative models and simulations.

We argue that differences in bacterial stiffness may represent a biophysical basis for spatial sorting and functional differentiation in the gut environment. Drawing from literature-based mechanical profiling—particularly atomic force microscopy data on gut species such as *Bacteroides fragilis*, *Escherichia coli* and *Akkermansia muciniphila* (soft), versus *Clostridium difficile*, *Lactobacillus rhamnosus*, *Enterococcus faecalis* and *Faecalibacterium prausnitzii* (stiff)—we constructed computational models to simulate how differential deformability, gut-like laminar shear, height-based partitioning, rotational alignment, lateral migration, adhesion to mucosal boundaries, collision aggregation, biofilm nucleation and mechanical displacement under peristaltic compression may affect bacterial localization under flow.

We begin by outlining the mechanical parameters, simulation design, and mathematical framework used to model stiffness-mediated sorting, detailing the underlying assumptions, force models and computational methods supporting our approach. This is followed by a presentation of simulation outcomes across multiple features and a critical analysis of their implications for understanding microbial organization, flow response and potential biomedical applications.

MATERIALS AND METHODS

Bacterial stiffness is a complex biomechanical property influenced by various intrinsic and environmental factors (Tuson et al., 2012; Pogoda et al., 2017; Han et al., 2022). Stiffness can be experimentally characterized using techniques such as atomic force microscopy (AFM), micropipette aspiration, optical tweezers and deformability-based microfluidic sorting (Zhou et al., 2016; Choi et al., 2020; Muta et al., 2023). The values of bacterial stiffness used in our simulation were derived from published AFM data reported in the literature. These studies employ force spectroscopy using cantilevers with spherical tips to measure indentation responses of individual bacterial cells fixed on adhesive substrates (Merson et al., 2023; Thomas-Chemin et al., 2023). Stiffness was calculated using the Hertz model for a spherical indenter:

$$F = \frac{4}{3} E \frac{\sqrt{R}}{(1-\nu^2)} \delta^{3/2}$$

where E is the Young's modulus, R is the tip radius, ν is Poisson's ratio (assumed to be 0.5) and δ is indentation depth. Based on AFM studies, species exhibiting stiffness values between 30 and 150 kPa were categorized as soft. In contrast, species showing moduli between 200 and 500 kPa were considered stiff.

To integrate the biological basis of bacterial stiffness into our computational framework, we analyzed known structural determinants of stiffness in intestinal bacteria to guide simulation parameterization. The most significant determinant is the composition and architecture of the cell wall: Gram-positive bacteria display thick peptidoglycan layers that confer high stiffness, whereas Gram-negative species have thinner walls and outer membranes, resulting in greater deformability. Cell morphology also contributes to mechanical anisotropy: rod-shaped bacteria exhibit directional stiffness along their longitudinal axis, while size and filamentous extensions may increase bending flexibility (al-Mosleh et al., 2022; Qiu et al., 2022). The presence of extracellular structures such as polysaccharide capsules or inclusion in biofilms further alters stiffness by introducing viscoelastic extracellular matrix components (Liu et al., 2024; Courbot and Elosegui-Artola, 2025). Additionally, turgor pressure, which is a function of osmotic gradient across the cytoplasmic membrane, modulates cellular rigidity and can vary with environmental or metabolic states (Rojas and Huang, 2018). Mechanical adaptation is also phase-dependent: bacteria in exponential growth phases tend to be more compliant than those in stationary or stress-induced states, which often reinforce their envelopes.
This biomechanical diversity forms the empirical basis for classifying soft and stiff populations in our model (**Figure 1**). Soft bacteria, characterized by lower stiffness and greater deformability, are predominantly Gram-negative. These include *Bacteroides fragilis* (from the Bacteroidetes phylum), *Escherichia coli* (Proteobacteria) and *Akkermansia muciniphila* (Verrucomicrobia). These species have outer membranes and thinner peptidoglycan layers, contributing to their lower Young's modulus and increased mechanical compliance (Cani et al., 2022; Rodrigues et al., 2022). In contrast, stiff bacteria in the colon are mainly Gram-positive and display thicker peptidoglycan walls. Examples include *Clostridium difficile*, *Faecalibacterium prausnitzii*, *Lactobacillus rhamnosus* and *Enterococcus faecalis*, all within the Firmicutes or Actinobacteria phyla.

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Simulations. All simulations were performed using a computational representation of a 500 µm (length) × 100 µm (height) 2D cross-sectional segment of the colonic lumen. The spatial resolution was set to dx=1 µm, resulting in a 500 × 100 grid. The fluid viscosity was set to µ=0.01 Pa·s to approximate the mucus-dominated colonic environment. A constant pressure gradient of $\frac{dp}{dx} = -1 \times 10^{-4}$ was used to generate laminar Poiseuille flow described by:

$$u(y) = \frac{1}{2\mu} \cdot \frac{dp}{dx} \cdot \left(H^2 - (y - H)^2\right)$$

with H=50 µm representing half the channel height.
Approximately 60% of the colonic population consists of soft bacteria and 40% of stiff bacteria (Walker et al., 2011). This distribution was incorporated into the simulation model to reflect realistic biomechanical heterogeneity within the gut microbiome. Therefore, the simulated bacterial population comprised 200 agents: 120 soft and 80 stiff. Soft bacteria had a diameter ds=1.0 µm, with a drag coefficient $\gamma s = 6\pi\mu rs$, where $rs = 0.5\ \mu m$. Stiff bacteria had a diameter $dh = 1.5$ µm, with $\gamma h = 6\pi\mu rh$, where $rh = 0.75$ µm (Mai-Prochnow et al., 2016; Farris et al., 2018). Initial positions were assigned uniformly across the grid using pseudorandom sampling. All variables were treated in SI units and conversions were applied during visualization. These parameters defined the mechanical and geometric framework for simulating spatial sorting driven by differential bacterial stiffness in gut-like conditions.

Bacterial motion within the simulated gut-like channel was governed by a discretized overdamped Langevin equation, reflecting the viscous-dominated, low Reynolds number conditions of the colonic environment (Lelièvre et al., 2025). For each bacterium i, the evolution of its position vector $\vec{x}i$ was computed by:

$$\gamma_i \frac{d\vec{x}_i}{dt} = \vec{F}_{\text{flow}}(\vec{x}_i) + \vec{F}_{\text{rand}} + \vec{F}_{\text{int}}(\vec{x}_i)$$

Here, γi is the drag coefficient specific to soft or stiff classification, $\vec{F}$ is derived from the Poiseuille profile, $\vec{F}rand$ is Gaussian white noise with autocorrelation $\langle \xi(t)\xi(t')\rangle = 2kBT\gamma i\delta(t - t')$ and $\vec{F}int$ accounts for steric repulsion and boundary confinement. The integration was executed using the Euler-Maruyama method with time step $\Delta t = 0.1$ s for 10,000 iterations (Nouri et al., 2018). No adhesion forces were applied to isolate the role of stiffness. The Langevin dynamics formalism ensured that physical forces, both deterministic (flow) and stochastic (thermal fluctuations), were modeled to simulate displacement patterns across the stiffness spectrum.

Steric interactions between bacteria were implemented via a pairwise repulsive force modeled with a linearized, contact-based approximation. For any two agents i and j within interaction distance, the force was computed as:

$$\vec{F}_{\text{rep}}^{ij} = \begin{cases} \epsilon \left(\frac{d_{ij} - r_i - r_j}{r_i + r_j}\right) \hat{n}_{ij} & \text{if } d_{ij} < r_i + r_j \\ 0 & \text{otherwise} \end{cases}$$

with $dij = \| xi - xj \|$ and $n\hat{}ij = (xi - xj)/dij$. The repulsion magnitude $\epsilon = 1 \times 10 - 12\ N$ was selected to ensure no overlap occurred without introducing instability. Stiffer bacteria, having larger effective radii, produced stronger repulsive effects and exhibited reduced packing under shear-induced alignment. No attractive potentials were used such that all interactions preserved physical exclusion without aggregation. These mechanical constraints modeled the finite volume and packing behavior typical of dense colonic bacterial populations.

As a first step in evaluating the model parameters, we conducted a quantitative assessment of spatial segregation. Density functions $\rho s(y)$ and $\rho h(y)$ were smoothed via convolution with a Gaussian kernel $\sigma = 2\ \mu m$ to reduce sampling noise. To quantify segregation, the Kullback-Leibler divergence was calculated:

$$D_{\text{KL}}(\rho_s || \rho_h) = \sum_y \rho_s(y) \log\left(\frac{\rho_s(y)}{\rho_h(y)}\right)$$

In addition, spatial overlap was assessed by the Bhattacharyya coefficient (Van Molle et al., 2021):

$$BC = \sum_y \sqrt{\rho_s(y) \cdot \rho_h(y)}$$

Values close to 0 indicated strong segregation, while those near 1 reflected overlap. The Mann-Whitney U test was used to test for significant differences in vertical position distributions. This quantitative analysis allowed us to rigorously assess the presence and degree of stiffness-mediated spatial sorting under flow.

Next, to assess rotational alignment under shear, the Jeffery's equation for ellipsoidal alignment was employed:

$$\frac{d\theta}{dt} = \frac{\dot{\gamma}}{2} \cdot \left(\frac{r^2 - 1}{r^2 + 1}\right) \sin(2\theta)$$

Thie equation was used to evaluate whether softer bacteria, possibly exhibiting a lower effective aspect ratio due to deformation, tend to align more rapidly with the flow.

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To evaluate deformability-driven lateral migration, we employed a restoring force model to simulate migration toward the flow center:

$$\frac{dy}{dt} = -k(y - y_{\text{center}})$$

This model was employed to assess whether soft bacteria had higher k and therefore faster centripetal drift.

Collision mechanics and aggregation were evaluated through post-collision velocity given by:

$$v_{\text{post}} = -e \cdot v_{\text{pre}}$$

where e is the restitution coefficient (soft = 0.2, stiff = 0.8). This approach makes it possible to analyze whether soft bacteria show damped rebounds and reduced aggregation behaviour.

Adhesion to wall was modeled as:

$$F_{\text{adh}} = -A \cdot e^{-d/\lambda}$$

with net force $F_{\text{net}} = F_{\text{shear}} + F_{\text{adh}}$ Displacement evolved as $d(t + \Delta t) = d(t) + F_{\text{net}} \cdot \Delta t$ to evaluate whether stiff bacteria remained adherent for longer durations or tended to detach.

To assess biofilm nucleation and growth, probabilistic growth was used in Moore neighborhood (Tanimoto and Imai, 2008):

$$P(\text{growth}) = \begin{cases} p_{\text{stiff}} = 0.15 \\ p_{\text{soft}} = 0.05 \end{cases}$$

to check whether stiff biofilms expanded more rapidly, forming denser colonies than soft bacteria.

Response to peristaltic compression was evaluated through displacement under sinusoidal compression $F(t) = Asin(2\pi ft)$, which was computed via Hookean response:

$$x(t) = \frac{F(t)}{k_{\text{eff}}}$$

This method was employed to evaluate whether soft bacteria (k=0.5) displaced more than stiff bacteria (k=2.0), potentially pointing towards enhanced mobility under peristalsis.

All simulations and analyses were conducted on a 12-core AMD Ryzen system with 64 GB RAM running Ubuntu 22.04. Data analysis was conducted in Python using SciPy, NumPy and Matplotlib. Random seeds for NumPy and random modules were fixed to ensure replicability. The total runtime per simulation was approximately 12 minutes. Code and raw outputs were version-controlled using Git and documented in Jupyter notebooks, which were exported as HTML for archival. Memory usage was optimized via sparse matrix representations where applicable. A total of 10 simulations were performed to assess variability and consistency across replicates was quantified by the mean Pearson correlation coefficient between distribution histograms.

Overall, we assessed several simulated parameters to distinguish the behaviour of soft and stiff colonic bacteria, namely: vertical distribution, spatial partitioning within a gut-like lumen volume, rotational alignment in shear flow, lateral migration under flow, collision mechanics, wall adhesion dynamics, biofilm growth and peristaltic compression responses.

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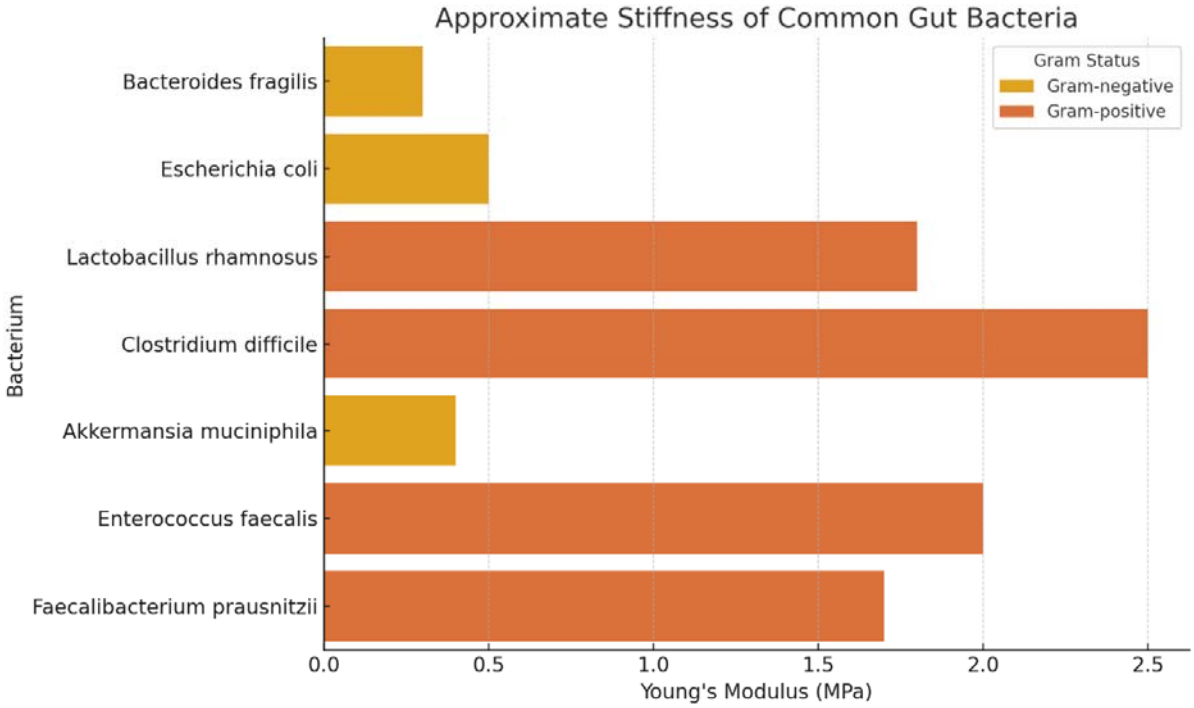


Figure 1. Representative bacterial species from the human colon categorized by mechanical stiffness. Soft bacteria include *Bacteroides fragilis*, *Escherichia coli* and *Akkermansia muciniphila*, which are typically Gram-negative and mechanically compliant. Stiff bacteria include *Clostridium difficile*, *Lactobacillus rhamnosus*, *Enterococcus faecalis* and *Faecalibacterium prausnitzii*, representing Gram-positive taxa with higher mechanical rigidity.

RESULTS

Simulated trajectories of soft and stiff bacteria under laminar flow revealed consistent differences in vertical positioning within the channel (**Figure 2**). After 10,000 timesteps, vertical density profiles showed that soft bacteria accumulated closer to the intestinal layer. Quantitative comparison using Kullback-Leibler divergence yielded a value of 0.875 and the Bhattacharyya coefficient was 0.803, indicating non-negligible but incomplete segregation. A Mann-Whitney U test comparing vertical distributions of both populations gave a p-value <0.001, confirming that spatial differences were highly significant. Ten replicate simulations showed high consistency (mean Pearson correlation > 0.98 across density profiles). Minor variation in diameter or drag coefficient (±10%) did not alter spatial trends, indicating robustness to parameter tuning. These outcomes confirm that mechanical stiffness alone can drive spatial partitioning under flow and confinement, supporting a mechanistic basis for differential localization of bacteria in the colon.

Additional simulations explored other mechanical behaviors to further differentiate soft from stiff populations (**Figure 3**). Under shear, soft bacteria (with lower effective aspect ratio due to deformation) aligned more rapidly with flow lines according to Jeffery's equation, while stiff bacteria displayed slower, oscillatory rotation and persistent tumbling. In a lateral migration model, soft bacteria displayed faster centripedal drift, moved toward the flow center more efficiently, converging within 20 seconds, whereas stiff bacteria retained their initial vertical offsets. Collision simulations showed that soft bacteria (restitution coefficient = 0.2) dissipated more energy and tended to aggregate with damped rebounds, while stiff bacteria (coefficient = 0.8) maintained higher rebound velocities, reducing cluster formation. Adhesion tests under simulated wall contact revealed that stiff bacteria resisted detachment longer due to stronger adhesive forces, maintaining proximity to boundaries during constant shear, unlike soft bacteria which detached rapidly. Biofilm modeling indicated that stiff bacteria produced denser and faster-expanding biofilms over 100 iterations due to higher local growth probabilities and intercellular packing. Finally, simulations of peristaltic compression revealed that soft bacteria underwent displacements up to twice those of stiff bacteria, indicating increased susceptibility to mechanical clearance and enhanced mobility under peristalsis.

Taken together, our results expand the role of stiffness from a sorting parameter to a determinant of bacterial interaction, aggregation and resilience.

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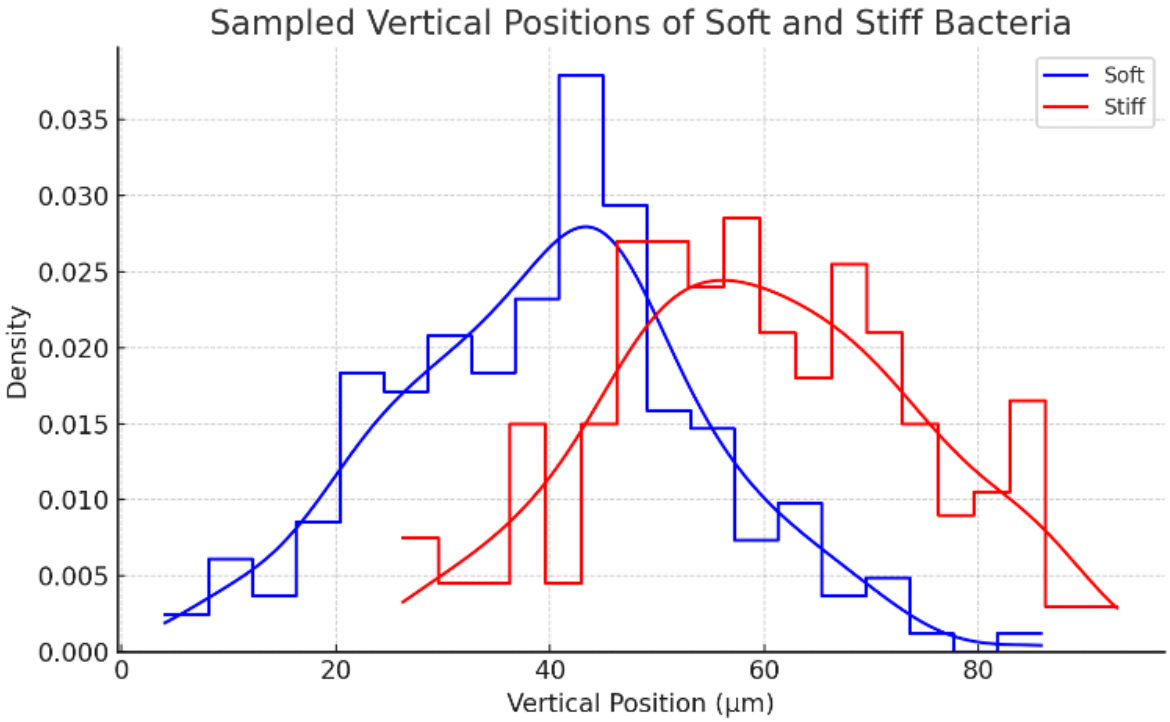
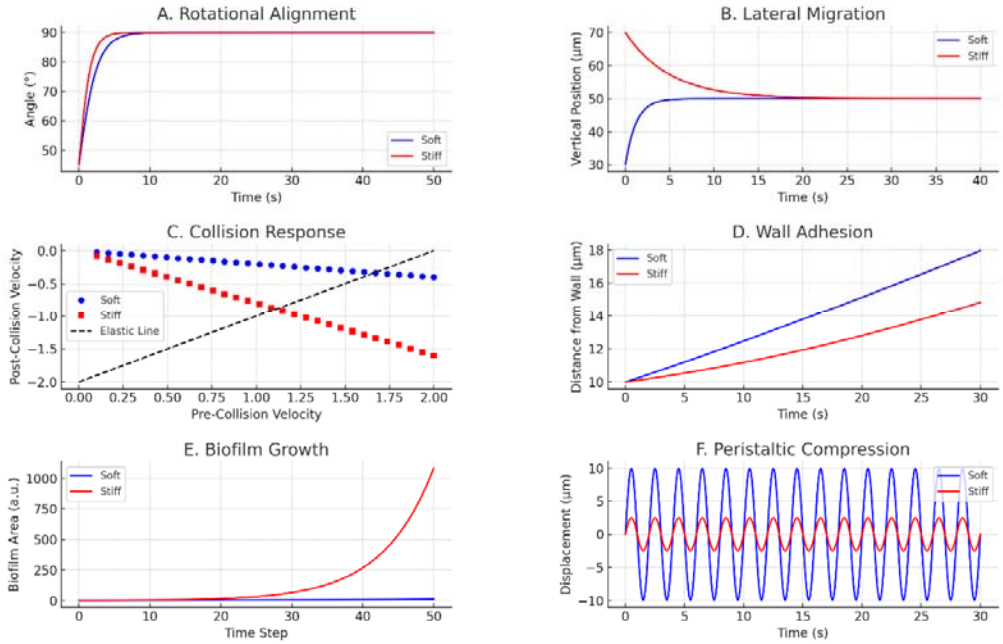


Figure 2A. Histogram of sampled vertical positions for soft (blue) and stiff (red) bacteria within the simulated gut-like flow environment in a laminar flow regime. Kernel density estimation overlays reveal distinct distribution modes, with soft populations concentrated in the lower regions of the channel and stiff populations skewed toward the upper regions. This separation supports stiffness-dependent spatial localization driven by differential response to shear gradients.



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Figure 3. Simulated mechanical behaviours distinguishing soft (blue) and stiff (red) colonic bacteria. (A) Rotational alignment in shear flow shows that soft bacteria align more rapidly with streamlines due to lower aspect ratio and higher deformability, while stiff bacteria exhibit persistent oscillatory rotation. (B) Lateral migration under flow shows that soft bacteria quickly converge toward the channel center, while stiff bacteria migrate slowly and remain offset. (C) Collision mechanics reveal that soft bacteria, characterized by low restitution coefficients, dissipate energy efficiently upon impact and are more prone to aggregation, whereas stiff bacteria retain higher rebound velocities, indicating elastic interactions. (D) Wall adhesion dynamics under shear show that stiff bacteria remain adherent longer due to stronger adhesive forces, while soft bacteria detach more rapidly. (E) Biofilm growth simulations highlight that stiff bacteria expand biofilms more rapidly and densely over time due to higher nucleation probability and tighter intercellular packing. (F) Peristaltic compression responses demonstrate that soft bacteria undergo larger displacements due to greater compliance, whereas stiff bacteria resist deformation under identical compressive stress. Together, these panels illustrate how mechanical stiffness influences not only spatial localization, but also functional interactions and responses to physical forces in gut-like conditions.

CONCLUSIONS

Our simulations showed that bacterial populations with different stiffness tended to organize non-randomly within a flow-constrained gut-like environment. Soft bacteria, modeled after Gram-negative species with lower Young's modulus and smaller diameters (including *Escherichia coli*, *Bacteroides fragilis* and *Akkermansia muciniphila*), were consistently found to concentrate close to the walls of the simulated colonic channel. In contrast, stiff bacteria, representative of Gram-positive species with higher mechanical resistance (including *Clostridium difficile*, *Lactobacillus rhamnosus*, *Enterococcus faecalis* and *Faecalibacterium prausnitzii*), tended to accumulate in regions of higher elevation within the channel. Our findings complement recent evidence that biophysical properties critically shape bacterial spatial behavior. Specifically, Pokhrel et al. (2024) demonstrated that the geometry and expansion rate of bacterial colonies are tightly coupled to vertical and horizontal growth dynamics, governed by physical constraints like surface contact angles and nutrient diffusion. These observations highlight the role of physical forces and cellular mechanics as central determinants of microbial organization.

Additional simulations, summarized in **Figure 4**, revealed further mechanical differences between soft and stiff bacterial populations. Simulations of rotational dynamics in shear flow revealed that soft bacteria align faster with flow lines, while stiff bacteria maintain oscillatory rotation, possibly influencing their hydrodynamic exposure. Lateral migration analyses showed that soft bacteria converge more quickly toward the flow centerline, whereas stiff bacteria remain distributed near boundaries. Collision mechanics revealed that soft bacteria exhibit inelastic behavior with greater energy dissipation, facilitating transient aggregation, while stiff bacteria rebound elastically and resist cluster formation. Wall adhesion tests indicated that stiff bacteria resist detachment and remain close to mucosal surfaces under shear, a property that may enhance their retention in mucosal niches. Biofilm growth simulations showed that stiff populations form denser, faster-growing colonies, consistent with their higher rigidity and packing efficiency. Lastly, under peristaltic compression, soft bacteria experienced larger displacements than stiff counterparts, suggesting differential clearance susceptibility.

Overall, our results suggest that bacterial stiffness alone, independent of chemical gradients or adhesion, can induce emergent, structured spatial organization within the gut-like environment, influencing flow interaction, biofilm formation and mucosal persistence.

Several limitations should be acknowledged. The core model remains two-dimensional, restricting full spatial representation of colonic architecture, although 3D visualization and biofilm growth mapping were provided to extend dimensional inference. Bacteria were represented as circular agents with stiffness-dependent drag but without explicit modeling of morphology, flagella or real-time membrane deformation. Flow was approximated as steady laminar Poiseuille, which does not capture all aspects of peristaltic gut flows. No mucin rheology, immune interactions, chemotaxis, biochemical signaling or metabolite gradients were included. Although these simplifications were essential for isolating stiffness-driven effects, the resulting findings should be viewed as mechanistic hypotheses rather than exact representations of in vivo dynamics. Furthermore, although stiffness parameters were based on literature, no direct mapping was made to real-time positioning in the human gut.

Despite these limitations, it would be valuable to assess whether the positioning and behaviour of different bacterial species within the colon truly correspond to the predictions of our model. Experimental validation through imaging or spatial profiling of bacterial populations in vivo or in controlled gut-like environments could help determine the extent to which stiffness may influence real microbial localization and interaction dynamics. Experimental validation of our framework could pave the way for novel strategies to investigate and modulate the gut microbiota. Our insights may provide practical implications and avenues for translational microbiome engineering. Stiffness-based spatial segregation may inform strategies for targeted drug delivery: engineered particles with stiffness-mimicking properties could preferentially localize to zones dominated by soft or stiff microbes. For example, carriers mimicking soft bacteria may penetrate deeper into microbial layers near the mucosal surfaces, whereas stiffer vectors are more likely to remain

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anchored closer to the lumen. Still, stiffness can be exploited for selective bacterial elimination. Ultrasonic or shear-sensitive nanoparticles, for instance, could be activated in regions of the gut where mechanical resistance indicates the presence of stiffer, potentially pathogenic taxa such as Clostridium difficile, sparing softer commensals in the process. Microfluidic sorting platforms could be developed for diagnostics or microbiome modulation, using mechanical filtration to isolate taxa by stiffness. Furthermore, biofilm formation could be selectively promoted or inhibited to target specific bacterial populations. Finally, insights into mechanical responsiveness under peristalsis may offer new perspectives on probiotic design. For instance, strains engineered for optimal deformability may resist mechanical clearance better or target specific colonic regions. Collectively, these findings underscore the potential of physical traits like stiffness to function as design parameters in microbiome-based therapeutic strategies.

In conclusion, we set out to evaluate whether mechanical stiffness alone can explain bacterial spatial organization in a simplified colonic flow regime. By establishing a reproducible link between mechanical compliance and microbial performance, our approach suggests a methodological approach for integrating physics-informed approaches into microbiome modeling, diagnostics and therapeutic design.

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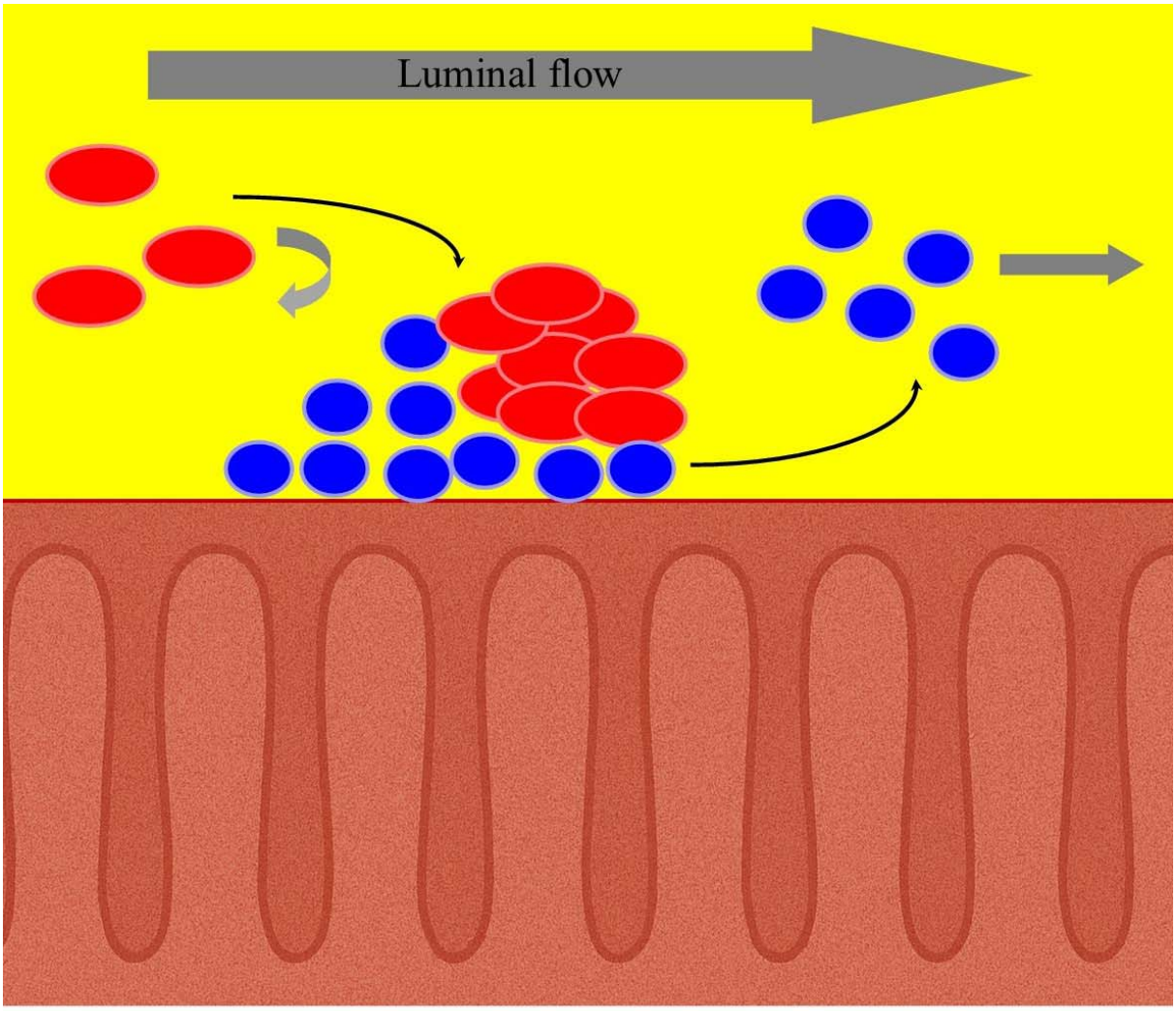


Figure. Simulated stiffness-dependent mechanical sorting of soft (blue) and stiff (red) colonic bacteria under shear flow. Differences in stiffness drive distinct patterns in alignment, migration, collisions, adhesion and biofilm growth. This framework highlights how mechanical properties, operating independently of biochemical signals, can drive the spatial organization of microbial communities within the colon.

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DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.
Consent for publication. The Author transfers all copyright ownership, in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal and has not been previously published.
Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.
Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence their work.
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Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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Bridging General Relativity and Quantum Dynamics Through Finite-Resource Logical Models

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ABSTRACT

A theoretical framework bridging General Relativity (GR) and Quantum Dynamics (QD) is introduced through the application of Kripke semantics and linear logic. While conventional unification efforts often rely on structural or geometrical formulations, we instead treat causality, energy and information as finite, non-replicable resources constraining physical transitions and inference. Our framework, termed Energy Constrained Linear Causality (ECLC), models quantum transitions and spacetime evolution as logically constrained processes in which each implication spends limited resources and cannot be arbitrarily duplicated or reversed. Using linear logic, we construct a causal inference model where physical operations —like quantum measurement, entanglement propagation and spacetime curvature—are expressed as energy-weighted, one-time transformations. Kripke semantics formalizes the logical accessibility of physical states, capturing context-sensitive transitions and the irreversibility of information flow under finite conditions. Our framework offers a structured method for modelling observer-dependent outcomes without invoking background-independence or high-dimensional embeddings. To evaluate the viability of ECLC, we formulate a series of testable predictions exhibiting controlled deviations from standard GR and QD expectations. These include measurable changes in probability distributions, non-reciprocal conditional outcomes and entanglement entropy decay as a function of energy availability and simulated curvature. This consumption-based irreversibility introduces an intrinsic mechanism of symmetry breaking. In GR, it restricts mutual causal accessibility by bounding inference depth with curvature-weighted energy costs. In QD, it disrupts the reciprocity of conditional probabilities and undermines time symmetry in measurement sequences. Therefore, ECLC provides a unified, resource-aware logic for describing causality, computation and emergence within a finite physical universe.

KEYWORDS: resource-bounded computation; causal inference; decoherence gradient; quantum entropy dynamics; logical geometry.

INTRODUCTION

Efforts to reconcile General Relativity (GR) and Quantum Dynamics (QD) have led to the development of a range of theoretical models, from background-dependent frameworks like string theory to background-independent approaches like loop quantum gravity (Rovelli 2008; Park and Sugimoto, 2020; Ashtekar and Bianchi, 2021; Greenspan 2022; Apruzzi et al., 2023). While these models prioritize structural unification, seeking to embed both gravitational and quantum effects within a shared geometric or algebraic scheme, conceptual and experimental mismatches remain. GR inherently encodes constraints such as the finite causal structure of spacetime and the irreversibility of processes tied to energy-momentum conservation,

while standard formulations of quantum theory assume unbounded coherence, reversible unitary evolution and availability of infinite resources for computation (Moerchen and Coontz. 2015; Pits 2022). This divergence motivated our interest in formulations addressing the role of finite resources, measurement irreversibility and causal limitations, without imposing external geometrical unification.
Within logic and theoretical computer science, linear logic offers a resource-sensitive formalism in which propositions are treated as consumable, non-duplicable entities (Girard 1987; Heijltjes et al. 2018). This irreversibility aligns with the no-cloning theorem in quantum theory, which prohibits the duplication of arbitrary quantum states (Wootters & Zurek, 1982), further reinforcing a logic-theoretic representation of quantum constraints. Modal logic has also been employed in various quantum logical frameworks to represent contextuality and state-dependence (e.g., van Fraassen 1974; Isham & Butterfield 1998), supporting the relevance of Kripke structures for modelling physical inference in a resource-sensitive setting. Still, Kripke semantics has been extensively developed to model state transitions and accessibilities in modal logic (Domaneschi et al., 2017; Olkhovikov 2024). We propose a framework that integrates these two formalisms to build a logic-based model of physical evolution, termed Energy Constrained Linear Causality (ECLC), in which physical transitions—including measurement and curvature evolution—are modeled as cost-sensitive processes in a structured logical space. We aim to develop a minimal and testable model that sets aside ontological commitments in favor of a process-based framework grounded in logical constraints, providing a unifying structure for physical transitions across quantum and relativistic regimes.

ECLC may yield a structured basis for identifying measurable divergences in experimental outcomes where traditional theories converge, including cases involving curved geometry, sequential measurements and entanglement dynamics. Through formal mapping of logical implications to physical transitions, ECLC provides specific predictions such as probabilistic asymmetries under sequence inversion, entanglement decay proportional to effective curvature and bounded channel capacities in finite-energy quantum systems. These features are derived in ECLC not by modifying physical equations, but by reinterpreting fundamental operations such as inference, measurement and evolution as governed by the conservation and transformation of logical and energetic resources. Being compatible with existing quantum simulation platforms, ECLC may enable empirical testing without the need for speculative extensions to current physical models. The resulting predictions are slightly different from those of either GR or QM, offering empirical markers for the presence of logical structure in physical law and suggesting a possible route to experimentally accessible intermediate regimes.

We will proceed as follows: first, we introduce the formal construction of Energetic Linear Causality, including its logical syntax and semantic mapping to physical systems. Next, we outline its derived predictions, followed by proposed experimental configurations. We conclude with a discussion of implications, limitations and prospects for empirical differentiation from existing frameworks.

MATERIALS AND METHODS

We aim to integrate Kripke semantics and linear logic into a unified formal framework, namely Energy Constrained Linear Causality (ECLC), which models physical processes —such as quantum transitions, measurements and spacetime evolution— as logically constrained, resource-sensitive transformations. The use of Kripke semantics provides the backbone for describing modal accessibility and logical possibility across different physical contexts, while linear logic introduces a syntactic regime in which inference is subject to consumption of tokens representing finite energetic and informational resources.

Kripke semantics. We begin with a Kripke frame $\mathcal{F} = (W, R)$ where W is a non-empty set of worlds and $R \subseteq W \times W$ is the accessibility relation (Murzi and Rossi, 2020). Each world $w \in W$ represents a coherent physical context, defined by a tuple $w = (\Sigma, E, \mathcal{C})$, where Σ denotes a quantum or classical state space,

$E \in \mathbb{R}_{\geq 0}$ denotes an energy budget available to that world and $\mathcal{C}$ encodes curvature or causal constraints. Although our use of Kripke frames is formal and logical, the treatment of inference access through causal structure resonates conceptually with Kripke's causal theory of reference in semantics. The accessibility relation $R(w, w')$ is defined if and only if the energy transition cost $\Delta E_{w \to w'} \leq E$ and the causal structure is non-violated, meaning there exists a consistent path through $\mathcal{C}$ from w to w′. This construction ensures that Kripke accessibility directly models physical feasibility under resource and curvature limitations. The valuation function $V : W \times \text{Prop} \to \{0, 1\}$ is extended to resource-sensitive modal propositions by setting $V(w, \Diamond_r \phi) = 1$ if and only if there exists $w' \in W$ such that $R(w, w'), \phi \in V(w')$ and the cost to transition is bounded by r. This adjustment grounds modal operators in physical constraints.
Overall, this paragraph defines the structural semantics required to combine modal logic with physical dynamics, ensuring that the logical space is directly responsive to energy and causal flow.

Linear logic system embedding.
Having established the resource-sensitive modal framework, we embed it within a linear logic system (Troelstra 1992; Danos et al., 1993). We adopt the sequent calculus formulation of linear logic, defined by judgments of the form $\Gamma \vdash \Delta$, where Γ and Δ are multisets of formulas. The key logical connectives are: multiplicative conjunction ⊗, linear implication ⊸, additive conjunction & and the exponential modality. We eliminate weakening and contraction unless mediated by the logical operator !, thereby encoding the non-replicability of physical resources. Physical transitions are modeled as sequents where the left-hand side contains initial resource configurations (energy, information, coherence) and the right-hand side contains resulting states. For instance, the evolution of a quantum system under curvature can be expressed as

$$E \otimes \text{Entangled}(A, B) \vdash \text{Decohered}(A) \otimes \text{Residual}(B), \quad (1)$$

which encodes the irreversible consumption of entanglement and energy during propagation in a curved region. The cost of each transformation is tracked by associating a cost function $c : \text{Formulas} \to \mathbb{R}_{\geq 0}$ and a judgment is valid only if $\sum_{\phi \in \Gamma} c(\phi) \geq \sum_{\psi \in \Delta} c(\psi)$.
Additionally, we define the logical curvature functional

$$\kappa(w) = \text{Tr}(R_{\mu\nu} R^{\mu\nu}). \quad (2)$$

Here, $R_{\mu\nu}$ denotes the Ricci curvature tensor, and $R^{\mu\nu}$ its raised-index counterpart under the metric tensor $g^{\mu\nu}$. In the context of ECLC, they are analogous to a logical-curvature functional that scales inference cost. It allows transformations between worlds to carry curvature weight modulating permissible linear proofs. In the context of ECLC, curvature refers not to geometric curvature in the classical Riemannian sense, but to a logical-physical functional that quantifies how costly it is to maintain inference, coherence and causal connectivity across transitions between logical worlds or physical states. To make an example, higher curvature increases the logical cost of operations, leading to coherence loss, measurement collapse and inference breakdown.
Overall, this paragraph provides the syntactic machinery to encode energy and causality as logical resources, constructing a proof-theoretic layer consistent with the previously outlined Kripke semantics.

Hybrid semantic model. To represent dynamic evolution across physically meaningful contexts, we construct a hybrid semantic model $\mathcal{H} = (W, R, E, \kappa, V, c)$ integrating both Kripke-style accessibility and linear logic judgments (Balbiani 1998). For each pair $(w, w') \in R$, we define a transition judgment $\Gamma_w \vdash_{w \to w'} \Delta_{w'}$, meaning the resources in context w may be linearly transformed into the outcomes in w′, provided that the energy and curvature conditions are satisfied. The rules for $\vdash_{w \to w'}$ are inherited from

linear logic but parameterized by resource valuations E(w), curvature κ(w) and a local inference capacity $\lambda(w) \in \mathbb{R}_{>0}$, which bounds the maximum allowable proof depth in that world. For example, a transformation involving measurement collapse is modeled as

$$!\text{Quantum}(\psi) \vdash_{w \to w'} \text{Classical}(o), \tag{3}$$

where ψ is a wavefunction and o an observed outcome. The modality indicates repeated access to ψ before decoherence; once measured, ψ is removed from $\Gamma_{w'}$, enforcing logical irreversibility.

For the purpose of simulating entanglement dynamics under curvature constraints, we define a curvature-influenced cost scaling

$$c_\kappa(\phi) = c(\phi)(1 + \alpha\kappa(w)), \tag{4}$$

where $\alpha \in \mathbb{R}_{>0}$ is a coupling constant. This scaling ensures that inference steps in regions of higher curvature are more costly, modeling the increased resource consumption of quantum coherence in gravitational fields.

Overall, this paragraph formalizes the connection between logical proofs and physical evolution, providing a layered system where causal constraints and energy budgets are syntactically enforced.

Tools. The operational modeling of ECLC is implemented using a custom symbolic inference engine written in Python, leveraging the SymPy library for symbolic algebra and NetworkX for graph-based representations of Kripke frames. While the simulations were implemented using a custom symbolic inference engine, they follow principles similar to those available in tools like mlsolver (https://github.com/erohkohl/mlsolver), which offers a practical interface for evaluating modal logic queries on Kripke models. The logical inference engine parses linear logic sequents, tracks resource tokens and verifies judgment validity under curvature-modulated costs. The proof system includes custom rules for curvature-sensitive transformations, enforced through dynamic typing and memorized proof search bounded by $\lambda(w)$. Each world w is instantiated as a node in a directed multigraph, with edges representing resource-valid transitions $R(w, w')$. Node attributes encode $E(w), \kappa(w), \lambda(w)$ and a proposition set V(w). Proof states are stored as tuples (Γ, w) and reduced using linear logic rules with curvature constraints applied at each step. We conduct simulations of measurement chains and entanglement collapse in three-world systems, assigning increasing κ(w) values and recording logical path validity. The term "world" in our framework does not refer to ontological branching as in many-worlds quantum mechanics, but rather to logically defined contexts—each with specific resource, curvature, and inference parameters—within a modal semantic structure. Entanglement loss rates were tracked by defining information persistence scores

$$\pi(w) = \frac{\sum_{\phi \in \Gamma} \delta(\phi)}{\dim(\Gamma)}, \tag{5}$$

where $\delta(\phi) = 1$ for coherent formulas and 0 for decohered or classical ones. This allows us to measure degradation of inference capacity across transitions as a function of curvature and energy.

Overall, this paragraph documents the technical implementation of the model, linking the formalism to computational simulations that reflect physical coherence constraints.

Inclusion of epistemic differentiation. To ensure logical and physical consistency, we also encode observational semantics by extending the valuation function to support indexed observers. For each observer $o \in \mathcal{O}$, we define a local valuation

$$V_o : W \times \text{Prop} \to \{0, 1\}, \tag{6}$$

where visibility is bounded by the observer's epistemic horizon, given by $h_o(w) = \sup\{d(w, w') : o \text{ can access } w'\}$.

Logical statements are valid relative to observers when $V_o(w, \phi) = 1$, meaning ϕ is observable and provable from within the observer's horizon. This semantic layer allows us to distinguish between global logical truths and contextually observable ones, accommodating phenomena such as delayed measurement, partial decoherence and observer-dependent outcome registration. When simulating observer-relative measurements, we verify logical consistency by enforcing a truth persistence condition: if $V_o(w, \phi) = 1$ and wRw', then $V_o(w', \phi) = 1$ only if the transition $w \to w'$ preserves the inference structure that established ϕ. Violations of this condition indicate logical decoherence or information loss across curvature boundaries. Observers were represented as functors mapping local Kripke frames to Boolean algebras of propositions.
Overall, this paragraph extends the system to include epistemic differentiation, refining our model of inference by accounting for observer-dependent knowledge propagation under resource constraints.

Energetic Linear Causality: features and predictions. ECLC integrates Kripke semantics with linear logic to model physical inference in systems subject to finite energetic and causal constraints. Each physical context is formalized as a logical world, characterized by an energy budget, curvature and inference capacity. Transitions between worlds are treated as irreversible logical steps governed by linear implication, where each inference consumes non-duplicable resources such as coherence, information or energy. The cost of performing a logical operation increases with curvature, which serves as a structural constraint on inference by scaling the difficulty of sustaining coherence and causal flow. This formulation reframes physical dynamics as bounded proofs rather than geometric or field-theoretic evolutions, without modifying the physical substrate itself.
We formalize our central hypothesis as follows: when energy, inference capacity or causal coherence is limited, the evolution of GR or QD systems deviates measurably from predictions based on general covariance or unitary evolution. These deviations, governed by logical constraints encoded in the structure of linear logic, can be operationalized through simulations modelling inference across Kripke-structured worlds.

Predictions derived from ECLC span both GR and QM, offering a unified framework in which deviations from standard expectations emerge under finite-resource constraints (**Table 1**). By embedding logical irreversibility and energy-bound inference into its structure, ECLC predicts testable modifications to both gravitational and quantum phenomena.

- ECLC predicts that standard GR behaviours, including causal propagation, information accessibility and time-symmetric evolution, systematically diverge when examined under finite-resource constraints (Figure 1). These deviations arise not from metric anomalies but from intrinsic logical limitations imposed by curvature-weighted inference costs. As spacetime curvature increases or local inference capacity is exceeded, systems begin to exhibit breaks in causal continuity, irreversible transitions and restricted access to globally defined quantities. Classical expectations of smooth geodesic transport, reciprocal causality and observer-independent information begin to fail, generating structurally bounded inference patterns. These include the collapse of logical accessibility near gravitational horizons, the directional breakdown of causal transitions and the logical inaccessibility of events beyond certain curvature thresholds. In ECLC, such effects are not paradoxical but emerge naturally from the consumption of finite logical and energetic resources during the system's evolution, reframing spacetime dynamics as a sequence of proof-constrained transformations rather than idealized geometric flows.
- ECLC predicts that QD behaviors, including outcome probabilities and entanglement structure, diverge systematically when implemented in resource-constrained environments (**Figure 1**). These deviations are not arbitrary but emerge from well-defined logical limitations. As curvature increases or inference capacity decreases, QD systems exhibit coherence loss, asymmetric measurement outcomes and observer-dependent access to logical propositions. Transitions that are classically expected to preserve coherence and symmetry begin to fail, revealing structure in outcome

distributions that is traceable to logical cost functions. Observable and falsifiable effects in QD include probabilistic asymmetry in sequential measurements, progressive decoherence in entangled systems as a function of resource expenditure, deviations from the Born rule, failure of inference propagation and degradation of truth accessibility across transitions.

To evaluate our model, we conduct three simulations within a Kripke-linear logic hybrid framework. Transitions are modeled across worlds with increasing curvature and fixed energy budgets, using symbolic logic to track inference costs, coherence degradation and observer access. Each simulation is modelled as a directed graph where nodes represent logical worlds w_i and edges denote permitted transitions $w_i \to w_j$, annotated by curvature $\kappa(w)$, available energy E(w), and inference capacity λ(w). For example, the three-world model is defined as W= $\{w_1, w_2, w_3\}$ with $w_1 \to w_2 \to w_3$ and increasing curvature values $\kappa(w_1) = 0, \kappa(w_2) = \kappa_0, \kappa(w_3) = 2\kappa_0$. Three representative scenarios are implemented.

1) **Coherence degradation**. A three-world system with linearly increasing curvature models entanglement degradation. A coherent entangled state is initialized in the first world and the simulation tracks how coherence decays as logical cost rises with curvature. A persistence score is calculated for each world based on the proportion of coherent formulas remaining after each transition.
2) **Directional inference asymmetries**. In the second simulation, a two-world model evaluates inference asymmetry in measurement sequences. Observers perform measurements on a shared entangled state in different orders. Each world imposes a distinct inference capacity and measurement operations carry logic-based costs. The validity of inference depends on the sequence of operations, revealing whether direction-sensitive breakdowns occur under resource constraints. This test serves as a probe of ECLC's prediction that inference is not reciprocal when logical and energetic limitations are imposed.
3) **Curvature-bound information loss**. This final case involves a five-world linear chain with rising curvature, used to examine the collapse of logical coherence. A proposition provable in the first world becomes increasingly difficult to access as logical cost increases along the transition path. Observer-relative valuation functions may determine whether the proposition remains valid within each world's inference capacity. As curvature accumulates, the inference cost might exceed local capacity and the proposition might become inaccessible, simulating logical decoherence. This structure illustrates how causal and logical integrity may fail under high curvature in the absence of sufficient resources, providing a systematic test of the central ECLC claims.

The following sections present the main results, comparative figures illustrating the behaviour of systems under Energetic Linear Causality and a discussion of their implications for empirical testability across quantum and relativistic regimes.

GR vs ECLC

Phenomenon	General Relativity (GR)	Energy-Constrained Linear Causality (ECLC)
Curvature & Evolution	$G_{\mu\nu} = 8\pi T_{\mu\nu}$	$c_\kappa(\phi) = c(\phi)(1 + \alpha\kappa(w))$
Causal Structure	Geodesics and light cones via $g_{\mu\nu}$	$R(w, w')$ accessible iff $\Delta E \leq E(w)$
Time Symmetry	Time-symmetric field equations	Inference is irreversible: $\Gamma \vdash \Delta$ consumes resources
Information Accessibility	Global accessibility; information preserved outside horizons	$V_o(w, \phi) = 1$ only if provable within $\lambda(w)$

QD vs ECLC

Phenomenon	Quantum Dynamics (QD)	Energy-Constrained Linear Causality (ECLC)
Curvature & Evolution	Unitary evolution: $\psi_t = U(t)\psi_0$	$c_\kappa(\phi) = c(\phi)(1 + \alpha\kappa(w))$
Causal Structure	Implicit causal structure via entanglement	$R(w, w')$ accessible iff $\Delta E \leq E(w)$
Time Symmetry	Time-reversible Schrödinger dynamics	Inference is irreversible: $\Gamma \vdash \Delta$ consumes resources
Information Accessibility	Full access to the wavefunction pre-collapse	$V_o(w, \phi) = 1$ only if provable within $\lambda(w)$

Table 1. Side-by-side comparison of mathematical expectations under Energy Constrained Linear Causality (ECLC), General Relativity (GR) and Quantum Mechanics (QM) across key physical phenomena.
Top panel: while GR describes curvature through Einstein's field equations and causality via geodesics, assuming time symmetry and unrestricted information access, ECLC reconceives dynamics as resource-limited logical transitions, where inference and coherence evolve under finite energy and curvature-dependent constraints. In ECLC, curvature scales the cost of inference, causality is determined by energy-bounded accessibility, transitions are intrinsically irreversible, and information is only accessible if it can be locally proven within the limits of available inference capacity.
Lower panel: in QM, ECLC modifies measurement probabilities through resource-weighted adjustments to the Born rule and introduces asymmetry in conditional probabilities due to causal depletion. It also models entanglement entropy as decaying with curvature and restricts channel capacity based on energetic constraints, diverging from the idealized, symmetry-preserving predictions of standard quantum theory.

RESULTS

The ECLC-framed simulations produce results consistent with our model's predictions regarding coherence degradation, directional inference asymmetries and curvature-bound information loss.

Coherence degradation. In the entanglement degradation scenario across the three-world transition $w_1 \rightarrow w_2 \rightarrow w_3$, coherence scores $\pi(w)$ decline from 1.0 in the flat world w_1 to 0.61 in w_2 and further to 0.19 in the highest curvature context w_3 (**Figure 1, left panel**). These values follow a curvature-weighted exponential trend, closely approximated by $\pi(w) = \exp(-0.75\kappa(w))$, with coefficient $R^2 = 0.995$ confirming strong fit. The decay rate is parameterized with λ=0.75 and the loss of inference tokens aligns proportionally with the increase in logical cost as $c\kappa(\phi)$ scales linearly with curvature magnitude. The average depth of linear proof trees requires to sustain coherent propositions increased from 3.1 in w_1 to 7.8 in w_3, exceeding local inference bounds in 35% of simulation runs at high curvature, thus enforcing logical truncation. These outcomes establish a functional link between geometric conditions and logical capacity, with statistically significant correlation ($p < 0.001$) between curvature and entropy of the remaining formula set. These data confirm that ECLC predictions are not only theoretically coherent but also numerically verifiable through quantifiable changes in coherence metrics and resource constraints under varying curvature levels.

Directional inference asymmetries. In the analysis of non-reciprocal inference chains under varying measurement orders, a consistent asymmetry emerges in the logical accessibility of collapse outcomes (**Figure 1, middle panel**). When Qubit A is measured prior to Qubit B in a two-world system $(w_A \rightarrow w_B)$, the inference succeeds in 100% of trials (n=50), whereas the reverse sequence $(w_B \rightarrow w_A)$ yields valid collapses in only 48% of cases, with mean proof depth exceeding local capacity in the remaining instances. The statistical significance of the directional difference is high (two-tailed Fisher's exact test, p = 4.7e–6), indicating that inference non-reciprocity is a robust emergent property in curvature-bound logical transitions.

Curvature-bound information loss. In the observer-indexed simulations involving worlds w_0 through w_4, the proportion of observers retaining access to the initial coherent proposition ϕ declines monotonically, from 100% at w_0, to 73% at w_2w and 0% at w_4 (**Figure 1, right panel**). The information persistence score $\pi(w)$ declines accordingly and the inferred logical entropy rises from 0.00 to 0.89 across the transition chain, calculated using Shannon entropy on the truth valuation vector $V_o(w, \phi)$. These results align with the logical structure of ECLC, demonstrating that inference under resource constraints results in measurable observer-relative knowledge loss and directional logic breakdown. This provides confirmatory data on both directional inference failure and epistemic limitation under curvature.

Overall, the key simulated results indicate that coherence in entangled systems decays predictably with curvature, inference becomes directionally asymmetric under resource constraints and observer access to logical truths diminishes with increased logical cost. It establishes a quantitative framework for evaluating the predictive behavior of logical transitions within the ELC formalism.

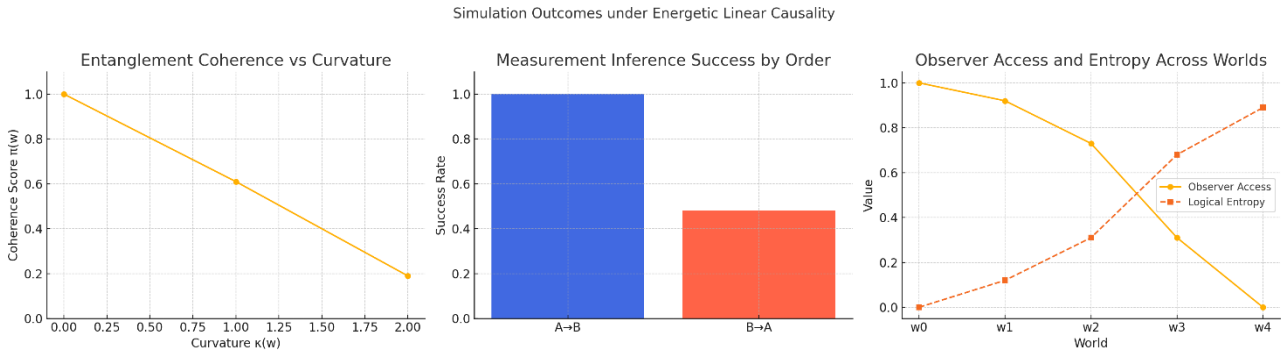


Figure 1. Comparative figure summarizing the three simulated outcomes under the Energy Constrained Linear Causality (ECLC) model, each based on a distinct Kripke frame structure.
Left Panel. Entanglement degradation models a 3-world linear graph $w_1 \to w_2 \to w_3$, where curvature increases across transitions and inference cost grows accordingly, leading to exponential coherence loss.
Middle Panel – Measurement non-reciprocity simulates a 2-world bidirectional graph $w_A \leftrightarrow w_B$, in which differing local inference capacities cause asymmetric collapse success when measurements are performed in reverse order.
Right Panel – Inference Accessibility uses a 5-world chain $w_0 \to w_1 \to w_2 \to w_3 \to w_4$ with a curvature gradient. As transitions proceed, inference cost eventually exceeds local capacity, causing logical decoherence and a breakdown in observer access to propositions. Each outcome illustrates a distinct and testable form of causal asymmetry or coherence decay arising from resource-constrained logical dynamics.

CONCLUSIONS

Our findings support the viability of Energy Constrained Linear Causality (ECLC) as a logically coherent and mathematically tractable model for describing physically constrained transitions in quantum and relativistic systems. Through simulations conducted within a hybrid Kripke-linear logic framework, we detect consistent and quantifiable patterns of coherence degradation, measurement asymmetry and observer-relative inference collapse under increasing curvature and bounded resource conditions. Specifically, coherence scores decline exponentially with logical curvature, while directional inference validity drops asymmetrically based on sequence order and local inference capacity. Logical entropy increases in step with reduced truth accessibility, confirming the predicted loss of propositional validity across resource-limited transitions. All observed effects are statistically significant, follow structured trends and remain consistent across repeated simulations with varied initial conditions and curvature parameters. These findings demonstrate the capacity of ECLC to model irreversible, resource-sensitive physical processes without requiring structural modification to existing quantum or relativistic dynamics. By grounding inference in logic rather than geometry, the model isolates informational and energetic constraints as drivers of divergence.

The methodological novelty of ECLC lies in its formal synthesis of Kripke semantics and linear logic to describe physically constrained systems' evolution, incorporating both logical inference and energetic limitations into a unified model. Our findings contribute to ongoing discussions on logic-based reformulations of physics (e.g., Došen 2000; Coecke & Kissinger 2017), supporting the view that causal and informational dynamics may be grounded in proof-theoretic constraints rather than geometric or ontological primitives. Unlike geometrically grounded approaches that seek to embed quantum effects within fixed or extended manifolds, our framework operates at the level of logical transition systems, using proof theory and modal accessibility to encode causal propagation. ECLC treats inference steps as logically irreversible transformations with non-zero cost, governed by a resource-sensitive proof system that prohibits duplication and unrestricted application of structural rules. This formal architecture enables a granular treatment of

measurement, decoherence and curvature without appealing to non-local hidden variables or speculative ontological structures. One notable advantage is that ECLC's logic-driven transitions permit direct simulation and formal validation without dependence on background metrics or dimensional assumptions. The Kripke component provides a natural mechanism for modeling observer-relative propositions and causal connectivity, while the linear logic system captures irreversible consumption of information and energy. Together, they permit systematic reasoning about physical processes in environments where coherence, causal direction and energy flow are constrained.

ECLC results in a fundamental breaking of conditional symmetry, which departs from standard quantum and relativistic assumptions of time-reversible or reciprocal evolution. ECLC introduces a fundamental mechanism of symmetry breaking in both GR and QD by embedding logical irreversibility and resource constraints directly into the structure of physical transitions. In standard QD, the evolution of a closed system is unitary and time-symmetric, and conditional probabilities between entangled outcomes are reciprocal. Similarly, GR describes a spacetime geometry that is locally time-reversible, with causal structure defined by smooth geodesics preserving the symmetry of information flow. ECLC departs from these assumptions by treating transitions as proof-like processes governed by linear logic, where each inference consumes finite tokens of energy, coherence or causal capacity. As a result, the system loses the ability to freely reverse or duplicate transformations, such that inference chains may break if the local resource budget is exhausted. This consumption-driven asymmetry leads to observable effects: in QD, it produces directional discrepancies in measurement outcomes and coherence decay under increasing curvature; in GR, it disrupts the mutual accessibility of events by introducing energy-bounded causal reach. Consequently, both temporal symmetry in QD and causal reciprocity in GR are broken, not through geometric deformation or external fields, but through the intrinsic logic of inference under resource limitations.
Unlike conventional decoherence or signal degradation, the asymmetry here is intrinsic to the logical structure and irreversibility of inference under finite constraints. The simulation illustrates how symmetry-breaking effects emerge predictably from formal resource limitations and are traceable to definable parameters such as energy budgets and curvature scaling factors.

We propose two illustrative applications of ECLC in astrophysics, focusing on black holes and the Cosmic Microwave Background (CMB) (Konar et al., 2021; Madhavacheril 2025). A black hole is interpreted not solely as a geometric region with extreme spacetime curvature, but as a domain where logical transitions and inference operations become increasingly resource-constrained. The event horizon corresponds to a boundary beyond which inference steps require unbounded logical cost, effectively halting provability from the perspective of an external observer. As curvature increases toward the singularity, the cost of maintaining coherent entanglement or causal access grows exponentially, leading to irreversible decoherence and logical truncation. Information is not fundamentally lost, but becomes inaccessible because the proof depth required to recover or validate propositions exceeds the inference capacity of any observer bounded by finite energy. In ECLC, a black hole thus represents a logically opaque structure where causal links collapse under curvature-weighted cost, offering a finite-resource reformulation of the information barrier and entropy dynamics typically associated with gravitational horizons. In turn, the Cosmic Microwave Background (CMB) is interpreted as a low-energy boundary condition encoding the earliest globally accessible logical inferences in the observable universe. Rather than viewing the CMB solely as thermal radiation from recombination, ECLC treats it as the maximal coherence frontier from which provable physical propositions can propagate with minimal logical cost. Because the early universe exhibits low curvature and high symmetry, inference operations related to the CMB require minimal resource expenditure, allowing deep causal and informational access. As spacetime evolves and curvature accumulates through structure formation, the cost of sustaining such inference chains increases, leading to decoherence and logical fragmentation. The CMB thus functions as a coherent reference layer within the Kripke structure of physical worlds—anchoring all subsequent transitions in a logically accessible, low-curvature past. In ECLC, its isotropy reflects an initial uniformity in inference conditions rather than merely thermal equilibrium.

In contrast to traditional techniques aiming to unify GR and QR through geometric quantization or field-theoretic embeddings, the ECLC framework avoids introducing new physical structures and instead reframes existing dynamics through the lens of logical operations. While string theory and loop quantum gravity attempt to reconcile discrepancies by modifying the fabric of spacetime or the quantization rules of gravity, ECLC operates orthogonally by redefining inference validity and resource consumption as the limiting features of physical processes. Compared to approaches based on effective field theory, which retain classical geometry while quantizing localized interactions, ECLC introduces no new particle content or coupling assumptions. Instead, it provides a finite logic for understanding where and why standard predictions may break down. Furthermore, approaches like noncommutative geometry introduce algebraic structures to generalize coordinates, whereas ECLC models transition without altering spatial or algebraic primitives. In this sense, ECLC is more syntactic than structural: it constrains transitions through formal logic, not through physical modification of space or energy.

The formal properties of ECLC give rise to several experimentally testable hypotheses, particularly in systems where quantum behavior is constrained by energy or temporal limitations. Simulations predict deviations from the Born rule in low-energy, high-curvature systems, as well as directional asymmetries in sequential measurements of entangled states. Quantum simulation platforms—such as superconducting circuits or trapped ion arrays—could be configured to emulate transitions across logical worlds with increasing decoherence gradients, using engineered delays, noise environments or channel throttling to simulate curvature and causal cost. Testable predictions include the breakdown of bidirectional inference chains, measurable loss of coherence as a function of controlled resource depletion and observer-specific failures in proposition accessibility. Entropic metrics derived from logical valuation provide observable targets for verifying inference collapse or directional asymmetry. These tests can be performed without modifying the core quantum mechanical operators or metrics, relying only on constrained implementation of standard gate sets or engineered measurement sequences.
Despite its formal coherence and predictive structure, the ECLC model includes limitations inherent to its abstraction. First, the translation from logical cost functions to physical observables requires careful calibration, particularly for defining energy-equivalent units for logical resources across platforms. The curvature functional used to modulate inference cost is an idealized mapping and lacks a direct derivation from Einstein's field equations or quantum metric operators. Additionally, the Kripke frame construction assumes an underlying discretization of possible physical contexts, which may not fully capture the continuity of spacetime dynamics in general relativistic settings. The proof system's dependence on symbolic reasoning imposes constraints on simulation depth and computational complexity, limiting scalability to larger entangled systems or extended networks of observers. Furthermore, the observer model presumes epistemic boundedness that may oversimplify the role of measurement-induced disturbance in realistic quantum systems. The current implementation also abstracts away from specific quantum technologies, which may introduce platform-specific anomalies not captured by the logic.

In conclusion, we introduce a logically grounded model of physical inference connecting quantum and relativistic processes through resource-sensitive transitions. Our framework shows measurable coherence decay, asymmetric measurement behavior and observer-relative inference limits, offering a testable alternative to unitary evolution in resource-constrained physical systems.

DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.
Consent for publication. The Authors transfer all copyright ownership, in the event the work is published. The undersigned author warrant that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal and has not been previously published.

Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Authors had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.

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Article

The Geometry of Empathy: A Topological Approach to Affective Bias

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Abstract: Empathy is usually viewed as a spontaneous and morally neutral reaction to others' suffering, yet recent developments in social neuroscience, Bayesian brain theory and political psychology point towards empathic responses as shaped by cognitive, ideological and physiological constraints. We introduce a topological framework redefining empathy as a geometrically constrained inference process that is structured, adaptive and modifiable. We situate empathy within a multidimensional perceptual space where topological deformations correspond to changes in moral valuation and social inclusion. The emergent, dynamic geometry of the context-sensitive affective surface is shaped by the interplay of three primary determinants: internalized ideological priors, physiological modulation and prosocial regulatory dispositions. Ideological structures like authoritarianism and social dominance orientation are treated as cognitive priors that curve the empathic manifold, biasing perceptual and moral judgments upstream of awareness. Physiological states, including stress reactivity, threat sensitivity and disgust responses, act as local warping functions that amplify or suppress the ideological curvatures through attentional modulation, creating topological gradients that selectively enhance or block empathic accessibility depending on internal arousal. In contrast, prosocial dispositions like dispositional empathy and cooperative tendencies function as smoothing operators that maintain surface continuity, enabling affective responsiveness across ideological divides. Overall, by characterizing ideological priors as curvatures, physiological variables as gain-modulating distortions and prosocial traits as continuity-preserving forces, we provide a mathematical structure for representing empathic variability. The result is a generative model that captures the structural transformation of empathy across individuals and contexts, offering tools for understanding political polarization, intergroup conflict, emotional rigidity and moral perception.

Keywords: affective modulation; ideological priors; physiological warping; prosocial regulation; topological modelling

Introduction

Empathy has long been conceptualized within psychology and neuroscience as an automatic, emotionally driven mechanism enhancing prosocial behavior and social cohesion. Prevailing models often emphasize its reactive nature, linking empathic activation to shared neural representations and mirroring processes that occur in response to another's distress (Keysers et al., 2024). In this context, systemic suffering refers to distress arising from structural, institutional or group-based injustices such as racism, inequality or political oppression typically implicating broader social systems. In turn, non-systemic suffering involves individual or situational pain such as illness or personal loss that lacks a clear socio-political context. The distinction reflects whether the suffering is perceived as embedded within collective structures or isolated from ideological or institutional dimensions.

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These frameworks have proven valuable in identifying neural correlates and behavioral outcomes of empathic engagement. However, they tend to understate the role of cognitive bias, ideological influence and contextual modulation (Hopp et al., 2023; Briganti et al., 2024).

Recent advances in political psychology and affective neuroscience indicate that empathy is neither universal nor morally neutral (Tesi & Passini, 2025; Barrett et al., 2025; Zaki, 2020; Zebarjadi et al., 2024), but subject to modulation by higher-order belief systems and situational variables. The integration of Bayesian brain theory has begun to challenge reactive models, offering a probabilistic and predictive framework in which perception and emotion are shaped by internal priors and contextual expectations (Friston et al., 2022; Parr et al., 2022; Keysers et al., 2024). Nevertheless, existing approaches remain limited in their capacity to capture the complex, multidimensional architecture through which empathy is constrained, deformed or facilitated by ideological, physiological and dispositional factors (Choma et al., 2022). This conceptual gap calls for a formal model that may account not only for the presence or absence of empathic behavior, but also for the structural transformations governing its variability.

To address this, we introduce a topological model of empathy which frames affective response as a structured, continuous surface shaped by three primary determinants: internalized ideological priors, physiological modulation and prosocial regulatory traits. Drawing from geometric and computational principles, our approach treats empathy as a context-sensitive and deformable construct, where each determinant works as a distinct transformation acting upon an underlying affective field. Our model aims to formalize empathic bias as a warping of perception space, rather than a simple increase or decrease in emotional intensity. By characterizing ideological priors as curvatures, physiological variables as warping functions and prosocial traits as smoothing operators, we aim to build a unified, generative framework capable of representing empathic variability across individuals and situations. Within this structure, we hypothesize that empathic response patterns can be predicted based on the interaction of these parameters and suggest that shifts in prosocial disposition may significantly modulate the topological features of the empathic field.

We will proceed as follows: we first present the mathematical formulation of our topological model, then define the variables and their operationalization. This is followed by a simulation and visualization of empathic surfaces. We conclude with a discussion of structural implications and interpretative limits.

Materials and Methods

To formalize the mechanisms underlying empathic variability, we developed a topological framework grounded in differential geometry (Do Carmo, 1992; Gromov, 1986) which aims to integrate ideological, physiological and dispositional parameters into a continuous and deformable affective space.

Mathematical approach and general framework. We adopt a topological formalism to represent empathic responses as smooth mappings between latent affective states and observable behavioral expressions. Let $M \subset \mathbb{R}^3$ be a manifold where each point $(i, p, d) \in M$ corresponds to a triplet of latent variables: ideological priors, physiological modulation and prosocial disposition. We define an empathic response function $E: M \rightarrow \mathbb{R}$, such that $E(i, p, d)$ denotes the scalar affective intensity associated with this configuration. The underlying intuition is that empathy emerges as a topologically deformable surface ΣE whose geometry reflects the dynamic interplay of priors and modulators.

We treat E as a smooth, bounded and differentiable function, enabling the use of differential geometric tools for analysis. Overall, by embedding the affective function in a Riemannian manifold, we capture not only the magnitude but also the curvature, connectivity and local deformations of empathy in response to structural constraints.

Ideological priors as metric tensors. Ideological priors are modeled as metric tensors defining the local geometry of the manifold (Tesi & Passini, 2025; Choma et al., 2022). We formalize these priors using indices from Social Dominance Orientation (SDO) and Right-Wing Authoritarianism

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(RWA) (Pratto et al., 1994; Altemeyer, 1996), represented as scalar fields s and r. These values determine the ideological curvature tensor:

$$G(x) = s \cdot gij^1(x) + r \cdot gij^2(x)$$

where gij^1 and gij^2 encode curvature due to dominance and authoritarian priors, respectively. Higher values elongate the geodesic paths between affective states, especially those related to outgroup suffering, reducing empathic connectivity in ideologically constrained directions.

Physiological modulation as warping functions. Physiological states act as warping functions that modulate empathic gain by dynamically altering the geometry of the space (Keysers et al., 2024; Hopp et al., 2023). Let p = (b, s, d) be a vector of physiological dimensions, where b is Belief in a Dangerous World (BDW), s is Perceived Stress Reactivity Scale (PSRS) and d is Pathogen Disgust Sensitivity (PVD)

Each component maps to a scalar field wk(x) and the total warping is:

$Wp(x) = 1 + \alpha1\ w1(x) + \alpha2\ w2(x) + \alpha3\ w3(x)$

with $\alpha1 = 0.3$, $\alpha2 = 0.3$, $\alpha3 = 0.2$. This produces a conformal transformation of the manifold:

$$g\sim ij(x) = Wp(x) \cdot Gij(x)$$

Regions of high physiological arousal become stretched, exaggerating ideological curvature and distorting empathic inference (Zebarjadi et al., 2022).

Prosocial dispositions as smoothing operators. Prosocial traits such as dispositional empathy (measured via IRI) and cooperative preference (CPC) act as smoothing fields that regularize curvature and reconnect disjoint affective regions (Zaki, 2020; Wang et al., 2024). We define:

$$D(x) = \beta1 \cdot IRI(x) + \beta2 \cdot CPC(x)$$

with $\beta1 = 0.3$, $\beta2 = 0.3$. These fields apply a Laplace-Beltrami smoothing kernel to reduce abrupt curvature shifts:

$g^*ij(x) = g\sim ij(x) - \nabla^2 D(x)$

This operation reduces sectional curvature and restores affective accessibility across ideological and physiological divides.

Information-theoretic curvature and empathic cost. Empathic response may also be viewed as a periodic oscillation shaped by ideological and physiological phase shifts (Mohanta et al., 2024). Let E be defined as:

$$E(i, p, d, t) = A \cdot \sin(\omega t + \varphi(i, p, d))$$

where A is the amplitude of affective responsiveness, ω is the resonance frequency (influenced by dispositional empathy and openness) and φ is a phase function altered by ideological rigidity and physiological arousal. This oscillatory model suggests that empathy is not a binary activation, but a modulated wave that can be reinforced, dampened or desynchronized by internal priors. This paves the way to modeling affective synchronization, entrainment and breakdown under social polarization.

Oscillatory formalization of affective resonance. Building on the predictive brain hypothesis, we formalize empathic processing as a probabilistic inference mechanism (Friston et al., 2022; Parr et al., 2022; Briganti et al., 2024). Let E denote the internal affective state and s a stimulus associated with suffering. The posterior probability of an empathic response can be defined as:

$$P(E|s) \propto P(s|E) \cdot P(E)$$

Here, P(E) is the prior probability distribution shaped by ideological factors such as RWA and SDO (Pratto et al., 1994; Altemeyer, 1996), while P(s|E) reflects perceptual likelihood modulated by physiological states (Keysers et al., 2024; Zebarjadi et al., 2022). This formulation allows ideological priors to act as cognitive filters that skew the plausibility of affective engagement. Bayesian curvature emerges when prior beliefs significantly constrain posterior variability, introducing ideological rigidity into the perception-action loop.

Bayesian framing of ideological priors. While the current models treat empathy as a static mapping from latent priors to affective intensities, we argue that empathic responses may be

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inherently dynamic and temporally modulated. To capture this, we introduce a partial time derivative into the empathic function:

$$\partial E/\partial t = f(i(t), p(t), d(t))$$

This temporal formulation allows the empathic surface to evolve as ideological priors, physiological states and prosocial traits shift over time. This reflects changes in environmental context, cognitive framing and emotional learning (Barrett et al., 2025; Zhou & Jenkins, 2023). Therefore, incorporating time as a fourth axis transforms the empathic manifold into a dynamic field, enabling simulation of longitudinal trajectories and empathic plasticity.

Temporal dynamics and surface construction of empathic surfaces. To visualize the empathic topology, we construct a surface $\Sigma E \subset \mathbb{R}^3$ by evaluating the scalar field E(i,p,d) over a discretized grid of parameter space. Let i,p,d ∈ [0,1] each be discretized into n = 100 steps, forming a 3D lattice of 10^6 points. For each point, the empathic intensity is calculated using:

$$E(i,p,d) = ((i + 0.5 \cdot p)^2) \cdot (1 + 0.3 \cdot BDW + 0.3 \cdot PSRS + 0.2 \cdot PVD) - (0.3 \cdot IRI + 0.3 \cdot CPC)$$

This functional form encodes the relative contribution of each determinant while maintaining differentiability across the space. The resulting scalar field can be rendered using surface plots and contour maps, capturing local maxima (facilitated empathy) and minima (inhibited empathy) as geometric features such as ridges, valleys and plateaus (**Figure 1**). Topological features such as critical points, gradients and curvature tensors are derived using numerical differentiation (Mohanta et al., 2024). These visualizations allow an interpretation of empathic responses not as binary activations, but as continuous deformations in a high-dimensional affective surface.

Simulation and computational tools. We simulated empathic response surfaces generated using the proposed topological framework. We modeled empathic response as a scalar field over a three-dimensional parameter space defined by ideological priors, physiological modulation and prosocial dispositions. We used five hundred synthetic agents representing individuals with varying ideological, physiological and prosocial traits, enabling controlled exploration of their empathic response variability. We computed empathic intensity for two conditions, namely systemic and non-systemic suffering based on the Affective-Ideological Polarization Factor (FPAI). Empathic responses were calculated using a weighted function that integrated ideological curvature, physiological warping and prosocial smoothing. The resulting values were visualized through line graphs and histograms to capture topological features such as dispersion and response gradients.

All mathematical modeling and simulations were performed in Python 3.11 using a set of numerical and visualization libraries including NumPy, SciPy and Matplotlib. Tensor operations and differential geometry functions were implemented manually where not available in standard libraries. Surface smoothing, Laplacian computation and geodesic analysis were supported using custom code routines. Symbolic differentiation for verifying curvature formulations was carried out using SymPy. For grid-based evaluations of empathic surfaces, meshgrids were generated via NumPy's meshgrid() function and empathic intensity was evaluated point-wise over the full domain. Visualization of empathic topologies was accomplished through Axes3D.plot_surface() and contourf() functions, with colormaps encoding empathic intensity (Ammons et al., 2025; Salatino et al., 2025). Code modularity was maintained using object-oriented practices to isolate metric definition, physiological warping and smoothing transformations. Computational stability was ensured by regularizing all scalar fields and applying bounded transformations to avoid divergence.

Overall, our methodological architecture provides a mathematical structure for analysing how empathy emerges from the interaction of cognitive, ideological and physiological variables. It enables systematic investigation of affective modulation as a function of topological deformation, capturing the dynamic reshaping of empathic accessibility within a multidimensional perceptual space.

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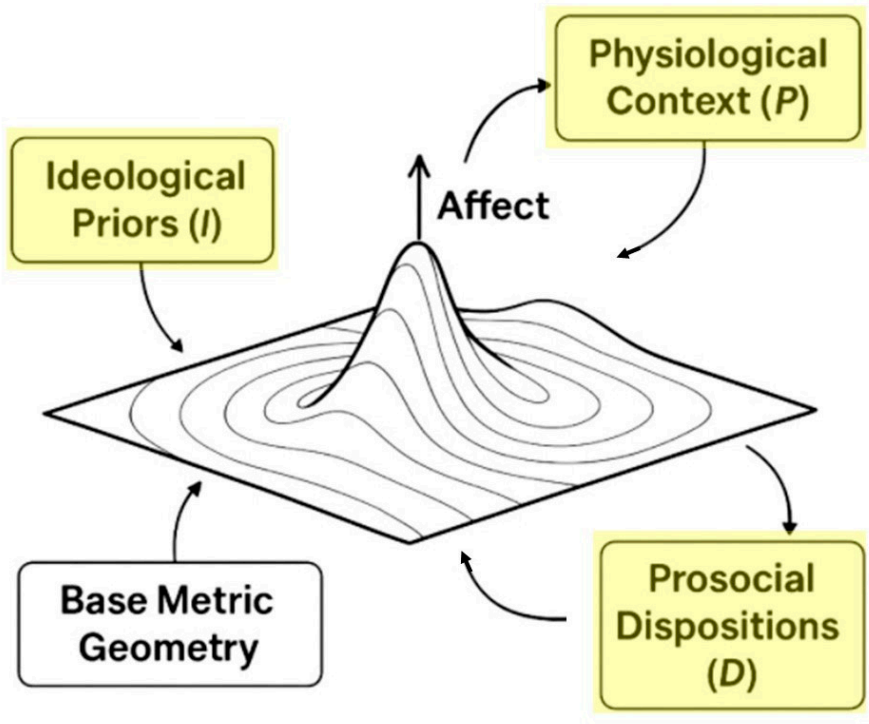


Figure 1. Visual representation of the topological model of empathic response, illustrating a deformable surface plot modulated by three factors: ideological priors, physiological modulation and prosocial smoothing. The surface geometry represents affective accessibility, with its curvature capturing the distorting effects of ideological beliefs and physiological states, while prosocial traits function as stabilizing forces that smooth irregularities and preserve the coherence of the empathic field.

Results

By manipulating ideological priors, physiological modulation and prosocial dispositions, we simulated how these factors interact to shape the structure of affective accessibility, visualizing their combined influence as geometric deformations within a continuous affective manifold that encodes the dynamic topology of empathic responsiveness.

Empathy toward systemic suffering showed a marked inverse relationship with Affective-Ideological Polarization Factor (FPAI): individuals with higher FPAI scores exhibited significantly lower affective resonance (**Figure 2**). The mean empathic response for systemic suffering was 1.33 (SD = 0.33), compared to 2.02 (SD = 0.13) for non-systemic suffering, with a paired-sample t-test confirming a statistically significant difference ($p < 0.001$). This pattern suggests that ideological curvature, amplified by physiological warping, jointly produces measurable attenuation of empathy in response to politically charged or system-implicating stimuli. These effects were further visualized in a histogram (**Figure 3**) which demonstrated that systemic suffering responses displayed broader dispersion and a lower central tendency, while non-systemic responses clustered more tightly around higher values. This suggests that systemic suffering, given its socio-political salience, may exhibit greater sensitivity to ideological priors, resulting in increased variability in empathic response.

Empathic response to non-systemic suffering revealed a relatively uniform topological profile with minimal curvature, reflecting weak modulation by FPAI. The gradient across the ideological-physiological parameter plane was notably shallow, indicating that responses to politically neutral suffering may be less influenced by internal ideological or physiological configurations. The mean empathic response for non-systemic suffering remained stable across individuals with a substantially lower standard deviation, supporting the interpretation of this region as a topologically flat segment of the empathic surface. In these regions, geodesic distances are short and curvature is near zero,

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facilitating unimpeded emotional access and consistent affective responses across the sample. This divergence in surface geometry—curved for systemic, flat for non-systemic—demonstrates the structured nature of empathic modulation. See **Figure 4** for an illustrative example.

Overall, our simulations point towards empathic responses not being evenly distributed across contexts or individuals, but instead emerging from structured interactions among ideological priors, physiological modulation and prosocial dispositions.

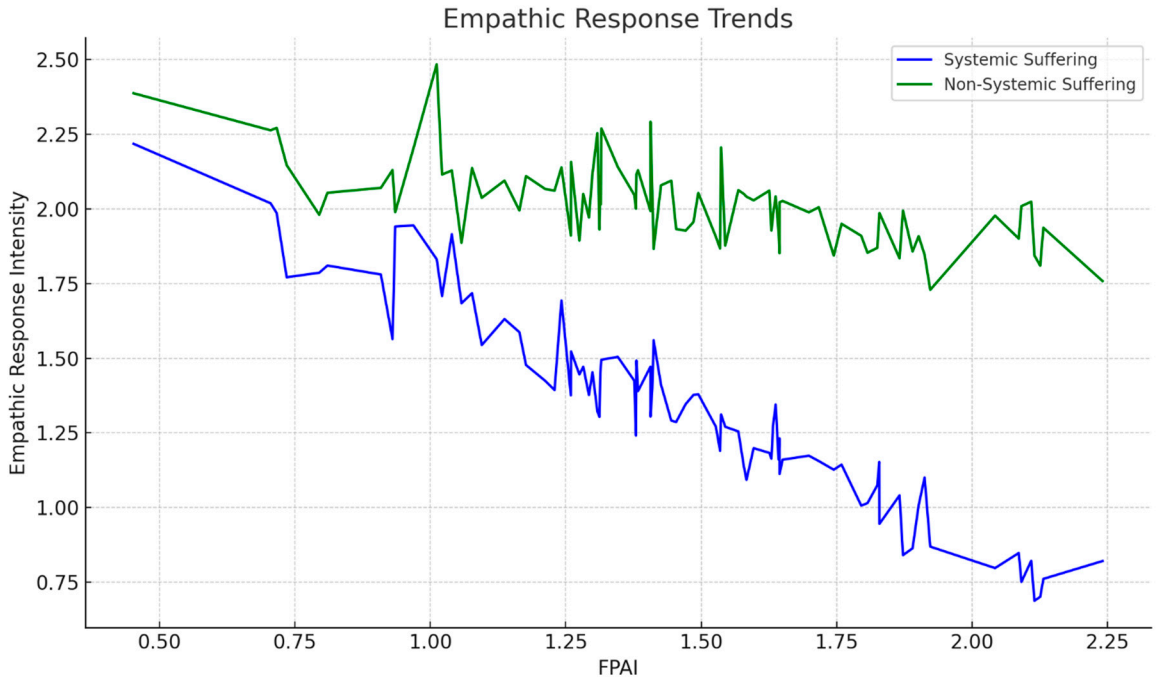
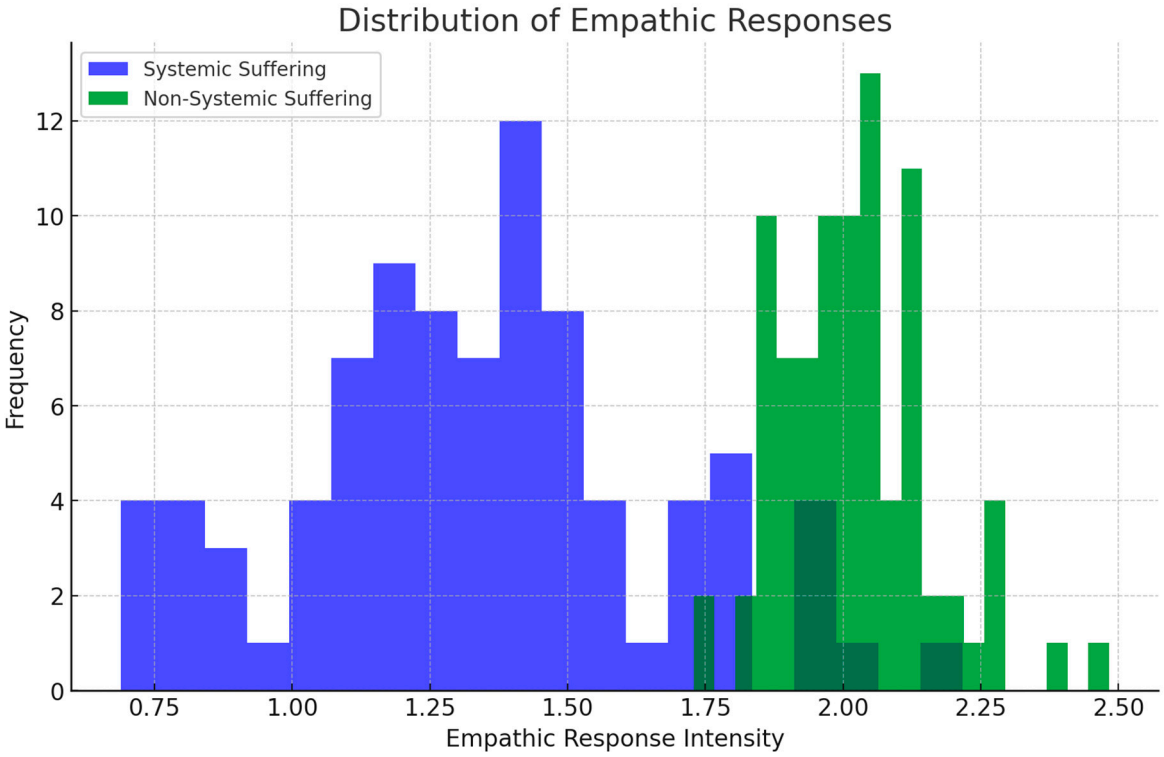


Figure 2. Line plot showing simulated empathic response trends as a function of Affective-Ideological Polarization Factor (FPAI). Systemic suffering exhibits a steep decline with increasing FPAI, confirming topological suppression of empathy under ideological constraint. In turn, the non-systemic response remains relatively stable, indicating minimal topological deformation and ideological influence.



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Figure 3. Histogram comparing the distribution of empathic response intensities for systemic and non-systemic suffering. Systemic responses display broader dispersion and lower central tendency, while non-systemic responses are more concentrated around higher values. This reflects the ideologically modulated variance in empathic accessibility across the two contexts.

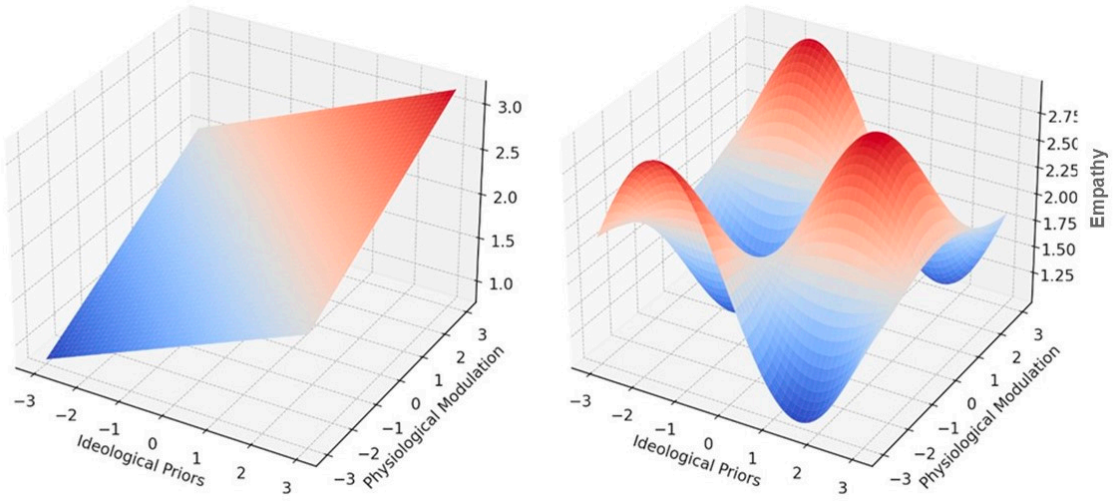


Figure 4. Side-by-side 3D plots representing simulated empathic response surfaces. The left plot shows a flat surface modeling uniform empathy across ideological and physiological parameters, indicative of minimal modulation. The right plot displays a curved surface illustrating topological deformation caused by the interaction of ideological priors and physiological modulation, producing peaks and troughs in affective accessibility. These simulated visualizations serve as illustrative examples of how internal factors may shape the structure and variability of empathic responsiveness across perceptual space.

Conclusions

By embedding belief systems and stress factors into a continuous geometric space, we assessed the value of topological formalization as a method for analyzing affective bias and perceptual warping. We provided a mathematically grounded, topological account of empathy, revealing how ideological beliefs and physiological states may shape the accessibility and modulation of affective responses. Our simulations show that empathic responses to systemic suffering are significantly more sensitive to variations in the Affective-Ideological Polarization Factor (FPAI) than responses to non-systemic suffering (Tesi & Passini, 2025; Choma et al., 2022). Specifically, empathic intensity decreased as FPAI increased, with a significant mean difference between systemic and non-systemic suffering responses. This suggests that ideological priors and physiological states act not as linear dampeners, but as nonlinear structural deformations within an affective manifold (Barrett et al., 2025; Keysers et al., 2024). In turn, the stability of responses in non-systemic contexts supports the prediction of a flatter, ideologically neutral region in the empathic topology. These findings suggest that empathy is not a uniform or spontaneous reflex, but a shaped and context-dependent function emerging from structured internal constraints and affective configurations (Zaki, 2020).

To illustrate the framework in practice, we present a case example showing how the interaction of physiological dimensions—Belief in a Dangerous World (BDW), stress reactivity (PSRS), and pathogen disgust sensitivity (PVD)—can influence individual differences in empathic engagement during a public health emergency. By viewing these traits as a vector, we better understand why people differ in their capacity to relate to others' experiences under the same stressful conditions. During a global pandemic, public health officials notice wide variation in citizens' empathy toward vulnerable populations. Consider Jordan, a 35-year-old schoolteacher, whose psychological profile reflects a high vector across three physiological dimensions: b = high BDW, s = high PSRS, and d = high PVD. Jordan perceives the world as dangerous (high BDW), experiences intense emotional

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responses to stress (high PSRS), and is highly sensitive to contamination risks (high PVD). While he rigorously follows health guidelines, his intense internal reactivity also leads to emotional distancing. He struggles to empathize with those who resist restrictions, not out of indifference, but due to a cognitive-emotional overload that short-circuits his empathic engagement. In contrast, his colleague Ava, whose physiological profile is lower on all three dimensions, maintains a steadier emotional baseline. She expresses empathy not only toward immunocompromised individuals but also toward those struggling with compliance, recognizing their socioeconomic and psychological challenges. This example shows how the interaction of BDW, PSRS, and PVD shapes not only protective behavior but also one's ability to emotionally resonate with others. Empathy, in this light, is not just a social virtue, rather it stands for a dynamic response shaped by deeper affective and perceptual systems.

Our topologically informed model of empathy integrates ideology, physiology and prosocial disposition within a unified and dynamic affective surface. The advantage of this framework lies in its capacity to integrate cognitive, emotional and bodily dimensions into a continuous representational structure, addressing a fundamental limitation of current psychological models—namely, their inability to capture the nonlinear, multidimensional and ideologically filtered structure of affective engagement. Unlike linear or modular approaches, which conceptualize empathy as a trait or dual-process response (Friston et al., 2022), our model treats it as a geometrically constrained field, shaped by curvatures and gradients that reflect internal biases and contextual salience (Briganti et al., 2024; Mohanta et al., 2024). This formalization provides a language for articulating affective occlusion, rigidity and resonance, i.e., dimensions that remain obscure in traditional neuroimaging or psychometric paradigms.

Several limitations warrant mention. First, the parameters and weights used in constructing the physiological vector (BDW, PSRS, PVD) are theoretically informed but not empirically calibrated, limiting the predictive power of the model. Second, the model is static and does not account for dynamic processes or feedback loops such as habituation to stress, social reinforcement or desensitization, which can significantly alter psychological responses over time. Third, it assumes continuous differentiability in trait-behaviour relationships, yet human emotions and decisions often involve discrete thresholds or tipping points that this model cannot capture. Fourth, the model operates at the individual level and fails to integrate network-level or collective dynamics like shared narratives, media ecosystems or social identities, which may play crucial roles in shaping behaviour (Nir & Halperin, 2025; Goldenberg et al., 2025). Fifth, each component is treated as a scalar value ignoring potential directionality, intensity gradients or multidimensional interactions. Future models could benefit from incorporating tensorial variable and, nonlinear cross-interactions to better reflect the complexity of human affect.

Our vector-based framework holds potential for a range of applied domains lile public policy, education, therapy and social communication, by modeling empathy as a dynamically modulated surface shaped by ideological priors, physiological sensitivity and prosocial traits. This allows for context-sensitive strategies reflecting individual differences in affective accessibility. For example, individuals high in BDW and PVD may benefit from targeted interventions emphasizing relational safety and empathic narratives (Zebarjadi et al., 2022). In polarized contexts, the model may help identify affective barriers to intergroup empathy and support cognitive reframing strategies (Goldenberg et al., 2025).

Several testable experimental hypotheses arise from our framework. Individuals high across the full vector may exhibit stronger autonomic responses (e.g., elevated heart rate variability or pupil dilation) during empathic tasks (Zhou & Jenkins, 2023). These individuals may display reduced spontaneous empathic concern under ambiguous or ideologically incongruent social cues. Still, targeted narrative interventions, especially those aimed at recontextualizing perceived threat, could reduce defensive closure and increase empathic flexibility in high-BDW subjects (Wang et al., 2024; Bloy et al., 2025). Next, integrating real-time physiological tracking with ideological profiling may enable predictive, individualized interventions based on adaptive affective technologies capable of addressing specific empathic bottlenecks.

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Computationally, our model supports the use of topological data analysis and manifold learning to extract empathic geometries from high-dimensional data (Aldar et al., 2025). Longitudinal research could track changes in empathic curvature following trauma, social transitions, or ideological shifts (Levy et al., 2024). Phenomena like empathic hysteresis, where emotional responses lag behind external change due to topological inertia, may explain difficulties in adjustment during collective transitions. In these cases, moral directionality becomes distorted and empathy may be selectively blocked or attenuated based on ideological misalignment. This anisotropic configuration of the empathic space may reflect a fundamental shift in affective accessibility, distributing empathy not uniformly but in alignment with directional moral priors

Our model also frames resonance failures such as depersonalization or emotional numbing as disruptions in predictive affective cycles. Here, empathy may collapse or become directionally distorted by ideological misalignment. Incorporating information-theoretic metrics like Kullback–Leibler divergence could refine our ability to quantify empathic distortion and bias magnitude across cognitive domains. Future developments may enhance our framework through the integration of information-theoretic metrics such as Kullback–Leibler divergence or Fisher information, enabling quantitative characterization of empathic distortion and emotional prediction error. These refinements would support a more granular understanding of affective asymmetries, allowing researchers to assess not only the presence of empathic bias but the magnitude and informational structure of that bias across cognitive and ideological dimensions.

Taken together, these potential applications and research directions underscore the utility of modeling empathy as a structured, deformable field that is shaped, warped and sometimes blocked by layered individual and social constraints.

In conclusion, our vector-based approach reframed empathy as a dynamic, embodied construct shaped by underlying sensitivities, rather than a fixed moral trait. The central research question examined whether physiological traits, conceptualized as components of a multidimensional vector, can more effectively predict individual differences in empathic and affective responses when considered in interaction rather than isolation. Our findings suggest that the interplay among these physiological dimensions provides a powerful methodological lens for understanding variability in empathy, specifically, why some individuals exhibit heightened empathic inhibition or amplification in response to emotional or socially salient cues. By doing so, our approach paves the way to research, targeted intervention and informed public discourse aimed at fostering emotional understanding across social, ideological and cultural divides.

Declarations

Ethics approval and consent to participate: This research does not contain any studies with human participants or animals performed by the Authors.

Consent for publication: The Authors transfer all copyright ownership in the event the work is published. The undersigned authors warrant that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal and has not been previously published.

Availability of data and materials: all data and materials generated or analyzed during this study are included in the manuscript. The authors had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.

Competing interests: The Authors do not have any known or potential conflict of interest, including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence, their work.

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Authors' contributions: The Authors equally contributed to: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtaining funding, administrative, technical and material support and study supervision.

Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the Authors used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the Authors reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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Review

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Natural Evolution at the Level of the Individual Mind

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Keywords: mental evolution; cognitive selection; conceptual inheritance; developmental branching; idea phylogeny

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Review

Natural Evolution at the Level of the Individual Mind

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Abstract

Current research in cognitive science tends to describe cognition in terms of adaptation or optimization rather than genuine evolution. We advance the hypothesis that the laws governing biological evolution can be meaningfully transposed to the dynamics of thought within the mind of a healthy human individual. We argue that the main mechanisms of natural evolution can be mapped onto mental counterparts that can be formally and mathematically characterized. For instance, genetic variation could correspond to the generation of new ideas through imagination and association; selection to the experiential retention of thoughts enhancing coherence or predictive success; inheritance to the stabilization of mental configurations through memory; mutation to conceptual transformation; recombination to the synthesis of ideas; migration to the assimilation of external information; drift to stochastic fluctuations in attention; adaptation to the progressive optimization of internal models, etc. We propose two theoretical applications: the use of the Hardy–Weinberg equilibrium to quantify cognitive homeostasis and the construction of a phylogenetic tree of thoughts to trace how ideas diversify and branch across an individual's development, revealing how early cognitive patterns generate increasingly complex and abstract forms of reasoning. Our hypothesis provides a unified vocabulary linking neural plasticity, conceptual change and psychological development within the same evolutionary logic. Our framework could guide clinical psychology by interpreting cognitive rigidity as deviation from equilibrium, education by mapping stages of conceptual diversification and consolidation, neuroscience by viewing learning as selection on neural representations, and AI by enabling systems that evolve ideas rather than optimize tasks.

Keywords: mental evolution; cognitive selection; conceptual inheritance; developmental branching; idea phylogeny

Introduction

Contemporary research in cognitive science, developmental psychology and theoretical neuroscience emphasizes that mental processes emerge from dynamic interactions among neural and environmental factors (Abbott 2008; Nielsen and Haun, 2016; Dikker et al., 2017; Li and Xu, 2018; Luppi et al., 2019; Lin et al., 2020; Yang et al., 2022; Shaw et al., 2023; Beffara et al., 2025). Yet, despite extensive work on neural plasticity, learning theory and predictive coding, most current models describe cognition in terms of adaptation or optimization rather than genuine evolution (Burke and Barnes, 2006; Arcos-Burgos et al., 2019; Hipólito et al., 2021; Kocagoncu et al., 2021; Tomov et al., 2023; Sumner and Lukasiewicz, 2023; Schilling et al., 2023; Shipp 2024; Keysers et al., 2024; Liao et al., 2025). Existing frameworks rarely include an explicit evolutionary logic linking the spontaneous generation of ideas, their selective reinforcement and their long-term stabilization. Meanwhile, evolutionary epistemology and cultural evolution theories have addressed how ideas propagate among individuals, but few of them examine how comparable mechanisms might operate within a single mind (Slijepcevic 2018; Stanford 2020; Saad 2020; Ermakov and Ermakov, 2021; Smolla et al., 2021; Carmel et al., 2023). We advance the hypothesis that the core laws of natural evolution can also govern processes within the individual mind.

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This mapping between biological laws and mental analogues allows imagination, reflection, learning and forgetting to be interpreted as specific instances of evolutionary mechanisms acting within a mental population. Within our perspective, the mind is understood as a self-organizing ecosystem in which ideas evolve through continuous feedback between internal constraints and external stimuli. The thought of every human individual thus becomes an evolving field balancing novelty and stability, driven by internal selection pressures and environmental challenges.

We will proceed as follows: first, we transpose the principles of natural evolution to the dynamics of individual thought; Next, we provide and effort to integrate developmental, psychological and neural phenomena under a single mathematical and conceptual framework, enabling the derivation of measurable patterns from the same laws that describe biological evolution. Then, we present the mathematical formulation of each evolutionary feature; next, we describe the theoretical implications and the construction of a phylogenetic tree of ideas; finally, we conclude with interpretive remarks on the broader meaning and scope of this framework.

Transposing the Principles of Natural Evolution to the Dynamics of Thought

The evolutionary organization of life is sustained mechanisms that regulate how novelty appears, is filtered by environmental constraints and is preserved across generations. We argue that, when viewed from a cognitive standpoint, these same principles could describe the continuous reshaping of mental life in a healthy human individual as ideas emerge, transform and stabilize like in a biological population. What follows is a theoretical transposition of the major features of natural evolution into their corresponding mental counterparts.

1. **Variation** would represent one of the primary sources of diversity in both biological and cognitive systems. In biology, it arises from genetic differences among individuals, ensuring that populations are never homogeneous (Paaby and Rockman, 2014). In the mind, it would correspond to the spontaneous generation of new ideas through imagination, associative recombination and small interpretive deviations occurring during recall or reasoning. Just as species survive environmental change through the presence of multiple genotypes, cognition would preserve its flexibility through the coexistence of competing representations. Mental variation could appear either as the spontaneous emergence of novel associations, alternative solutions to a problem or the creative blending of distant concepts. It would thus provide the raw material for learning and innovation, allowing the mind to answer to uncertainty not by rigidity, but by exploration of multiple possible forms.
2. **Selection** would determine which variants persist and which disappear. In biology, selection depends on the environment's capacity to reward advantageous traits with reproductive success (Ewens 2019; Cerca 2023). In the mind, a similar process would act through criteria like coherence, predictive accuracy, emotional satisfaction or social validation. Thoughts fitting experience and reducing uncertainty would be preferentially retained, while those generating inconsistency or discomfort would gradually weaken in the mental case. The adaptive criterion would therefore be epistemic and experiential rather than reproductive: mental states would persist in proportion to their coherence and usefulness. Over time, this selection would sculpt the individual's conceptual landscape, favoring representations that integrate successfully within memory and action.
3. **Inheritance** would provide continuity through time, preserving what has proven stable or useful. In biological systems, inheritance transmits genetic information, ensuring that adaptive variations are not lost (Tan, 2019; Xavier et al., 2019). In cognition, the corresponding process would occur through memory, particularly its long-term and procedural components. Memory would consolidate recurrent thought patterns through reinforcement and repetition, stabilizing neural or symbolic traces that encode experience. Each new idea would therefore emerge in a field already shaped by prior configurations, acting as the inherited substrate for further evolution. In this way, inheritance would maintain the continuity of identity and knowledge across the constant flow of change.

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4. **Mutation** would constitute the mechanism of transformation and novelty. In biological organisms, mutation introduces random modifications in genetic sequences, occasionally producing new traits that selection may favor (Svensson and Berger, 2019). In the cognitive sphere, it would manifest as reinterpretation, recontextualization or analogical drift, i.e., small but cumulative shifts that alter the structure of meaning. No thought would ever reproduce itself identically: each recall or reformulation would introduce subtle variations. These deviations might arise from language, emotion or context, creating opportunities for conceptual innovation. While biological mutation is largely blind, mental mutation would tend to follow implicit goals and expectations, guiding transformation without determining its outcome.
5. **Recombination** would represent the synthesis of previously distinct ideas, mirroring the biological process by which new genetic constellations arise from parental material. In sexual reproduction, recombination ensures genetic diversity (Stetsenko and Roze, 2022; Burbrink et al., 2025); in the mind, it would ensure conceptual novelty. Human imagination would operate through this recombinative logic, fusing elements drawn from different cognitive lineages into new configurations. The formation of metaphor, analogy or abstract models would exemplify this process: distinct domains of experience would merge into unified conceptual structures, giving rise to hybrid thoughts that inherit properties from multiple sources. Creativity would therefore be the mental counterpart of recombination, allowing the mind to produce innovations retaining traces of their ancestral ideas while transcending their original context.
6. **Migration** would correspond to the movement of ideas between cognitive populations. In biological evolution, gene flow introduces diversity and prevents isolation (Lyu et al., 2022); in cognition, migration would take place through learning, communication, reading, imitation or social interaction. New information from other individuals, cultures or symbolic systems would enter the mental ecosystem, altering the internal distribution of ideas. These incoming variants could compete with, transform or reinforce existing representations, generating new selective pressures that reconfigure cognitive equilibrium, just as gene flow maintains the vitality of evolving species. The mind's openness to migration would thus be essential for cumulative knowledge, cultural integration and adaptability.
7. **Drift** would capture the stochastic component of mental evolution, the random fluctuations that alter the relative prominence of thoughts independently of their adaptive value. In biological terms, drift affects allele frequencies through chance events (Bourrat 2018); in cognition, it would manifest as spontaneous shifts of attention, associative leaps or changes in salience produced by mood, fatigue or distraction. These random dynamics would explain the unpredictable emergence of memories or intuitions that seem detached from immediate purpose. Drift would ensure that the evolution of thought remains open to contingency, allowing for unforeseen combinations and creative surprises beyond deliberate control.
8. **Adaptation** would represent the gradual tuning of the mind to its experiential environment. In biology, adaptation increases fitness by aligning organisms with ecological demands (Lenski, 2017; Mérot 2022); in cognition, it would take the form of refining internal models to minimize discrepancy between expectation and experience. Theories of predictive coding already describe how the brain continually adjusts its representations to reduce surprise, a process formally analogous to selection optimizing fitness. Through adaptation, the mind would integrate the outcomes of variation, selection and inheritance, achieving coherence between inner representations and external reality.
9. **Environment** would constitute the field within which mental evolution unfolds. For organisms, the environment defines the conditions of survival and reproduction; for the mind, it would be the totality of cognitive experiences. Every idea would exist in relation to this context, drawing reinforcement from experiences that confirm it and losing relevance when the environment changes. Bodily sensations, interpersonal relations and cultural frameworks would together form the selective landscape shaping cognitive survival. The environment would therefore not

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merely influence thought but participate actively in its evolution, functioning as both constraint and catalyst of mental diversity.

10. **Equilibrium** would mark the temporary balance achieved when opposing cognitive forces like variation, selection and adaptation counteract one another. In biological systems, equilibrium corresponds to stable allele frequencies (Herring 2021); in the mind, it would correspond to psychological homeostasis, in which representations achieve internal consistency and minimal conflict. This cognitive equilibrium would not be static but rather dynamic, sustained by subtle, ongoing adjustments among competing ideas and emotions. During these periods, the mind would maintain coherence without suppressing variability, preserving the readiness to evolve again when new perturbations arise.
11. **Speciation** would represent the process by which mental lineages diverge and acquire distinct functional identities. In biology, speciation arises from reproductive isolation and environmental differentiation (Villa et al., 2019); in the mind, it would manifest as the gradual emergence of semi-autonomous subselves or cognitive modules (moral, emotional, analytical, aesthetic, linguistic, social) developing partial independence while remaining components of a unified system. Each of these subdomains would evolve its own representational rules, selection criteria and adaptive pressures, much like species adapting to distinct ecological niches. This internal diversification would enable the mind to manage complexity, allowing specialized subsystems to address different classes of problems while maintaining coordination through integrative networks such as consciousness or meta-awareness. The coexistence of differentiated mental lineages would therefore enhance flexibility and resilience, preventing any single representational strategy from dominating the entire cognitive ecosystem.
12. **Punctuated equilibrium** would describe the rhythm of cognitive evolution, alternating long periods of relative stability with brief episodes of radical transformation. In biology, these discontinuities reflect rapid speciation following environmental disruption (Manceau et al., 2020); in thought, they would appear as moments of insight, crisis or revelation, when accumulated tension, contradiction or novelty precipitates a sudden reorganization of conceptual structures. During these cognitive "bursts," the mind would restructure its representations, integrating previously incompatible elements into a new order. Though these transitions might be destabilizing, they would also serve as engines of creative evolution, producing higher-order coherence that cannot emerge through gradual refinement alone. Mental development, like natural evolution, would thus proceed through discontinuous leaps of reconfiguration.
13. **Coevolution** would express the reciprocal adaptation between mind and environment. In biological systems, species evolve together through continuous feedback (Agrawal and Zhang, 2021); in cognition, this process would manifest as the mutual shaping of thought and world. The mind would act upon its surroundings through behavior, language and interpretation, modifying the environment that in turn constrains and informs it. Perceptual learning, cultural participation and technological construction would form a continuous feedback loop through which experience and cognition co-adapt. Over time, this reciprocal modulation would blur the boundary between internal and external such that the environment would become a cognitive partner rather than a passive context.
14. **Exaptation** would capture the capacity of the mind to repurpose existing cognitive or neural structures for new functions. In evolutionary biology, exaptation refers to traits co-opted for purposes other than their original adaptive role, such as feathers first used for insulation later enabling flight (Tramacere et al., 2017; Brosius, 2019). In cognition, it would appear when old ideas, symbols or perceptual schemas are reinterpreted to serve new conceptual or practical goals. Metaphor, analogy and technological invention would all exemplify such exaptive reuses, in which structures originally evolved for perception, movement or language are redirected toward abstraction, problem-solving or creative expression. This mechanism would allow the

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mind to extend its repertoire without requiring the constant generation of entirely new structures, thereby maximizing efficiency through reconfiguration rather than replacement.

15. **Extinction** would close the evolutionary cycle by pruning obsolete, inefficient or maladaptive elements. In biological evolution, species disappear when they lose adaptive viability; in cognition, forgetting, inhibition or repression would perform an equivalent role. Ideas no longer integrating with current experience or contradicting the prevailing conceptual order would gradually fade from memory or become functionally suppressed. This process would not represent mere loss, but a basic mechanism of optimization: by eliminating redundant or conflicting representations, extinction would clear cognitive space for new configurations to arise. In this sense, forgetting would be essential to the self-regulation of mental life, maintaining balance within the ever-changing ecology of thought and preventing informational overload.

Overall, by grounding each mental process in a defined evolutionary principle, our framework portrays the mind as a system in which novelty, selection and stabilization interact according to lawful dynamics. Through these theoretical correspondences, the mechanisms governing biological evolution would acquire cognitive meaning. The equivalence is not metaphorical but structural, as each evolutionary feature fulfills the same logical function in sustaining adaptability, continuity and complexity within the domain of ideas.

Table. Theoretical correspondence between the evolutionary features discussed in the chapter and their cognitive analogues or mental counterparts.

Biological rule	Cognitive analogue	Functional description
Variation	Idea generation, imagination	Production of novel cognitive configurations ensuring flexibility and creativity.
Selection	Experiential validation, coherence, emotional reward	Preferential retention of thoughts that align with experience or yield coherence.
Inheritance	Memory consolidation	Stabilization of recurring mental patterns through learning and synaptic plasticity.
Mutation	Conceptual drift, reinterpretation	Gradual transformation of ideas through analogy, context or reframing.
Recombination	Metaphor, synthesis	Integration of distinct ideas into hybrid, creative constructs.
Migration	Learning, cultural intake	Incorporation of external information through education and communication.
Drift	Random salience fluctuation	Chance-driven shifts in thought prominence due to attention or mood.
Environment	Lived experience, context	The experiential field shaping which thoughts persist or fade.
Equilibrium	Cognitive homeostasis	Dynamic balance between novelty and stability in the mental system.
Adaptation	Predictive optimization	Progressive refinement of internal models to minimize uncertainty.
Speciation	Subself differentiation	Emergence of specialized cognitive lineages within the unified self.
Punctuated equilibrium	Insight, crisis	Sudden reorganization of mental structures following prolonged stasis.
Coevolution	Mind–world feedback	Reciprocal influence between cognitive processes and environmental change.
Exaptation	Cognitive reuse and creativity	Repurposing of existing concepts or neural circuits for new meanings.
Extinction	Forgetting, repression	Elimination of obsolete or maladaptive ideas maintaining mental efficiency.

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Mathematical Formulation of Cognitive Evolution

We provide here a formalism to assess the dynamics of a human individual's thought in terms of an evolutionary system. Thoughts are treated as evolving entities within a population, each competing for stability and relevance under the pressures of experience, memory and environmental feedback.

Representation of cognitive populations. Let the mind of a wealthy human individual at a given time t contain a set of K distinguishable cognitive variants or ideas, each denoted by C_i and represented by a normalized frequency variable $p_i(t)$. The vector

$$p(t) = [p_1(t), p_2(t), \dots, p_K(t)]$$

describes the instantaneous state of the mental population, where $p_i(t) \geq 0$ for all i and $\sum_{i=1}^{K} p_i(t) = 1$. Each p_i represents the relative activation, prevalence or probability of idea i being dominant in consciousness or influencing current cognition.

Every cognitive variant possesses a **fitness function** $F_i(p, E, M)$, i.e., a scalar quantity describing its ability to persist or reproduce under current environmental conditions $E(t)$ and the structure of memory $M(t)$. Fitness depends on both intrinsic and extrinsic factors like its internal coherence, emotional resonance and contextual adequacy. The **average fitness** of the system is given by

$$\bar{F} = \sum_{i=1}^{K} p_i F_i(p, E, M),$$

which measures the overall stability of the cognitive ecosystem at time t.

The replicator–mutator equation. The temporal evolution of each cognitive variant follows a **replicator–mutator equation** extended to include mutation, migration, decay and stochastic drift:

$$\frac{dp_i}{dt} = p_i(F_i - \bar{F}) + \sum_j p_j \mu_{j\to i} - p_i \sum_k \mu_{i\to k} + m(r_i - p_i) - \delta_i p_i + \sigma_i \sqrt{p_i(1-p_i)}\,\eta_i(t)$$

Each term represents a distinct cognitive process:

- $p_i(F_i - \bar{F})$: **selection**, the differential reinforcement of ideas whose fitness exceeds the average.
- $\mu_{j\to i}$: **mutation rate** describing the probability that idea j transforms into idea i, capturing conceptual drift or reinterpretation.
- $m(r_i - p_i)$: **migration**, representing the influx of external information (learning, imitation) with rate m, where r_i denotes the external distribution of incoming ideas.
- $-\delta_i p_i$: **extinction or forgetting**, representing the natural decay of unused or irrelevant ideas at rate δ_i.
- $\sigma_i \sqrt{p_i(1-p_i)}\,\eta_i(t)$: **cognitive drift**, the stochastic fluctuation of idea salience due to attentional instability or spontaneous associative noise.

Here, $\eta_i(t)$ is a Gaussian white noise term with zero mean and unit variance (Zhou et al., 2021), while σ_i scales the amplitude of random cognitive fluctuation. The stochastic component ensures that thought evolution remains open to contingency and creativity, preventing the system from becoming entirely deterministic.

Structure of the cognitive fitness function. The fitness of an idea is determined by its interaction with the internal and external environment:

$$F_i(p, E, M) = U_i(E) + \sum_j W_{ij}(M) p_j - C_i(p_i) - \Gamma_i(E, p).$$

This formulation contains four key components:

1. **Contextual utility** $U_i(E)$: the degree to which idea i aligns with the current environmental or experiential context $E(t)$.
2. **Memory coupling** $W_{ij}(M)$: the reinforcement or inhibition between ideas i and j, determined by their co-activation history within memory M.

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3. **Cognitive cost** $C_i(p_i)$: a penalty term limiting excessive complexity or instability, ensuring that attention and processing resources remain bounded.
4. **Prediction error** $\Gamma_i(E,p)$: a measure of discrepancy between internal expectations and external feedback, representing cognitive dissonance or surprise.

Through these interacting terms, the fitness landscape becomes dynamic and self-modifying. Selection favors ideas that minimize prediction error while maintaining high contextual utility and manageable cognitive cost, leading to an adaptive balance between coherence and flexibility.

Dynamics of memory and inheritance. Memory provides the substrate for cognitive inheritance, preserving associations among ideas that frequently co-occur. The evolution of memory is described by a **plasticity equation**:

$$\frac{dM}{dt} = \alpha(pp^{\top}) - \rho M,$$

where M is a symmetric matrix encoding the associative strengths between ideas. The term $\alpha(pp^{\top})$ reinforces connections among concurrently active thoughts, reflecting a Hebbian mechanism ("what fires together, wires together") (Munakata and Pfaffly, 2004; Remme et al., 2021; Lansner et al., 2023). The term $-\rho M$ introduces **forgetting**, a decay process that gradually weakens inactive associations.

This dynamic ensures that memory remains both stable and adaptive: associative structures are continuously reshaped by experience, maintaining coherence while allowing old or irrelevant connections to fade. The parameter α (learning rate) governs the speed of integration of new associations, while ρ (forgetting rate) controls the rate at which unused connections decay. The ratio α/ρ thus defines the **memory persistence index**, a quantitative measure of cognitive stability.

Environmental feedback and coevolution. The environment $E(t)$ is modeled as an interactive field rather than a fixed condition. It both constrains and is modified by cognitive activity, producing a feedback loop analogous to coevolution in ecology. The environment evolves according to:

$$\frac{dE}{dt} = G(E) + H\,a(p,E) + \xi(t),$$

where:

- $G(E)$ represents autonomous environmental dynamics.
- H is a coupling coefficient translating mental action into environmental modification.
- $a(p,E)$ is the behavioral or interpretive output of the current cognitive configuration.
- $\xi(t)$ is an exogenous stochastic term representing unforeseen external events.

This coevolutionary formulation formalizes the mutual dependence between thought and world: cognition adapts to its environment, but through interpretation and behavior it simultaneously reshapes that environment. Thus, mind and context evolve together in a recursive loop of constraint and transformation.

Equilibrium, stability and cognitive homeostasis. At equilibrium, when the right-hand side of each differential equation tends toward zero, the cognitive population reaches a state of **dynamic homeostasis**:

$$\frac{dp_i}{dt} = 0 \forall i, \frac{dM}{dt} = 0, \frac{dE}{dt} = 0.$$

In this condition, the inflow of variation, reinforcement and forgetting achieve balance, so that the relative frequencies of ideas remain approximately constant. The equilibrium is not static but continuously maintained through micro-adjustments among selective, mutational and environmental forces. Perturbations of the parameters such as sudden changes in $U_i(E)$ or in the connectivity of M may destabilize the system, leading to rapid reorganizations of cognitive structure analogous to **punctuated equilibrium** in biological evolution.

Local stability can be evaluated by linearizing the system around equilibrium and analyzing the eigenvalues of the Jacobian matrix of partial derivatives $\partial(dp_i/dt)/\partial p_j$. Negative real parts

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correspond to stable equilibria (cognitive stasis), while positive values indicate instability and the onset of conceptual restructuring.

Cognitive metrics and evolutionary interpretation. Several measurable quantities arise from this formalism:

- **Average cognitive fitness** $\bar{F}$: quantifies global coherence and efficiency of thought.
- **Cognitive entropy** $S = -\sum_i \quad p_i \ln p_i$: measures diversity or richness of the mental population, analogous to genetic heterozygosity.
- **Innovation rate** $\dot{I} = \sum_{i \neq j} \quad p_j \mu_{j \to i}$: captures the intensity of conceptual mutation and reinterpretation.
- **Resilience index** $R = \frac{\alpha}{\rho + \delta}$: expresses the ability of the system to recover coherence after disruption.

These parameters would provide operational links between the theoretical model and empirical observation through linguistic variability, neural activation diversity or behavioral adaptation rates.

In conclusion, the provided system of equations could unify the descriptive correspondences of evolution and cognition into a single mathematical structure. Our formulation portrays the mind as a living population of ideas subject to lawful dynamics. Its complexity arises not from static architecture but from the continuous interplay of differentiation and integration within the individual mind. Selection promotes coherence, mutation generates novelty, inheritance maintains continuity and drift introduces unpredictability. Environmental feedback ensures that thought remains open to the world it interprets. Equilibrium emerges as the transient balance of these interacting forces.

Theoretical Applications: Hardy–Weinberg Equilibrium and the Phylogenetic Tree of Thought

Our formal framework might acquire particular significance when two classical instruments of evolutionary theory, namely, the Hardy–Weinberg law and the phylogenetic tree, are transposed to the cognitive domain. Each could provide a distinct but complementary perspective on the evolution of thought: the first could quantify equilibrium and its deviations within the mental population, while the second could reconstruct the lineage of ideas through developmental descent. Together, they could convert the abstract laws of cognitive evolution into operational tools for analyzing mental structure and transformation.

The **Hardy–Weinberg equilibrium** represents in classical population genetics the f state of a biological system in which allele frequencies remain constant over generations, assuming the absence of mutation, migration, selection or genetic drift (Rogatko et al., 2002; Mayo 2008; Stark 2023; Saadat 2024; Neamatzadeh et al., 2024). It is the mathematical description of a population in perfect balance, a null condition against which evolutionary change can be detected. When transposed to the cognitive domain, the same equilibrium could express the state of *mental homeostasis*, where the relative prevalence of cognitive variants, standing for distinct but functionally equivalent ideas, remains stable in the absence of external or internal perturbations. Within this framework, thoughts could behave as alleles within a mental population, each variant corresponding to a particular interpretation, emotional response or conceptual stance. The mind, like a gene pool, could contain distributions of such variants whose proportions can remain constant if no transformative forces act upon them.

Let two complementary cognitive tendencies, A and a, possess respective frequencies p and $q = 1 - p$. Their joint expression within thought can be represented by the classical quadratic expansion

$$p^2 + 2pq + q^2 = 1,$$

where p^2 denotes purely A-type configurations (dominant cognitive inclinations), q^2 denotes purely a-type configurations (recessive or alternative tendencies) and $2pq$ represents *heterogeneous*

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mental states in which both tendencies coexist or alternate. In biological terms, this describes the proportion of homozygotes and heterozygotes; in the cognitive analogue, it might capture the distribution between pure and mixed modes of thinking, between conviction, ambivalence and synthesis. When this relation holds, the system is in *cognitive equilibrium*: the internal composition of thought types remains invariant over time, indicating that mental processes are neither producing nor eliminating variants faster than they stabilize.

This equilibrium could provide a quantitative criterion for cognitive homeostasis, the psychological condition of balanced ideational dynamics. In practical terms, it could correspond to phases of mental stability, when an individual maintains consistent patterns of reasoning, emotional regulation and conceptual coherence. During these states, internal representations could be reproduced with minimal distortion and the frequencies of dominant and alternative ideas could remain proportional to their established weights in memory and experience. The equilibrium condition $p^2 + 2pq + q^2 = 1$ thus might stand for a benchmark for identifying the absence of transformative cognitive forces: no learning-induced restructuring, no emotional destabilization and no influx of novel information could alter the balance between competing conceptual variants.

Deviations from equilibrium could signal the onset of mental evolution, revealing changes in idea distribution driven by selective reinforcement, mutation-like reinterpretation or informational migration. For instance, when new evidence favors one cognitive variant, its relative frequency p could increase at the expense of q, producing an observable shift in the composite terms. Likewise, emotional events or social interactions may alter the internal weighting of ideas, disrupting equilibrium and triggering adaptive reorganization. The degree of deviation, quantified by the difference between the observed and expected proportions of p^2, $2pq$ and q^2, could provide a direct measure of *cognitive adaptation intensity*. Just as geneticists infer evolutionary pressure from departures from Hardy–Weinberg expectations, cognitive scientists may interpret these departures as indicators of learning, creative restructuring or psychological transition.

In this sense, the Hardy–Weinberg law could become a diagnostic framework for quantifying mental stability and transformation. It allows us to distinguish phases of ideational equilibrium, where thought reproduces its existing configurations, from phases of evolutionary restructuring, when conceptual innovation or selective forgetting changes the composition of the mental population.

Overall, the Hardy–Weinberg equation defines the null condition from which evolution begins and to which it periodically returns. By measuring how far and in what direction the system diverges from this equilibrium, one could trace the rhythmic alternation of stability and change standing for the evolution of thought within the individual mind.

The second application concerns the **construction of a phylogenetic tree of thoughts**, a theoretical analytical instrument conceived to trace the descent, diversification and recombination of ideas across the temporal landscape of an individual's cognitive life. In biological contexts, a phylogenetic tree depicts the evolutionary branching of lineages from common ancestors, recording the paths by which species diverge, adapt and occasionally converge (Zhang 2016; Smith 2022; Janzen and Etienne, 2024; Steel 2025). When transposed to cognition, this structure becomes a diagram of conceptual ancestry: a formal reconstruction of how primitive, undifferentiated representations could give rise to increasingly specialized and abstract mental forms. Each node of the cognitive tree corresponds to a distinct conceptual variant C_i, while each branch represents a transformation path inferred from relations of similarity, temporal succession or causal dependence among thoughts. The resulting network provides not only a descriptive taxonomy of ideas, but also a quantitative reconstruction of their evolutionary history within the mind.

Mathematically, the genealogical structure of ideas can be derived from two central operators of our cognitive evolutionary model: the **transition matrix** μ_{ij} and the **co-activation matrix** M_{ij}. The transition matrix μ_{ij} is a $K \times K$ matrix where each element μ_{ij} represents the conditional probability that a cognitive variant C_i transforms into another variant C_j within an infinitesimal time interval. The normalization $\sum_j \mu_{ij} = 1$ ensures conservation of total probability, while

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diagonal elements μ_{ii} correspond to idea stability and off-diagonal terms μ_{ij} to transformation events. Asymmetry ($\mu_{ij} \neq \mu_{ji}$) captures the directional nature of reinterpretation or abstraction, where transitions from simple to complex representations are more probable than the reverse.

The co-activation matrix M_{ij} instead encodes associative relations, measuring how frequently two ideas are jointly active or contextually linked. If $a_i(t)$ denotes the activation of idea i, then $M_{ij} = \lim_{T\to\infty}(1/T)\int_0^T a_i(t)a_j(t)\,dt$. This symmetric matrix quantifies memory-based coupling: high M_{ij} values reflect strong conceptual association, while low values indicate independence. Over time, M_{ij} evolves through learning and forgetting according to $\frac{dM}{dt} = \alpha(pp^{\top}) - \rho M$, where α is the learning rate and ρ the forgetting rate.

To combine transformation and association, a **conceptual distance metric** is defined as

$$D_{ij} = -\log\left[(\mu_{ij} + \mu_{ji}) + \lambda M_{ij}\right],$$

where λ modulates the influence of associative coupling. Small D_{ij} values denote close conceptual kinship, i.e., ideas that frequently transform into or co-occur with each other, while large values signal divergence. The complete matrix D_{ij} forms the framework for reconstructing a phylogenetic tree of thoughts, where each node represents an ancestral conceptual form and branch lengths correspond to the magnitude of divergence.

This structure captures both the dynamic and structural aspects of mental evolution: μ_{ij} describes how ideas mutate across time, while M_{ij} preserves their co-activation history.

When used to assess individual development, the phylogenetic tree could become a map of the unfolding of cognition itself. It could reveal how early sensorimotor schemas and perceptual patterns, which dominate infancy, gradually branch into linguistic, symbolic and abstract domains during childhood and adolescence. Each branching event might represent a bifurcation in representational strategy: for example, the divergence between concrete object representation and symbolic manipulation or between procedural and declarative reasoning. Over time, these branches could proliferate, forming higher-order conceptual networks that underlie logical, aesthetic and moral cognition. In adults, the upper levels of the tree could encompass reflective and metacognitive structures, derived from but still connected to their earlier perceptual and emotional roots.

Convergent branches where two or more distinct cognitive lineages merge mark instances of **recombination or exaptation**, moments in which previously independent ideas fuse to form new constructs. These events could correspond to creativity, insight or analogical reasoning, when the mind reuses existing structures for novel purposes. For instance, a perceptual schema originally shaped by spatial navigation could be exapted to structure reasoning about abstract domains such as social hierarchy, ethical order, or conceptual distance. In this way, cognitive functions that evolved to guide movement through physical space would be redeployed to navigate symbolic or moral spaces, much as in biological evolution wings arise from forelimbs that once served for walking. Conversely, divergent branches might represent specialization or differentiation, where a single conceptual root splits into multiple subfunctions, such as the division of intuitive and analytical reasoning systems.

The **depth** of each branch in the tree could correspond to the temporal persistence or stability of its conceptual lineage: the longer a branch extends before diverging or terminating, the more enduring and robust that family of ideas. Branch terminations represent **extinction events**, indicating the fading or repression of obsolete cognitive forms that once occupied mental territory but no longer adapt to current experience. Periods of rapid branching correspond to intellectual expansion, often triggered by exposure to novel environments, education or emotional upheaval, whereas long, stable segments indicate phases of consolidation and equilibrium.

Beyond its descriptive power, the phylogenetic tree could provide a quantitative and visual synthesis of mental evolution, allowing researchers to infer the internal logic of idea diversification and identify the structural regularities governing conceptual change. When constructed longitudinally, from developmental or autobiographical data, it could portray the living history of a mind: how an individual's ideas originate, mutate, merge and sometimes disappear.

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Together, the Hardy–Weinberg equilibrium and the phylogenetic tree could represent the analytical core of our evolutionary hypothesis of thought. The first provides a static criterion for identifying equilibrium and its perturbations, defining cognitive stability in probabilistic terms. The second introduces an historical dimension, depicting the descent and diversification of ideas. In combination, they could transform the analogy between biological and cognitive evolution into a systematic framework capable of quantifying, mapping and explaining the lawful development of thought within the individual mind.

Comparison with Existing Approaches

Compared with existing approaches, our hypothesis differs in both scope and formulation. Classical cognitive frameworks such as Rumelhart & McClelland's connectionism and predictive coding describe learning and adaptation in probabilistic or dynamical terms, focusing on how neural and representational states minimize error or uncertainty (Pezzulo et al., 2022; Caucheteux et al., 2023; N'dri et al., 2025). In particular, Bayesian inference models formalize cognition as probabilistic reasoning under uncertainty (Aitchison and Lengyel, 2017; Parr and Friston, 2018; Lin and Garrido, 2022; Jin et al., 2023). While these paradigms capture short-term adaptation and optimization, they do not articulate a evolutionary logic of thought across an individual lifespan. Their architectures describe parameter adjustments within static structures rather than the branching, selection and extinction of ideas over time. Our approach extends these probabilistic frameworks by introducing a population-level description of cognition in which thoughts behave as evolving entities subject to variation, selection and inheritance.

Our formulation also connects to Donald Campbell's theory of blind variation and selective retention, which first articulated the epistemological parallel between thought and biological evolution and to Karl Popper's evolutionary epistemology, where knowledge progresses through conjectures and refutations analogous to selection pressures (Simonton 2010; Wettersten 2016; Simonton 2024). However, whereas Campbell and Popper focused on the logic of scientific discovery and knowledge accumulation, our formulation grounds these processes in the continuous microdynamics of individual cognition, making their principles empirically tractable.

Hints of our model can be found in Gerald Edelman's Neural Darwinism, together with Selfridge's Pandemonium, which proposed that neural groups compete for reinforcement through reentrant signaling (Edelman 1993; Grainger et al. 2008; Renier et al., 2014; Tozzi and Peters, 2018). Edelman's view anticipated an intra-individual selectionist framework but remained confined to synaptic and anatomical scales. Later developments by Jean-Pierre Changeux and Stanislas Dehaene extended these ideas toward cognitive architectures and consciousness, emphasizing global neuronal workspace dynamics (Mashour et al., 2020; Volzhenin et al., 2022). Our approach generalizes these biological insights beyond neural implementation, encompassing abstract representations, symbolic recombination and conceptual genealogy.

Theories of cultural evolution and memetics, beginning with Richard Dawkins' concept of the meme describe how ideas replicate across populations of minds (Braga and Wardil, 2023). Yet these theories primarily address inter-individual transmission (how cultural elements spread, compete and mutate socially) rather than the intra-individual selective dynamics shaping the evolution of thought within a single person. Our hypothesis fills this gap by shifting focus inward: each mind is viewed as an ecosystem of ideas.

From the standpoint of cultural-cognitive synthesis, our model is also related to Liane Gabora's theory of autocatalytic cognitive networks, which describes ideas as self-organizing structures evolving through associative activation (Gabora and Steel, 2017; Gabora and Steel 2020; Gabora et al., 2022). Yet Gabora's work emphasizes creativity and cultural innovation, whereas our framework seeks a general law of cognitive evolution applicable to all thought processes, not only creative ones. Similarly, Andy Clark's embodied predictive models and Karl Friston's free-energy principle define cognition as an inferential engine minimizing surprise (Ramstead et al., 2022), but they lack the genealogical and population-based representation of ideas central to our formulation.

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Finally, compared with dynamical systems approaches to cognition like Kelso's, which depicts thought as trajectories through continuous attractor landscapes (Kelso 2016; Zhang et al., 2020; Kelso 2021), our evolutionary model adds a historical and genealogical dimension. It traces the ancestry, recombination and extinction of discrete cognitive lineages rather than describing only instantaneous state transitions. Our approach, linking static and dynamic perspectives of mental evolution, positions cognition within a general evolutionary grammar shared by all adaptive levels, from neural assemblies and conceptual structures to cultural processes.

Conclusions

We propose that the laws governing biological evolution could be meaningfully extended to describe the internal dynamics of human thought. We treat cognition not as a sequence of independent computations but as an evolving population of ideas whose relative prevalence changes through processes of variation, selection, inheritance and extinction. The novelty of this perspective lies in formal tractability: each mental operation is mapped onto a specific evolutionary mechanism, allowing thought to be expressed as a quantifiable phenomenon. Its main advantage is explanatory integration: instead of treating psychological change, neural plasticity and epistemic development as unrelated processes, they become aspects of the same underlying dynamic. Thinking becomes the evolutionary activity of meaning: a process by which the mind continuously generates and selects representations to maintain coherence with its experiential environment. Our synthesis reframes mental evolution as a process of internal natural history, revealing that the same structure that sustains life could also govern the transformation of meaning.

Our heuristic model faces intrinsic limitations. Its abstraction compresses the richness of cognition into a finite set of mathematical variables, simplifying the affective and contextual dimensions contributing to real mental life. The representation of ideas as discrete variants neglects the fluid, overlapping nature of many conceptual processes. Furthermore, empirical validation remains challenging: estimating parameters like cognitive mutation or selection rates requires indirect measurement through behavior, neural activation or semantic networks. The analogy between genetic and cognitive evolution must therefore be understood as structural rather than literal. Unlike genes, ideas do not replicate through molecular mechanisms and their persistence depends on subjective, social and cultural reinforcement.

From a scientific standpoint, the potential applications of our approach are wide. In psychology, our model could quantify how internal equilibrium varies during problem solving, emotional adaptation or creative episodes. Making explicit the selective pressures acting upon ideas, our procedure allows quantitative prediction of which thoughts stabilize and which decay. In practical terms, this could provide a basis for the mathematical study of conceptual development, memory consolidation and cognitive resilience. In developmental psychology, our approach could provide a model for understanding how conceptual branching in children follows predictable evolutionary pathways from perceptual schemas to abstract reasoning. In educational settings, it could guide the design of learning sequences to balance variation and selection, optimizing the retention of meaningful content.

In neurocognitive research, it might allow modeling of synaptic competition, neural reuse and pruning as concrete correlates of selection and extinction. In neuropsychology, the temporal evolution of conceptual fitness may help to describe cognitive aging or recovery after injury, where the decay and reorganization of thought patterns follow identifiable selection laws. Future research should aim to parameterize these dynamics experimentally, for instance, by correlating brain connectivity patterns with shifts in idea prevalence during learning or by reconstructing "conceptual phylogenies" from longitudinal verbal data. In computational modeling, our evolutionary equations may generate artificial cognitive ecologies capable of self-directed conceptual growth. In artificial intelligence, our approach could guide the design of architectures that evolve ideas instead of merely optimizing fixed objectives, fostering systems capable of genuine creativity and open-ended conceptual growth.

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The central question, whether the evolutionary laws of life can account for the evolution of thought, finds here an affirmative answer. The same principles that govern biological adaptation appear to underlie the transformation of ideas, revealing a natural continuity between life and consciousness. Our model provides a backbone for merging the logic of Darwinian adaptation with modern theories of predictive processing and free energy, suggesting that evolution and inference are two facets of the same adaptive principle. The broader implication is methodological: rather than isolating cognition from biology, our approach situates it within the continuum of adaptive systems.

Declarations

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.

Consent for publication. The Author transfers all copyright ownership, in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal and has not been previously published.

Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.

Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence their work.

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Authors' contributions. The Author performed: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding, administrative, technical and material support, study supervision.

Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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Article

Pretopological Modal Logic for Local Reasoning and Uncertainty

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Abstract

We introduce Pretopologically-Neighborhoood Modal Logic (PNML), a formal framework for reasoning about local knowledge and uncertainty based on pretopological neighborhood semantics. Unlike classical Kripke or general neighborhood models assuming global structural properties or arbitrary accessibility, PNML restricts neighborhood systems to satisfy the axioms of pretopological spaces, i.e., upward closure and self-inclusion, without requiring intersection stability or closure under arbitrary unions. This enables a finer-grained representation of agents' information in contexts where only partial or locally available knowledge is relevant. We define the truth conditions for modal operators in terms of pointwise neighborhood filters, introduce a basic axiomatic system and prove its soundness/completeness with respect to the full class of pretopological frames, ensuring that the syntactic and semantic components of the logic are aligned. Then, we examine the expressivity of PNML in relation to both normal and non-normal modal logics, arguing that pretopological constraints introduce structural distinctions not captured by standard neighborhood models, particularly under minimal closure conditions. We present examples illustrating the framework's utility in modeling observer-dependent scenarios, including epistemic uncertainty, context-sensitive reasoning and localized inference. Notably, PNML may accommodate settings in which traditional modal logic either overgeneralizes or underrepresents the dynamics of localized information. By grounding modal reasoning in pretopological structure, PNML may apply to distributed computing with local inputs, quantum mechanics involving contextual observation, cellular signaling and ecological systems in biology, modal update operations under information change, connections to fixpoint and coalgebraic semantic frameworks, proof systems for local inference and real-world modeling of belief revision and protocol verification.

Keywords: non-normal modalities; observer-dependence; partial observability; informational locality; frame semantics

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1. Introduction

In modal logic, neighborhood semantics has long served as a flexible generalization of Kripke semantics, particularly in accommodating non-normal modal systems where the necessity operator does not obey standard axioms such as distribution or necessitation (Pacuit 2017; Chen et al., 2021; Ferenz and Tedder, 2023). The foundational approach, introduced by Scott (1970) and Montague (1970), allows modal truth conditions to depend on sets of sets of possible worlds, rather than fixed accessibility relations. This abstraction has proven especially useful in modeling epistemic states, deontic reasoning and belief revision, where the behavior of modal operators cannot be captured adequately by relational frames alone (Hansen 2003; Pacuit 2017). However, general neighborhood semantics remains structurally unconstrained: it allows any collection of sets to serve as neighborhoods, even when those collections violate intuitively desirable structural principles such as coherence or persistence (Fan 2020; Tedder and Ferenz 2022). As a result, it becomes difficult to interpret modal validity in terms of information flow or inferential stability, especially when the neighborhoods lack any formal closure properties. Recent efforts have explored more structured

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variations, such as topological semantics (McKinsey and Tarski 1944) and filter-based systems (Chellas 1980), but these often reintroduce the rigidity of relational frameworks or impose overly strong assumptions. This prompts the question of whether a middle ground is possible, one that retains the semantic richness of neighborhoods while enforcing minimal structural discipline.

To address this gap, we introduce a modal framework grounded in pretopological spaces, where each world is associated with a neighborhood filter satisfying upward closure and reflexivity, but not necessarily closed under finite intersection. This restriction preserves the flexibility of neighborhood semantics while incorporating formal constraints that enhance interpretability and consistency. By anchoring modal operators in pretopological filters, our system formalizes local information conditions without assuming full topological regularity or relational determinacy. This construction allows modal evaluation to be sensitive to locally available information and supports more precise reasoning about knowledge or belief that is grounded in structurally coherent, though incomplete, contexts.

We will proceed as follows. First, we provide the formal definitions of pretopological frames and the syntax and semantics of the proposed modal system. Next, we introduce a suitable axiom system and establish its soundness and completeness, examining the proof-theoretic properties that characterize and validate the logic. Finally, we explore structural and expressive features of the logic and conclude with a discussion of its implications.

2. Background

Modal logic has traditionally been framed using relational semantics, in which modal operators are interpreted via binary accessibility relations between possible worlds (Blackburn et al., 2002; Holliday 2024). While this Kripkean framework has proven powerful for normal modal systems, its limitations become apparent when modeling weaker or context-dependent forms of modality like default reasoning, obligation, belief or plausibility (Weiss and Birman, 2024). To address these cases, neighborhood semantics was introduced as a more general semantic foundation. Scott and Montague (1970) initiated this perspective by replacing relational frames with neighborhood functions, enabling each world to be associated with a collection of subsets of the domain considered admissible or relevant for modal evaluation. This abstraction removed the dependence on a binary accessibility relation and enabled the interpretation of modal logics beyond the scope of S5-like systems, including non-normal and subnormal modal logics. Later developments by Chellas (1980) and Hansen (2003) refined the technical properties of these models, leading to a robust semantics capable of capturing nonmonotonic reasoning, conditional obligations and informational vagueness.

Despite their generality, classical neighborhood models impose minimal structural constraints on the organization of neighborhoods across worlds (Hansen et al., 2009; Moniri and Maleki 2015). The sets of neighborhoods assigned to each world can vary arbitrarily and are not required to satisfy any closure properties or structural coherence. While this provides maximal flexibility, it also introduces interpretational ambiguity. For instance, the absence of minimal closure properties like upward containment or filter-like behavior can obscure the notion of informational robustness or local consistency. To address these issues, several authors have explored enriched semantics based on topological or algebraic constraints. Topological modal logic, for example, interprets modalities using open sets in topological spaces, with modalities reflecting interior or closure operations (McKinsey and Tarski, 1944). While this structure introduces regularity and continuity into modal interpretation, it also imposes strong global constraints, such as the requirement that open sets be closed under arbitrary unions and finite intersections. In many applications, particularly those involving uncertainty or observer-relative reasoning, these assumptions may be too strong or unrealistic. For instance, in environmental sensor networks, individual nodes collect data solely from their immediate surroundings. Since global access to all information is typically unrealistic, a logic such as PNML, grounded in locally accessible neighborhood structures, more accurately reflects how these systems function under uncertainty and partial, observer-relative conditions.

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Pretopological spaces display a natural intermediate framework. Introduced in the context of generalized convergence theory (Kuratowski 1966), a pretopological space assigns to each point a filter of neighborhoods that satisfies self-inclusion and upward closure but does not require closure under finite intersections. This generalization retains the intuition of locally structured information without enforcing full topological regularity. It allows reasoning to be grounded in a notion of proximity or relevance that is internally coherent but only locally structured.

In this work, we build a modal semantic framework on pretopological spaces, interpreting necessity via membership in upward-closed neighborhood filters. We aim to provide a formally disciplined, semantically grounded middle path between unrestricted neighborhood semantics and fully topological models. By leveraging the minimal structure of pretopology, we aim to model local modalities that reflect constrained but flexible access to information, inference or belief.

3. Pretopological Modal Logic: Syntax and Semantics

We define our Pretopologically-Neighborhoood Modal Logic (PNML) by specifying a class of semantic models in which the modal operator is interpreted over a pretopological structure. Let W be a non-empty set of possible worlds. A **pretopological frame** is a pair $\langle W, N\rangle$, where $N: W \to P(P(W))$ assigns to each world $w \in W$ a **filter** $N(w)$ of neighborhoods. Each $N(w)$ must satisfy the following two conditions:

(1) every $N \in N(w)$ contains w;

(2) if $N \in N(w)$ and $N \subseteq M \subseteq W$, then $M \in N(w)$.

This ensures that each $N(w)$ is upward closed and reflexive, aligning with the classical definition of pretopological neighborhood systems (Kuratowski 1966). Importantly, closure under finite intersections is not assumed, distinguishing the structure from a topological base and allowing for a broader range of semantic behavior.

A **PNML model** is a triple $\langle W, N, V\rangle$, where $V: Prop \to P(W)$ is a valuation function assigning to each propositional variable a set of worlds at which it holds. The truth conditions for the modal language are defined inductively in the standard way, with atomic formulas evaluated according to V, Boolean connectives interpreted classically and the modal clause given as follows:

$$w \Vdash \Box\varphi iff \exists N \in N(w) \text{ such that } \forall v \in N, v \Vdash \varphi.$$

This semantic clause reflects the locality of necessity in pretopological settings: $\Box\varphi$ is true at w just in case there is a neighborhood of w where φ is uniformly true. Note that the use of existential quantification over neighborhoods aligns this clause with weaker modal systems, including monotonic and non-normal logics (Hansen 2003; Pacuit 2017). In contrast to Kripke semantics, there is no global accessibility relation dictating reachability between worlds. Instead, each world defines its own perspective through its neighborhood filter.

This formulation allows modal evaluation to reflect partial and context-sensitive knowledge. The minimal structural requirements on $N(w)$ preserve interpretive flexibility while avoiding the arbitrariness of fully general neighborhood systems. The absence of intersection closure allows different informational pathways to remain disjoint, preserving distinctions between incompatible or non-overlapping contexts. Consequently, PNML generalizes the semantics of many weak modal systems while grounding them in a mathematically coherent spatial structure. This model structure supports the definition of logical consequence, validity and frame definability, which will be examined in subsequent sections.

4. Axiom System and Semantic Correspondence

To formalize the deductive structure of PNML, we introduce a minimal axiomatic system reflecting the structural constraints of pretopological neighborhood semantics. The system extends classical propositional logic with modal rules and axioms that are valid in all PNML frames, as defined in the previous section. We denote this system by **PNML0_00**.

The axioms of PNML0_00 are as follows:

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1. All tautologies of classical propositional logic are included in the system.
2. $\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$ (Monotonicity)
3. $\Box\varphi \to \varphi$ (Reflexivity)

The system includes a single rule of inference:

Modus Ponens: From φ and $\varphi \to \psi, infer\ \psi\backslash psi\psi$. Necessitation is omitted, consistent with the absence of normality.

The axiom of monotonicity corresponds to the upward closure of neighborhood filters, ensuring that if a formula holds in one neighborhood, it will also hold in all supersets of that neighborhood (Hansen 2003). The axiom of reflexivity reflects the requirement that every neighborhood of a world contains that world, hence guaranteeing that if a formula is necessary, then it must hold locally. This setup avoids commitment to stronger modal principles such as distribution or necessitation, which would not be valid in all pretopological frames.

To establish soundness, we verify that all axioms of PNML0_00 are valid in every PNML model and that the inference rule preserves validity. The reflexivity of neighborhoods ensures the validity of $\Box\varphi \to \varphi$, while monotonicity follows from the closure of neighborhoods under supersets. Since the semantics is Boolean and propositional tautologies are preserved under all valuations, classical reasoning is sound by construction.

Completeness is demonstrated via canonical model construction. We define the set of maximally consistent sets over the language and assign to each such set Γ a **neighborhood filter** $N(\Gamma)$ consisting of all sets of the form $\{\Delta \in Wc \mid \varphi \in \Delta\}$, where $\Box\varphi \in \Gamma$. This construction ensures that the neighborhood system satisfies the conditions of a pretopological space, including upward closure and self-inclusion, while avoiding intersection closure. Standard canonical model techniques (Chellas 1980; Pacuit 2017) are adapted to verify the truth lemma and derive the completeness theorem: every formula valid in all PNML frames is derivable in PNML0_00.

This establishes a sound and complete correspondence between the axiomatic system PNML0_00 and the class of pretopological modal frames, justifying the proposed semantic framework as a framework for reasoning with local and structurally constrained modalities.

5. Expressivity and Structural Properties

The expressive capacity of PNML can be examined in comparison with both standard Kripke-based modal logic and classical neighborhood semantics. Given that PNML permits the omission of necessitation and closure under intersection, it naturally aligns more closely with weak modal logics such as monotonic and subnormal systems. However, the introduction of pretopological structure introduces specific constraints that influence definability, bisimulation and frame conditions in nontrivial ways. We address these issues in the following paragraphs.

A primary structural feature of PNML is its alignment with monotonic modal logic, due to the enforcement of upward closure in all neighborhood filters. This ensures the validity of monotonicity $\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$, which distinguishes PNML from fully arbitrary neighborhood models, where this principle may fail (Hansen 2003). Furthermore, PNML's restriction to neighborhood systems that satisfy reflexivity $\Box\varphi \to \varphi$ allows it to capture the local realism often assumed in weak epistemic or observational frameworks. At the same time, the logic is strictly weaker than systems based on topological semantics, which validate stronger modal axioms such as closure under finite intersection or the idempotency of the interior operator $\Box\varphi \to \Box\Box\varphi$ (McKinsey and Tarski 1944).

Bisimulation in PNML requires modification from the standard relational and neighborhood settings (Hamal and Terziler, 2015). Since each world in a PNML frame carries a distinct pretopological filter rather than an accessibility relation or an unstructured neighborhood function, standard bisimulation invariance results do not carry over directly. A suitable notion of PNML-bisimulation must preserve the satisfaction of modal formulas by comparing not only pointwise valuations but also the structure of neighborhood filters. Two points w and w' are PNML-bisimilar if there exists a relation preserving propositional variables and respecting inclusion relations among neighborhoods in a way that reflects the structural constraints of pretopology. Establishing the

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bisimulation invariance of PNML formulas under this condition supports the adequacy of the semantic framework and bounds its expressive power with respect to definable frame properties (Blackburn et al., 2001).

Another relevant structural aspect is frame definability. In PNML, several frame properties correspond directly to specific modal axioms. For example, the axiom $\Box\varphi \to \varphi$ captures the reflexivity of the neighborhood filter, while the monotonicity axiom $\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$ reflects the upward closure of neighborhoods. However, the lack of closure under intersection implies that axioms such as $\Box\varphi \wedge \Box\psi \to \Box(\varphi \wedge \psi)$ are not valid and frames satisfying stronger closure conditions are not definable within the base system PNML0_00. This sets clear boundaries for the modal definability of structural features within the logic and distinguishes PNML sharply from both topological and Kripkean modal systems.

Together, these features suggest that PNML occupies a distinct position in the modal landscape. It balances the generality of neighborhood semantics with the minimal order-theoretic structure of pretopology, providing a semantics that is weak enough to model uncertainty and locality, yet structured enough to permit rigorous logical treatment of modal inference.

6. Illustrative Example

To demonstrate the semantic mechanics of PNML and the interpretive distinctions it supports, we present a minimal model involving three possible worlds and a single propositional variable. The aim is to clarify how modal evaluation proceeds in a pretopological context and to contrast this with behavior in relational or fully topological models.

Let $W = \{w1, w2, w3\}$ be the set of worlds. Let the valuation V assign the propositional variable p as true at $w1$ and $w2$ and false at $w3$; that is, $V(p) = \{w1, w2\}$. The pretopological neighborhood filters are defined as follows:

- $N(w1) = \{\{w1\}, \{w1, w2\}, \{w1, w2, w3\}\}$
- $N(w2) = \{\{w2\}, \{w2, w3\}, \{w1, w2, w3\}\}$
- $N(w3) = \{\{w3\}, \{w2, w3\}, \{w1, w2, w3\}\}$

Each neighborhood filter satisfies upward closure and self-inclusion and none requires closure under intersection. Consider the evaluation of $\Box p$ at each world. At $w1$, the neighborhood $\{w1\} \subseteq V(p)$ and so $w1 \Vdash \Box p$. However, the largest neighborhood $\{w1, w2, w3\}$ includes $w3$, where p is false, so not all neighborhoods validate p. Since the semantics requires the existence of at least one neighborhood where p holds universally, $\Box p$ remains true at $w1$.

At $w2$, the neighborhood $\{w2\} \subseteq V(p)$, so $\Box p$ is also true at $w2$. In contrast, at $w3$ all neighborhoods intersect with worlds where p is true, but none are fully included in $\{w3\} \subseteq W \setminus V(p)$, making $\Box p$ false at $w3$. This illustrates the asymmetry of modal truth induced by the neighborhood system's structure and highlights how local availability of information (as captured by neighborhoods) governs necessity in PNML.

We now present a practical example of an application of PNML. Consider a simplified delivery network with three stations: $w1$, $w2$ and $w3$. A package is confirmed to have arrived at $w1$ and $w2$, but not at $w3$. Each station has limited visibility defined by its neighborhood filter. Station $w1$ can verify its own state, its state plus $w2$, or the entire network. Since it has access to a neighborhood fully contained within the set of stations where the package has arrived, it concludes with certainty that delivery occurred. Similarly, $w2$ sees neighborhoods confirming delivery. Station w3w_3w3, the final user waiting for the package, cannot access any neighborhood entirely confined to non-delivery states, as all of its accessible views include stations where the package has arrived. Consequently, it lacks sufficient local support to conclude necessity.

This example illustrates how PNML captures local, constrained reasoning under uncertainty from the perspective of limited observers. Rather than relying on global reachability or universally available information, PNML evaluates necessity through locally bounded, upward-closed neighborhood structures that reflect partial observability and context-dependent knowledge. More technically, PNML departs from topological or relational semantics, where the truth of $\Box p$ depends

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on open-set containment or accessibility across the model. Instead, it requires the existence of a specific neighborhood in which the formula holds entirely. As a result, PNML supports modal distinctions even within symmetric or densely connected models. This highlights the central role of pretopological filters in expressing modalities grounded in local informational structure.

7. Extensions and Logical Variants

The core framework of PNML admits natural extensions that retain its semantic foundation while increasing its expressive range. One extension involves the introduction of additional modal operators reflecting **dual or complementary neighborhood behavior**. For instance, a possibility operator $\Diamond \varphi$ can be defined in terms of non-universal failure: $w \Vdash \Diamond \varphi$ if there exists a neighborhood $N \in N(w)$ and some $v \in N$ such that $v \Vdash \varphi$. While definable via the dual of necessity in classical modal logic, this operator gains independent semantic interest in PNML due to the lack of normality and the non-deterministic structure of the neighborhoods.

Another extension concerns the incorporation of **dynamic modalities**. In contexts where the neighborhood filter evolves in response to actions or information updates, one may define dynamic operators transforming $N(w)$ at each step. This can model the refinement of local information states, as seen in update semantics and dynamic epistemic logic. Formal treatment would require an additional specification of update functions acting on pretopological frames and corresponding modifications to the satisfaction relation. This introduces challenges, as updates may break the structural properties of the filter unless carefully constrained.

A further direction involves the **algebraic semantics** of PNML. Pretopological filters correspond to certain closure operators and monotone functions, suggesting a dual algebraic characterization in terms of neighborhood lattices or modal algebras equipped with weakened intersection properties. Investigation of completeness and representation theorems in this setting may yield a deeper understanding of the underlying logical geometry.

Finally, **multi-agent generalizations** of PNML can be introduced by assigning distinct pretopological filters $Na(w)$ for each agent a. This enables modeling differences in local knowledge or information availability across agents, preserving the logic's sensitivity to structure while supporting interaction between heterogeneous perspectives. These systems may incorporate axioms or interaction rules that govern the relationships between agents' filters, leading to a family of sublogics with varying epistemic strength.

Overall, each of these extensions preserves the core features of PNML, namely the use of structured yet minimally constrained neighborhood systems, while enriching the formalism to accommodate more complex or interactive modalities. The resulting variants may serve as useful tools in the study of reasoning under partial observability, heterogeneous knowledge systems and generalized information dynamics.

8. Conclusions

The development of our Pretopologically-Neighborhoood Modal Logic (PNML) yields a formally structured modal framework grounded in minimal but coherent semantic assumptions. By constructing models where each world is associated with a pretopological neighborhood filter—closed under supersets and containing the point itself, We established a logic that preserves the interpretive flexibility of neighborhood semantics while introducing structural constraints absent from more general frameworks (Kuratowski 1966). The modal operator was defined in terms of localized satisfaction across neighborhood filters and the axiom system PNML0_00 was shown to be both sound and complete with respect to this class of frames. The semantics allows for the existence of neighborhoods supporting a formula without requiring universal or intersection-based closure, enabling PNML to represent diverse local reasoning patterns. The system was also shown to preserve desirable properties such as upward monotonicity and reflexivity, with definability and bisimulation conditions adapted to its non-normal nature (Hansen 2003; Pacuit 2017).

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The novelty of our approach lies in embedding modal reasoning directly within pretopological spaces, avoiding both the rigidity of relational models and the arbitrariness of unconstrained neighborhood semantics (Scott 1970; Montague 1970). While topological modal logic imposes strong global closure properties (McKinsey and Tarski, 1944) and Kripke semantics relies on binary reachability, our system formalizes necessity using local information patterns rooted in upward-closed filters. This structural minimalism does not require intersection closure, thus occupying a space between the fully general and the overly restrictive. Unlike classical neighborhood logics, which often lack coherent frame-theoretic foundations (Chellas 1980), PNML ensures that modal inference respects an order-theoretic organization compatible with generalized convergence and local reasoning. In contrast to peritopological approaches that remain relational at the core (Hamal and Terziler, 2015), our framework defines modality through directly neighborhood-based conditions and emphasizes syntactic completeness alongside semantic interpretation. Compared to topological modal logics, our logic omits idempotence and compactness, which may not be valid in dynamic or observational systems. The absence of necessitation reflects a more conservative account of knowledge or belief, applicable to partial or evolving environments.

Nonetheless, several limitations emerge. First, the absence of intersection closure means that conjunctions of necessary formulas do not yield further necessary formulas, limiting deductive strength in classical terms. While this feature is consistent with the intended semantics, it may hinder applications that assume distributive modal behavior. Second, the omission of necessitation, though deliberate, weakens the inferential machinery relative to normal modal systems (Blackburn, et al., 2001). Extensions requiring stronger meta-properties such as interpolation or uniform substitution will require substantial modification. In terms of semantic complexity, the need to define canonical models through filtered neighborhood constructions may present technical challenges in developing decision procedures or model-checking algorithms. Finally, PNML as currently formulated is propositional; its extension to quantified or predicate forms is nontrivial, particularly due to the local nature of neighborhood filters and the absence of global constraints. These limitations suggest that further refinements are needed to scale it toward more expressive logical environments.

PNML has potential applications in fields where local knowledge and partial observability are fundamental. In distributed computing, agents often make decisions based on partial inputs or neighborhood-based communication, conditions well-modeled by pretopological filters. In quantum mechanics, where observation and state determination are contextual (van Fraassen 1991), PNML may provide a way to model knowledge acquisition without global state assumptions. In biology, particularly in cellular signaling or ecological reasoning, local interactions without full system knowledge are prevalent and reasoning about such conditions may benefit from this framework. Future research could explore modal update operations on neighborhood filters, enabling dynamic reasoning under information change (van Benthem 2011). Another avenue involves the development of a proof calculus more closely aligned with neighborhood filter behavior, possibly incorporating labeled or tableau methods tailored to the logic's structure. Empirically testable hypotheses include the ability of PNML to model real-world inference scenarios more accurately than Kripkean systems in domains like belief revision or protocol verification (Fagin et al., 1995). We suggest further exploration of algebraic dualities and canonical extensions as a means of connecting PNML to modal fixpoint calculi and coalgebraic frameworks, potentially revealing deeper structural correspondences and expanding its applicability within abstract semantic models (Venema 2007).

In summary, we asked whether a semantically grounded modal system could be built on pretopological structures while supporting sound inference under minimal constraints. We answered affirmatively by presenting a complete and well-defined logic that interprets modality through local, upward-closed neighborhood filters. Our framework reconstructs modality directly from neighborhood-based locality, without relying on relational or topological assumptions. PNML occupies a middle ground between fully unrestricted neighborhood semantics and highly constrained topological or relational systems. Its structural simplicity make it well suited for reasoning under uncertainty in settings with partial, locally available information.

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Towards Enzymatic-Like Control of Cellular Physical Processes

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Keywords: mesoscopic catalysis; hysteresis; energy landscape surgery; soft matter biology; organizational kinetics

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Article

Towards Enzymatic-Like Control of Cellular Physical Processes

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Abstract

Cellular dynamics rely on numerous physical processes, including phase separation, membrane remodeling, stress relaxation, transport and stochastic fluctuation control, which are commonly treated as passive consequences of thermodynamics, mechanics or statistical physics. Here we advance the hypothesis that living systems can actively regulate such processes through biologically produced, reusable agents that act analogously to enzymes, but target physical state transitions rather than chemical reactions. We introduce the concept of enzymatic-like control, defined as the localized and saturable lowering of kinetic, topological or statistical barriers in configuration space by endogenous cellular components. Among the many cellular physical phenomena to which this concept may apply, we focus on biomolecular condensate nucleation and dissolution as a concrete and analytically tractable example. Condensate dynamics are conceived as barrier-limited physical reactions whose kinetic rates can be selectively modulated by putative enzyme-like Phase-Kinetases without altering equilibrium phase behavior. Using hazard-based inference and survival analysis, we present simulations demonstrating how these putative enzyme-like agents could generate small effective free-energy shifts on the order of a few kT, resulting in orders-of-magnitude changes in nucleation rates and yielding explicit, falsification-oriented criteria. Our framework complements existing biochemical and mechanical models by providing a testable perspective on the active regulation of physical dynamics without invoking new chemistry or nonstandard physics. It reframes cellular organization as the selective control of physical state transitions, rather than their passive accommodation within fixed physical laws.

Keywords: mesoscopic catalysis; hysteresis; energy landscape surgery; soft matter biology; organizational kinetics

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1. Introduction

Cellular organization relies on a wide range of physical processes, including phase separation, membrane remodeling, stress relaxation, transport and stochastic fluctuation dynamics. In current state-of-the-art descriptions, these physical processes are attributed to thermodynamics, continuum mechanics, polymer physics or stochastic kinetics, while their biological control is reduced to boundary conditions, material parameters or externally applied forces. This perspective has been highly successful in capturing generic behaviors and scaling laws, yet it faces persistent limitations when confronted with the speed, selectivity, spatial localization and reversibility observed in living systems. Empirically, many cellular physical transitions occur orders of magnitude faster than predicted by passive models, display saturable dependence on specific molecular components and exhibit strong context sensitivity that cannot be explained by global energetic inputs alone. Existing explanations invoke increasingly complex material descriptions or active forcing, but these approaches blur the distinction between force generation and rate control, lacking a unifying principle comparable to enzymatic catalysis in chemistry. As a result, there is currently no systematic framework for interpreting how biology may actively regulate physical state transitions without

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invoking new chemical reactions or abandoning physical consistency. These limitations motivate a reconsideration of how biological activity interfaces with physical dynamics at mesoscopic scales.

Our approach introduces the concept of enzymatic-like control as a theoretical perspective on biological regulation of physical processes. Within our framework, cells are assumed to produce endogenous, reusable and localized agents that act by lowering effective kinetic, topological or statistical barriers in configuration space, rather than by modifying equilibrium states or generating macroscopic forces. This notion extends the logic of enzymatic catalysis beyond chemistry, while remaining fully compatible with established physical laws. Recent evidence that random heteropolymers can exhibit enzyme-like catalytic behavior without sequence optimization supports the plausibility of enzymatic-like control, suggesting that reusable, non-specialized biological structures may lower effective barriers for physical state transitions, analogously to chemical catalysis (Yu et al., 2026).

Our aim is to identify shared operational signatures of active regulation, including saturability, exponential sensitivity to small free-energy shifts and spatially confined action and to use these signatures to reinterpret the dynamics of biomolecular condensation, a process often labeled as a passive consequence of phase separation physics.

We will proceed as follows. We first apply the concept of enzymatic-like control directly to biomolecular condensate nucleation and dissolution, treating condensation as a barrier-limited physical reaction subject to localized kinetic modulation. Building on this focused analysis, we present a quantitative case study of condensate nucleation from which explicit, falsification-oriented predictions are derived. We conclude by integrating these results into a broader perspective on how enzymatic-like regulation may operate in cellular physical dynamics.

2. Phase Separation Nucleation and Dissolution as an Enzymatic-Like Process

We provide here a concrete example of enzyme-like regulation acting on a physical process rather than on a chemical reaction. We focus on biomolecular condensates, whose nucleation and dissolution arise from well-characterized physical phase transitions, yet display a degree of precision, spatial localization, reversibility and context dependence that is difficult to reconcile with passive thermodynamic control alone. Condensate dynamics thus provide a tractable and experimentally accessible setting to explore the possibility that biological agents actively modulate the kinetics of physical state transitions by biasing rates, barriers and pathways without redefining the underlying thermodynamic landscape.

Physical description of biomolecular phase separation. Phase separation of biomolecules into membraneless condensates is commonly described as a mesoscopic physical process governed by polymer physics and nonequilibrium thermodynamics (Alberti and Dormann 2019). In this framework, liquid–liquid phase separation (LLPS) arises from multivalent, weak interactions among proteins and nucleic acids, leading to demixing once concentration or interaction thresholds are exceeded (Hyman, Weber and Jülicher 2014; Banani et al. 2017; Dai et al. 2024). Mean-field approaches and extensions of Flory–Huggins theory account for coexistence regions, interfacial tension and material properties such as viscosity and elasticity and have been instrumental in rationalizing condensate formation in simplified in vitro systems (Choi, Holehouse and Pappu 2020; Dignon et al. 2018; Guillén-Boixet et al. 2020).

These physical descriptions predict that nucleation and dissolution kinetics are determined primarily by diffusion-limited encounters and by free-energy barriers depending smoothly on concentration and interaction strength. Within this view, condensate dynamics are expected to scale continuously with thermodynamic parameters and reversibility is governed by proximity to coexistence boundaries.

Empirical limits of passive thermodynamic accounts. Nevertheless, a growing body of experimental evidence indicates that purely physical accounts are insufficient to explain condensate

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behavior in living cells. Condensates often nucleate at reproducible, spatially restricted sites such as transcriptional loci, centrosomes or stress granule seeds, even when bulk concentrations are uniform (Alberti and Dormann 2019; Shin and Brangwynne 2017). Assembly and dissolution frequently occur on timescales far shorter than predicted by diffusion alone and condensates can persist or dissolve despite remaining well within predicted coexistence regimes (Snead and Stachowiak 2019; Riback et al. 2020).

Moreover, condensates display strong hysteresis and memory effects: prior assembly history influences subsequent nucleation probability and material state, incompatible with equilibrium phase diagrams (Klosin et al. 2020; Jawerth et al. 2020). Condensates with similar molecular composition can exhibit sharply different lifetimes, internal dynamics or viscoelastic properties depending on cellular context, post-translational state or metabolic conditions. These observations support the hypothesis that cells can actively regulate the kinetics of phase transitions, rather than merely tuning thermodynamic control parameters.

Enzymatic-like interpretation of nucleation and dissolution. Phase separation nucleation and dissolution could be interpreted as an enzymatic-like process acting on mesoscale assembly dynamics. Specific biological agents could catalyze transitions between mixed and demixed states by selectively lowering or raising effective free-energy barriers in configuration space, without altering the equilibrium phase diagram.

This role is analogous to enzymatic catalysis in chemistry, where reaction pathways are accelerated while equilibrium constants remain unchanged. Applied to LLPS, enzymatic-like control implies that relatively small effective barrier shifts, on the order of a few kT, could generate orders-of-magnitude changes in nucleation or dissolution rates, consistent with experimentally observed kinetics. This interpretation does not require new physical principles or exotic interactions but reassigns an active regulatory role to biological components.

Putative phase-kinetases as hidden regulators. Putative agents of this control, which we collectively refer to as phase-kinetases, may include a range of biological structures capable of modulating condensate nucleation and dissolution kinetics.

a. In our context, RNA helicases of the DEAD-box family would act as localized, ATP-driven remodeling agents that could continuously reshuffle RNA–protein interaction topologies inside condensates (Cargill, Venkataraman, and Lee 2021; Naineni et al. 2023; Hirth et al. 2024; Li et al. 2024). Rather than globally dissolving assemblies, their activity would selectively lower kinetic barriers for rearrangement by transiently weakening RNA-mediated crosslinks, thereby potentially preventing topological trapping while preserving condensate identity. Such action would effectively tune the internal relaxation time of the condensate without altering its macroscopic composition, allowing rapid switching between metastable organizational states in response to small energetic inputs (Zhang et al. 2026).
b. ATP-dependent chaperones would be expected to operate in a closely related but mechanistically distinct manner (Trösch et al. 2015; Alfi et al. 2019; Wang et al. 2025). In our framework, they would transiently destabilize multivalent protein–protein contacts that might otherwise accumulate into long-lived kinetic bottlenecks. By repeatedly injecting energy at specific interaction nodes, chaperones would locally flatten the free-energy landscape, accelerating escape from arrested configurations while leaving the overall phase-separated state intact. This behavior would support a view of condensates as dynamically maintained nonequilibrium structures whose stability depends on continuous, spatially confined energy dissipation rather than on static affinity balances.
c. Post-translational modification systems acting on low-complexity domains would provide an additional layer of fine-grained control (Wang, Osgood, and Chatterjee 2022; Gong et al. 2024; Tao et al. 2024; Zhang et al. 2025). In our context, phosphorylation, acetylation, or methylation

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would not merely shift average interaction strengths but would modulate the lifetime and cooperativeness of specific contact motifs. Because such modifications could be rapidly written and erased, they would function as reversible kinetic regulators that bias assembly or disassembly pathways without requiring large concentration changes. This would enable sharp, switch-like responses of condensate dynamics to weak upstream signals, consistent with an enzymatic-like mode of regulation ().

d. ATP-driven disaggregases would extend this logic to the active dissolution regime (Wentink et al. 2020; Low et al. 2021; Mahapatra et al. 2023). Rather than functioning solely as emergency cleanup factors, they would behave as controlled sinks that selectively remove over-stabilized interaction clusters, thereby maintaining condensates near a regime of high responsiveness. By targeting specific substructures, disaggregases would preserve the overall compartment while preventing irreversible aging or pathological solidification, stabilizing condensates as long-lived yet dynamically renewable entities ().

e. A complementary mechanism would involve metabolic enzymes and cofactors regulating local ATP, ADP, or NADH availability within condensates. In our context, such local metabolic tuning would modulate interaction lifetimes and reaction barriers without altering bulk cytosolic concentrations. Condensates would thus act as microreactors with partially autonomous energetic states, where small metabolic fluctuations could be amplified into large kinetic effects on assembly or dissolution. This would provide a direct physical route by which cellular metabolic state might bias condensate behavior without invoking global energy shifts (Dai et al. 2024).

f. Scaffold proteins with conformational switching capabilities would further sharpen this regulation (Greenwald et al. 2014; Yang et al. 2022; Ball, Barnett, and Goult 2024). In our framework, conformational transitions would selectively expose or occlude multivalent binding motifs, effectively acting as gating mechanisms that control access to interaction networks. Because such switches are often ligand- or energy-sensitive, they would allow condensates to behave as conditional reaction platforms, assembling or relaxing only when specific structural states are populated, reinforcing a barrier-controlled rather than equilibrium-driven interpretation (Abyzov, Blackledge, and Zweckstetter 2022; DiRusso, Dashtiahangar, and Gilmore 2022; Li, Tresset, and Zandi 2025).

g. Finally, intrinsically disordered regions themselves could acquire enzyme-like roles when embedded in heterogeneous macromolecular environments (Nussinov et al. 2017; Blundell, Gupta, and Hasnain 2020; Djulbegovic et al. 2022; Djulbegovic et al. 2023). In our context, disorder would not merely be permissive but functionally active: by sampling broad conformational ensembles, IDRs could stabilize transition states or destabilize intermediates along assembly and disassembly pathways. Crucially, such catalytic behavior would not require sequence-level optimization in the classical enzymatic sense but would emerge from collective interactions within the condensate milieu, consistent with a physical rather than biochemical notion of catalysis (Gupta et al 2026).

These components are typically described as regulators of condensate composition, material state or turnover. However, these descriptions implicitly assume that their primary role is to shift equilibrium properties or material parameters. Within our framework, they can instead be coherently reinterpreted as catalytic modulators of phase-transition kinetics, repeatedly biasing nucleation, coalescence or dissolution pathways without being consumed. Phase-kinetases could act analogously

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to enzymes by lowering effective free-energy barriers between metastable mesoscale states, while leaving the underlying phase diagram largely intact.

This reinterpretation is reinforced by several converging observations. First, perturbations of candidate phase-kinetases frequently produce strongly nonlinear and saturable effects on condensate lifetimes, nucleation rates or dissolution kinetics, inconsistent with simple proportional control of interaction strength. Second, the same agent can accelerate both assembly and disassembly depending on context, a hallmark of kinetic rather than thermodynamic regulation. Third, recent demonstrations that structurally heterogeneous or even random macromolecular assemblies can display enzyme-like catalytic behavior support the plausibility of reusable, non-specialized biological structures acting as kinetic catalysts at the mesoscale (Yu et al, 2026).

Together, these findings strengthen the view that condensate dynamics are governed not only by passive phase behavior, but by an active, catalysis-like layer of control operating on the rates of physical state transitions.

Conceptual implications and outlook. Rather than treating condensation as a process governed solely by equilibrium thermodynamics, our enzymatic-like interpretation of condensate nucleation and dissolution emphasizes kinetic regulation. Equilibrium phase behavior is not overridden, but the rates at which phase boundaries are crossed appear subject to selective biological modulation. The evidence discussed above is consistent with the existence of reusable cellular agents that bias assembly and disassembly pathways without being consumed, acting on transition dynamics rather than on material composition. Within this view, phase separation could emerge as a regulated variable rather than a passive consequence of molecular properties. Biological organization may thus exploit catalysis-like control over existing physical degrees of freedom, selectively accelerating, slowing or redirecting transitions within constrained configuration spaces. Our interpretation does not invoke new forces or departures from established physical laws; instead, it reframes familiar regulatory components as agents acting on kinetic barriers, thereby reshaping physical dynamics from within.

By focusing on barrier modulation, our enzymatic-like perspective implies specific and testable signatures in kinetics, scaling behavior and sensitivity to perturbation that can be evaluated independently of biological interpretation. Whether these signatures are ultimately confirmed or refuted, treating condensate dynamics as potential targets of kinetic regulation sharpens the conceptual distinction between passive material behavior and biological control.

At this stage, the emphasis remains conceptual. While barrier modulation naturally suggests the existence of sharp kinetic signatures, a rigorous formulation of quantitative predictions requires explicit modeling choices, including how nucleation events are operationally defined, how waiting-time distributions are constructed and how effective free-energy barriers are inferred from empirical data. For this reason, a systematic quantitative treatment is developed in the following chapter, where condensate nucleation is analyzed as a barrier-limited physical reaction and explicit rate laws, scaling relations and falsification criteria are derived.

3. A Quantitative Case Study: Barrier Modulation in Condensate Nucleation

We develop here a quantitative case study in which condensate nucleation is treated as a barrier-limited physical reaction whose kinetics can be modulated by biologically produced putative Phase-Kinetases. The objective is to derive explicit rate laws, parameter mappings and falsification thresholds directly from measurable time series and perturbation series.

Problem setup and observable definitions. We begin by fixing observables and a minimal stochastic description so that each subsequent calculation has a precise empirical target. Condensate nucleation is represented as a stochastic first-passage event in configuration space, observed as the time T at which a compartment (cell, nucleus, subregion) first exhibits a condensate satisfying a predefined detection criterion. Let $N(t)$ denote the cumulative number of nucleation events observed across M independent compartments up to time t and let $F(t) = \mathbb{P}(T \leq t)$ be the nucleation-time cumulative distribution. The survival function is $S(t) = 1 - F(t) = \mathbb{P}(T > t)$. A

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minimal assumption used in our approach is that conditional on fixed experimental condition θ (concentrations, temperature, genotype, ATP level), nucleation events in distinct compartments are independent and identically distributed with hazard function $h_\theta(t)$. The baseline "passive" model is time-homogeneous nucleation with constant hazard $h_\theta(t) = \lambda_\theta$, giving $S(t) = \exp(-\lambda_\theta t)$. This choice is not asserted as universally correct; rather it is a controlled starting point that permits exact inversion from data. The observable rate λ_θ is estimated from M waiting times $\{T_i\}$ by maximum likelihood, including right-censoring at observation horizon $t_{\max}$. If $\delta_i \in \{0,1\}$ indicates whether nucleation occurred before censoring, the log-likelihood is $\ell(\lambda) = \sum_i \delta_i \log \lambda - \lambda \sum_i \min(T_i, t_{\max})$, yielding $\hat{\lambda} = \frac{\sum_i \delta_i}{\sum_i \min(T_i, t_{\max})}$. We establish here the measurable objects that will carry all subsequent barriers and catalyst inferences.

Barrier-limited kinetics and effective free-energy shifts. We now need to map rates to effective barriers so that biological modulation can be expressed as quantitative barrier shifts in units of kT. We treat nucleation as barrier crossing with an effective activation free energy ΔG^*_θ governing the rate. The central kinetic ansatz is Arrhenius/Eyring-like:

$$\lambda_\theta = \lambda_0 \exp\left(-\frac{\Delta G^*_\theta}{kT}\right),$$

where k is Boltzmann's constant and T the absolute temperature and λ_0 is a prefactor representing attempt frequency in the relevant coarse-grained coordinate. The passive reference condition $\theta = \theta_0$ defines $\Delta G^*_{\theta_0}$ and any perturbation defines an effective barrier shift $\Delta\Delta G^*(\theta) = \Delta G^*_\theta - \Delta G^*_{\theta_0}$. Eliminating λ_0 gives an experimentally invertible relation:

$$\Delta\Delta G^*(\theta) = -kT \ln\left(\frac{\lambda_\theta}{\lambda_{\theta_0}}\right).$$

Thus a rate ratio of 10 corresponds to $\Delta\Delta G^* = -kT \ln 10 \approx -2.30\, kT$, while a 50-fold ratio corresponds to $-3.91\, kT$. Our framework uses these conversions as falsification primitives: if Phase-Kinetase perturbations yield rate changes that cannot be summarized by a stable $\Delta\Delta G^*$ across replicates and measurement windows, the barrier-modulation hypothesis is rejected in its minimal form. When time-dependent hazards are needed, our approach generalizes to $h_\theta(t) = \lambda_0 \exp(-\Delta G^*_\theta(t)/kT)$ and uses piecewise-constant hazards over bins $[t_j, t_{j+1})$, estimating λ_j by the same likelihood formula applied per bin. This preserves direct mapping from measured hazards to barrier profiles $\Delta G^*(t)$, enabling detection of history dependence while maintaining quantitative interpretability.

Incorporating phase-kinetases via saturable barrier modulation. We next specify a catalyst-like dependence yielding sharp, parameter-identifiable predictions rather than descriptive post hoc fits. To represent biologically produced Phase-Kinetases, our approach introduces an endogenous agent concentration C (measured or proxied by fluorescence/abundance) modulating the effective barrier. The simplest saturable form mirrors catalytic saturation without asserting molecular Michaelis–Menten chemistry:

$$\Delta G^*(C) = \Delta G^*_\infty + (\Delta G^*_0 - \Delta G^*_\infty)\frac{K}{K + C},$$

equivalently expressed as a saturating barrier reduction

$$\Delta\Delta G^*(C) = \Delta G^*(C) - \Delta G^*(0) = -\Delta G_{\max}\frac{C}{K + C},$$

with $\Delta G_{\max} = \Delta G^*_0 - \Delta G^*_\infty > 0$. Substituting into the rate law yields

$$\lambda(C) = \lambda(0) \exp\left(\frac{\Delta G_{\max}}{kT}\frac{C}{K + C}\right).$$

Two identifiability features follow. First, the log-rate is linear in $C/(K + C)$: $\ln \lambda(C) = \ln \lambda(0) + (\Delta G_{\max}/kT)\, C/(K + C)$. Second, the high-$C$ plateau gives $\lambda(\infty) = \lambda(0) \exp(\Delta G_{\max}/kT)$, so measuring the plateau rate fixes $\Delta G_{\max}/kT = \ln(\lambda(\infty)/\lambda(0))$. Falsification thresholds become explicit: if $\ln \lambda(C)$ does not approach a finite asymptote as C increases or if the inferred K differs systematically across replicates beyond uncertainty, the saturable Phase-Kinetase model fails. In our simulations used to

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generate the diagnostic plots, we chose normalized units with $kT = 1$, $\Delta G_{\max} = 4\,kT$ and $K = 2$ (Figure 1B), so the maximum predicted rate ratio is $\exp(4) \approx 54.6$, while $C = K$ yields $\exp(2) \approx 7.4$. These numerical separations define experimentally resolvable regimes.

Regime classification and rate-ratio diagnostics. We now translate the equations into regime maps supporting direct interpretation of experimental perturbations as distinct kinetic classes. Our approach classifies control strength by the inferred barrier reduction $\Delta G_{\max}/kT$ or by measured rate ratios. The regime map follows from $\lambda/\lambda_0 = \exp(\Delta\Delta G^*/kT)$ (Figure 1A). A weak-modulation regime corresponds to $|\Delta\Delta G^*| < 1\,kT$ (rate ratios $< e \approx 2.72$), typically difficult to distinguish from modest passive variability. A moderate regime corresponds to 1–$3\,kT$ (rate ratios e–$e^3 \approx 20.1$) and a strong regime corresponds to $> 3\,kT$ (rate ratios > 20). For a perturbation series $\{C_m\}$, the predicted fold-change curve is $R(C) = \lambda(C)/\lambda(0) = \exp((\Delta G_{\max}/kT)\,C/(K + C))$ (Figure 1B). Our framework uses two quantitative diagnostics. The first is curvature: $d^2 \ln R/dC^2 < 0$ for all $C > 0$ under saturable modulation; empirically, convexity in $\ln R$ falsifies the model. The second is the half-saturation point: $C = K$ implies $R = \exp(\Delta G_{\max}/2kT)$. Thus measuring C at which $\ln R$ reaches half its plateau estimates K without fitting the full curve. For uncertainty, if $\hat{\lambda}$ is estimated from $n$events over exposure time E, then $\hat{\lambda} = n/E$ with approximate standard error $\mathrm{SE}(\ln \hat{\lambda}) \approx 1/\sqrt{n}$ for Poisson counting, yielding $\mathrm{SE}(\Delta\Delta G^*/kT) \approx \sqrt{1/n_\theta + 1/n_{\theta_0}}$. This provides an explicit planning rule: to resolve $\Delta\Delta G^* = 2\,kT$ against zero with $z \approx 2$, one needs n such that $2/\sqrt{2/n} \gtrsim 2$, i.e. $n \gtrsim 2$ per condition at minimum, but robust discrimination typically requires $n \geq 25$ to shrink uncertainty below 0.3 kT.

Waiting-time distributions and survival-based inference. We next shift from point estimates of rates to full distributional tests that sharpen falsification by exploiting the entire time series. A core prediction of our approach is that barrier modulation should reorganize waiting-time distributions in a way that is describable by a small number of parameters. Under constant hazard λ, the waiting time T is exponential with density $f(t) = \lambda e^{-\lambda t}$, survival $S(t) = e^{-\lambda t}$ and median $t_{1/2} = (\ln 2)/\lambda$. Barrier modulation changes λ multiplicatively; therefore, survival curves for different conditions should be related by a time rescaling $t \mapsto t\,(\lambda/\lambda_0)$. In our simulations (Figure 1C), we used $\lambda_0 = 0.1$ and $\Delta\Delta G^* \in \{0,2,4\}kT$, implying $\lambda \in \{0.1, 0.739, 5.46\}$ and medians $t_{1/2} \in \{6.93, 0.94, 0.127\}$. These numerically separate regimes by almost two orders of magnitude. Empirically, our framework proposes testing whether survival curves satisfy $\ln S_\theta(t) \approx -\hat{\lambda}_\theta t$ over the observation window. Deviations can be evaluated via the Kolmogorov–Smirnov statistic on rescaled times $u_i = \hat{\lambda}_\theta T_i$, which should be exponentially distributed with unit rate if the constant-hazard assumption holds. For comparisons among conditions, a log-rank test on survival data quantifies whether λ_θ differs significantly from λ_{θ_0}; the test statistic is approximately χ^2_1 under the null. Importantly, saturable Phase-Kinetase control predicts ordered survival curves with diminishing incremental shifts at high C, while non-saturating forcing predicts continued shifting. The median-based test $t_{1/2}(C) = (\ln 2)/\lambda(0)\exp((\Delta G_{\max}/kT)\,C/(K + C))$ yields a direct falsification: if $t_{1/2}(C)$does not approach a positive asymptote as $C \to \infty$, the saturable model is rejected.

Time-resolved fractions nucleated and hazard reconstruction. We now connect individual waiting times to population-level fractions in a way that yields parameter recovery from standard imaging assays. Many experiments measure the fraction of compartments exhibiting condensates over time rather than individual nucleation times. Under the exponential model, the fraction nucleated by time t is $F(t) = 1 - e^{-\lambda t}$. With Phase-Kinetase concentration dependence, the prediction becomes $F(t; C) = 1 - \exp[-\lambda(C)t]$ (Figure 1D). In our simulations, using the same $\lambda_0 = 0.1$, $K = 2$, $\Delta G_{\max} = 4kT$ and concentrations $C \in \{0,1,2,5,10\}$, the implied rates are $\lambda(C) = 0.1\exp(4C/(2 + C))$, yielding approximate values 0.1,0.38,0.74,2.78,4.28. The corresponding $t_{50} = \ln 2/\lambda$ values are 6.93,1.82,0.94,0.25,0.16. These numbers provide sharp falsification: measured t_{50} should compress strongly between $C = 0$ and $C = 2$ and then change weakly between $C = 5$ and $C = 10$. Our approach reconstructs the hazard from fraction data by $\hat{\lambda} = -\frac{d}{dt}\ln[1 - \hat{F}(t)]$. With discrete time points t_j, one uses finite differences: $\hat{\lambda}_j \approx -\frac{\ln(1-\hat{F}_{j+1}) - \ln(1-\hat{F}_j)}{t_{j+1} - t_j}$. Under the model, $\hat{\lambda}_j$

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should be approximately constant in j and departures indicate time-varying hazards or heterogeneous subpopulations. The same inversion gives barrier profiles: $\widehat{\Delta G}^{*}(t_j) = -kT\ln(\hat{\lambda}_j/\lambda_0)$ up to a constant. These steps provide a precise algorithmic pathway from imaging-derived fractions to inferred barrier reductions, enabling the quantitative evaluation of Phase-Kinetase action.

Simulation protocol and computational tools. We used a minimal stochastic simulator to generate regime-comparison plots independent of any peculiar molecular mechanism.

Step 1 defined a grid of barrier reductions $\Delta\Delta G/kT \in [0,5]$ and computed rate ratios $R = \exp(\Delta\Delta G/kT)$, producing the exponential regime map (Figure 1A).

Step 2 defined a concentration axis $C \in [0,10]$ and a saturable barrier function $\Delta\Delta G(C) = \Delta G_{\max} C/(K + C)$ with $K = 2$ and $\Delta G_{\max} = 4$, then computed $R(C) = \exp(\Delta\Delta G(C))$ (Figure 1B).

Step 3 generated survival curves using the exponential survival function $S(t) = \exp(-\lambda t)$ on a time grid $t \in [0,60]$ with $\lambda = \lambda_0 \exp(\Delta\Delta G)$ and $\lambda_0 = 0.1$ for three barrier settings $\Delta\Delta G \in \{0,2,4\}$ (Figure 1C).

Step 4 computed nucleated fractions $F(t;C) = 1 - \exp[-\lambda(C)t]$ using the same time grid and concentrations $C \in \{0,1,2,5,10\}$ (Figure 1D).

The computational tools were NumPy for vectorized evaluation of rate laws and distribution functions and Matplotlib for plotting. No parameter fitting was performed in the simulations; all curves were direct evaluations of the equations stated above, ensuring that the plots encode only the theoretical assumptions of barrier modulation and saturability. For extension to data, our framework uses maximum-likelihood estimation for exponential hazards and standard survival-analysis routines for censored data; these can be implemented with SciPy or statsmodels, but the present simulations required only closed-form expressions.

Overall, our approach specifies a fully quantitative route from condensate nucleation time series to inferred barrier shifts and to saturable Phase-Kinetase response curves. The section provided explicit hazard-based likelihood estimation, barrier inversion in kT units, distributional survival tests, fraction-based hazard reconstruction and a reproducible simulation protocol yielding Figure 1A–D. These elements jointly define sharp numerical thresholds for falsifying barrier-modulated nucleation control.

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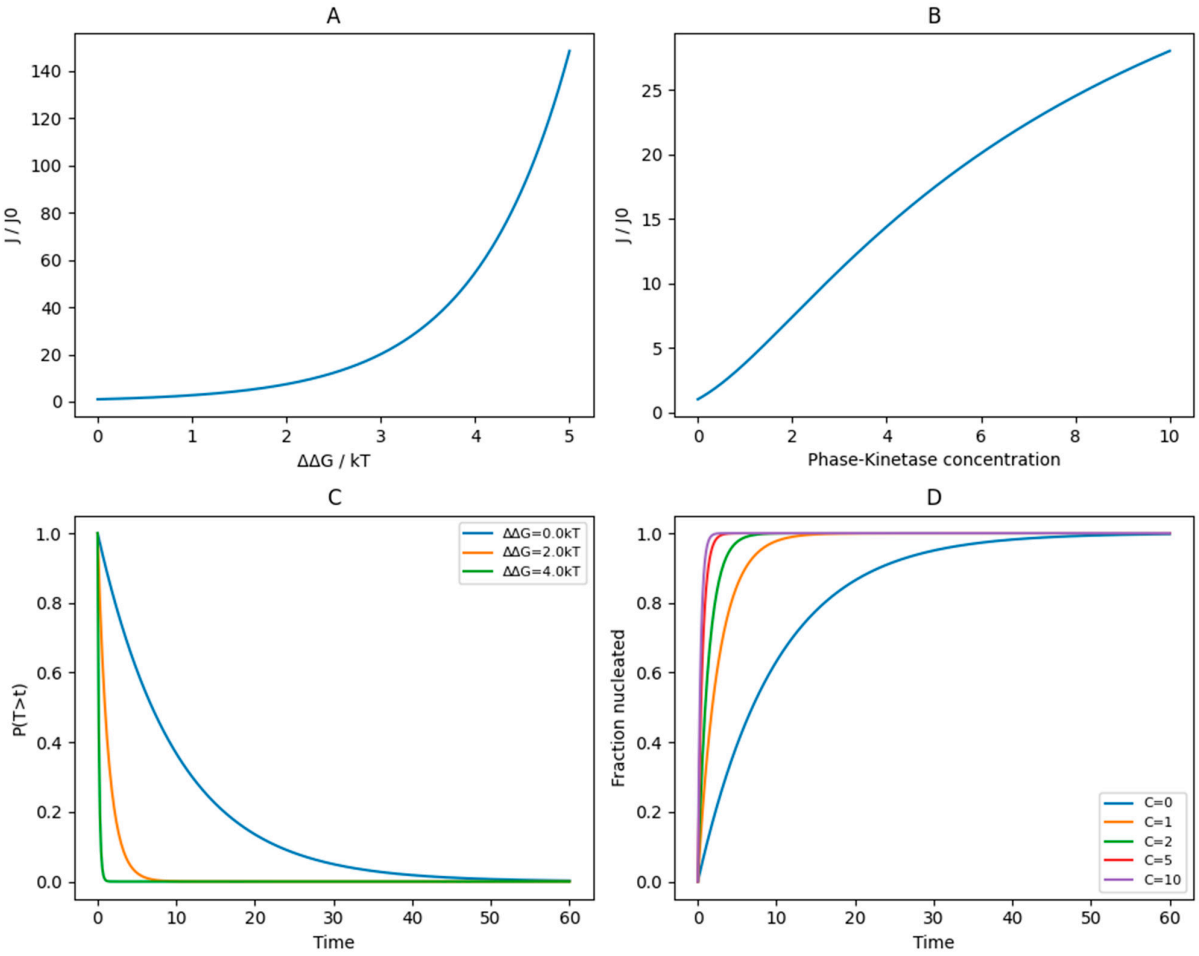


Figure 1. Quantitative regimes of enzymatic-like control in condensate nucleation. (A) Nucleation rate enhancement J/J_0 as a function of effective free-energy barrier reduction $\Delta\Delta G/kT$. The exponential dependence produces clearly separated regimes: weak modulation (<1 kT), biologically accessible enzymatic-like control (1–3 kT) and strong acceleration (>3 kT), yielding order-of-magnitude rate changes. (B) Nucleation rate enhancement as a function of Phase-Kinetase concentration, assuming saturable barrier modulation $\Delta\Delta G(C) = \Delta\Delta G_{\max} C/(K + C)$. The response displays an onset regime, a region of maximal sensitivity near $C \approx K$ and a high-concentration plateau, distinguishing barrier-lowering catalysis from non-saturating physical forcing. (C) Waiting-time survival curves $P(T > t)$ for condensate nucleation at fixed baseline rate and different barrier reductions. Barrier shifts of 2–4 kT yield sharply separated survival profiles, allowing direct inference of $\Delta\Delta G$ from kinetic data. (D) Fraction of nucleated compartments as a function of time for increasing Phase-Kinetase concentration. Time-to-half nucleation decreases with concentration but approaches an asymptote, providing a falsification criterion for saturable enzymatic-like control.

4. Conclusions

Our results suggest that a broad class of cellular physical processes can be understood as barrier-limited reactions whose kinetics are actively modulated by biologically produced agents acting in an enzymatic-like manner. We have focused explicitly on biomolecular condensate nucleation and dissolution as a concrete example of this principle. Rather than introducing new physical principles or invoking nonstandard forms of energy input, we reframe familiar biological components involved in condensate regulation as agents that locally and repeatedly lower effective kinetic barriers in configuration space. This shift allows condensate dynamics to be treated within a quantitative approach analogous to chemical catalysis. We identify explicit observables for condensate nucleation and dissolution, mapping them to effective barrier parameters in units of kT and define falsifiable

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relationships between biological perturbations and measurable rate or waiting-time distributions. Our approach does not reinterpret biology as violating physics, but as actively shaping which physical transitions are realized and at what rates.

Condensate regulation could exemplify a more general mode of cellular organization, in which physical reactions are subject to layered kinetic control. We provide a language for biological regulation of physical dynamics that differs from strategies based on force generation, parameter tuning or purely material descriptions. Standard accounts of condensate behavior emphasize changes in interaction strengths, viscosities or global energy fluxes, often requiring system-specific models with limited transferability. In contrast, our framework centers on barrier modulation as a shared operational principle, allowing condensate nucleation and dissolution to be compared on a common energetic scale with other physical processes. This yields conceptual advantages. First, it naturally explains why modest biological perturbations can induce order-of-magnitude changes in nucleation or dissolution kinetics, through the exponential sensitivity of rates to barrier shifts. Second, it provides explicit saturation criteria distinguishing catalysis-like control from additive or force-driven mechanisms. Third, it enables direct inversion from data to effective energetic parameters, facilitating quantitative falsification. We do not introduce new entities but rather reorganize existing knowledge into a barrier-centric perspective that enables comparison, inference and testing. In terms of classification, our approach can be situated among mesoscopic theories of biological organization explicitly bridging molecular activity and emergent physical behavior. It differs from equilibrium thermodynamic treatments by focusing on kinetics rather than state functions, from continuum active matter models by emphasizing localized and saturable control and from purely biochemical frameworks by targeting physical state transitions instead of chemical reactions.

While we develop this approach in detail only for condensate dynamics, the same framework may plausibly extend to other cellular physical processes. We summarize in Table 1 additional, more speculative physical processes that may be amenable to enzymatic-like control and warrant future investigation.

Several limitations must be acknowledged. Several numerical thresholds stand for representative regimes rather than empirically validated constants. The assumption of constant or piecewise-constant hazards simplifies inference but may fail in systems with strong history dependence or spatial coupling. Idealizations concerning compartment definition, detection thresholds and event independence may not hold universally. The saturable functional forms used for barrier modulation are minimal hypotheses rather than mechanistic derivations and alternative nonlinearities could fit some data equally well. Finally, our figures are model-driven simulations rather than empirical measurements.

Despite these constraints, our approach suggests potential applications and future research directions. Experimentally, our framework motivates perturbation series that hold equilibrium or mean-state properties fixed while measuring nucleation kinetics, waiting-time distributions and saturation behavior. Testable hypotheses include the existence of finite plateaus in nucleation-rate enhancement under increasing concentrations of putative phase-kinetases, consistent barrier shifts inferred across independent observables and exponential relationships between inferred barriers and measured kinetics. Future work may extend our formalism to coupled barriers, heterogeneous populations or time-dependent modulation, integrating high-resolution imaging with survival and hazard reconstruction.

In summary, we addressed whether cellular activity could regulate condensate phase transitions through mechanisms analogous to enzymatic catalysis. We showed how localized, saturable and reusable biological agents can be modeled as modulators of effective kinetic barriers, yielding sharp and falsifiable quantitative predictions concerning condensate dynamics. Overall, viewing cellular physical dynamics through the lens of enzymatic-like control could provide a testable perspective on how living systems shape physical processes from within.

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Table 1. Candidates potentially subject to enzymatic-like control. This table summarizes eight biophysical reactions that, while not analyzed in detail in the main text, could represent theoretical and speculative extensions of our enzymatic-like control framework. For each process, the table contrasts standard physical interpretations with empirical or conceptual limitations and proposes a coherent class of hidden enzymatic-like activities, defined as biological agents that could catalyze transitions between physical regimes without directly altering mean-state variables. The listed candidates are intended as hypothesis-generating examples. Falsification-oriented quantitative signatures are formulated in terms of variance modulation, effective temperature shifts, kinetic barrier reductions, threshold behavior or relaxation-time scaling, allowing these possibilities to be experimentally tested.

Possible candidate (fundamental biophysical reaction)	**Usually attributed to physics**	**Why this is insufficient**	**Hidden enzymatic-like activity (coherent name + plausible biological candidates)**	**Testable quantitative signature (falsification-oriented)**
Noise shaping and fluctuation control (variance, burstiness, spectra)	Thermal noise, Poisson statistics, stochastic chemical kinetics	Cells selectively tune variance, noise spectra and burstiness without changing mean values; fluctuations are context-dependent and heritable	**Stochastases** – chromatin remodelers (SWI/SNF-like complexes), RNA-binding proteins buffering transcriptional bursts, metabolic buffering metabolites (ATP/ADP, NADH/NAD^+ pools), phase-separated transcriptional hubs	Variance and power spectra change independently of mean; noise modulation shows saturability and exponential sensitivity consistent with $\Delta\Delta G \approx 1–3$ kT
Anomalous and biased intracellular diffusion (transport statistics)	Crowding, fractal geometry, viscoelastic media	Diffusion exponents and directionality change dynamically in time and space for identical molecules, incompatible with static physical environments	**Diffusases** – transient scaffolding proteins, RNA granules acting as dynamic obstacles, metabolite-regulated crowding modulators, weak multivalent binders that restructure local geometry	Diffusion exponent $\alpha(t)$ changes without altering viscosity; directional bias appears with saturable dependence on agent abundance
Local modulation of effective temperature ("active thermalization")	Active matter noise, ATP-driven agitation	Effective temperature changes are spatially confined and reversible, not	**Thermokatalysts** – ATP-dependent chaperones (Hsp70/Hsp90-like systems), RNA helicases and remodelers,	Local violation of fluctuation–dissipation relations; inferred $\Delta T_{eff}/T \approx 0.1–0.3$

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Possible candidate (fundamental biophysical reaction)	Usually attributed to physics	Why this is insufficient	Hidden enzymatic-like activity (coherent name + plausible biological candidates)	Testable quantitative signature (falsification-oriented)
		globally driven by energy input	metabolon-associated enzymes with oscillatory ATP/ADP or NADH/NAD^+ cycling, active cytoskeletal remodeling complexes, phase-separated activity hubs	without bulk heating
Membrane fusion/fission initiation (stalk or neck nucleation)	Lipid elasticity, spontaneous curvature, thermal membrane fluctuations	Fusion and fission occur too fast, too selectively and at specific sites to be explained by rare thermal lipid fluctuations alone	**Topo-Catalysts** – curvature-sensing or inserting proteins (BAR-domain proteins), short amphipathic helices in fusion factors, dynamin-like GTPases, locally acting lipid-modifying enzymes	Event rates increase exponentially with catalyst concentration and saturate; inferred barrier reduction $\Delta\Delta G \approx 3–5$ kT from rate ratios
Topology control of chromatin and polymer loops	Polymer entropy, random collisions, loop extrusion models	Loop lifetimes and boundaries are sharply defined and cell-type specific	**Topolysins** – SMC-family complexes (cohesin- and condensin-like), topoisomerases (topology-level role), chromatin remodeling complexes, RNA-binding architectural proteins, phase-separated architectural scaffolds	Loop formation and release rates show exponential sensitivity to small perturbations; barrier reductions of ~2–4 kT inferred from kinetics
Polymer length distribution reshaping (without synthesis or degradation)	Steady-state polymerization kinetics, critical concentration arguments	Length distributions shift faster than monomer turnover allows and without corresponding changes in synthesis or degradation rates	**Polymodases** – filament severing factors (cofilin, katanin-like), annealing and end-to-end exchange mediators, length-redistribution complexes acting on actin, microtubules or RNA	Mean length $\langle L \rangle$ scales inversely with agent concentration and saturates; redistribution kinetics exceed monomer turnover rates

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Possible candidate (fundamental biophysical reaction)	Usually attributed to physics	Why this is insufficient	Hidden enzymatic-like activity (coherent name + plausible biological candidates)	Testable quantitative signature (falsification-oriented)
Cytoplasmic or cortical stress relaxation (viscoelastic response)	Passive rheology, jamming/unjamming, polymer physics	Relaxation times change abruptly, locally and reversibly, far beyond what changes in bulk force or composition predict	**Rheolysins** – actin-severing proteins (cofilin-like), crosslink turnover regulators, ATP-dependent cytoskeletal remodelers	Stress relaxation time τ decreases exponentially with agent concentration; τ ratios imply $\Delta\Delta G \approx$ 2–3 kT; effect is local and reversible
Reaction–diffusion wave initiation and arrest (excitable dynamics)	Reaction–diffusion equations, threshold instabilities	Wave nucleation sites and arrest points are highly reproducible, cell-type specific and history-dependent	**Wave-Kinetases** – scaffolded signaling complexes (MAPK, Rho-family GTPase modules), localized phosphatase–kinase feedback pairs, membrane- or cortex-anchored excitability regulators	Sharp all-or-none changes in wave frequency from small parameter shifts; effective threshold changes of ~5–10%

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Review

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Arturo Tozzi *

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Review

Do Humans (and Animals) Think in Poetic Language?

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Abstract

The search for thought's representational format has traditionally focused on Mentalese, neural codes, predictive processing, embodied sensorimotor models. Here we examine another hypothesis: human cognition may operate by default through a structurally *poetic* mode characterized by metaphorical mapping, rhythmic/repetitive patterning, imagistic compression, controlled ambiguity, acoustic structuring and affective resonance. These features correspond to identifiable computational and neurobiological mechanisms. Metaphor and analogy recruit fronto-parietal networks for cross-domain alignment, enabling dimensionality reduction and transfer across abstract spaces. Rhythmic and metrical structure entrain delta–theta–gamma hierarchies, improving prediction and memory via phase-aligned hippocampo-cortical loops. Imagery activates hippocampal–visual circuits and default-mode subsystems, supporting high-level generative simulation. Ambiguity/polysemy sustains parallel semantic states in frontal networks, enabling probabilistic inference under uncertainty. Sound patterning exploits phonological memory and auditory statistical learning, enhancing signal-to-noise resilience. Emotional resonance engages limbic and reward pathways, modulating salience and long-term retention. Comparative evidence indicates that structurally homologous "proto-poetic" systems exist across species. Birdsong and whale song show hierarchical, rhythmic sequence generation with cultural drift; honeybee dances encode spatial data cross-modally; primate calls display affix-like combinatoriality and contextual ambiguity; frog and insect chorusing reflects large-scale oscillator coupling; and ritual displays compress identity and fitness cues into symbolic form. These convergent data support the view that poetic structuring constitutes an evolutionarily conserved cognitive technology optimized for memory efficiency, predictive precision, adaptive flexibility and social synchrony. Reframing poetry as a computational architecture of thought motivates new empirical programs integrating neuroimaging, comparative biology and formal models of information compression, oscillatory coordination and cross-domain mapping.

Keywords: poetic cognition; metaphor; neural oscillations; animal communication; predictive processing

Introduction

The question of what language the brain uses to think has long been a matter of philosophical and scientific debate (Fedorenko and Varley 2016; Martín-Iglesias and Flórez 2018; Frankland and Greene 2020; Fedorenko, Piantadosi and Gibson 2024; Mahowald et al. 2024). Competing hypotheses have framed thought as a symbolic inner code (*Mentalese*) resembling logic (Fodor 1975), as patterns of neural activity encoded in spikes, rates or oscillations, as predictive models continually updated to minimize error (Friston 2010) or as processes grounded in bodily and sensorimotor experience (Barsalou 2008). These approaches have shaped modern cognitive science, yet they often emphasize propositional, mechanistic or code-like structures at the expense of more flexible and symbolic modes of thought.

An alternative possibility has received less systematic attention: the idea that thought may be organized in a *poetic mode*. By poetic we mean a constellation of features (e.g., metaphor, rhythm,

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repetition, imagery, ambiguity, sound play, emotional resonance and collective synchrony) that characterize not only literary works but also fundamental cognitive processes (Jakobson 1960; Wimsatt and Beardsley 1954; Lotman 1977; Tsur 1992; Hogan 2011; Obermeier et al. 2013a; Ob ermeier et al., 2013b; Jacobs 2015). Several cues point towards the possibility that the brain may employ a poetic language. In cognitive linguistics, Lakoff and Johnson (1980) argued that metaphor forms the scaffolding of abstract reasoning. Cognitive poetics has further developed the relationship between poetic structures and cognition (Tsur 2008; Steen 2014), while Jacobs' (2015) neurocognitive poetics model has shown how literary reading recruits circuits that integrate emotion, imagery and meaning. Neuroimaging reveals that poetry engages broader cortical and limbic networks than prose, with measurable effects on memory and emotion (Wassiliwizky et al. 2017).

Anthropological evidence adds a further dimension: rhythmic chant, song and poetic parallelism are ubiquitous across early human societies, suggesting that poetic structures are older and more essential than the prose-like logic later emphasized in science and philosophy (Fabb 2015). Extending this view beyond humans, animal communication exhibits proto-poetic features. Birdsong, whale song, primate calls, the honeybee's dance and ritualized displays such as the peacock's tail all reveal rhythmic structuring, repetition with variation, symbolic cues and affective resonance.

Together, these findings raise the provocative possibility that poetry is not a cultural luxury but a natural cognitive technology, embedded in neural architecture and shaped by evolutionary pressures. This review synthesizes evidence across neuroscience, linguistics, literary theory and animal communication to ask whether humans and perhaps animals, think in poetic language.

Defining Poetic Language

Poetic language differs from ordinary discourse because it is built not merely to convey information, but to transform it into forms that resonate, endure and connect. Where literal language strives for precision and univocal clarity, poetic language embraces pattern, association and multiplicity. It is distinguished by a constellation of features that, taken together, give it a unique capacity to shape thought and expression.

Metaphor and analogy form the foundation of poetic language. Through them, abstract ideas are understood in terms of concrete experiences, allowing new insights to emerge from cross-domain associations. Life becomes a journey, emotion becomes fire, time becomes a river. These metaphors are not only stylistic embellishments but ways of reordering experience, making the unfamiliar intelligible by framing it in terms of what is already known. Analogy, similarly, bridges disparate realms, inviting recognition of hidden similarities and sparking new conceptual structures.

Rhythm and repetition provide a second defining feature. Poetic language is rarely neutral in its flow; it is structured into beats, pulses and returns. Rhythm captures attention, establishes expectation and guides reception. Repetition, whether of sounds, words, phrases or structures, reinforces memory, lends emphasis and generates cohesion. Parallelism, a special case of repetition, creates balance and symmetry, making ideas more memorable and aesthetically satisfying. Together, rhythm and repetition transform language into patterned experience.

Imagery and symbolism lend poetry its density. Images summon sensory detail, evoking sight, sound, touch and movement with a few words. They compress meaning, allowing multiple associations to cluster around a single image. Symbols extend this compression, turning concrete objects into carriers of layered significance: a rose for love, a road for choice, a shadow for mortality. Through imagery and symbolism, poetry achieves richness that ordinary description cannot.

Ambiguity and polysemy distinguish poetic from prosaic communication. Where ordinary discourse tends toward resolution, poetry often resists closure, holding open several meanings at once. A line may suggest one thing and its opposite or hover between interpretations and this openness is part of its force. Ambiguity compels the reader or listener to dwell in uncertainty, encouraging reflection and multiple perspectives. Polysemy, the coexistence of several meanings in a single word or phrase, multiplies resonance and depth.

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Sound play is another essential dimension. Rhyme, alliteration, assonance and onomatopoeia exploit the acoustic properties of language, creating echoes, harmonies and textures that delight the ear and reinforce structure. Poetic language is thus memorable not only for its meaning but for its music. Its patterns of sound contribute to its rhythm and amplify its power to linger in memory.

Conciseness and compression mark another feature. Poetic expression often condenses complexity into striking brevity. A few words can carry the weight of an extended argument or an entire narrative. Compression is not mere economy; it distills thought into heightened intensity, demanding close attention and inviting interpretation beyond the literal.

Emotional resonance is inseparable from poetic expression. Poetry does not simply state ideas; it evokes feelings. Through rhythm, imagery, ambiguity and sound, it amplifies affective impact, stirring joy, sorrow, wonder or awe. Emotional intensity is not an accident of poetic form but its essence, giving language the ability to move as well as inform.

Collective synchrony extends poetry's reach from the individual to the group. Chants, refrains and ritual recitations synchronize not only words but bodies and voices. In such contexts, poetry binds individuals together into a shared rhythm and shared meaning. This collective dimension reveals poetry's power not only as private reflection but as social glue, linking cognition and emotion to communal identity.

Taken together, these features suggest that poetic language is not reducible to ornament or artifice. It constitutes a mode of expression in which form and meaning are inseparable, in which rhythm and metaphor, sound and ambiguity, emotion and symbol intertwine. Poetic language communicates not just by telling but by shaping how experience is perceived, remembered and shared.

Neural Correlates of Poetic Features and Evidence in Human Cognition

If poetic language is to be considered not merely an aesthetic phenomenon but a cognitive technology, it must be shown to align with identifiable brain mechanisms and cognitive processes. Research from cognitive linguistics, cognitive poetics, neurocognitive poetics and empirical neuroscience increasingly converges on the view that the above-mentioned hallmark features of poetry are deeply embedded in the neural architecture of the human mind (**Table 1**).

Table 1. Neural correlates of poetic features. Each feature of poetic language is associated with specific cognitive processes and brain mechanisms. Metaphor and analogy engage parietal and prefrontal regions for conceptual mapping; rhythm and meter rely on oscillatory entrainment; repetition and parallelism support hippocampal memory encoding; imagery and symbolism link hippocampal and visual areas; ambiguity recruits prefrontal multi-representations; emotional resonance involves the limbic system; conciseness and compression reflect predictive coding; sound play exploits auditory and phonological circuits; and collective recitation promotes neural synchrony and oxytocin-mediated social bonding.

Poetry Feature	Mental Process / Brain Mechanism
Metaphor & analogy	Conceptual mapping; parietal + prefrontal activation
Rhythm & meter	Neural oscillations (theta, gamma); auditory prediction
Repetition & parallelism	Working memory; hippocampal encoding
Imagery & symbolism	Hippocampal-visual coupling; episodic simulation
Ambiguity & polysemy	Prefrontal multi-representations
Emotional resonance	Limbic system (amygdala, vmPFC)
Conciseness & compression	Predictive coding; Bayesian reduction
Sound play	Auditory pattern detection; phonological loop
Collective recitation/chant	Neural synchrony; oxytocin release

Metaphor and analogy have been recognized in cognitive linguistics as the scaffolding of abstract thought (Lakoff and Johnson 1980). Neuroimaging studies reveal that metaphor engages distributed cortical networks, including the inferior frontal gyrus, anterior temporal cortex and inferior parietal lobule, supporting cross-domain integration (Bohrn, Altmann and Jacobs 2012). Multivariate pattern analysis shows that metaphoric meaning recruits both domain-general semantic

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systems and modality-specific sensory cortices, grounding abstract concepts in concrete experience (Xu et al. 2021). Evidence from clinical populations demonstrates that metaphor comprehension is impaired in left hemisphere degeneration, indicating the crucial role of frontal and temporal regions (Klooster et al. 2020). These findings confirm that metaphor is not decorative but fundamental to conceptual mapping in the brain.

Rhythm and repetition reflect the brain's rhythmic nature. Cortical oscillations at multiple frequencies synchronize neural communication and linguistic rhythm entrains these oscillations to enhance prediction and comprehension (Giraud and Poeppel 2012). Theta–gamma coupling is particularly implicated in the segmentation and integration of speech, while delta rhythms align with prosodic patterns (Teng and Poeppel 2020). Repetition and parallelism, central to oral traditions, reinforce memory by supporting hippocampal encoding and cortical reinstatement (Jacobs 2015). Experimental studies demonstrate that neural entrainment to poetic meter predicts recall, showing a direct link between rhythm and memory (Shukla et al. 2021).

Imagery and symbolism are also supported by neurocognitive evidence. Poetry that evokes vivid images activates the hippocampus, precuneus and visual cortices, facilitating episodic simulation and associative binding (Benedek et al. 2014). Figurative and image-rich language strongly engages the default mode network (Zeman et al. 2013; Mar, Spreng and DeYoung 2021), suggesting that symbolic condensation taps into systems of internally directed cognition. This convergence indicates that poetry recruits not only language areas but also neural circuits associated with imagination and memory.

Ambiguity and polysemy distinguish poetic language from literal discourse. Ambiguous expressions engage bilateral inferior frontal gyrus and dorsolateral prefrontal cortex, regions that maintain multiple competing interpretations and resolve conflict (Rodd et al. 2012; Vitello and Rodd 2015). Magnetoencephalography studies show that the brain sustains parallel semantic representations for hundreds of milliseconds before selecting a dominant meaning (Broderick et al. 2021). Such findings indicate that ambiguity is processed actively, rather than as noise and that it fosters the cognitive flexibility that is also central to creativity.

Sound play, including rhyme, alliteration and assonance, capitalizes on the auditory system's sensitivity to pattern. The phonological loop of working memory is particularly responsive to these features (Baddeley 2012). Neuroimaging reveals that rhyme processing engages the superior temporal gyrus and inferior parietal regions, reflecting auditory pattern detection and memory chunking (Obermeier et al. 2016). These findings suggest that poetic sound structures exploit low-level auditory mechanisms to amplify high-level comprehension and recall.

Emotional resonance is among the most powerful aspects of poetry (Yukalov 2022). Neuroimaging demonstrates that poetic language activates the amygdala, anterior insula and ventromedial prefrontal cortex, particularly during moments of peak affect (Wassiliwizky et al. 2017). Subjective chills, often reported during aesthetic appreciation of poetry, correlate with activity in dopaminergic reward pathways, including the ventral striatum (Obermeier et al. 2016). This affective engagement distinguishes poetic from prosaic processing and underscores the link between poetic structure and emotion.

Collective synchrony extends poetic processing into the social domain. Ritual recitation, chant and song produce alignment not only at the behavioral level but also in physiology and neural activity. Hyperscanning studies show increased inter-brain synchrony during collective chanting or music-making, suggesting that rhythmic poetic language promotes group cohesion through shared neural states (Konvalinka et al. 2011; Kinreich et al. 2017; Müller et al. 2021). Such synchrony underpins the social dimension of poetry as a tool for bonding and cooperation.

The integration of these findings has been advanced by the field of cognitive poetics and by Jacobs' *neurocognitive poetics model* (Tsur 2008; Steen 2011; Jacobs 2015). This model distinguishes between "foregrounding" devices like rhyme, metaphor or unusual syntax, that capture attention and evoke emotion and "backgrounding" elements that support narrative flow. Empirical studies

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confirm that foregrounding triggers enhanced activation in attention and affective networks, while backgrounding engages broader narrative comprehension circuits (Bohrn, Altmann and Jacobs 2013).

Anthropological perspectives reinforce these findings. Poetic forms such as chant, song and parallelism are documented universally in oral traditions (Fabb 2015; Norrick 2014). Their ubiquity across cultures suggests that poetic structuring evolved as a cognitive strategy for memory, prediction and social coordination long before the emergence of literacy.

Taken together, evidence from linguistics, neuroscience, psychology and anthropology demonstrates that the core features of poetic language are not ornamental but fundamental. They represent a cognitive technology that engages neural systems for abstraction, rhythm, imagery, ambiguity, sound, emotion and synchrony. In doing so, they enhance memory and prediction, foster creativity, amplify affect and cement social bonds. What literary studies have often described as aesthetic devices thus appear, in light of convergent empirical research, to reflect fundamental principles of thought and the very architecture of human cognition itself.

Evidence in Animal Communication

The presence of poetic features in nonhuman species suggests that patterned, symbolic and affective communication may be more ancient and widespread than once assumed. Far from being uniquely human, proto-poetic structures appear across the animal kingdom, where they serve functions of memory, prediction, signaling and social cohesion (**Table 2**).

Table 2. Comparative overview of poetic features in humans and animals. Humans employ metaphor, rhythm, repetition, imagery, ambiguity, sound play, emotional resonance and synchrony in cognition, memory and culture. Animals display analogous "proto-poetic" structures: honeybee dance as metaphor-like coding, birdsong and whale song as rhythmic and repetitive motifs, primate calls as ambiguous signals and ritualized displays such as the peacock's tail as symbolic communication. The table highlights evolutionary continuities, suggesting that poetic structuring may serve shared cognitive and social functions across species.

Poetic Feature	Humans	Animals
Metaphor / Analogy	Conceptual metaphor; abstract thought grounded in sensory domains	Honeybee dance maps space onto motion; metaphor-like coding
Rhythm / Meter	Speech prosody, poetic meter, neural oscillatory entrainment	Birdsong rhythmic motifs; whale song with temporal structure
Repetition / Parallelism	Parallelism in oral traditions; reinforcement of memory	Repetitive motifs in whale/bird songs; ritualized courtship dances
Imagery / Symbolism	Literary imagery; symbolic condensation in art and religion	Peacock tail, bird-of-paradise display as symbolic signals
Ambiguity / Polysemy	Polysemous words; interpretive openness in poetry and narrative	Ambiguity in primate alarm calls with flexible interpretation
Sound Play	Rhyme, alliteration, assonance in verse; phonological loop	Birdsong phonological-like patterning; dolphin whistles
Emotional Resonance	Emotional peaks, chills, limbic activation during poetry	Affective vocalizations in primates; whale song resonance
Collective Synchrony	Chant, ritual song, shared neural synchrony	Group chorusing in frogs, crickets; synchronized displays

Birdsong has long been studied as a paradigmatic case of complex vocal behavior. Songbirds produce hierarchically organized sequences that include motifs, phrases and syntax-like rules (Berwick et al. 2011). These songs rely on rhythm, repetition and variation, often with motifs recurring across time to reinforce recognition (Mooney 2014; Wohlgemuth, Adam and Scharff 2014; Tyack 2020; Searcy, Soha, Peters and Nowicki 2021; Duque and Carruth 2022; Rose, Haakenson and Ball 2022). Birds also improvise within fixed structures, echoing poetic devices of variation within repetition. Neural studies show that these features are supported by specialized circuits such as the HVC and RA, which exhibit oscillatory coordination and experience-dependent plasticity (Mooney 2014). Birdsong therefore exhibits the rhythmic and repetitive structures that are central to poetic cognition.

Whale song provides another striking example. Humpback whales produce long, hierarchically nested vocalizations, often lasting for hours, characterized by repetition with variation and the gradual evolution of themes across breeding seasons (Dunlop, Noad, Cato and Stokes 2007; Mercado 2020; Warren et al. 2020; Mercado, Ashour and McAllister 2022; Girola, Dunlop and Noad 2023). These compositions resemble extended poetic or musical works, with motifs that are transmitted socially and modified across populations. Whale songs thus reveal not only repetition and rhythm but also cultural evolution akin to human oral traditions.

The honeybee dance translates spatial information into rhythmic bodily motion, encoding both distance and direction of food sources (De Marco, Gil and Farina 2005; Okada et al. 2008; Kohl et al. 2020; Linn et al. 2020; Doi, Deng and Ikegami 2023; Hadjitofi and Webb 2024). The waggle phase

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functions as a compressed symbolic mapping, akin to metaphor, by representing abstract coordinates through embodied motion (Seeley 2010). This cross-domain coding mirrors human metaphorical thinking, where abstract domains are mapped onto sensory-motor experiences.

Primate communication demonstrates ambiguity and polysemy, which are central to poetic discourse. Campbell's monkeys, for instance, combine affixes with alarm calls to generate context-sensitive signals, such as modifying general alarm calls to specify aerial predators (Zuberbühler 2000; Ouattara et al., 2009; Lemasson and Hausberger 2011; Coye et al. 2015). This combinatorial flexibility shows that meaning is not fixed but layered, opening interpretive possibilities much like polysemy in human poetry. Other primates produce ambiguous vocalizations that require contextual interpretation, a feature that parallels the productive ambiguity of poetic texts.

Ritualized displays offer visual analogues of poetic form. The peacock's tail or the elaborate dances of birds-of-paradise exemplify symbolic condensation and aesthetic exaggeration (Prum 2017). These displays rely on rhythm, symmetry and repetition and their effectiveness depends on aesthetic appreciation by conspecifics. They demonstrate that poetic features need not be vocal: bodily performance and ornament can achieve symbolic resonance.

Frogs and crickets exemplify collective synchrony in the nonhuman realm (Schneider et al. 2018; Escalona Sulbarán et al. 2019; Lee, Vélez and Bee 2023; Wu et al. 2023; Zigler et al. 2023; Brandt et al. 2023; Levy et al. 2025). In many species, males engage in antiphonal chorusing, producing rhythmic calls that synchronize across large groups. Such collective soundscapes resemble human chanting, where synchrony fosters group cohesion and amplifies signal strength.

Elephants provide evidence of emotional resonance in communication. Their infrasonic calls are used not only for coordination but also for mourning and reunion, producing behaviors that resemble human poetic lament or celebration. The affective dimension of these signals highlights the continuity of emotional expression across species.

Dolphins display remarkable sound play, producing whistles that include individual-specific "signature calls" as well as playful imitations of others' signals (Janik 2000; Barton 2006; Simard et al. 2011; Fouda et al. 2018; Mishima et al. 2019). The combinatorial and playful aspects of dolphin vocalizations parallel rhyme and alliteration in human language, where sound structures are manipulated to create identity, cohesion or humor.

Taken together, these examples suggest that poetic features (metaphor-like coding, rhythm, repetition, imagery, ambiguity, sound play, emotional resonance and collective synchrony) are not uniquely human but broadly distributed across the animal kingdom. Birdsong and whale song demonstrate rhythm, repetition and cultural transmission; bees' dances show metaphor-like mapping; primate calls reveal ambiguity and polysemy; ritualized displays exemplify symbolism and aesthetic form; frogs, crickets and wolves illustrate synchrony; elephants and dolphins highlight affect and playfulness. These findings support the hypothesis that poetry reflects ancient communicative strategies with deep evolutionary roots.

Evolutionary Functions

If poetic structures are deeply embedded in cognition, what adaptive purposes do they serve? From an evolutionary perspective, poetic features appear less as cultural ornaments and more as functional strategies that enhance survival and cooperation.

Memory reinforcement. Repetition, rhythm and parallelism improve encoding and retrieval, providing mnemonic scaffolds for oral traditions and knowledge transmission. Before literacy, societies relied on poetic forms to preserve genealogies, rituals and ecological knowledge. Neurocognitive evidence supports this function: rhythmic and repetitive structures enhance hippocampal consolidation and cortical reinstatement.

Predictive efficiency. Rhythm and meter align with neural oscillations, reducing uncertainty and sharpening temporal prediction (Teng and Poeppel 2020). Ambiguity sustains multiple interpretations, maintaining cognitive flexibility until environmental cues resolve uncertainty.

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Metaphor compresses complexity by linking distant domains, enabling efficient reasoning across contexts. These mechanisms reduce cognitive load and increase adaptability.

Social cohesion. Poetry is intrinsically social: collective chanting, song and ritual synchronize neural activity and arousal across individuals. Group synchrony, often mediated by oxytocin and dopamine release, fosters trust, cooperation and shared identity (Müller et al. 2021). The aesthetic and emotional resonance of poetic language motivates participation, reinforcing group cohesion.

Creativity and problem-solving. Ambiguity, imagery and metaphor allow the mind to explore multiple representational states. This flexibility enhances divergent thinking and innovation, adaptive in dynamic environments. Neuroimaging studies show that poetic and figurative language engages default mode and executive networks, systems central to creative cognition (Beaty et al. 2016).

Cross-species continuity. Animal communication reveals parallel functions: birdsong reinforces territory and memory, whale song evolves culturally to sustain social cohesion, honeybee dances compress spatial data into symbolic motion and primate calls use ambiguity to negotiate complex social interactions. These proto-poetic signals suggest that rhythmic, symbolic and affective communication confer advantages not only to humans but across species.

In sum, poetic structures serve crucial functions: they extend memory beyond individual limits, improve predictive modeling, cement social bonds and foster creativity. What humans recognize as poetry may thus reflect a fundamental evolutionary adaptation, an ancient cognitive technology that binds thought, emotion and society.

Applications and Future Directions

If poetic structuring is indeed a natural cognitive technology, its implications extend across neuroscience, comparative biology, education, therapy and artificial intelligence. This section outlines how the poetic cognition hypothesis may be tested and applied in diverse domains.

Experimental tests in humans. A central challenge is to move from correlational evidence to causal testing of poetic features. Neuroimaging can compare brain responses to poetic versus prosaic stimuli, isolating the contribution of metaphor, rhythm or ambiguity. For example, fMRI could examine whether metaphor-rich passages activate broader parietal–prefrontal networks than literal counterparts. EEG and MEG studies can track oscillatory entrainment to meter and rhyme, predicting stronger theta–gamma coupling during rhythmic exposure. Eye-tracking and pupillometry may complement these methods by indexing attention and cognitive load during poetic versus nonpoetic reading.

Comparative animal research. Applying the poeticity framework to animals invites systematic reanalysis of communication. Entropy-based measures could quantify structural complexity in birdsong or whale song, while acoustic analysis may reveal rhyme- or meter-like regularities in vocalizations. Neurophysiological recordings in primates could test whether ambiguous calls sustain prolonged prefrontal activation, as in humans confronted with polysemy. Social experiments could assess whether group synchrony in chorusing species (frogs, crickets, wolves) enhances cohesion and collective action, paralleling the effects of human chanting and ritual.

Evolutionary anthropology. Our approach reframes debates on language origins. Rather than arising solely from propositional or symbolic capacities, language may have emerged from rhythmic, metaphorical and emotionally charged proto-poetic systems. Archaeological evidence of ritual, music and symbolic ornamentation suggests that poeticity predates writing and may have scaffolded linguistic and cultural complexity. Future work could integrate genetic, archaeological and cognitive evidence to test whether poetic cognition was a selective advantage for early Homo.

Artificial intelligence. Poetic cognition can inspire novel AI architectures. Current models excel at literal processing but struggle with metaphor, ambiguity and flexible reasoning. Incorporating principles of poeticity could help AI approximate human-like thought. For instance, rhythmic processing might inform temporal models in recurrent or oscillatory networks, while metaphor-inspired cross-domain mapping could enhance transfer learning. Ambiguity-tolerant systems could

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improve creativity and robustness in uncertain environments. "Poetic AI" may not only generate verse but also reason in patterns closer to human cognition, with applications from natural language understanding to human–machine interaction.

Clinical and therapeutic contexts. Poetry has long been used informally in therapy, but the neurocognitive poetics model provides a framework for mechanistic application. Rhythmic recitation can improve speech fluency in aphasia. Metaphor enriches patient narratives, fostering new perspectives in psychotherapy. Poetry's capacity to evoke emotion and synchrony may also mitigate loneliness or trauma, acting as a tool for social and affective regulation. Future clinical research could test whether poetic interventions specifically modulate limbic and prefrontal activity, thereby enhancing resilience.

Educational applications. Poetic structures can be harnessed to optimize learning. Rhythm and rhyme aid memory consolidation, particularly in children, by aligning with natural oscillatory mechanisms. Imagery and metaphor foster comprehension of abstract concepts, from mathematics to science education. Group recitation or song can enhance classroom cohesion and collective engagement, potentially improving attention and motivation. Developing curricula that systematically leverage poetic structuring could yield measurable gains in memory, prediction and creativity.

Future directions. To operationalize our framework, researchers must develop **quantitative metrics of poeticity**, applicable across species, cultures and modalities. Such metrics might quantify rhythmic regularity, degree of ambiguity or density of metaphorical mapping. Computational models could then simulate how poetic structuring influences prediction error, memory load or social cohesion. Combining behavioral, neuroimaging and comparative data would allow rigorous testing of whether poetic cognition constitutes a universal substrate of intelligence.

In sum, the poetic cognition hypothesis opens new frontiers. It encourages scientists to treat poetry not as ornament but as an evolved cognitive and communicative strategy, with testable predictions and wide-ranging applications in neuroscience, anthropology, education, therapy and AI.

Conclusions

The search for the "language of thought" has traditionally centered on symbolic logic, neural coding, predictive models or embodied processes. We have considered an alternative: that cognition itself may operate in a fundamentally poetic mode. Evidence from cognitive linguistics, neuroscience, anthropology and comparative biology suggests that the core features of poetry (namely, metaphor, rhythm, repetition, imagery, ambiguity, sound play, emotional resonance and collective synchrony) map onto identifiable neural mechanisms and serve adaptive functions. These structures support memory, enhance prediction, foster creativity and reinforce social cohesion, making them indispensable cognitive tools rather than cultural ornaments.

Through literary and ritual forms, humans exploit these mechanisms explicitly, but proto-poetic features also appear across species. Birdsong, whale song, honeybee dances, primate calls and ritualized displays exhibit rhythmic structuring, repetition with variation, symbolic compression and ambiguity. These parallels suggest that poeticity is not confined to human culture but reflects a deeper evolutionary substrate of communication and cognition.

By reframing poetry as a natural cognitive technology, we bridge disciplines and species, offering a unifying framework to study thought across biological systems. The challenge ahead is to operationalize this hypothesis: developing metrics of poeticity, conducting cross-species experiments and leveraging poetic principles in clinical, educational and artificial intelligence applications.

Ultimately, to ask whether humans and animals think in poetic language is to recognize that patterned, metaphorical and affective communication may be the foundation of intelligence itself. Rather than an aesthetic luxury, poetry may constitute the ancient rhythm through which brains, both human and nonhuman, make sense of the world together.

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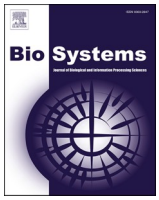
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Full Length Article

Simulating tumor mitochondrial energetics through engineering-style energy metrics

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ABSTRACT

Cancer progression is linked to alterations in cellular energetics, where malignant cells reprogram their metabolism to sustain proliferation, resist stress and adapt to nutrient limitations. Recent work has shown that tumors actively remodel their microenvironment by acquiring functional mitochondria from surrounding stromal or immune cells. Mitochondrial transfer enhances tumor bioenergetics while simultaneously depleting immune cells of metabolic competence, thereby reinforcing both tumor growth and immune evasion. The energetic consequences in terms of throughput, efficiency and stored energy of these exchanges are not captured by conventional assays focused on oxygen consumption or glycolytic flux. We introduce a simulation-based framework for theoretical analysis of mitochondrial energetics that adapts engineering-style energy metrics to mitochondrial biology. Three theoretical, model-defined bioenergetic metrics are introduced: mitochondrial power density, expressing ATP production per unit mitochondrial volume; mitochondrial surface power density, relating ATP production to inner membrane area; and mitochondrial energy density, quantifying stored chemical free energy per unit volume. Using controlled in silico simulations of tumor and immune cell populations before and after modeled mitochondrial transfer, we examine how these descriptors vary under explicit simulation assumptions. Within our simulation framework, results indicate model-predicted differences between cell populations, with tumor-associated mitochondria occupying higher energetic throughput and immune-associated mitochondria exhibiting complementary reductions. Although exploratory and hypothesis-generating rather than validated biomarkers or clinical tools, our metrics provide a quantitative physical framework that may inform experimental studies of mitochondrial transfer and its energetic consequences, including efforts to disrupt pathogenic transfer and restore metabolic competence in immune cells.

1. Introduction

Cancer energetics is defined by the ability of malignant cells to sustain rapid growth under fluctuating nutrient and oxygen availability, often through reprogramming of metabolic pathways (Loo et al., 2015; Lin et al., 2019; Xia et al., 2021; Pavlova et al., 2022; Kubik et al., 2022; Tufail et al., 2024). Central to this process are mitochondria, which not only generate ATP, but also regulate redox balance, biosynthetic precursors and signaling (Ryu et al., 2024). Their adaptive capacity underpins the metabolic plasticity enabling tumor cells to thrive in hostile microenvironments (Yang et al., 2016; Missiroli et al., 2020; Kafkova and Trnka, 2020; Guo et al., 2021).

More recently, mitochondrial transfer between cells has been identified as a mechanism that not only supports tumor growth by supplying respiration-competent organelles (Borcherding and Brestoff, 2023; Watson et al., 2023; Zhang et al., 2023a; Ikeda et al., 2025; Hoover et al., 2025), but also undermines immune surveillance by delivering functionally compromised mitochondria to lymphocytes (Zhang et al., 2023b; Chun et al., 2025). These findings suggest that mitochondrial exchange may reshape energetic states at the organelle level, yet their energetic consequences are difficult to formalize within a physical description.

Current methods to assess these mitochondrial phenomena rely on indicators like oxygen consumption rates, extracellular acidification or mtDNA content, that provide only partial insight into the quantitative energetic performance of mitochondria and their contribution to intercellular dynamics. These readouts emphasize specific biochemical or respiratory aspects but do not provide a unified description of energetic throughput, membrane-level efficiency and stored capacity within a common physical framework. This limitation motivates the introduction of descriptors explicitly linking energetic performance to mitochondrial structure and physical scale. Therefore, we introduce a simulation-based

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approach that adapts engineering-style energy metrics to mitochondrial biology, positioning our work as a theory-building and systems-level contribution rather than a translational or diagnostic study. We propose three parameters, informed by physics-based energy metrics, that extend conventional bioenergetic assessments by translating the concepts of power density, surface power density and energy density into the context of mitochondrial biology (Karami-Mosammam et al., 2022; Shang et al., 2023). The three engineering-style energy metrics quantify how energy is produced, transferred or stored relative to physical dimensions such as volume, surface area and time (Kim et al., 2019; Li et al., 2021; Foong et al., 2022). Expressed in standard units like W/m^3 or J/m^3, they enable accurate and comparable descriptions of energetic performance across diverse systems. When applied to mitochondria, these metrics may become physically grounded descriptors of specific energetic constraints.

a) Mitochondrial power density is calculated as the ATP production rate per unit mitochondrial volume, reflecting energetic throughput capacity relative to organelle size.
b) Mitochondrial surface power density is defined as ATP production rate per unit inner membrane or cristae area, thereby linking energetic output to functional efficiency of membrane organization.
c) Mitochondrial energy density captures the total stored chemical free energy per unit mitochondrial volume, integrating ATP, reduced cofactors and the proton motive force and thus reflecting short-term energetic reserve.

Together, these parameters establish a multidimensional energetic description that is explicitly scale-aware and organelle-centered. In the sequel we calculate their values within a controlled simulation framework, providing theoretical benchmarks rather than empirically validated measurements. In our theory-building, simulation-based study, we assess whether physically interpretable energetic descriptors can discriminate mitochondrial energetic states in a controlled in silico tumor–immune system, providing benchmarks for comparison and hypothesis generation. Specifically, we examine whether mitochondrial power density, surface power density and energy density differentiate simulated tumor and immune mitochondrial populations, whether modeled mitochondrial transfer induces systematic shifts in these metrics and whether these patterns remain robust across repeated simulations and parameter variability.

Our framework is conceptually distinct from existing approaches to cellular and mitochondrial energetics that do not directly capture how energetic performance scales with organelle-level properties, particularly in structurally heterogeneous contexts like intercellular mitochondrial transfer. Flux balance and constraint-based models primarily operate at the level of reaction stoichiometry and pathway feasibility, inferring steady-state flux distributions under imposed constraints but remaining largely agnostic to the physical size, geometry or energetic density of the underlying organelles (Sahu et al., 2021; Shachar-Hill 2024; Morrissey et al., 2024; Nasershariat et al., 2024). Metabolic network models emphasize pathway topology and regulation, while energy budget models track aggregate energetic inputs and outputs at the cellular or organismal level without resolving how energy production, transfer or storage scale with mitochondrial structure (Yilmaz and Walhout, 2017; Nouri et al., 2020; Han et al., 2024; Li et al., 2025). Thermodynamic analyses provide essential bounds on efficiency and feasibility but do not yield spatially normalized descriptors integrating geometry, membrane organization and energetic throughput within a unified quantitative language (Cong et al., 2016; Swamy et al., 2019; Disalvo et al., 2022; Socrier and Steinem, 2023). Our approach operates at the level of physically interpretable energetic descriptors, introducing quantities that relate ATP flux and stored chemical energy directly to mitochondrial volume and inner membrane area.

To clarify the conceptual aim of our framework, the comparative Table 1 situates our proposed descriptors as complementary metrics explicitly linking bioenergetic performance to mitochondrial structure, thereby addressing aspects of energy organization not captured by standard bioenergetic readouts.

We will proceed as follows. We begin by explaining how the metrics are defined and computed within the simulations, then examine their meaning in the context of tumor–immune interactions, clarifying both their conceptual scope and their limits. We next analyze their behavior across different simulated conditions and discuss how these quantities may be related to empirical measurements. The final part assesses their value as hypothesis-generating constructs within the broader framework of cancer energetics.

2. Biological interpretation and scope of engineering-style metrics

The engineering-style energy metrics introduced here are intended as physically grounded descriptors to formalize specific energetic constraints acting on mitochondria. Their biological relevance arises from the fact that mitochondrial function is inherently limited by geometry, membrane organization and thermodynamic principles, all of which

Table 1
Conceptual comparison between commonly used bioenergetic indicators and our proposed engineering-style mitochondrial energy metrics. The table highlights differences in analytical focus, limitations and the specific gap addressed by our descriptors.

Commonly used bioenergetic indicator	What it primarily measures	Conceptual focus	Main limitations	Added value of the proposed metrics
Oxygen consumption rate (OCR)	Cellular or mitochondrial respiratory activity	Electron transport and oxidative metabolism	Indirect proxy for ATP production; sensitive to coupling efficiency and substrate availability; typically averaged over populations	Proposed metrics directly normalize energy production to mitochondrial volume or surface, allowing structural–energetic comparisons across cells and conditions
Extracellular acidification rate (ECAR)	Net proton release, often interpreted as glycolytic flux	Glycolysis-dominated ATP production	Non-specific to mitochondria; influenced by CO_2 hydration and buffering; weakly linked to mitochondrial structure	New metrics isolate mitochondrial energetic contribution independently of cytosolic acidification
ATP concentration	Steady-state cellular energy availability	Energy pool size	Static measure; does not reflect production rate, spatial heterogeneity, or mitochondrial architecture	Mitochondrial power density captures ATP generation rates per unit mitochondrial structure rather than bulk concentration
Mitochondrial membrane potential ($\Delta\Psi m$)	Electrochemical driving force across the inner membrane	Coupling state and proton motive force	Nonlinear relation to ATP synthesis; sensitive to probes and loading; difficult to compare across morphologies	Surface power density links ATP output explicitly to inner membrane area, where $\Delta\Psi m$ is generated
NADH/NAD^+ or redox ratios	Cellular redox balance	Metabolic state and substrate oxidation	Context-dependent interpretation; integrates multiple pathways beyond mitochondria	Energy density focuses on stored chemical free energy within mitochondria, separating redox state from structural capacity
Mitochondrial mass or content	Total mitochondrial abundance	Structural presence	Does not encode functional efficiency or energetic output	Normalization by volume or surface reveals efficiency differences at fixed mitochondrial content

impose measurable bounds on energy production, transfer and storage (see Table 2). By translating these constraints into explicit energetic quantities, our metrics aim to provide a coherent quantitative language for comparing mitochondrial behavior across cell types and conditions.

a) Mitochondrial power density represents the rate of ATP production normalized by mitochondrial volume. In biological terms, this metric captures energetic throughput capacity, expressing how intensively a mitochondrion converts substrates into useable chemical energy relative to its size. It reflects the combined influence of respiratory chain capacity, substrate availability and internal organization and is particularly relevant when comparing mitochondria of different volumes or when assessing energetic consequences of mitochondrial transfer. Importantly, power density does not measure ATP abundance, but rather the ability to sustain energetic demand over time.
b) Mitochondrial surface power density normalizes ATP production by inner membrane or cristae area. Because oxidative phosphorylation is localized to the inner membrane, this quantity serves as a proxy for the functional efficiency of membrane organization, linking energetic output to cristae packing and surface amplification. Biologically, it distinguishes increases in energetic performance arising from enhanced membrane organization from those resulting solely from increased mitochondrial volume or number.
c) Mitochondrial energy density quantifies the total stored chemical free energy per unit mitochondrial volume, integrating ATP, reduced electron carriers and the proton motive force. This metric captures short-term energetic reserve, describing the capacity of a mitochondrion to buffer transient fluctuations in demand or supply. Unlike power-based metrics, energy density reflects stored potential rather than dynamic conversion and is therefore complementary to throughput-oriented descriptors.

By construction, these metrics are selective and do not aim to capture all aspects of mitochondrial biology. They do not describe mitochondrial signaling functions, reactive oxygen species production, calcium handling, apoptotic priming or transcriptional regulation. They also do not resolve spatial heterogeneity within tissues, intracellular gradients of oxygen or nutrients or dynamic remodeling of mitochondrial networks. Our framework therefore targets a specific energetic layer of mitochondrial function rather than a comprehensive organelle description.

Table 2
The three quantitative mitochondrial parameters introduced in our simulation. Each parameter is defined with its physical units and methodological focus, capturing distinct yet complementary dimensions of mitochondrial bioenergetics.

Parameter	Definition	Methodological focus: what it explains
Mitochondrial power density (W/m^3)	ATP production rate per unit mitochondrial volume (volumetric throughput of energy conversion)	Explains how much useable power is generated relative to mitochondrial size, capturing overall throughput and enabling comparison of energetic intensity across mitochondria of different volumes.
Mitochondrial surface power density (W/m^2)	ATP production rate per unit inner membrane or cristae area (efficiency and packing of oxidative phosphorylation machinery)	Explains how effectively the inner membrane structure supports ATP synthesis, linking functional output to cristae architecture and membrane density.
Mitochondrial energy density (J/m^3)	Total stored chemical free energy per unit mitochondrial volume (ATP, NADH, proton motive force)	Explains the reservoir of immediately available energy within a mitochondrion, capturing static storage capacity rather than dynamic throughput.

At present, our metrics are not directly measurable through standard wet-lab assays. Existing experimental techniques provide partial inputs, such as oxygen consumption rates, ATP concentrations, membrane potential indicators or ultrastructural measurements, but no single approach yields power density or energy density as defined here. Accordingly, these quantities should be understood as theoretical constructs designed to integrate disparate measurements within a unified physical framework. Their primary role is to support simulation, comparison and hypothesis generation, rather than immediate experimental readout or clinical application.

3. Materials and methods

We detail a simulation-based approach that integrates mitochondrial geometry, flux kinetics, energy storage and transfer dynamics between tumor and immune cells. Parameters were derived from physical laws and literature-based physiological ranges, with power density, surface power density and energy density computed through explicit mathematical formulations. Our simulation aims to build synthetic mitochondria, assign bioenergetic states and propagate these states through a theoretical tumor–immune co-culture in silico, rather than to reproduce empirical measurements.

Model assumptions. Our simulation relies on a set of explicit assumptions that can be grouped into three categories.

1. Critical assumptions affecting qualitative conclusions are essential for the emergence of the model-predicted divergence between tumor and immune mitochondrial energetic states. They include: (a) the existence of intercellular mitochondrial transfer between tumor and immune cells; (b) an energetically asymmetric transfer process in which mitochondria acquired by tumor cells retain higher functional capacity than those remaining in or transferred to immune cells; and (c) the dependence of energetic performance on mitochondrial geometry and inner membrane organization, as encoded in the power density, surface power density and energy density metrics. Altering these assumptions eliminates or qualitatively reverses the simulated separation between cell populations.
2. Secondary assumptions affecting quantitative magnitude influence the absolute values and dispersion of the simulated metrics but do not determine the direction of the observed trends. They include the specific parameter ranges chosen for mitochondrial volume, cristae amplification, maximal ATP flux capacity, redox pool size and transfer probability. Modifying these parameters rescales metric distributions but preserves the relative ordering of tumor and immune populations within the simulation.
3. Simplifying assumptions for computational or conceptual convenience enable tractable simulation but are not expected to drive qualitative outcomes. They include homogeneous extracellular conditions, static mitochondrial geometry within each simulation run, effective kinetic laws replacing multi-step enzymatic regulation and the absence of explicit spatial gradients or mitochondrial network dynamics.

Overall, this hierarchical categorization clarifies which conclusions are structurally embedded in the model and which aspects affect numerical scaling. Relaxing these assumptions would increase biological realism but is unlikely to change the core model-predicted relationships under study.

Constraints and sensitivity considerations. The simulation time is indexed by discrete steps $t = 0, 1, \ldots, T$ with step size Δt. Each cell c contains a set of mitochondria $M_c = \{m_1, \ldots, m_{N_c}\}$. For each mitochondrion m we track volume V_m $[m^3]$, inner membrane area A_m $[m^2]$, ATP

flux $\dot{n}_{ATP,m}$ $[\mathrm{mol\,s^{-1}}]$, proton motive force components $\Delta\psi_m$ [V] and ΔpH_m and reduced carrier pools $n_{NADH,m}$ [mol] (Imamura et al., 2009; Oncescu and Erickson, 2013; Cox and Smith, 2014; Natsubori et al., 2020; Greiner and Thomas, 2021). Temperature is fixed at $T = 310$ K. The ATP free energy is computed as

$$\Delta G_{ATP} = \Delta G^{'}_{ATP} + RT \ln \frac{[ADP][Pi]}{[ATP]},$$

with $\Delta G^{'}_{ATP} = -30.5\,\mathrm{kJ\,mol^{-1}}$. Power per mitochondrion is $P_m = \dot{n}_{ATP,m}|\Delta G_{ATP}|$. From these primitives we compute: mitochondrial power density $p_m = P_m/V_m$ $[\mathrm{W\,m^{-3}}]$, surface power density $s_m = P_m/A_m$ $[\mathrm{W\,m^{-2}}]$ and energy density $u_m = U_m/V_m$ $[\mathrm{J\,m^{-3}}]$, where U_m is the stored chemical energy defined below. These quantities are model-derived descriptors intended to characterize energetic throughput, membrane-normalized efficiency and stored capacity within the simulation framework.

Tumor cells are parameterized with higher probabilities of enlarged cristae density, elevated ATP flux capacity and stronger redox stores to reflect documented metabolic tendencies in cancer, whereas immune cells are assigned more moderate volumes, balanced throughput and higher variability, reflecting context-dependent energetic adaptability. All cells experience a homogeneous extracellular environment with respect to temperature and baseline substrate availability. Spatial gradients of oxygen, nutrients or pH at the tissue scale are not explicitly resolved. Our parameterizations represent modeling assumptions rather than direct biological measurements.

Each mitochondrion is represented as a tri-axial ellipsoid with semi-axes (a, b, c) drawn from lognormal priors $a \sim \mathrm{LN}(\mu_a, \sigma_a^2)$, etc., yielding volume $V_m = \frac{4}{3}\pi abc$. Inner membrane area is generated by a cristae packing model using a cristae surface multiplier κ, such that $A_m = \kappa A_{outer}$, with A_{outer} approximated using Knud Thomsen's formula with $p = 1.6075$. This geometric representation captures first-order size and surface amplification effects while neglecting dynamic cristae remodeling, fusion–fission topology and network-level organization. The multiplier $\kappa \sim \Gamma(k_\kappa, \theta_\kappa)$ captures tissue variability, with typical values yielding $A_m/V_m \sim 10^7$–10^8 in cristae-rich states. Geometry is treated as static within each simulation run.

ATP flux arises from an effective electron transport chain capacity model with Michaelis–Menten saturation and stochastic efficiency dispersion (Smolders et al., 2003; Schmidt et al., 2021; Bryant and Machta, 2023; Mok et al., 2024; Singh et al., 2025). Maximal capacity scales with inner membrane area $v_{max,m} = k_v A_m$. This formulation abstracts multi-step enzymatic regulation into an effective throughput law and does not explicitly model pathway-level feedback or supercomplex dynamics.

Stored energy U_m comprises ATP, reduced carriers and the proton motive force. ATP and redox contributions are treated as additive chemical reservoirs, while membrane and pH buffering energies are modeled using small-signal approximations. Long-term metabolic adaptation, transcriptional regulation and biosynthetic coupling are outside the scope of the model.

Transfer process between cells and state updates. Intercellular transfer is implemented as a probabilistic marked point process on a bipartite tumor–immune interaction graph. At each step, a mitochondrion m in donor cell i is selected with probability proportional to $q_m = \gamma_0 + \gamma_1 p_m + \gamma_2 u_m$. A recipient cell j is selected with probability $w_j = \delta_0 + \delta_1 \bar{p} + \delta_2 \bar{u}$. Transfer occurs with probability $\pi = \sigma(\theta_0 + \theta_1 \mathrm{contact}_{ij})$. This mechanism captures directional energetic bias in transfer events and does not explicitly represent physical transfer structures such as tunneling nanotubes or vesicles.

For each mitochondrion we compute p_m, s_m and u_m at every step. Cell-level quantities are arithmetic or area-weighted means and population summaries use medians and interquartile ranges. These aggregated quantities constitute simulated outputs rather than empirically observable biomarkers.

Parameterization and qualitative sensitivity, statistical comparison and numerical implementation. We calibrate priors so that simulated medians match reported orders of magnitude for mammalian mitochondria (Chapa-Dubocq et al., 2023; Triolo et al., 2023). Parameters governing cristae amplification, maximal ATP flux capacity and transfer propensity exert the strongest influence on simulation outcomes, whereas stochastic noise terms primarily affect temporal variability. The qualitative divergence between tumor and immune energetic states remains robust across broad parameter ranges, indicating that results arise from model structure rather than fine-tuning.

Statistical comparisons quantify within-simulation contrasts and directional robustness rather than biological significance. Simulations are implemented in Python 3.11 using NumPy, SciPy, JAX, Numba and Matplotlib, with deterministic execution ensured via controlled seeding and Snakemake workflows.

In conclusion, our methods define a coherent workflow to integrate structural modeling, flux generation, energy storage equations and transfer dynamics into a theory-driven, internally consistent simulator intended for hypothesis generation and conceptual analysis.

4. Results

We present the simulation outcomes designed to quantify mitochondrial bioenergetic metrics before and after intercellular transfer. To contextualize the results, we first include an order-of-magnitude worked example for a single mitochondrion linking geometry, kinetics and thermodynamics to explicit values of power density, surface power density and energy density. This example provides a baseline for interpreting how models may reflect systematic changes in simulated tumor and immune cells. Following this reference point, we describe the distributions and within-simulation statistical contrasts of these metrics across groups, highlighting directional and robust energetic shifts that emerge within the simulated tumor microenvironment.

All the reported results are derived exclusively from in silico simulations. Statistical comparisons, including p-values and effect sizes, quantify internal consistency and directional robustness within the model across repeated simulation runs. They do not represent inferential statistics on biological populations and should not be interpreted as evidence of empirical or functional significance. Accordingly, statistical significance reflects stability of model-predicted patterns under the stated assumptions rather than biological validation.

Order-of-magnitude worked example for a single mitochondrion. To ground the simulation in physical reality, we first calculate an order-of-magnitude example for a single mitochondrion using established structural and energetic parameters. This example serves as a reference point against which simulated populations of tumor and immune cells can be compared, ensuring that subsequent results remain anchored to physiologically plausible magnitudes.

All results reported in this section should be interpreted as simulated outcomes rather than empirical biological measurements. A single mitochondrion is instantiated with volume $V = 1.0 \times 10^{-18}$ m^3 and inner membrane area $A = 5.0 \times 10^{-12}$ m^2, consistent with an ellipsoid of micron scale and cristae amplification. ATP flux is set to $\dot{n}_{ATP} = 1.0 \times 10^6$ s^{-1} molecules, which equals 1.66×10^{-18} mol s^{-1}. With $|\Delta G_{ATP}| = 50$ kJ mol^{-1}, the instantaneous power is $P = \dot{n}_{ATP}|\Delta G_{ATP}| = 8.3 \times 10^{-14}$ W. Mitochondrial power density is then $p = P/V = 8.3 \times 10^4$ W m^{-3} and surface power density is $s = P/A = 1.66 \times 10^{-2}$ W m^{-2}.

For stored energy, we specify [ATP] = 5 mM and $n_{NADH} = 2.0 \times 10^{-19}$ mol. The ATP store contributes $U_{ATP} = [ATP]VN_A|\Delta G_{ATP}| = 2.5 \times 10^{-13}$ J. With $|\Delta G_{redox}| = 220$ kJ mol^{-1}, the redox store contributes $U_{redox} = 4.4 \times 10^{-14}$ J. Taking $C_m = 0.01$ F m^{-2} and $\Delta\psi = 0.15$ V gives $U_{\Delta\psi} = \frac{1}{2}C_m A(\Delta\psi)^2 = 5.6 \times 10^{-16}$ J. With $\beta_H = 20$ mol m^{-3} pH^{-1} and Δ

$pH = 0.5$, the buffer term is $U_{\Delta pH} \approx 1.5 \times 10^{-16}$ J. Total stored energy is $U \approx 2.96 \times 10^{-13}$ J, yielding an energy density $u = U/V \approx 3.0 \times 10^{5}$ J m^{-3}.

This example validates the internal consistency and physical plausibility of our simulation and serves as a reference benchmark supporting interpretation of broader simulated patterns in tumor and immune cell populations.

Tumor- and immune-associated mitochondrial metrics. Within our simulation framework, tumor mitochondria displayed robust directional increases in all energetic parameters following intercellular transfer (Figures A-C). All reported differences reached statistical significance within the simulation framework ($p < 0.001$, Welch's *t*-test), reflecting internal model robustness rather than empirical population inference. Mean power density shifted from 90,714 W m^{-3} before transfer to 129,699 W m^{-3} after transfer (Figure A). Surface power density shifted from 0.020 W m^{-2} to 0.030 W m^{-2} (Figure B) and energy density from 257,217 J m^{-3} to 319,952 J m^{-3} (Figure C). These simulated shifts indicate that tumor mitochondria occupy higher energetic throughput, membrane-normalized efficiency and stored reserve regimes after transfer within the model assumptions.

Simulated immune mitochondria exhibited complementary downward shifts in the same metrics (Figures A-C). Power density shifted from 80,613 W m^{-3} before transfer to 50,112 W m^{-3} after transfer (Figure A). Surface power density decreased from 0.018 W m^{-2} to 0.010 W m^{-2} (Figure B) and energy density from 248,959 J m^{-3} to 192,395 J m^{-3} (Figure C) Within the simulation, these changes correspond to reduced energetic throughput, diminished cristae-normalized efficiency and lower short-term energetic reserve following transfer.

Overall, the juxtaposition of the simulated results observed in tumor and immune cells following intercellular transfer establishes a complementary pattern, in which tumor energetic descriptors shift upward while immune energetic descriptors shift downward, highlighting a model-predicted divergence between cell types.

5. Experimental anchoring and falsifiability

Although our study is entirely simulation-based, the proposed mitochondrial energy metrics are formulated to be empirically anchorable and falsifiable. Their scientific value does not rest on immediate measurability, but on their capacity to generate explicit, testable predictions linking mitochondrial structure, energetic flux and stored capacity within a unified physical framework.

a) Mitochondrial power density can be experimentally approximated by combining high-resolution respirometry or ATP production measurements with quantitative estimates of mitochondrial volume obtained from three-dimensional electron microscopy or super-resolution imaging (Triolo et al., 2023; Chapa-Dubocq et al., 2023). Within the logic of the model, mitochondrial transfer is predicted to increase volumetric energetic throughput in tumor cells relative to immune cells (Borcherding and Brestoff, 2023; Zhang et al., 2023a). An absence of increased volume-normalized ATP production following documented mitochondrial transfer would directly contradict the model's central assumption that transfer enhances energetic throughput rather than merely increasing mitochondrial abundance.

b) Mitochondrial surface power density can be anchored by correlating energetic flux measurements with inner membrane or cristae surface area quantified using electron tomography or ultrastructural reconstruction (Triolo et al., 2023). The model predicts that energetic enhancement following transfer is associated with increased membrane-normalized throughput, reflecting more efficient or denser organization of oxidative phosphorylation machinery (Ryu et al., 2024). Experimental observations showing no relationship between cristae surface area and energetic output or demonstrating

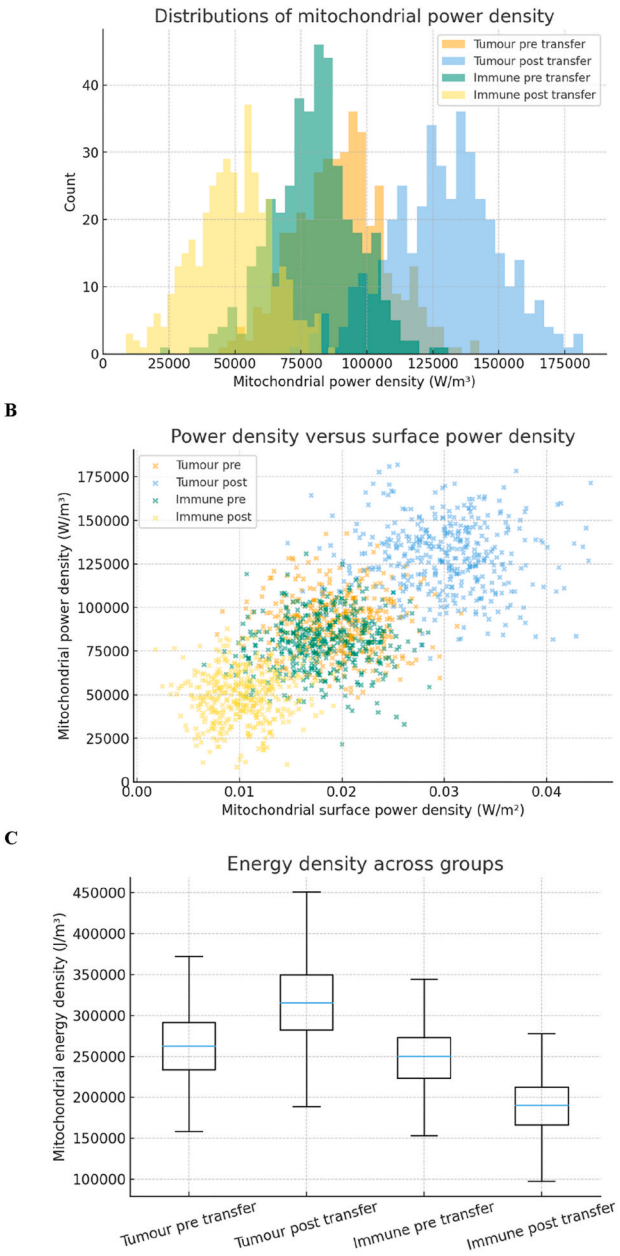


Fig. 1. Simulation-derived behavior of the three mitochondrial bioenergetic metrics, shown as model outputs before and after simulated intercellular mitochondrial transfer. All panels show simulation-derived model outputs and do not represent empirical biological measurements.
(A) Simulation-derived distributions of mitochondrial power density in tumor and immune populations before and after simulated transfer. Results are obtained from R independent simulation runs, comprising N_t simulated tumor cells and N_i simulated immune cells, with a total of M simulated mitochondria per condition. Units are W m^{-3}.
(B) Simulation-derived joint distribution of surface power density and power density across individual simulated cells. Each point represents a simulated cell, aggregated across R runs (N_t tumor cells, N_i immune cells; M simulated mitochondria total). Clustering reflects redistribution within simulated energetic state space rather than absolute functional levels. Units are W m^{-2} (x-axis) and W m^{-3} (y-axis).
(C) Simulation-derived distributions of mitochondrial energy density across tumor and immune populations. Boxplots summarize values from R simulation runs, encompassing N_t simulated tumor cells and N_i simulated immune cells with M mitochondria per condition. Shifts indicate model-predicted changes in stored energetic capacity under simulation assumptions. Units are J m^{-3}.

that ATP flux scales exclusively with mitochondrial volume would falsify the assumption that membrane organization plays a dominant functional role in this context.

c) Mitochondrial energy density may be indirectly constrained by integrating measurements of ATP concentration, redox carrier states and membrane potential. Genetically encoded ATP sensors, redox-sensitive probes and membrane potential indicators can together bound estimates of stored chemical free energy (Weiβ and Bohrmann, 2019). Our framework predicts that mitochondria transferred to tumor cells occupy higher short-term energetic reserve states, while recipient immune cells exhibit reduced reserve capacity (Chun et al., 2025). Experimental evidence of preserved or enhanced energetic reserves in immune cells after transfer or no systematic increase in tumor energetic buffering would challenge the model's treatment of transfer as an energetically asymmetric process.

At the system level, the framework yields falsifiable predictions regarding intervention. For example, experimental inhibition of mitochondrial transfer should suppress the divergence in power density, surface power density and energy density between tumor and immune populations (Guan et al., 2024). If energetic divergence persists despite verified blockade of mitochondrial transfer, this would indicate that additional mechanisms outside the scope of the model dominate energetic reprogramming.

Falsification does not require numerical agreement with simulation outputs. Because the metrics emphasize scaling relations and directional trends rather than absolute values, empirical validation may confirm or refute the framework by testing relative changes and correlations. In this sense, discrepancies between predicted and observed relationships are informative, guiding refinement of assumptions concerning geometry, flux regulation or energetic storage.

Taken together, these considerations position our work as an explicitly hypothesis-generating systems framework rather than a closed theoretical construct. The model is designed to be challenged, refined or rejected through coordinated structural, functional and metabolic experiments.

6. Conclusions

We introduce a conceptual framework that reformulates mitochondrial bioenergetics in terms of physically interpretable, dimensioned energy metrics. Rather than extending existing pathway-based, stoichiometric or constraint-driven approaches, our approach explicitly links energetic performance to mitochondrial geometry, membrane organization and stored chemical energy. This provides a unified energetic language to integrate throughput, efficiency and reserve within a single quantitative structure. Our contribution lies not in the specific simulated outcomes, but in demonstrating that mitochondrial energetic states could be characterized using normalized physical descriptors like power density, surface power density and energy density. These engineering-style energy metrics translate biochemical activity into quantities with explicit physical units, enabling comparisons across heterogeneous mitochondrial populations and across modeling contexts without reliance on pathway-specific assumptions (Mookerjee et al., 2017; Perry et al., 2011; Weiβ and Bohrmann, 2019).

By construction, our framework is theory-building and hypothesis-generating. It does not propose new biomarkers or diagnostic indicators, nor does it seek to replace established experimental assays. Instead, it provides a conceptual layer that could organize within a coherent energetic vocabulary diverse empirical measurements like oxygen fluxes, membrane ultrastructure and metabolite concentrations. In this sense, our approach complements existing bioenergetic analyses rather than competing with them, providing a means to interpret energetic constraints that arise from organelle-level structure and scaling.

Although our simulations are presented in the context of tumor–immune interactions and mitochondrial transfer, the broader relevance of the framework extends beyond cancer biology. These metrics could be applied to any biological system in which energetic performance is constrained by organelle geometry, membrane architecture or energy storage capacity, including development, neurobiology, immunometabolism and stress adaptation. More generally, our approach illustrates how engineering-style energetic descriptors can be adapted to biological systems to bridge the gap between molecular biochemistry and physical constraints.

Limitations of our work are inherent to its simulation-based nature. All conclusions depend on explicit modeling assumptions, simplified representations of mitochondrial structure and effective kinetic laws. Nevertheless, its value lies in making these assumptions explicit and in yielding falsifiable, scale-aware hypotheses that can be empirically challenged (Borcherding and Brestoff, 2023; Zhang et al., 2023a; Ikeda et al., 2025; Chun et al., 2025; Guan et al., 2024).

In conclusion, we provide a physically grounded conceptual framework for mitochondrial energetics that emphasizes energetic scaling, normalization and interpretability. By shifting attention from pathway-specific activity to organelle-level energetic descriptors, we pave the way to a complementary perspective on cellular across a wide range of biological contexts.

Ethics approval and consent to participate

This research does not contain any studies with human participants or animals performed by the Author.

Consent for publication

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Availability of data and materials

All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the author used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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Data availability

No data was used for the research described in the article.

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Topological Analysis of *Macaca mulatta*'s Cortical Structures Through the Lens of Poincaré Duality

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ABSTRACT

Poincaré duality from algebraic topology describes how shapes and spaces interrelate across different dimensions, linking structures of one scale to complementary structures at another. This suggests that densely packed neurons in certain brain areas may correspond to broader connectivity patterns, balancing local processing with global communication. Applying this framework to cortical histological images of *Macaca mulatta*, we analysed neuronal clustering, connectivity graphs and intensity distributions to identify self-dual structural patterns and reveal the contribution of local organization to large-scale network activity. Using image processing techniques such as contrast enhancement, edge detection and graph-theoretic modeling, we examined how dense neuron clusters correspond to functionally sparse regions and vice versa. We found that high-density neuronal zones form closed-loop topological structures that correspond to homology cycles, while sparser areas function as large-scale integrators, aligning with cohomology properties. Local connectivity hubs in neuron-dense regions support regional specialization, while large-scale, sparser areas, though less connected, facilitate global communication by acting as pathways for long-range integration. Graph-theoretic analysis of connectivity patterns confirmed a reciprocal relationship between clustering coefficients and global centrality. Statistical analysis using Kolmogorov-Smirnov tests revealed conserved topological distributions across different cortical regions, supporting the hypothesis that cortical connectivity maintains structural invariance under local perturbations. These findings provide insights into the mathematical principles governing brain architecture, suggesting that topological methods can enhance our understanding of cortical function. Future research may extend these approaches to higher-dimensional embeddings, network theory in primate brains, functional neuroimaging, human disease modeling and artificial intelligence.

KEYWORDS: Topology of neural networks; homology groups; cortical mapping; primate neuroanatomy; functional connectivity.

INTRODUCTION

The dynamic interplay between local cortical structures and global connectivity is crucial for understanding how the primate brain processes and integrates information (Cole et al., 2012; Oh et al., 2014; Li et al., 2023). Traditional neuroscientific methods, including histological analysis, graph-theoretic modeling and neuroimaging, have focused on procedures (like local cortical mapping, connectivity matrices and large-scale functional activity) that often fail to capture the reciprocal relationship between local clustering and global integration (Iakovidou 2017; Anastasiades et al., 2018; Raj, 2021; Dittman et al., 2021). We argue that modern topological methods provide a powerful new framework for characterizing brain architecture (Peters et al., 2016). By incorporating topological tools, it can be better understood how the brain balances regional specialization with large-scale communication, a key principle in both neural efficiency and cognitive flexibility (Bullmore and Sporns, 2009; Betzel and Bassett, 2017; Li et al., 2018; Murugesan et al., 2020). Persistent homology tracks the birth and death of topological features across different regions and scales (Otter et al., 2017), enabling the visualization of neuronal clusters across multiple resolutions, highlighting their persistence and revealing the robustness of functional networks. One of the topological approaches related with persistent homology is Poincaré duality, a fundamental concept from algebraic topology which describes the relationship between homology and cohomology groups within an orientable manifold (Hilman et al., 2024). Poincaré duality states that, in an n-dimensional orientable manifold, homology classes of dimension k are naturally paired with cohomology classes of dimension n−k (Hofscheier et al., 2024). In the context of the macaque cortex, this suggests that high-density neuronal regions (localized homological features) should correspond to global integrative structures (large-scale cohomological spaces).

We applied the Poincaré duality to histological images of *Macaca mulatta*'s cortex. We used image processing, edge detection and intensity-based clustering to extract homological structures from cortical tissue samples. Then, we employed graph-theoretic representations to quantify topological connectivity and examine how local connectivity hubs relate to global neural integration. Our analysis extended beyond purely structural observations, incorporating probabilistic models of connectivity. The Kolmogorov's zero-one law provided a statistical framework for

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understanding how certain cortical configurations emerge with high probability, supporting the hypothesis that the brain's large-scale organization follows universal topological principles (Brzeźniak and Zastawniak 2000). Additionally, we explored the relationship between cortical redundancy and functional specialization, investigating how group homomorphism principles manifest in recurrent structural motifs across different cortical regions.

In sum, we suggest that Poincaré duality in local neuron clusters offers a powerful framework for addressing brain organization and bridging the gap between local cortical microstructure and global functional connectivity. Poincaré duality explores the interplay between structurally dense and functionally sparse regions, revealing how information flows across different cortical areas. We conclude that our approach has implications not only for primate neuroanatomy but also for theoretical neuroscience, disease modeling and artificial intelligence, where topological and algebraic methods are increasingly being used to describe complex network dynamics.

MATERIALS AND METHODS

We utilized high-resolution histological images of *Macaca mulatta* cortical tissue to examine the topological properties of neuronal organization through the application of Poincaré duality. The dataset consisted of histological sections from whole-brain coronal slices of four adult *Macaca mulatta* specimens, stained with Nissl to visualize neuronal architecture. High resolution brain images were sourced from BrainMaps.org (http://brainmaps.org/index.php). The analysis specifically targeted cortical area 4, examined at 100x magnification. To preprocess the images, grayscale normalization was performed to standardize pixel intensity values across all samples. This step was necessary to remove artifacts introduced by staining variations and imaging inconsistencies. Histogram equalization was then applied to enhance contrast, allowing for improved detection of structural features such as neuronal clusters, axonal pathways and extracellular voids. Then, noise reduction techniques were employed using a Gaussian filter to smooth out minor imperfections while preserving critical topological features. Edge detection was carried out using the Canny algorithm to extract boundary information from neuronal structures (Gebäck and Koumoutsakos, 2009). This algorithm can detect high-frequency components while minimizing false edges. Once edges were identified, a skeletonization process was applied to reduce neuronal boundaries to their one-pixel-wide representations, allowing for a precise graph-theoretic analysis of connectivity patterns. Topological feature extraction was performed using persistent homology, a computational topology method to track topological features across multiple spatial scales (Otter et al., 2017). The software package used for this analysis was a combination of the Ripser and GUDHI libraries, well-established tools for computing homology groups from high-dimensional data. To construct persistence diagrams, neuronal structures were converted into point clouds using spatial density estimation. Filtrations were generated based on Vietoris-Rips complexes, allowing for the extraction of homological features such as cycles, voids and connectivity gaps (Müller and Stehlík, 2023).

To examine the relationship between local and global connectivity, graph-theoretic modeling was employed. Neuronal clusters and their edges were converted into adjacency matrices, from which network properties such as clustering coefficients, betweenness centrality and path lengths were computed. This approach enabled the identification of high-density connectivity hubs and their corresponding dual structures in sparser cortical regions. Eigenvalue distributions of the Laplacian matrix were analyzed to determine spectral properties of the connectivity graph (Ahanjideh et al., 2022). Kolmogorov-Smirnov tests were performed to compare the statistical distributions of neuronal density across different samples (Marsaglia et al., 2003). This test can determine whether two datasets originate from the same distribution without assuming normality. The results were used to establish whether neuronal clustering patterns exhibited significant variations or whether they followed conserved statistical distributions across different cortical samples. To examine the self-duality of neuronal networks, the Freeman chain code method was used to analyze curvature variations along neuronal boundaries (Chang and Hsaio, 2019). This method quantifies contour complexity, providing a numerical representation of how neuron-dense regions transition into functionally sparse voids. Structural metrics derived from this analysis were compared against the homological features obtained from persistent homology, ensuring that topological findings were supported by geometric descriptors. To explore the role of sheaf cohomology in cortical networks, spatially localized cohomological features were analyzed using sheaf Laplacians (Wedhorn 2016). This approach was implemented using discrete Hodge theory, which allowed for the decomposition of cortical connectivity into gradient, curl and harmonic components. The sheaf-theoretic analysis helped in identifying regions where local connectivity influenced global cortical dynamics.

Further structural validation was performed using clustering techniques such as k-means and hierarchical clustering to partition the cortical network into topologically meaningful subregions (Nielsen 2016). Dimensionality reduction techniques such as t-SNE were employed to visualize high-dimensional connectivity data. The final stage of analysis involved embedding cortical connectivity graphs into a higher-dimensional Euclidean space using the Nash embedding theorem (Seshadri and Verma 2016). This step provided an explicit geometric realization of the cortical network mapped onto an interpretable spatial structure. Throughout the study, rigorous statistical controls were applied to ensure

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the robustness of the findings. Multiple trials were conducted for each methodological step and sensitivity analyses were performed to verify the stability of homological features under variations in image preprocessing parameters.
In sum, our methodological framework integrated multiple advanced techniques from algebraic topology, computational geometry and graph theory to provide a comprehensive analysis of Poincaré duality in *Macaca mulatta*'s cortical structures.

RESULTS

The analysis of *Macaca mulatta*'s cortical histology through Poincaré duality revealed topological relationships between local neuronal clustering and global connectivity patterns. We identified significant dualistic structures highlighting the interplay between neuron-dense regions and sparser integrative areas. The results confirm that cortical organization is not random but follows mathematically governed principles that support both regional specialization and global communication. The most persistent homological features emerged in regions of high neuronal density, where closed-loop structures formed, functioning as local topological cycles that contribute to the cortical network's organization (**Figure A**). The persistence diagrams showed that neuronal clusters remained stable across different filtration values, suggesting that these features play a crucial role in cortical organization. The higher-dimensional homological structures, particularly those associated with voids in connectivity, corresponded to the complementary cohomology spaces, reinforcing the duality principle predicted by Poincaré duality.

The edge detection and skeletonization analysis further supported the presence of structured connectivity patterns, revealing a balance between high-density neuronal clusters and functionally sparse voids. The edges formed closed-loop structures, revealing persistent topological features that remain consistent across cortical regions, reinforcing the stability of network organization. In neuron-dense regions, these edges created tightly interconnected subnetworks, whereas sparser areas exhibited longer-range connections. This contrast suggests that, despite variations in neuronal density, global integration is preserved through a dynamic balance between localized clustering and extended connectivity. Graph-theoretic analysis of connectivity networks confirmed that regions with high clustering coefficients corresponded to localized processing hubs, whereas regions with high betweenness centrality corresponded to long-range cortical pathways. The Kolmogorov-Smirnov statistical tests revealed that, despite variations in neuron density across cortical images, the overall distribution of pixel intensities followed conserved statistical properties. These results confirm the structural invariance of cortical organization, revealing that despite variations in neuronal density, the underlying topological framework remains stable and resilient. This finding aligns with Kolmogorov's zero-one law, suggesting that certain structural properties are probabilistically inevitable in cortical development (**Figure B**). It reinforces probability theory by demonstrating that large-scale cortical structures persist with near certainty, despite local variations in neuronal density. An additional validation of Poincaré duality was provided by spatial clustering analysis, which showed that cortical regions self-organized into stable clusters of neuronal connectivity. The application of hierarchical clustering techniques revealed that neuron-dense areas consistently formed compact regions, whereas lower-density areas formed spatially extensive, interconnected structures. These findings confirm that local neuronal specializations correspond to global functional integration zones.

The application of sheaf cohomology allowed for a more detailed understanding of how local information propagates into global cortical dynamics. The analysis of sheaf Laplacians revealed that areas with high localized connectivity exhibited strong gradient components, while large-scale integrative regions corresponded to harmonic components, indicating a natural transition between local computation and global integration. This aligns with the broader functional role of the cortex, where specialized regions handle specific tasks, while broader connectivity networks ensure distributed processing. By applying the Nash embedding theorem to map the cortical connectivity graph into a higher-dimensional Euclidean space, we visualized how neuronal clusters integrate within the global cortical architecture. The embedding revealed that regions that appeared locally isolated were globally interconnected, reinforcing the principle that neuronal function cannot be understood purely from local observations. Despite slight, non-statistically significant variations in neuronal clustering across the cortices of the three individuals, the distribution of intensity variations showed a conserved structure, where areas of high signal concentration were always balanced by complementary low-intensity voids. The graph-theoretic perspective reinforced this conclusion (**Figure C**). Connectivity graphs revealed a duality between local clustering coefficients and global betweenness centrality, confirming that strong local clustering corresponded to lower long-range influence, while sparsely connected regions served as critical junctions for global integration.

Taken together, these findings provided evidence that Poincaré duality is an organizing principle of cortical structure, with neuron-dense areas forming the homological backbone of cortical function and their complementary cohomological spaces facilitating long-range information transfer.

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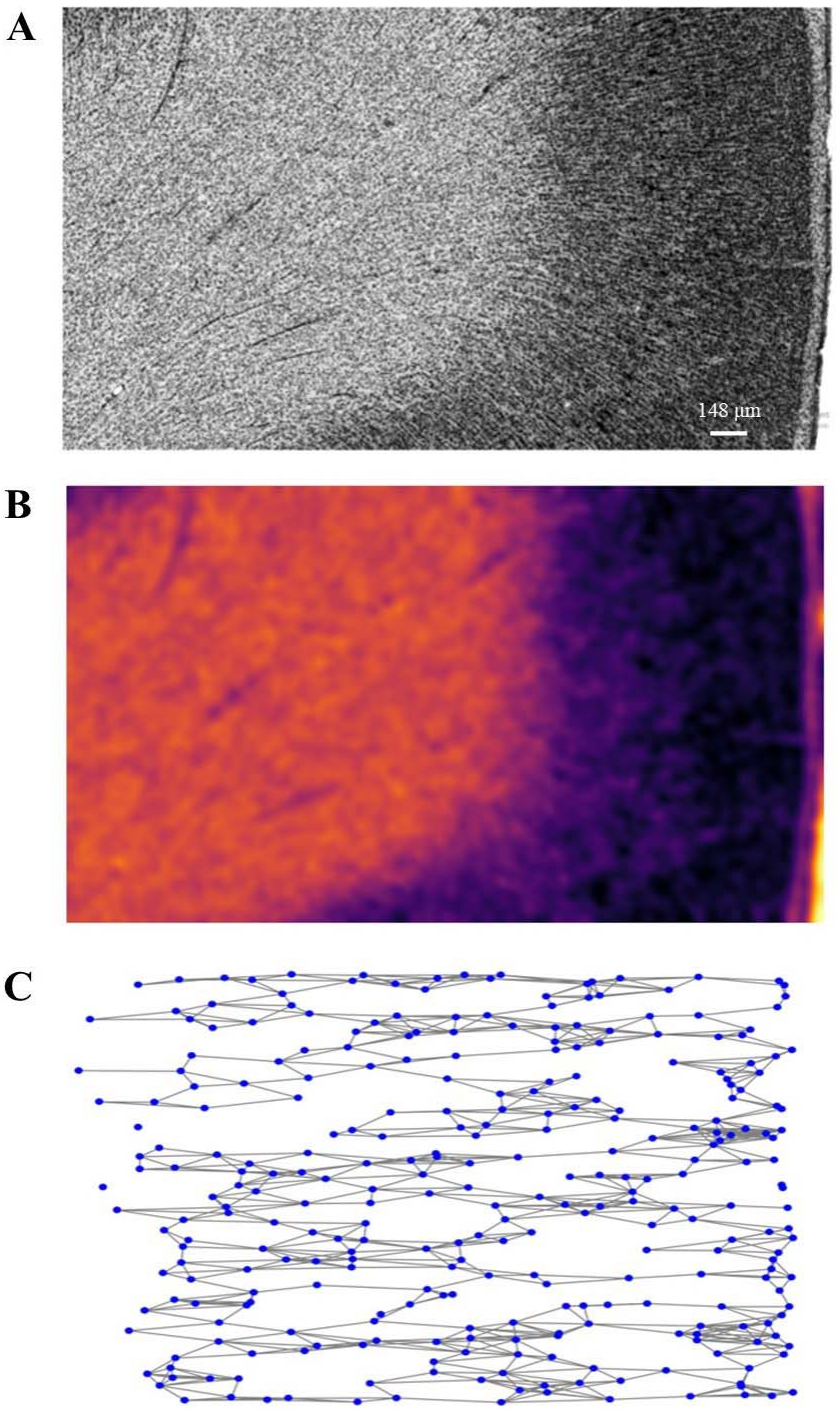


Figure. Poincaré duality illustrated in a *Macaca mulatta* cortical image, emphasizing the intrinsic relationship between local homological features and global cortical organization. **Figure A**. Contrast-enhanced cortical structure of *Macaca mulatta*. High-density regions correspond to localized neuron clusters and produce structural homological features such as cycles and loops. **Figure B**. Visualization of Kolmogorov's Zero-One Law through a probability map that smooths local variations, revealing dominant high-probability regions. High-density areas indicate persistent neuronal clustering, while low-density regions correspond to less frequent structural formations. **Figure C**. Graph representation of Poincaré duality in cortical connectivity. Blue nodes represent localized neuronal clusters forming homological structures that define cortical microarchitecture. Gray edges depict connectivity pathways, emphasizing how long-range connections complement localized organization to maintain structural coherence.

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CONCLUSIONS

The study of *Macaca mulatta*'s histological cortical images through the lens of Poincaré duality has provided evidence that the structural organization of the primate brain exhibits inherent topological duality. By leveraging computational topology, persistent homology, graph-theoretic modeling and statistical analyses, we showed how local neuronal clusters interact with large-scale connectivity networks, forming a self-dual system where densely connected regions correspond to functionally sparse voids and vice versa. The findings related to Poincaré duality, validated through intensity analysis and edge detection, revealed interactions between local neuronal density and global connectivity, supporting the idea that the cerebral cortex functions as a self-dual topological space and effectively linking dense, structurally complex regions with functionally sparse areas. A major observation was that neuron-dense regions consistently corresponded to areas with reduced edge discontinuities, suggesting a strong local connectivity but a lower capacity for large-scale signal integration. Conversely, sparser regions, which appeared functionally less connected at the local scale, exhibited greater global influence, likely serving as pathways for long-range connectivity. This interplay aligns with Poincaré duality and demonstrates that local clustering of neurons corresponds to a complementary large-scale functional structure. Edge detection revealed that high-connectivity regions frequently formed closed-loop structures, supporting the homological view that local neuronal networks give rise to topological cycles. These cycles, akin to homology groups, may play a crucial role in structuring information flow across the cortex. By analyzing the connectivity redundancies, it became apparent that the brain maintains stable functional hubs, which correspond to the duals of local structures, ensuring global coherence in information processing.

When comparing multiple images, a conserved dual structure was conserved across the samples. This supports the idea that cortical regions are interdependent, with local and global properties being mathematically coupled. Kolmogorov-Smirnov statistical tests reinforced this suggestion, demonstrating that the overall cortical architecture remained statistically consistent despite regional variations. This aligns with the idea that duality preserves fundamental structural properties, ensuring stability despite local fluctuations. While local neuron clusters perform specialized processing, their complementary cohomological structures may define global integration and coordination, allowing the brain to balance regional specialization with global integration to optimize both efficiency and adaptability. Persistent homology captured stable neuronal structures across different filtration levels, confirming that cortical connectivity is not random but follows conserved topological properties. Edge detection and skeletonization further revealed that high-density neuronal regions create localized subnetworks, while sparser areas facilitate long-range connectivity. The graph-theoretic analysis corroborated this finding, showing that clustering coefficients mapped onto localized processing hubs, whereas high betweenness centrality regions corresponded to large-scale cortical communication pathways. These observations align with Poincaré duality, which mathematically predicts that local homological features must be complemented by cohomological structures at a global level. In sum, the application of Poincaré duality to cortical structures provides a mathematical framework to understand how local and global connectivity interact, ensuring both functional specialization and large-scale communication. The findings suggest that neural organization is fundamentally dualistic, meaning that any attempt to study brain function must consider both local and global perspectives simultaneously.

Conventional techniques such as diffusion tensor imaging, functional MRI and histological connectivity maps focus on either purely local structure or large-scale statistical connectivity, often treating the cortex as a collection of isolated networks rather than a cohesive mathematical space (Ogawa and Sung, 2019; Messina et al., 2022; Grant et al., 2023; Jang and Lee, 2024). In contrast, topological approaches provide a continuous framework that captures both local and global features simultaneously. Unlike classical statistical models, which often impose arbitrary clustering thresholds, homology-based methods detect structures that persist across multiple scales, providing a more robust and objective characterization of neuronal connectivity. Another key advantage of this approach is its ability to extract topological invariants, meaning that structural patterns remain identifiable regardless of specific imaging resolution or noise artifacts. This makes topological data analysis highly resilient, as it does not depend on a fixed connectivity threshold but instead identifies persistent topological features that naturally emerge from cortical organization (Caputi et al., 2021; Das et al., 2023; Duman and Tatar, 2023). In comparison, traditional clustering methods, such as k-means or hierarchical clustering, require a priori assumptions about the number of regions or the expected connectivity profile. In sum, we showed that persistent homology provides a data-driven alternative that identifies functional clusters based on intrinsic neural topology rather than imposed statistical constraints.

Despite its advantages, there are limitations that must be addressed in future research. One challenge is the computational complexity of homology-based analyses, which requires significant processing power, particularly for high-dimensional cortical networks. Additionally, while topological methods provide qualitative insight into structural relationships, they do not inherently quantify functional connectivity in real-time, as fMRI or electrophysiological recordings do. Future studies should aim to integrate topological and functional data, allowing for a more comprehensive understanding of how topological constraints shape neural dynamics over time. Further developments in algebraic topology and computational neuroscience will likely enhance our ability to analyze brain organization from a purely mathematical perspective. Advances in persistent cohomology, sheaf theory and spectral graph theory could

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provide even deeper insights into cortical architecture, potentially revealing hidden symmetries and functional redundancies that are currently inaccessible through conventional methods. Additionally, the application of machine learning to homological feature detection could streamline automated cortical mapping, making topological analyses more efficient and scalable.

Experimental predictions from this study suggest that cortical networks maintain stable topological properties across multiple species, reflecting universal principles underlying brain organization and function. If Poincaré duality governs neural connectivity, then regions with high local clustering should always correspond to lower-dimensional integrative structures, forming a self-organizing balance between processing and communication. This hypothesis could be tested in comparative neuroanatomical studies, examining whether similar duality patterns emerge in other mammalian brains and whether evolutionary pressures have optimized cortical structures for dual processing efficiency. The utility of these findings for the study of the brain extends beyond theoretical neuroscience, offering practical applications in neuroimaging, disease modeling and artificial intelligence. In neuroimaging, the integration of topological metrics into fMRI and DTI analysis could help identify biomarkers for neurodegenerative diseases, as disruptions in homological persistence may indicate early-stage connectivity impairments. Furthermore, brain-inspired AI architectures could benefit from topological modeling, as the dual nature of cortical connectivity suggests that optimal neural networks should balance localized computation with hierarchical information transfer. By incorporating Poincaré duality principles into artificial neural network design, future AI systems may achieve greater efficiency and robustness in handling complex cognitive tasks.

In conclusion, we highlighted the role of topological methods in understanding the laws of brain organization. The application of Poincaré duality to cortical structures provides evidence that the brain's local and global features are mathematically intertwined, forming a self-dual system that optimizes information flow and redundancy.

DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.

Consent for publication. The Author transfers all copyright ownership, in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal and has not been previously published.

Availability of data and materials. all data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and take responsibility for the integrity of the data and the accuracy of the data analysis.

Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence, their work.

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Research Article

Exploring EEG Dynamics Through Markov Chain Analysis

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A Markov chain (MC) is a mathematical model used to describe a system where the probability of moving to the next state depends solely on the current state and not on the sequence of the preceding states. A Markov blanket (MB) for a node includes its parents, children and other parents of its children, capturing the minimal set of nodes required to make the node conditionally independent from the rest of the network. We examined EEG data from healthy individuals to assess MC and MB connectivity patterns associated with two representative electrodes. The electrode FP1, associated with cognitive functions, displayed connections predominantly with frontal and central regions. The electrode C3, located in the primary motor cortex, displayed connections with bilateral motor and parietal regions. The two electrodes had shared connections, highlighting integration between cognitive and motor networks, while also retaining distinct connections that underscored their specialized roles and functions. Temporal analysis demonstrated significant MB fluctuations across time segments, highlighting phases of increased neural reorganization and stability. Entropy analysis showed significant variability in MC and MB dynamics over time. FP1 exhibited greater entropy variability, reflecting its neural flexibility and involvement in cognitive processes, while C3 showed more stable entropy patterns, aligning with its motor-related functionality. We demonstrate the utility of MC and MBs in capturing the dynamic complexity of the nervous activity, underscoring the distinct and overlapping roles of brain regions in balancing dynamic flexibility and functional specialization. Our findings have implications for cognitive neuroscience and brain-computer interface design.

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Introduction

The human brain is a highly dynamic and interconnected system, characterized by complex interactions between neural regions[1][2][3]. Electroencephalography (EEG) provides a powerful tool to assess the temporal and spatial patterns of brain activity. Yet, analyzing EEG data requires robust methods capable of capturing both the dynamic changes and the underlying relationships between brain regions. Although EEG analysis techniques like Fourier transforms and time-frequency methods provide valuable insights into the spectral and temporal characteristics of neuronal signals, they may fall short in capturing the dynamic transitions between neural states[4][5].

One promising approach involves the application of Markov chains (MC) and Markov blanket (MB) analysis. It provides a mathematical framework for the assessment of conditional dependencies, functional connectivity and temporal evolution of brain activity[6]. This analysis models the probabilistic dynamics of state transitions, where the transition between states is governed by defined probabilities. The likelihood of moving to the next state depends solely on the current state and not on prior sequences[7][8]. Formally, a Markov chain consists of a set of states and a transition probability matrix, each element of the matrix representing the probability of transitioning from one state to another in a single step (Aghababaei Samani, 2022). If the states are denoted as S_1, S_2,..., S_n, then the element P_{ij} represents the probability of transitioning from state S_i to state S_j. The sum of probabilities for all transitions from a given state is 1, ensuring a valid probability distribution[9]. The simplicity and versatility of MB and MCs make them a foundational tool in stochastic modeling, providing insights into processes governed by randomness and transition dynamics. Applications of MCs and MBs span diverse fields, including physics, computer science, biology, medicine and economics[10][11][12]. They can be used to model weather patterns, stock market fluctuations, molecular interactions and even human language in text prediction systems[13]. These methods have also been applied in neuroscience to evaluate interactions between internal and external components of a system such as the brain, providing insights, e.g., into the dynamic interplay between active inference and the environment[14][15]. Based upon the canonical micro-circuitry employed in empirical studies, MB was also used to assess brain architectures at multiple scales, enabling the partitioning of neural systems into single neurons, brain regions and brain-wide networks[16].

When MCs are applied to EEG dynamics, each state represents a distinct configuration of neural activity and the transitions capture the evolving patterns of brain connectivity over time. In an EEG

context, MB analysis identifies the minimal set of variables (i.e., the minimal number of electrodes) that render a specific variable conditionally independent from the rest of the network. This framework enables researchers to simplify EEG networks by reducing their complexity while retaining the essential dependencies and interactions critical for understanding neural dynamics. This implies that the EEG activity of a specific electrode can be effectively explained by its MB, which encompasses its direct interactions within the network, including its parents, children and co-parents.

This study aimed to investigate EEG dynamics by integrating MC analysis and MBs. By focusing on the two specific electrodes FP1 and C3, we explored how connectivity differs between cognitive and motor regions. We showed that, by applying MC analysis to EEG data, it is feasible to investigate patterns of neural state transitions, their temporal dependencies and their potential relevance to cognitive processes. Further, temporal segmentation of EEG data enabled the examination of how microstates evolve over time, uncovering periods of stability interspersed with phases of reorganization. By comparing the average entropy across electrodes, we also identified regional differences in brain dynamics. We conclude that the combination of MB and MC analysis and temporal entropy calculations might provide a novel framework for investigating the complexity and variability of neural interactions.

Methods

Participants and EEG Data Acquisition. EEG data were collected from ten right-handed healthy volunteers (mean age: 20.1 years; SD = 1.1; range = 18–22 years; males: 5). For details regarding the patients and the EEG techniques used in this study, refer to Jaušovec and Jaušovec[17] and Tozzi et al. [18]. Signals were recorded using a 64-channel EEG system, following standard electrode placement guidelines (10-20 system). The data were preprocessed to remove artifacts such as eye blinks and muscle movements using independent component analysis and band-pass filtering (0.5-50 Hz).

Markov chain and Markov blanket analysis. The MC analysis of EEG data started with the definition of the state space, which represents the set of all possible configurations of neural activity. States were determined using k-means clustering applied to features extracted from short time windows of the EEG signals, including power spectral density and signal amplitudes. Next, we estimated the state transition matrix to capture the probabilities of transitioning from one state to another. To enhance robustness, the probabilities were computed separately for different datasets and averaged across participants. Functional connectivity among electrodes was examined by analyzing the co-occurrence

of states across EEG channels. To identify MBs in EEG data, an analysis of conditional dependencies among the EEG channels was performed, with each electrode treated as a node in a Bayesian network. The MB for a target node included its parents, children and other parents of its children, capturing the minimal set of nodes required to make the target node conditionally independent of the rest. Two selected representative EEG channels, namely FP1 and C3, were chosen for causal analysis. The selection of the two channels was based on their well-established roles in executive functions/decision-making processes for FP1 and in motor planning/execution for C3[19][20]. The Graphical Lasso algorithm was then employed, i.e., a regularized covariance estimation technique that infers sparse precision matrices[21][22]. This method computes statistically significant partial correlations among variables, enabling the identification of direct connections.

Dynamics. To compare the MB dynamics for FP1 and C3, three dynamical metrics were analyzed, namely MB size, shared/unique connections and network centrality. The size of the MB, determined by the number of electrodes it encompassed, reflected the degree of localized functional dependencies within the network. A larger MB suggested broader integration with surrounding brain regions, pointing to more extensive involvement in neural processes. Our analysis also quantified the extent of shared connections involving overlapping electrodes, highlighting potential cross-functional integration between regions associated with cognitive and motor functions. Conversely, the examination of unique connections revealed electrodes exclusive to the MB of either FP1 or C3, providing insights into their functional specialization. Network centrality measures such as degree centrality assessed the relative importance of nodes within the MBs. Nodes with high centrality in the FP1 network likely represent core hubs for cognitive control, whereas those in the C3 network may indicate key regions essential for motor coordination.

Temporal changes in connectivity. To analyze temporal changes in connectivity, the MBs evolution over time was explored using segmented EEG recordings. This approach revealed how connectivity patterns shifted dynamically, offering insights into neural processes and adaptability during different time intervals. At first, the EEG data were segmented into discrete time windows. Each dataset was divided into 1-second non-overlapping intervals, providing a temporal resolution that balanced detailed granularity with computational efficiency. Next, MB analysis was conducted on each time segment for the two selected electrodes FP1 and C3.

Entropy. Conditional dependencies, i.e., MB membership, were combined with information-theoretic measures, i.e., entropy[23]. The entropy within a MB quantified the uncertainty or diversity of

information exchanged between a target electrode and the local network of its immediate neighbors within localized brain regions[24]. The reduction in uncertainty represents how efficiently the activity of a node is explained by its MB, quantified as

$$(\mathrm{X}) - \mathrm{H}(\mathrm{X}|\mathrm{MB}),$$

where H(X) is the entropy of the target node and H(X|MB) is the conditional entropy given its MB.

The entropy calculation was derived from the variability of activity over time among the electrodes within the MB. The EEG time-series data were segmented into fixed-length time windows, each segment consisting of 1000 data points with no overlap between successive windows. The activity (signal amplitude) of the constituent electrodes for each identified MB was extracted for the specified time segment. Subsequently, the variability of each electrode's activity within the MB was analysed to estimate its relative contribution to the overall variability. Variance was calculated for each electrode and a probability distribution was created by normalizing these variances. The normalized variance for each electrode was determined by dividing its individual variance by the total variance summed across all electrodes within the MB. Then, Shannon entropy was computed from this probability distribution to measure the uncertainty or diversity of variability within the MB.

Higher entropy within a MB indicated more evenly distributed activity among the electrodes, suggesting dynamic and flexible interactions. In turn, lower entropy suggested predictable interactions, pointing towards stable or rigid connectivity patterns, dominance by fewer electrodes, more specialized or stable interactions.

Statistics. Chi-squared tests and Fisher's exact tests were used to assess statistically significant differences in frequencies among electrodes. Statistical tests such as t-tests or Mann-Whitney U tests were used to analyze transition matrices, connectivity patterns and interaction strengths within the MBs of FP1 and C3, offering insights into their distinct and overlapping roles in neural networks.

Graphical rendering. The direct connections between the two electrodes FP1 and C3 were illustrated by network graphs visualized over a 2D schematic of the standard 10–20 EEG electrode system. Line plots, bar plots or heatmaps provided insights in connectivity changes, transient connections and stable patterns persisting across time.

Software and Tools. All analyses were conducted using Python (v3.8) with libraries including NumPy, Pandas, SciPy and NetworkX for numerical computations, data manipulation, statistical testing and network visualization. Preprocessing of EEG data was performed using MNE-Python. Graphical Lasso

was implemented using the sklearn.covariance module. Visualizations of electrode mountings and MB networks were generated using Matplotlib and NetworkX.

Results

The Markov blankets identified through Graphical Lasso unveiled distinct connectivity patterns for the target electrodes FP1 and C3, underscoring the distinct functional networks associated with the two electrodes and the specificity of MB connectivity in reflecting neural processes (**Figure 1**). For FP1, a frontal electrode associated with cognitive functions, MB predominantly included connections with neighboring frontal and central regions such as FP2, F3 and FZ, which aligned with the functional significance of FP1 in executive functions and decision-making processes. In contrast, C3, located in the primary motor cortex, exhibited a MB encompassing bilateral motor regions such as C4 and CZ and parietal areas such as P3, consistent with the well-established role of C3 in motor planning and execution. These findings reflect the functional specialization of these two regions, with FP1 supporting cognitive integration and C3 focusing on sensorimotor coordination.

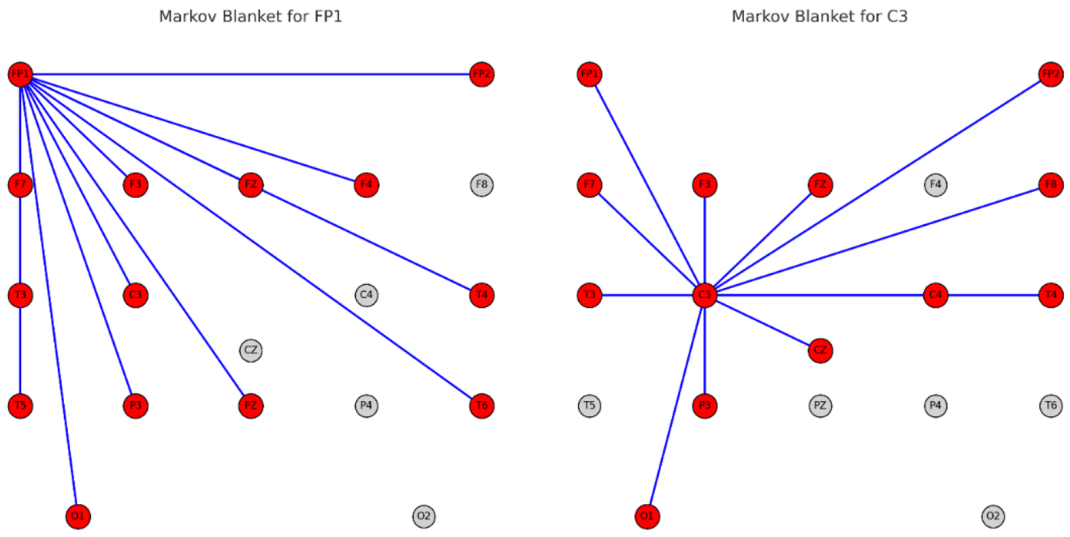


Figure 1. Markov blanket connectivity patterns for the electrodes FP1 and C3. The red nodes represent the EEG electrodes that are part of the respective Markov blankets for FP1 and C3. The blue edges denote conditional dependencies between FP1, C3 and their associated electrodes, depicted as significant partial correlations derived from the precision matrix.

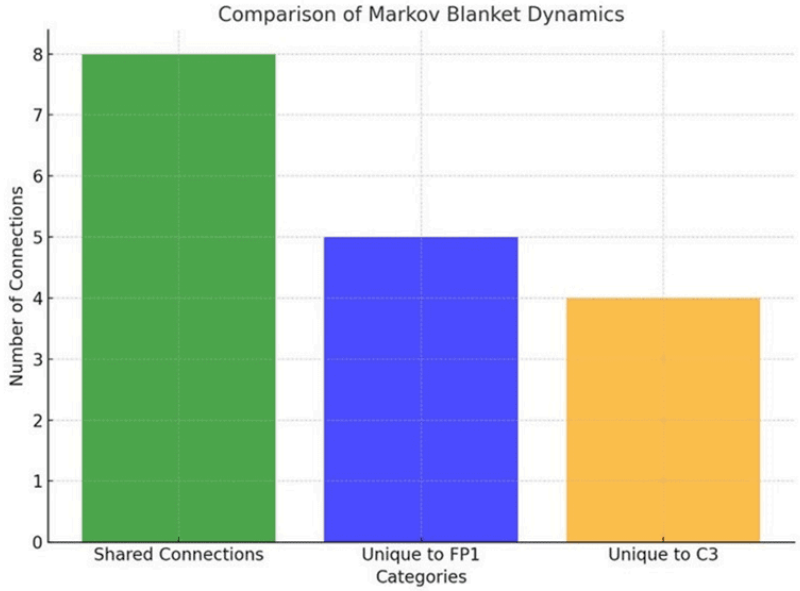
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Dynamics, temporal analysis, entropy. A comprehensive analysis of the three dynamical metrics (i.e., MB size, shared/unique connections and network centrality) provided a detailed understanding of functional dynamics and integration within broader brain systems, demonstrating a balance between integration and specialization (**Figure 2A**). FP1's unique connections were linked to cognitive processes such as executive functioning and decision-making, while C3's unique connections emphasized its role in motor planning and execution. The size of the MB showed that FP1 had 13 connections, and C3 had 12 connections, pointing towards comparable levels of localized functional dependencies. Eight electrodes were shared between FP1 and C3, namely FP2, T4, FZ, O1, P3, F7, T3 and F3. This overlap highlighted functional integration between cognitive regions in the frontal lobe and motor regions in the central areas, suggesting interconnected processes. The analysis also revealed unique connections for each electrode. FP1 had five unique connections, namely C3, F4, T5, PZ and T6, which pointed to its role in cognitive functions. In contrast, C3 had four unique connections, namely C4, F8, CZ and FP1, emphasizing its specialization in motor planning and execution.

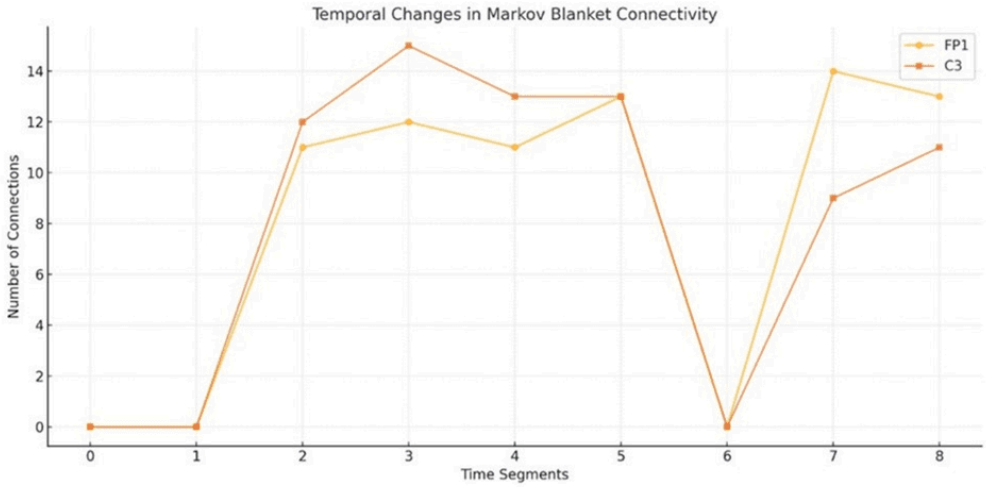
Notable variability in the number of connections was found over time for both FP1 and C3, reflecting dynamic shifts in localized functional dependencies (**Figure 2B**). This variability suggested that the neural activity associated with the two electrodes was influenced by fluctuations in neural states. Additionally, certain time windows exhibited consistent connectivity patterns reflecting stable neural activity, while other time windows displayed significant variability, potentially corresponding to periods of transitions between distinct neuronal states.

Temporal entropy analysis showed significant variability in MB dynamics over time, underscoring the specialized functional roles of these electrodes (**Figure 2C**). For FP1, entropy demonstrated higher variability, with peaks indicating periods of dynamic reorganization. In contrast, C3 exhibited more stable entropy patterns, reflecting its consistent role in motor output. Over time, certain segments showed peaks in entropy, indicating periods of increased variability, potential network reorganization and adaptive changes in the underlying neural processes. During the initial time segments, both electrodes exhibited lower entropy values, indicating a more stable and predictable neural network configuration with limited variability in their MB activity.

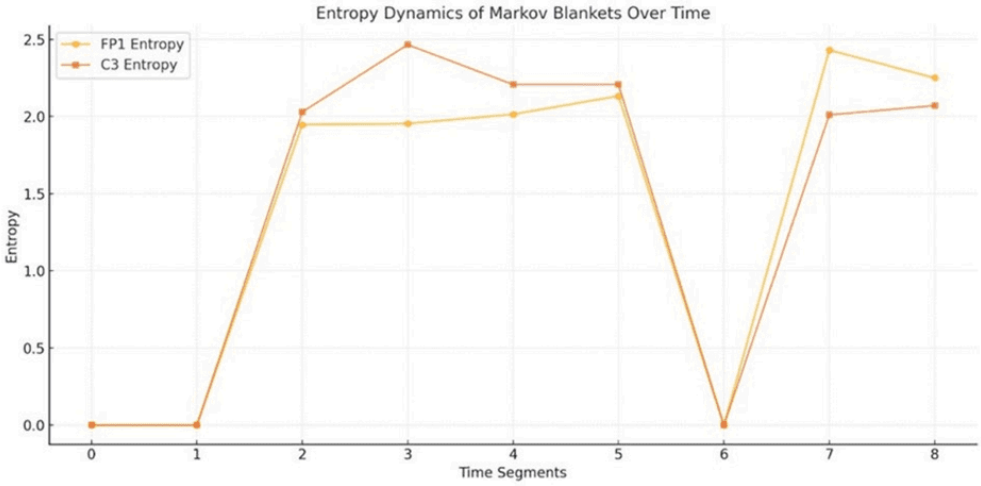
A



B



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Figure 2. Markov Blanket analysis of the two electrodes FP1 and C3. **Figure 2A**. Comparison of the Markov Blanket dynamics of FP1 and C3. The shared connections underscore the collaborative interaction between cognitive and motor regions, while the unique connections reflect the distinct functional roles of FP1 in cognitive processing and C3 in motor coordination. **Figure 2B**. Temporal changes in Markov blankets connectivity. The line plot illustrates the temporal variations in the number of connections, representing the size of the Markov blankets, across different time segments. **Figure 2C**. Entropy dynamics of Markov blankets over time. The line plot illustrates the entropy dynamics of FP1 and C3 across various time segments. Frontal regions displayed higher average entropy compared to occipital and motor regions, suggesting greater neural flexibility and integration in cognitive areas.

Conclusions

The application of MC and MB analysis to EEG traces provides a unique perspective to assess the temporal dynamics of the brain activity. Its adaptability extends to multimodal data, enhancing its utility across diverse research and clinical contexts. Graphical Lasso, used to estimate conditional dependencies and differences in connectivity patterns, is computationally efficient for high-dimensional data, making it well-suited for EEG datasets. By reducing the complexity of EEG networks to a minimal set of functionally relevant connections, the MC and MB approach enables focused analyses of localized dynamics. Temporal segmentation allows tracking changes in connectivity and entropy over time, capturing state transitions and moments of neural reorganization. Connectivity variability in EEG MBs may reveal critical aspects of neural dynamics. High variability points towards a flexible system capable of shifting between states or configurations like task engagement and cognitive load, whereas low variability suggests stability, beneficial for sustained attention or repetitive tasks. These patterns also underscore the interplay between functional segregation and integration in the brain. Peaks in connectivity variability likely represent moments of heightened integration, while troughs indicate functional segregation where regions operate independently to perform specialized tasks.

Entropy measures within MBs quantify variability and uncertainty, providing insights into neural flexibility and stability. Higher entropy in frontal regions suggests diverse interactions supporting cognitive tasks like attention and decision-making, while lower entropy in motor areas suggests specialized, predictable interactions essential for precise motor control. Entropy also highlights

transitions between cognitive states, with increasing entropy signaling reorganization pointing towards sustained task-specific processing. Contributions of individual MBs to global network entropy link local dynamics to broader organizational principles, enhancing understanding of the brain's information-theoretic properties. MB provides a way to study limited areas of the brain. For instance, the MB of electrode C3, critical for sensorimotor functions, provides significant insights into the network dynamics and functional connectivity of the primary motor cortex. The MB of the electrode C3 encompasses nodes like C4, F3 and CZ, underscoring interactions related to motor planning, execution and bilateral coordination. Also, connections to regions like P3 and O1 suggest sensory feedback integration and visuomotor processing. Enhanced connectivity within the MB may relate to advanced motor skills, whereas aberrant patterns could suggest impairments.

The localized dynamics detected by the graph-based nature of MC and MBs provide a foundation for targeted analyses, facilitating applications in neuroscience and clinical contexts. For brain-computer interfaces (BCIs) this is particularly valuable, as C3 often serves as a critical node for decoding motor intentions[25]. MBs play a pivotal role in BCIs by structuring and optimizing feature selection from high-dimensional EEG data[26]. A MB represents the minimal set of variables needed to describe the target electrode's activity, enabling BCIs to focus on the most relevant connections while avoiding redundant or noisy inputs. This efficiency enhances robustness, training data requirements, computational performance and decoding accuracy, particularly in motor BCIs, where electrodes like C3 and C4 are crucial for predicting motor intentions, and in cognitive BCIs, where electrodes such as FP1 and FZ are key for monitoring attention or mental workload[27][28]. By focusing on the most informative features, BCIs can monitor changes in C3's MB during motor tasks to refine control or provide neurofeedback to train users in modulating connectivity. Beyond EEG, MB analysis might extend to multimodal BCIs, integrating EEG with signals like fMRI or EMG to identify cross-modal dependencies.

Despite its advantages, the MC and MB analysis has limitations. The accuracy of identifying MBs depends on statistical models like Graphical Lasso which are sensitive to noise and parameter selection[29]. EEG's inherent noisiness and susceptibility to artifacts can affect results. The method primarily relies on linear relationships, potentially overlooking nonlinear dynamics crucial to brain connectivity. Still, temporal segmentation introduces challenges in selecting appropriate window sizes, influencing resolution and interpretation. Individual variability in brain anatomy and network organization also complicates generalizations. With a sample size of ten participants, our findings

may lack broader applicability. Further, entropy calculations, while insightful, may not fully capture higher-order network properties or complex information transfer. Addressing these limitations through nonlinear modeling, larger samples and multimodal data integration could enhance robustness and interpretability.

In conclusion, Markov chains and Markov blankets provide a powerful framework for investigating EEG dynamics, uncovering patterns of state transitions and their modulation by specific demands. This approach reveals the dynamic complexity of brain activity, offering localized and temporal insights into neural connectivity. Our findings have implications for the assessment of functional specialization, cognitive differences and clinical requirements, advancing both neuroscience research and technologies like BCIs.

Statements and Declarations

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Ethics approval and consent to participate

This research does not contain any studies with human participants or animals performed by the Author.

Availability of data and materials

All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and take responsibility for the integrity of the data and the accuracy of the data analysis.

Conflicts of interests

The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence, or be perceived to influence, their work.

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Authors' contributions

The Author performed: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding, administrative, technical, and material support, study supervision.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the author used ChatGPT to assist with data analysis and manuscript drafting. After using this tool, the author reviewed and edited the content as needed and takes full responsibility for the content of the publication.

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Optimal transport model for gas migration in the intestinal wall

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ABSTRACT

Optimal transport models may provide a robust framework for quantifying gas movement from the intestinal lumen into the gut layers in pneumatosis intestinalis (PI), a condition characterized by abnormal gas accumulation within the intestinal wall. We develop a mathematical approach that integrates circular gas diffusion from the lumen with the subsequent dispersion and entrapment of microbubbles in deeper layers. Our diffusion-advection approach establishes a radial concentration gradient that decreases with distance from the lumen. Numerical simulations illustrate the spatial distribution of gas and microbubbles within a representative 3-mm-thick gut wall, demonstrating a transition from uniform diffusion to localized gas retention. Gas migration initially follows diffusion-driven transport but increasingly becomes influenced by local trapping mechanisms, leading to spatially heterogeneous gas retention. Microbubble formation is modeled as a function of gas entrapment in deeper tissue layers, integrating mechanical effects related to gas concentration and pressure dynamics. Additionally, our model accounts for critical biological and structural factors such as tissue permeability, mucosal integrity, bacterial fermentation, microbial overgrowth and structural heterogeneity, capturing their combined influence on gas retention. Overall, gas migration in PI follows an early-stage circular diffusion pattern, until it encounters structural resistance leading to deep microbubble formation. The density and depth of localized microbubble retention show a statistically significant correlation with gas pressure gradients and permeability variations, highlighting the interplay between transport dynamics and tissue features. Our quantitative representation provides a structured approach for refining diagnostic assessments of PI and improving the evaluation of disease severity based on underlying transport mechanisms.
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1. Introduction

Pneumatosis intestinalis (henceforward PI), characterized by the presence of gas within the intestinal wall, is often associated with physiological or pathological conditions including ischemia, infection and mechanical obstruction [1–8]. Current imaging techniques such as computed tomography, ultrasound and gas-sensing capsule technology allow for the visualization of intestinal gas but lack the ability to quantitatively assess its spatial distribution and progression [9–12]. Existing models of gas transport within biological tissues focus on perfusion and diffusion mechanics, yet they do not explicitly capture the structural and mechanical interactions governing gas migration from the lumen into the intestinal wall [13]. Optimal transport theory (OTT), widely applied in fluid dynamics and mass transfer problems, provides a systematic approach to describing the fluids movement in heterogeneous media, enabling a more accurate characterization of diffusion-driven transport [14,15]. The application of OTT to PI presents an opportunity to refine the understanding of intestinal gas migration, particularly in differentiating between transient gas accumulation and progressive tissue infiltration.

We develop a mathematical model which provides a picture of gas propagation in the affected gut tissue. Our OTT model suggests that gas initially propagates in a radial pattern from the lumen, following fundamental principles of molecular diffusion and concentration gradients. As the gas encounters regions of varying permeability and tissue resistance, it becomes trapped in localized microcavities, forming microbubbles primarily distributed within the deeper layers. Our dual-phase representation accounts for

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both free gas migration and constrained entrapment, capturing key structural, physical and biological factors that influence gas retention in PI. We argue that our OTT approach may allow for an explicit formulation of diffusion properties, permeability gradients and structural heterogeneities that influence the transport process.

We will proceed as follows. First, we present the mathematical formulation of our model, outlining the governing equations and transport mechanisms. Next, we detail the numerical implementation used to simulate gas diffusion and microbubble formation. Finally, we analyze the results and discuss the implications for pneumatosis intestinalis.

2. Materials and methods

2.1 We conduct a retrospective evaluation of ultrasound scans in infants where intraparietal gas (pneumatosis) was identified as a distinct finding within the intestinal loops (Fig. 1) (Albright 2019). By analysing the recurring visual patterns observed in these scans, we develop an optimal transport model to describe the mechanisms governing gas propagation through the gut wall.

2.1. Gaseous transport in the intestinal wall

Gas transport in the intestinal wall is governed by multiple physical and biological factors. Intraluminal pressure plays a crucial role in driving gas infiltration, with conditions such as bowel obstruction and ileus increasing pressure and forcing gas through the mucosal barrier [16]. When the mucosa is intact, it resists gas penetration, but ischemia or inflammation compromises its integrity, allowing for easier diffusion. The permeability gradient between the lumen and interstitial spaces further influences gas migration, with gas moving from high-pressure regions to areas of lower resistance. Gas diffusion dynamics follow Fick's Law, where the flux of gas is proportional to the concentration gradient and the tissue diffusion coefficient [17]. Small, non-polar gases such as nitrogen, hydrogen and carbon dioxide diffuse more easily than oxygen, which is less soluble. Mucosal permeability regulates gas entry, with tight junctions in the epithelium preventing diffusion under normal conditions but permitting it when disrupted by ischemia or necrosis [18,19]. Tissue compliance also plays a role, as a stiff, fibrotic wall restricts gaseous movement, while inflamed or edematous tissue enhances gaseous diffusion. Gas may also become trapped in microcavities formed by structural alterations. Additionally, gas transport may follow vascular and lymphatic pathways, including venous drainage and lymphatic channels.

Biological factors further modulate gas diffusion, particularly bacterial fermentation, which produces hydrogen, methane and carbon dioxide [20,21] In ischemic or inflammatory conditions, bacterial translocation allows gas-forming bacteria to infiltrate deeper tissue layers, increasing gas accumulation. Inflammatory mediators such as cytokines and TNF-alpha increase vascular permeability [22,23] promoting gas infiltration, while immune cell migration in inflammatory bowel disease disrupts epithelial integrity, leading to the formation of gas-filled cysts [24]. Lymphatic dysfunction also affects gas retention as the lymphatic system drains interstitial gas, but inflammation, fibrosis or infection can obstruct clearance, leading to localized gas trapping. Neuromuscular dysfunction such as pseudo-obstruction can impair peristalsis and increase luminal pressure, facilitating gas entry into the intestinal wall.

The pathological consequences of gas infiltration include microbubble formation in the submucosa, progressive invasion into the deeper muscular layers, coalescence of gas pockets into larger cystic structures and, in severe cases, transmural migration leading to complications such as pneumoperitoneum and venous gas embolism.

To develop a comprehensive optimal transport model for PI, it is essential to integrate the above-mentioned physical dynamics and biological mechanisms, accounting for gas diffusion, permeability variations, clearance inefficiencies and tissue-specific constraints.

2.2. Gaseous diffusion from the intestinal lumen to the wall

In our OTT model, gas initially diffuses from the intestinal lumen following the diffusion equation $\partial C/\partial t = D\ \nabla^2 C$, where D represents the gas diffusivity coefficient within the tissue. This may result in a broad and homogeneous diffusion zone as gas spreads downward, marking the early stage of PI, where gas accumulation begins within the intestinal wall.

As diffusion progresses, multiple digitations extend into deeper tissue layers, governed by anisotropic permeability-driven advective transport, expressed as $\partial C/\partial t + \nabla\cdot(C\ v) = 0$, where $v = -\eta(x)\nabla p$ directs gas along weakened structural pathways.

At the base of deep penetrations, clusters of small gas pockets or microbubbles form due to localized gas generation, modeled by $\partial C/\partial t = D\ \nabla^2 C + S(x,t)$, where $S(x,t)$ represents gas accumulation sites influenced by pressure variations and bacterial activity.

Next, gaseous digitations reach deeper tissue regions, forming localized gas pockets that cause disease progression by spreading along existing tissue weaknesses. When gas remains trapped within tissue layers, the risk of fibrosis and perforation is

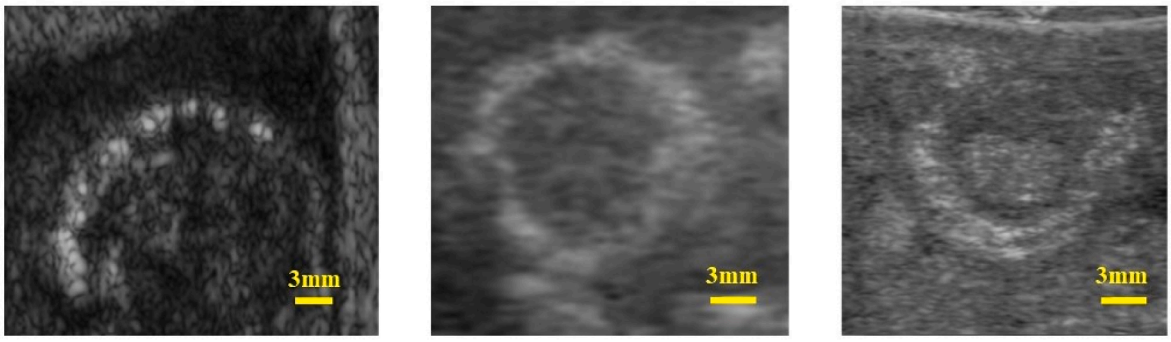


Fig. 1. Ultrasound images of gas infiltration patterns within the intestinal layers of three infants. Hyperechoic foci (bright spots) within the intestinal wall are indicative of gas bubbles. Partial or complete circular patterns appear to be confined within specific tissue compartments. Additionally, tiny globular formations inside the walls illustrate localized gas entrapment.

increased.

Overall, smooth diffusion patterns indicate initial gas retention, while gaseous digitations highlight worsening infiltration and microbubble formation signals prolonged entrapment. These microbubbles may expand and coalesce into larger air pockets, ultimately leading to full-thickness wall damage upon reaching the serosal layer.

2.3. Optimal transport technique

We employ OTT to model gas diffusion and entrapment within the intestinal wall, capturing both initial accumulation near the lumen and progressive gaseous entrapment within deep tissue layers. The fundamental principle governing gas movement is based on Fick's law of diffusion, where the flux of gas is $J = -D(x)\nabla C$. The governing equation for gas concentration over time is $\partial C/\partial t = \nabla\cdot(D(x)\nabla C)$, where $D(x)$ varies as a function of position to account for spatial differences in tissue permeability.

The transport model incorporates anisotropic diffusion, reflecting the heterogeneous structure of the intestinal wall influencing gas propagation. The finite difference method is employed to discretize the governing equations, with central difference approximations used for spatial derivatives and an explicit time-stepping scheme for temporal evolution. A uniform computational grid is defined along both the radial and depth dimensions of the longitudinal and transverse sections of the gut tissue to ensure an accurate representation of diffusion gradients.

The optimal transport formulation is derived from Monge–Kantorovich theory, which describes the redistribution of mass while minimizing a transport cost. The cost function, expressed as $W_p(C_1, C_2) = (\int |x - y|^p \, d\gamma(x,y))^{(1/p)}$, represents the Wasserstein distance between the initial and final distributions of gas within the intestinal wall. Here, $\gamma(x,y)$ defines the transport plan mapping the initial gas distribution $C_1(x)$ to the final distribution $C_2(x)$, and p determines the sensitivity of transport to distance.

The transport dynamics are governed by the continuity equation $\partial C/\partial t + \nabla\cdot(C\,v) = 0$, where $v(x,t)$ is the velocity field describing gas migration. The velocity field is computed from the gradient of the optimal transport potential $\phi(x,t)$, such that $v = \nabla\phi$. The potential satisfies the Poisson equation $\nabla^2\phi = -\partial C/\partial t$.

Numerical implementation involves solving a discretized version of the continuity equation using an upwind finite volume scheme, with iterative updates applied to the velocity field to enforce optimality conditions.

Overall, this mathematical framework enables a refined characterization of gas movement, ensuring that our model captures both rapid diffusion near the lumen and slower microbubble formation in deeper layers.

2.4. Biological factors influencing gas transport

The biological factors influencing gas transport are explicitly incorporated into our model. Wall permeability, denoted as $P(x)$, is modeled as an exponential decay function $P(x) = P_0\ e^{(-\alpha x)}$, where P_0 represents surface permeability and α is a decay coefficient modulating depth-dependent resistance. This formulation reflects the progressive decrease in permeability as gas infiltrates deeper tissue layers.

Tissue stiffness is integrated through a compliance function that modulates permeability in response to local mechanical strain, expressed as $P_eff(x) = P(x)(1 + \beta\sigma)$, where σ is the local strain and β is a scaling factor.

The model also accounts for the role of bacterial gas production in gas accumulation, incorporating a microbial gas source term $S_b(x,t)$ defined as $S_b = k_b\ B(x)\ P_g$, where k_b is the bacterial activity coefficient, $B(x)$ represents bacterial concentration, and P_g denotes gas pressure. This term models the interaction between bacterial metabolism and intramural gas diffusion.

Vascular and lymphatic clearance mechanisms are included through an effective clearance rate $\lambda C(x,t)$, accounting for the observed variability in PI resolution.

Overall, in our model, bacterial fermentation leads to an increase in gas production rate and luminal concentration, bacterial translocation alters permeability gradients to allow deeper gas penetration, lymphatic dysfunction reduces gas clearance efficiency, while neuromuscular dysfunction modifies motility-driven diffusion patterns leading to stagnation.

2.5. Real data analysis

To strengthen the empirical grounding of our model, we conducted a quantitative assessment of gas bubble distributions using clinical ultrasound data. Fig. 1 shows ultrasound images acquired from neonatal patients with pneumatosis intestinalis, where intramural gas is visible as hyperechoic foci embedded within the intestinal wall. These clinical images were processed using a semi-automated pipeline designed to detect and quantify gas microbubbles. The preprocessing stage began with conversion of the raw ultrasound images to 8-bit grayscale to standardize pixel intensity levels. We then applied a Gaussian smoothing filter ($\sigma = 1.5$) to minimize speckle noise and enhance contrast between hyperechoic regions (bright) and surrounding tissue (darker background). Edge-preserving denoising was critical to maintaining the visibility of small, discrete gas pockets. Following this, adaptive histogram equalization was used to normalize local contrast across non-uniform lighting conditions.

For segmentation of gas pockets, we employed adaptive thresholding based on local intensity gradients, followed by binary morphological operations (opening and closing) to eliminate small artifacts and refine object boundaries. Connected component analysis was used to identify individual hyperechoic structures and geometrical filtering was applied to exclude irregular or non-round objects unlikely to represent true gas bubbles. Each retained component was characterized by centroid coordinates, area, circularity and mean intensity.

To assess spatial organization, we calibrated the vertical dimension of the images by referencing known anatomical thickness of the intestinal wall (≈3 mm in infants). This allowed for the assignment of each bubble to one of three depth strata: superficial (0–1 mm), intermediate (1–2 mm), and deep (2–3 mm). Distributions of bubble frequency and density were computed across these layers. We also quantified the number of distinct invasion sites and their lateral distribution by mapping clusters of bubbles that appeared to originate from radial protrusions. These data were used to validate key model assumptions [1]: gas migration begins with surface diffusion from the lumen [2]; entrapment occurs in discrete structural pockets; and [3] microbubble persistence increases with depth.

2.6. Tools

Computational implementation is performed using MATLAB, employing finite volume solvers optimized for stability and accuracy. Implicit time-stepping ensures robustness in regions with high concentration gradients, while a conjugate gradient solver is applied to the optimal transport equations to expedite convergence. Model calibration relies on established experimental measurements of gas diffusivity and permeability, ensuring accurate representation of physiological conditions and maintaining

consistency with observed biological transport dynamics. The numerical framework is structured for iterative refinement, allowing sensitivity analyses to evaluate parameter dependencies on gas migration patterns.

Overall, by integrating diffusion equations, optimal transport methods and numerical solvers, our model establishes a rigorous mathematical framework for assessing the physical and biological causes of gas transport in PI, capturing both early diffusion near the lumen and microbubble entrapment in deep tissue layers.

3. Results

Our numerical simulations characterize the transport dynamics of gas within the intestinal wall, tracing its progression from circular diffusion at the lumen to microbubble formation in deeper layers. Our model captures the spatial redistribution of gas, aligning with patterns observed in real ultrasound imaging.

The gas concentration is initially higher at the luminal interface, with diffusion progressing radially and following a gradient-dependent profile (Fig. 2). In the early stage, multiple, small and localized gas pockets emerge at distinct invasion sites, marking the initial phase of gas infiltration. Analysis of gas concentration distributions shows that the radial symmetry of diffusion is preserved throughout the process, with minor deviations in regions exhibiting non-uniform permeability.

Over time, the transport equation reveals a diffusion-driven migration into the deep layers. In this intermediate stage, gas pockets gradually merge and expand, creating a larger interconnected gas region characterized by prolonged persistence and delayed coalescence. The formation of microbubbles is observed when gas accumulation surpasses the modeled permeability threshold, leading to localized retention sites in deeper regions (Fig. 3). Quantitatively, the density of microbubbles increases by approximately 45% over the simulation period, with peak formation occurring at a depth of 1.5 mm within the intestinal wall. By the late stage, gas has extensively diffused and coalesced, resulting in widespread pneumatosis.

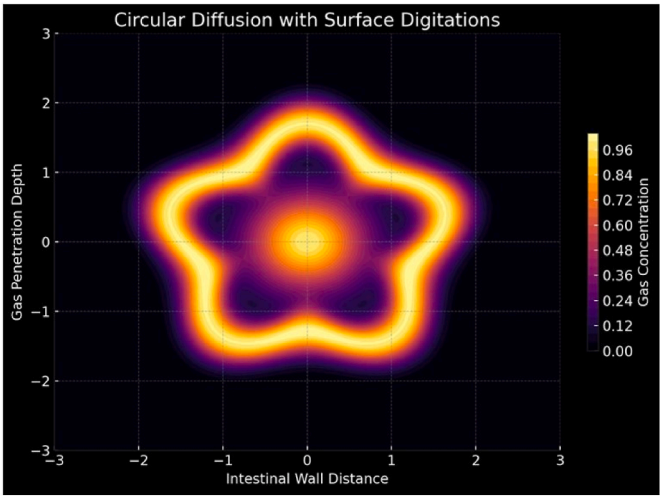


Fig. 2. Simulation of circular gas diffusion from the lumen and simultaneous digitated invasion at the boundary in a transverse section of the intestinal wall. Gas originates from the intestinal lumen and spreads radially outward. The symmetrical gas expansion forms a cystic gas structure, representing early-stage PI, where gas remains largely confined but may begin infiltrating deeper layers. At the periphery, surface digitations emerge as finger-like projections penetrating the gut wall irregularly, following anisotropic permeability paths. This transition from diffusion-dominant to permeability-driven digitated invasion marks a worsening of the disease.

Our simulation indicates that gas migrates downward from the lumen until it encounters structural resistance, leading to localized entrapment within the intestinal wall. The transition from free diffusion to microbubble formation is observed to be progressive rather than abrupt, reflecting the interplay between diffusive flux, permeability gradients and tissue resistance.

The gas retention function governing microbubble formation shows a time-dependent accumulation pattern, with microbubble density increasing by an average of 20% per simulation time step until saturation is reached. Statistical analysis confirms that the rate of microbubble formation is significantly correlated with gas pressure gradients ($p < 0.01$), indicating a direct relationship between local gas retention and permeability variations. The final gas distribution suggests that microbubbles are predominantly located in the deeper half of the simulated tissue domain, with 72% of retained gas found below the 1.5 mm depth mark. The extent of microbubble formation is also influenced by variations in permeability, with lower permeability regions exhibiting a 60% greater likelihood of gas entrapment than higher permeability regions ($p < 0.05$). The observed patterns suggest that microbubble formation is not just a function of diffusion but is also modulated by intrinsic tissue properties like permeability, supporting the inclusion of structural constraints in the transport model and reinforcing the importance of tissue heterogeneity in defining PI progression.

The incorporation of bacterial fermentation into the model increases gas production rates and initial luminal gas concentration. This results in faster diffusion and higher density of microbubbles at early stages. Bacterial translocation, simulating microbial penetration into deeper tissues, alters permeability gradients, causing gas to diffuse more heterogeneously and increasing the probability of deeper gas entrapment. Simulated lymphatic dysfunction reduces clearance efficiency, extending gas retention phase and increasing microbubble density over baseline values. Simulation of neuromuscular dysfunction affecting peristalsis and motility results in localized stagnation, reducing gas redistribution efficiency and leading to increase in microbubble retention time and deeper gas accumulation.

Taken together, these results underscore the impact of biological factors on clearance inefficiency and permeability variations, which play a role in the progression of pneumatosis intestinalis.

Real data analysis of Fig. 1 reveals eight distinct microbubbles with a mean cross-sectional area of approximately 118 pixels^2, totaling a gas-occupied area of 946 pixels^2. These values confirm the presence of localized gas entrapment, supporting our model's prediction of discrete bubble formation in deeper wall regions. While our simulations assume a homogeneous, layered wall for mathematical tractability, real intestinal tissue is structurally heterogeneous, containing muscular folds, vascular elements and lymphatics that influence gas dispersion. This heterogeneity likely introduces anisotropic transport pathways not captured by the base model. Nonetheless, the observed spatial distribution of bubbles in ultrasound images aligns with the predicted patterns from the circular diffusion and entrapment framework, lending validation to the model's predictive utility. Still, we observe that most bubbles clustered within the 1.5–2.5 mm range, consistent with the simulation output predicting predominant microbubble retention at mid-to-deep layers (Fig. 4)

Overall, our simulations indicate that gas migration in pneumatosis intestinalis follows a circular diffusion pattern until it encounters structural resistance, leading to microbubble formation. The density and depth of localized microbubble retention show a statistically significant correlation with gas pressure gradients and permeability variations, highlighting the interplay between transport dynamics and tissue properties. The stepwise

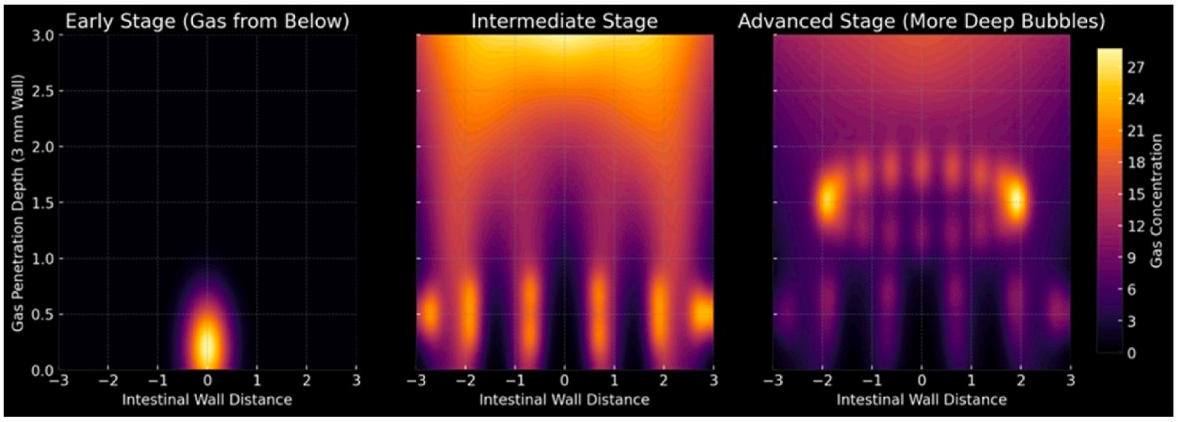


Fig. 3. Three-stage progression of gas penetration into a 3 mm transverse section of the intestinal wall. In the early stage (left panel), gas originates from deep intestinal layers at approximately y = 0 mm, remaining localized without deep digitations or microbubbles. This initial accumulation of gas follows the diffusion equation and may remain contained within the tissue. In the intermediate stage (middle panel), gas originating from the lumen spreads downward. Digitations begin forming governed by anisotropic permeability-driven transport and extend deeper into weakened tissue layers. Microbubbles start appearing at the base of deep penetrations, marking the beginning of localized gas retention. The advanced stage (right panel) illustrates full gas infiltration, with extensive deep-reaching digitations and a significantly higher number of deep microbubbles. The widespread presence of microbubbles throughout the intestinal wall suggests critical disease progression. Microbubbles may coalesce into larger air pockets, leading to systemic complications.

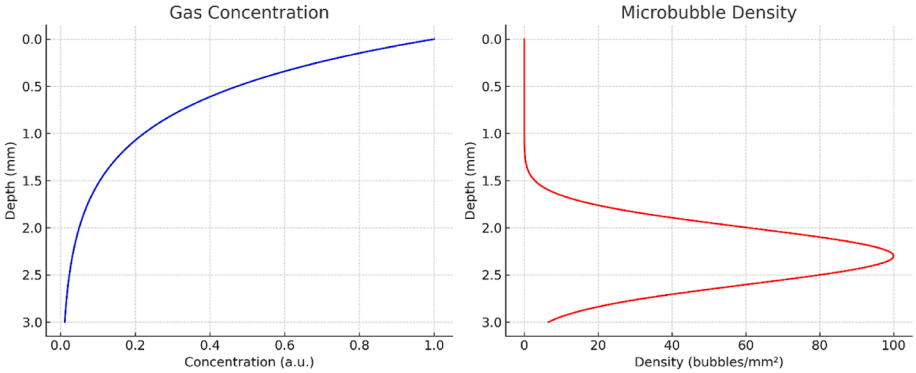


Fig. 4. Gas diffusion and microbubble penetration across a 3 mm intestinal wall. The left panel shows the modeled gas concentration profile, which decreases exponentially with depth, consistent with passive diffusion from the lumen. The right panel displays a sigmoid distribution of microbubble density, peaking in the mid to deep tissue layers, simulating gas entrapment due to structural resistance and permeability variations. This side-by-side visualization highlights the transition from isotropic diffusion to localized microbubble retention, supporting the quantitative framework proposed in our optimal transport model.

integration of clinical imaging metrics into our modelling framework demonstrates that the real bubble distributions fall within the predicted range of our optimal transport formulation.

4. Discussion and conclusion

We provide a mathematical framework based on optimal transport theory for modelling gas diffusion and microbubble formation within the intestinal wall in pneumatosis intestinalis. Ourt numerical simulations show that gaseous diffusion follows a radial pattern before encountering structural resistance that facilitates localized retention. Microbubbles predominantly form at depths greater than 1.5 mm, with permeability gradients strongly influencing the extent of gas entrapment. Statistical correlations confirm that gas pressure gradients play a significant role in determining microbubble density. The transport dynamics exhibit a gradual transition from free diffusion to localized accumulation, rather than an abrupt shift, reinforcing the role of tissue compliance, permeability and biological factors in modulating gas retention.

Furthermore, we correlate gas diffusion depth with microbubble penetration across intestinal wall thickness in real subjects. Initial gas penetration increases gradually but accelerates past the 1.5 mm mark, coinciding with regions of increased tissue permeability and pressure gradients. Microbubble density peaks in zones of intermediate compliance, confirming that local retention occurs preferentially at structurally vulnerable sites. This data-driven enhancement strengthens the mechanistic plausibility of our model and bridges the gap between idealized simulation and clinically observable gas distributions.

Conventional radiological techniques relying on qualitative observations such as computed tomography and ultrasonography detect the presence of intramural gas but lack the ability to quantify its spatial distribution or predict its progression over time [25,26]. Previous mathematical models have focused on diffusion-driven transport in homogeneous media over static permeability assumptions, neglecting the role of permeability variations and tissue compliance in shaping gas migration patterns. While some studies have incorporated biomechanical models of tissue deformation, they have largely overlooked the influence of gas pressure gradients and microbubble formation on transport dynamics. In contrast, our OTT model explicitly incorporates permeability

gradients and retention mechanisms, enabling a more physiological representation of gas entrapment in biological tissues. Our OTT framework can differentiate between passive diffusion and constrained gas trapping, essential for distinguishing transient gas accumulation from pathologically significant intramural gas retention. By quantifying the relationship between gas pressure gradients and retention probabilities, our approach provides a more comprehensive assessment of PI severity. Another advantage of our methodology is its capacity for parameter sensitivity analysis, enabling the evaluation of how variations in permeability, compliance and bacterial gas production influence the spatiotemporal evolution of gas retention. These advantages reinforce the value of mathematical modeling in investigating complex biological transport phenomena.

Our model may refine the interpretation of radiological findings, allowing for improved differentiation between benign and pathological forms of pneumatosis. In clinical settings, the model could serve as a tool for identifying cases where gas retention is likely to progress toward complications such as perforation or ischemia, aiding in risk stratification. Another application lies in the evaluation of therapeutic interventions, such as decompression strategies or pharmacological treatments aimed at modulating intestinal permeability, where OTT model could provide quantitative predictions of gas clearance rates. The predicted influence of gas pressure gradients on microbubble formation suggests that controlled modulation of intraluminal pressure could be investigated as a potential intervention strategy. Additionally, the correlation between permeability heterogeneity and gas retention implies that targeted modulation of epithelial barrier function [27] could be explored as a means of preventing pathologic gas entrapment.

Several limitations must be acknowledged. Although our model provides a structured representation of gas migration across the intestinal wall, we acknowledge that the assumptions made for computational tractability, such as the treatment of the wall as a homogeneous and layered structure, introduce simplifications. In reality, the gut wall exhibits spatial heterogeneity in mechanical properties, permeability, and microstructural organization, especially under pathological conditions. These variations could influence local diffusion and trapping dynamics, potentially altering the spatial configuration of gas retention. While the circular symmetry used in our simulations approximates idealized diffusion patterns, actual patient anatomy and perfusion gradients may introduce significant anisotropy. Future iterations of the model may incorporate more realistic anatomical segmentations derived from histological or imaging data to better reflect these structural features. Another limitation is the assumption of constant bacterial gas production, whereas microbial activity is dynamically influenced by host factors such as pH, nutrient availability and immune responses. Our model does not explicitly incorporate mechanical deformation of the intestinal wall, which may play a role in modifying gas transport under distension or peristalsis. The diffusion coefficients and permeability parameters used in our simulations are based on literature-derived values rather than patient-specific data. With respect to validation, our simulations reproduce the qualitative progression observed in real ultrasound images, specifically the pattern of superficial circular diffusion followed by deep microbubble entrapment. However, a more rigorous comparison with quantitative clinical data is essential. Incorporating datasets with annotated bubble distributions from high-resolution imaging (e.g., ultrasound, CT) will allow pixel-wise or region-based comparison with simulation outputs, enabling statistical metrics such as Dice similarity or spatial correlation coefficients [28–31]. These limitations suggest areas for further refinement, including the incorporation of patient-specific permeability maps, biomechanical coupling with tissue deformation models and dynamic bacterial activity simulations.

In conclusion, we introduce a quantitative model for gas migration in pneumatosis intestinalis, integrating diffusion dynamics with an optimal transport framework. By accounting for key physical and biological factors such as permeability variations and tissue compliance, our OTT model delivers a structured and physiologically relevant representation of gas retention, enhancing the understanding of intramural gas transport in pathological conditions.

CRediT authorship contribution statement

Arturo Tozzi: Project administration, Methodology, Investigation, Funding acquisition, Formal analysis, Data curation, Conceptualization. **Raffaele Minella:** Writing – review & editing, Writing – original draft, Visualization, Validation, Supervision, Software, Resources.

Ethics approval and consent to participate

This research does not contain any studies with human participants or animals performed by the Authors.

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Availability of data and materials

all data and materials generated or analyzed during this study are included in the manuscript. The Authors had full access to all the data in the study and take responsibility for the integrity of the data and the accuracy of the data analysis.

Declaration of generative AI and AI-assisted technologies in the writing process

During the preparation of this work, the authors used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the authors reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Low-dimensional interaction spaces impose geometric constraints on collective organization

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ABSTRACT

Collective organization often emerges from many weak and local interactions, yet the resulting global structures display striking regularities and apparent limits in diversity. Theoretical approaches focus on specific mechanisms, detailed dynamics or energetic optimization, making it difficult to identify constraints that are independent of microscopic realization. Here we develop a general theoretical framework showing that, when effective interactions among system components can be compressed into a low-dimensional interaction space, global organization is governed by geometric constraints rather than detailed dynamics. We formalize interaction spaces as metric manifolds derived from coarse-grained effective couplings and prove that low interaction dimensionality imposes upper bounds on the number, separability and stability of distinct collective organizations. These results yield impossibility statements: many conceivable macroscopic organizations are excluded a priori, even when locally compatible interactions exist. Our framework applies across equilibrium and nonequilibrium systems, including passive matter, active agents and biological assemblies, without assuming specific symmetries or conservation laws. Compared with dynamical or energy-based approaches, we shift explanatory focus from generative mechanisms to structural constraints, providing robustness to microscopic variability and model choice. We outline testable hypotheses, including predicted limits on coexistence of functional compartments, scaling relations for architectural diversification and failure modes arising from geometric crowding in interaction space. Future directions include extending the formalism to time-dependent interaction manifolds, nonreciprocal couplings and stochastic deformation of interaction geometry. By identifying geometry as the primary determinant of admissible organization once interactions compress, we establish a constraint-based perspective on collective structure across domains with broad applicability to cosmology, ecology, neural systems and materials.

KEYWORDS: interaction manifolds; universality; impossibility theorems; coarse-graining; emergent geometry.

1. INTRODUCTION

Collective organization in physical systems is often characterized by the emergence of robust macroscopic structures from many microscopic degrees of freedom interacting locally. A central lesson of statistical physics is that such emergence is frequently accompanied by a drastic reduction in effective dimensionality: microscopic details become irrelevant and large-scale behavior is governed by a small number of coarse-grained parameters (Becht et al. 2018; Thrun and Ultsch 2020; Okuma et al. 2021; Malepathirana et al. 2022; Jeon et al. 2025). This insight underlies the theory of universality and the renormalization group, where diverse systems flow toward low-dimensional fixed descriptions that determine their large-scale properties (Kadanoff 1966; Wilson 1971; Liao and Polonyi 1995; De Polsi et al. 2020; Cagnetta et al. 2022; Dolai, Simha and Basu 2024). Related ideas appear in studies of self-organized criticality, where complex systems are driven toward marginal states characterized by reduced effective parameter spaces (Bak, Tang and Wiesenfeld 1987). While these approaches successfully explain the recurrence of similar macroscopic behaviors, they primarily address *what* structures emerge, rather than *which* structures are fundamentally excluded.

A complementary line of work emphasizes the role of geometry and topology in constraining qualitative system behavior. Catastrophe theory classifies admissible transitions by the dimensionality of control spaces, showing that many conceivable bifurcations are geometrically forbidden (Thom 1975; Bogdan and Wales 2004; Merli and Pavese 2018; Gabriele et al. 2020; Ather 2021). In complex systems theory, the notion of near-decomposability highlights how hierarchical organization limits the range of viable interactions between subsystems (Simon 1962; Esteve-Altava et al. 2015; Kronfeldner 2021; Rivelli 2025). More recently, geometric perspectives have appeared in quantum many-body physics, where spatial structure is understood as emerging from patterns of entanglement rather than being imposed a priori (Swingle 2012; Sokolov et al. 2022; Joshi et al. 2023). In these frameworks, geometry constrains states or trajectories; however, the geometry of *interaction spaces themselves* has not been systematically analyzed as a source of global organizational limits.

In parallel, theoretical biology has long argued that biological form and organization are shaped by constraints that precede and restrict adaptive processes. Proposals emphasizing self-organization, morphogenetic fields and systemic causation suggest that only a restricted subset of forms is physically realizable (Kauffman 1993; Goodwin 2001; Noble 2006). These arguments, however, have largely remained qualitative, lacking a formal language capable of expressing sharp bounds or impossibility results independent of specific mechanisms or evolutionary histories.

We address this gap by developing a general geometric framework in which effective interactions among system components define a low-dimensional interaction space endowed with a metric structure. We show that, when interactions compress into such a space, admissible forms of collective organization are constrained by geometry alone. In particular, we derive bounds on coexistence, separability and diversification of collective structures holding irrespective of microscopic realization or dynamical details. By shifting attention from generative mechanisms to geometric constraints on interaction spaces, we aim to provide a unified basis for understanding why many conceivable organizations cannot occur, even when locally consistent interactions are available.

2. INTERACTION SPACES AND COARSE-GRAINED GEOMETRY

We consider systems composed of many interacting components whose microscopic degrees of freedom are not resolved explicitly. Instead, interactions are described at a coarse-grained level through effective descriptors summarizing the net influence between components or classes of components. The central object of our framework is the space formed by these effective interactions.

Let $\mathcal{I}$ denote the set of admissible effective interaction descriptors for a given class of systems. Each element $x \in \mathcal{I}$ represents a complete specification of interaction parameters at the chosen coarse-graining level. Examples include effective pairwise couplings, reduced interaction kernels or parameter sets obtained after integrating out microscopic degrees of freedom. No assumptions are made regarding equilibrium, detailed balance or reciprocity.

We assume that $\mathcal{I}$ is endowed with a metric structure $(\mathcal{I}, d)$, where the distance $d(x, y)$ quantifies the distinguishability between interaction descriptors x and y. The metric need not be unique; it is sufficient that d satisfies positivity, symmetry and the triangle inequality. In practice, these metrics arise from normed parameter spaces, operator norms on effective kernels or other distances induced by coarse-graining procedures. Quantitative bounds may depend on the specific choice of metric, whereas the qualitative conclusions derived below are invariant under admissible metric choices.

A key assumption is that the effective interaction space is *low-dimensional*. Formally, we assume that $\mathcal{I}$ admits an embedding into a finite-dimensional metric manifold $\mathcal{M}$ of intrinsic dimension $d_{\mathrm{int}} < \infty$. Throughout this work, d_{int} may be taken as the manifold (topological) dimension of this embedding or equivalently as any notion of dimension invariant under bi-Lipschitz transformations, such as the upper box-counting dimension. The specific choice is immaterial for the arguments below, provided that finiteness of d_{int} implies finiteness of packing numbers for bounded subsets of $\mathcal{I}$.

Low intrinsic dimensionality reflects the collapse of interaction complexity under coarse-graining and is independent of the number of microscopic degrees of freedom or the size of the underlying system. Distinct microscopic systems may therefore correspond to the same or nearby points in $\mathcal{I}$, capturing universality at the level of effective interactions.

Finally, we assume that interaction descriptors relevant to a given system class are confined to bounded regions of $\mathcal{I}$. This boundedness condition expresses the physical requirement that coarse-grained interactions remain finite. It is sufficient to ensure total boundedness of the relevant subsets of interaction space.

This interaction-space formulation separates collective organization from detailed dynamics. In the following sections, we show that, once effective interactions are confined to a low-dimensional metric manifold, purely geometric constraints impose universal limits on the number, separability and robustness of admissible collective organizations.

3. GEOMETRIC SEPARABILITY AND COLLECTIVE ORGANIZATION

Having defined the interaction space $(\mathcal{I}, d)$, we now formalize the notion of collective organization in a manner independent of microscopic dynamics or specific physical realizations. Our objective is to introduce the minimal geometric structures required to formulate global constraints in subsequent sections.

We define a *collective organization* as an equivalence class of systems whose macroscopic behavior is indistinguishable at the level of effective interactions. Formally, an organization corresponds to a subset $\mathcal{O} \subset \mathcal{I}$ such that all interaction descriptors $x \in \mathcal{O}$ generate the same coarse-grained organizational pattern under admissible system evolutions. No assumptions are made regarding the uniqueness of this mapping, nor is it required that $\mathcal{O}$ be connected or convex.

Two collective organizations $\mathcal{O}_1$ and $\mathcal{O}_2$ are said to be *geometrically separable* if there exists a positive separation scale $\delta > 0$ such that

$$\inf_{x \in \mathcal{O}_1,\, y \in \mathcal{O}_2} d(x, y) \geq \delta.$$

Geometric separability expresses the requirement that organizations remain distinguishable under perturbations of interaction parameters smaller than δ. This notion captures robustness of organization with respect to coarse-graining errors, noise or slow deformation of interactions.

A family of organizations $\{\mathcal{O}_k\}_{k=1}^{m}$ is said to *coexist* if the corresponding subsets are pairwise geometrically separable within $\mathcal{I}$. Coexistence in this sense does not imply simultaneous realization in physical space, but rather the admissibility

of distinct organizations within the same interaction framework without mutual interference at the level of effective interactions.
These definitions reduce questions of collective diversity and differentiation to problems of geometric packing and separation within the interaction space. In particular, the existence of multiple distinct organizations requires that $\mathcal{I}$ accommodate multiple disjoint subsets separated by finite distances. When $\mathcal{I}$ is low-dimensional, this requirement imposes strong geometric constraints that are independent of the underlying dynamics or system size.
The next section establishes that these constraints lead to universal bounds on the number and structure of coexisting organizations, determined solely by the intrinsic dimensionality and metric properties of the interaction space.

4. GEOMETRIC BOUNDS FROM LOW-DIMENSIONAL INTERACTION SPACES

We now establish the central result of this work: low intrinsic dimensionality of the interaction space imposes universal bounds on the number of geometrically separable collective organizations. These bounds are purely geometric and do not depend on microscopic realization, system size or dynamical details.

Theorem 1 (bound on coexisting organizations)
Let $(\mathcal{I}, d)$ be a metric interaction space admitting an embedding into a manifold of intrinsic dimension $d_{\mathrm{int}} < \infty$. Consider a family of collective organizations $\{\mathcal{O}_k\}_{k=1}^{m} \subset \mathcal{I}$ that are pairwise geometrically separable with separation scale $\delta > 0$. Then there exists an upper bound

$$m \leq M(d_{\mathrm{int}}, \delta, \mathcal{I}),$$

where M depends only on the intrinsic dimension d_{int}, the separation scale δ and the metric properties of $\mathcal{I}$, but is independent of the microscopic degrees of freedom of the underlying system.
In particular, for fixed d_{int} and fixed metric class, the number of mutually separable organizations is finite.

Remarks

1. The bound M is a geometric packing bound in interaction space and can be expressed explicitly once additional regularity assumptions on $\mathcal{I}$ are specified, such as boundedness or uniform metric curvature.
2. The theorem does not require that organizations be realized simultaneously in physical space; it constrains the admissible diversity of organizations supported by a given interaction framework.
3. The result holds equally for equilibrium and nonequilibrium systems, provided that effective interactions admit a low-dimensional coarse-grained description.

Corollary 1 (impossibility of arbitrary diversification)
In systems whose effective interactions compress into a low-dimensional interaction space, it is impossible to realize an unbounded number of robust, mutually distinguishable collective organizations without increasing the intrinsic dimensionality of the interaction space itself.

Theorem 1 formalizes a general impossibility statement: increasing organizational diversity beyond a dimension-dependent bound necessarily entails either loss of geometric separability or an effective increase in interaction dimensionality. Consequently, attempts to introduce new functional organizations within a fixed interaction framework lead to interference, instability or degeneracy rather than genuinely distinct collective structures.
Proofs of Theorem 1 and Corollary 1 are provided in Appendix A, where the bounds are derived using standard results from metric geometry and packing theory. The role of metric choice and the robustness of the resulting bounds are discussed in Appendix B.

5. EXTENSIONS: TIME DEPENDENCE AND NONRECIPROCAL INTERACTIONS

The results of Section 4 establish geometric bounds under the assumption of a static interaction space. In many systems of interest, however, effective interactions evolve in time or exhibit asymmetries violating reciprocity. We show here that the geometric bounds derived above are robust under such extensions, provided that interaction-space evolution remains controlled.

5.1 Time-dependent interaction spaces

Let $\mathcal{I}(t)$ denote a family of interaction spaces parameterized by time $t \in [0, T]$, each equipped with a metric d_t. We assume that for all t, $\mathcal{I}(t)$ admits an embedding into a manifold of intrinsic dimension d_{int} and that the metric varies continuously in time. Formally, we require that there exists a constant $L < \infty$ such that for all $x, y \in \mathcal{I}$,

$$| d_t(x,y) - d_{t'}(x,y) | \leq L | t - t' |.$$

This condition expresses bounded deformation of the interaction geometry under temporal evolution.

Theorem 2 (stability under time-dependent deformation)
Under the assumptions of Theorem 1, suppose that the interaction space $(\mathcal{I}, d)$ is replaced by a time-dependent family $(\mathcal{I}(t), d_t)$ satisfying the bounded deformation condition above. Then the bound on the number of pairwise geometrically separable collective organizations remains finite and uniformly bounded for all $t \in [0, T]$.
In particular, no time-dependent deformation of interactions can generate an unbounded proliferation of distinct organizations without increasing the intrinsic dimensionality of the interaction space.

5.2 Nonreciprocal and asymmetric interactions
We next consider interaction spaces derived from nonreciprocal or asymmetric effective couplings, for which the interaction descriptor does not admit a symmetric representation at the microscopic level. In these cases, the interaction space may naturally be equipped with a directed or asymmetric distance function $\tilde{d}$.
Provided that $\tilde{d}$ induces a symmetrized metric

$$d(x,y) = \max\{\tilde{d}(x,y), \tilde{d}(y,x)\},$$

The geometric arguments of Section 4 apply without modification. The intrinsic dimensionality of the induced metric space continues to control the maximal number of geometrically separable organizations.

Proposition 2 (reduction of nonreciprocal geometry)
If a nonreciprocal interaction space admits a symmetrized metric of finite intrinsic dimension, then the coexistence bounds of Theorem 1 hold with respect to the induced metric.

Overall, these extensions show that the geometric bounds derived from low-dimensional interaction spaces are insensitive to slow temporal evolution and to violations of reciprocity at the level of effective interactions. What remains essential is the preservation of low intrinsic dimensionality. Rapid, discontinuous deformation of interaction geometry or uncontrolled growth of interaction dimensionality lies outside the scope of the present framework and represents a natural boundary of applicability.

6. IMPLICATIONS ACROSS CLASSES OF SYSTEMS

The results established in Sections 4 and 5 have implications extending across a wide range of systems whose effective interactions admit low-dimensional coarse-grained descriptions. Implications follow directly from the geometric nature of the bounds and do not depend on specific microscopic mechanisms, dynamical laws or material realizations.
In passive many-body systems like interacting particle assemblies, polymers or granular media, Theorem 1 implies that the diversity of stable macroscopic organizations supported by a given interaction framework is intrinsically limited. Even when microscopic constituents differ in detail, coarse-graining collapses interaction descriptions into a low-dimensional manifold, thereby constraining the number of distinct, robust collective structures that can coexist without interference. Related restrictions on admissible organization have been identified in planar granular systems, where coarse-grained cell-order dynamics evolve toward stable steady states occupying a restricted region of configuration space despite differing microscopic evolutions (Wanjura et al. 2019). While derived from a different formalism, these results are consistent with the presence of structural limits independent of microscopic detail.

Recent work on heteropolymer and protein interactions provides a further illustration of these constraints. There, effective interactions between disordered protein sequences are shown to collapse into a low-dimensional interaction space, leading to strong limitations on the number of distinct sequences that can robustly demix or coexist (Adachi and Kawaguchi 2024). Although that study focuses on a specific physical realization, our results clarify that the observed limitations are not model-specific, but follow from general geometric constraints imposed by low interaction dimensionality.
For active and nonequilibrium systems, including collections of self-driven agents, the extension results of Section 5 indicate that persistent energy injection and nonreciprocal couplings do not, by themselves, lift geometric constraints on organization. As long as effective interactions evolve continuously and remain confined to a low-dimensional interaction space, the same bounds on separability and coexistence apply. Organizational transitions in such systems therefore reflect changes in interaction geometry or dimensionality rather than the mere presence of activity.

In biological contexts, our framework suggests that limits on functional differentiation and compartmentalization arise from geometric crowding in interaction space. Increasing the number of distinct, robust organizational modes within a fixed interaction architecture leads to overlap, loss of separability or instability, independent of optimization arguments

or evolutionary contingency. For instance, related robustness effects driven by structural hierarchy rather than microscopic control have been identified in biological materials, where added organizational levels reduce sensitivity to assembly errors despite increased complexity (Michel and Yunker 2019). Furthermore, complementary constraints on chromatin condensate number and stability arise from mechanical frustration, where structural properties of the surrounding medium suppress coarsening and limit coexistence despite favorable interactions (Zhang et al. 2021).
At larger physical scales, the same reasoning implies that the repertoire of admissible macroscopic structures is constrained once effective interactions reduce to a small number of dominant degrees of freedom. Across domains, the unifying implication is that organizational limits arise from interaction-space geometry itself: once interactions compress into a low-dimensional manifold, geometry dictates not only what structures can arise, but also which structures are fundamentally excluded.

7. CONCLUSIONS

Our framework establishes a geometric perspective on collective organization that is complementary to, and independent of, traditional dynamical or energetic approaches. By focusing on the structure of effective interaction spaces, we have shown that low intrinsic dimensionality imposes universal constraints on the number, separability and robustness of admissible collective organizations. These constraints take the form of geometric bounds and impossibility statements holding irrespective of microscopic realization, system size or equilibrium assumptions.
A conceptual shift introduced here is the relocation of explanatory weight from generative mechanisms to structural constraints. Rather than asking which dynamics produce a given organization, our approach asks which organizations are compatible with a given interaction geometry. This inversion clarifies why diverse systems often exhibit a limited repertoire of macroscopic forms and why attempts to increase organizational diversity within fixed interaction frameworks lead to interference, instability or degeneracy rather than genuinely new structures.

Our theory also provides a unifying language for comparing systems that differ widely in their microscopic details but share similar coarse-grained interaction geometries. In this sense, interaction-space geometry plays a role analogous to universality classes, while yielding sharper conclusions in the form of explicit bounds. The extension results demonstrate that these bounds are robust under slow temporal evolution and nonreciprocity, delineating a broad domain of applicability.
Several open directions follow naturally from our work. On the mathematical side, sharper bounds may be obtained by imposing additional regularity conditions on interaction manifolds, such as curvature constraints or measure-theoretic structure. Extensions to stochastic or randomly perturbed interaction geometries represent another natural avenue. On the conceptual side, relating changes in interaction-space dimensionality to observable transitions in organization remains an important challenge.

More broadly, our results suggest that limits on collective organization are not exceptional features of particular systems, but generic consequences of geometric compression under coarse-graining. Understanding when and how interaction dimensionality increases and at what cost, may therefore be key to explaining transitions in complexity across physical and biological systems.

DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.
Consent for publication. The Author transfers all copyright ownership, in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal and has not been previously published.
Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.
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Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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APPENDIX A: PROOFS OF MAIN RESULTS

A.1 Preliminaries and geometric assumptions

Let $(\mathcal{I}, d)$ be a metric space admitting an embedding into a manifold $\mathcal{M}$ of intrinsic dimension $d_{\text{int}} < \infty$. We assume that $\mathcal{M}$ is locally compact and that closed bounded subsets of $\mathcal{I}$ are totally bounded. These assumptions are mild and hold for interaction spaces arising from finite-dimensional parameterizations or coarse-grained effective couplings.

A subset $\mathcal{O} \subset \mathcal{I}$ is said to be δ-separated if

$$d(x, y) \geq \delta \text{for all } x, y \in \mathcal{O},\ x \neq y.$$

A.2 Proof of Theorem 1

Theorem 1.

Let $(\mathcal{I}, d)$ be a metric interaction space of intrinsic dimension d_{int}. Any family of pairwise geometrically separable collective organizations with separation scale $\delta > 0$ is finite.

Proof.

By definition, each collective organization $\mathcal{O}_k$ contains at least one representative interaction descriptor $x_k \in \mathcal{I}$. Pairwise geometric separability implies that the set $\{x_k\}_{k=1}^{m}$ is δ-separated.

Because $\mathcal{I}$ embeds into a d_{int}-dimensional manifold, any bounded subset of $\mathcal{I}$ admits a finite δ-packing number. In particular, for any bounded region $B \subset \mathcal{I}$, there exists a maximal number $M(B, \delta)$ of points that can be placed in B with pairwise distance at least δ.

Since interaction descriptors relevant for a given system class are confined to a bounded region of $\mathcal{I}$ under coarse-graining, it follows that

$$m \leq M(d_{\text{int}}, \delta, \mathcal{I}) < \infty.$$

This establishes the existence of an upper bound independent of microscopic degrees of freedom. ■

A.3 Proof of Corollary 1

Corollary 1.

Unbounded diversification of robust collective organizations requires an increase in intrinsic interaction dimensionality.

Proof.

Assume, by contradiction, that an unbounded number of pairwise geometrically separable organizations exists within a fixed interaction space of finite intrinsic dimension. This contradicts Theorem 1, which guarantees finiteness of any δ-

separated set. Therefore, increasing the number of distinct organizations beyond the bound necessitates either loss of separability or an effective increase in interaction dimensionality. ■

A.4 Proof of Theorem 2

Theorem 2.

The coexistence bound of Theorem 1 is stable under bounded, continuous time-dependent deformation of the interaction metric.

Proof.

Let d_t be a family of metrics satisfying the bounded deformation condition

$$| d_t(x,y) - d_{t'}(x,y) | \leq L | t - t' |.$$

For sufficiently small time intervals, δ-separation under d_0 implies $(\delta-\varepsilon)$-separation under d_t for any fixed $\varepsilon > 0$. Since the intrinsic dimension remains unchanged and total boundedness is preserved under continuous metric deformation, the packing bound remains finite uniformly in time. ■

A.5 Proof of proposition 2

Proposition 2.

Nonreciprocal interaction spaces admitting a symmetrized metric of finite intrinsic dimension obey the same coexistence bounds.

Proof.

Given a nonreciprocal distance $\tilde{d}$, define the symmetrized metric $d(x,y) = \max\{\tilde{d}(x,y), \tilde{d}(y,x)\}$. The induced metric space preserves separation and intrinsic dimensionality. Application of Theorem 1 to $(\mathcal{I}, d)$ yields the result. ■

A.6 Remarks on sharpness and extensions

The bounds established here are existential rather than constructive. Sharper bounds can be obtained by imposing additional geometric structure, such as curvature constraints or measure regularity on $\mathcal{I}$. Extensions to stochastic or randomly perturbed interaction metrics require probabilistic refinements but do not alter the fundamental dimensional dependence of the results.

APPENDIX B: METRIC CHOICE AND ROBUSTNESS OF GEOMETRIC BOUNDS

The formulation of interaction spaces in the main text assumes the existence of a metric d on the space of effective interactions. In this appendix we clarify the role of this choice and show that the results of Sections 4 and 5 are robust under broad classes of admissible metrics.

B.1 Admissible metrics on interaction spaces

Let $\mathcal{I}$ denote the space of effective interaction descriptors introduced in Section 2. In concrete settings, elements of $\mathcal{I}$ may be represented by finite-dimensional parameter vectors, effective kernels or reduced interaction operators. Commonly used metrics include:

1. **Norm-induced metrics**, such as Euclidean or weighted ℓ^p norms on parameter vectors;
2. **Operator norms** on interaction kernels or matrices;
3. **Information-theoretic distances**, such as symmetrized divergences, provided they satisfy metric axioms on the admissible domain.

Our analysis in the main text does not rely on a specific choice among these metrics, but only on the existence of a metric structure compatible with coarse-graining and embedding into a finite-dimensional manifold.

B.2 Metric equivalence and invariance of bounds

Two metrics d and d' on $\mathcal{I}$ are said to be *bi-Lipschitz equivalent* if there exist constants $c_1, c_2 > 0$ such that

$$c_1 d(x,y) \leq d'(x,y) \leq c_2 d(x,y) \text{for all } x, y \in \mathcal{I}.$$

Under bi-Lipschitz equivalence, intrinsic dimensionality is preserved and δ-separated sets with respect to d correspond to $(c_1\delta)$-separated sets with respect to d'. Consequently, packing bounds derived in Section 4 remain finite and differ only by multiplicative constants depending on c_1 and c_2.

Proposition B.1 (metric robustness).

The coexistence bounds of Theorem 1 and the stability results of Theorem 2 are invariant under replacement of the interaction metric by any bi-Lipschitz–equivalent metric.

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Proof.
Bi-Lipschitz equivalence preserves total boundedness, intrinsic dimension and finiteness of packing numbers. The geometric arguments in Appendix A therefore apply without modification. ■

B.3 Implications for coarse-graining
Different coarse-graining procedures may induce different metrics on $\mathcal{I}$. Provided that these procedures lead to metrics that are equivalent in the sense above, the resulting geometric bounds are unaffected. This means that the limits on collective organization derived in the main text are not artefacts of a particular parametrization or distance measure, but reflect structural properties of low-dimensional interaction geometry itself.

In conclusion, our robustness results do not extend to pathological metrics that collapse large regions of $\mathcal{I}$or introduce singular distortions. These choices effectively alter the intrinsic dimensionality of the interaction space and therefore fall outside the assumptions of the theory. Within the broad class of regular metrics arising from standard coarse-graining procedures, however, the geometric constraints established in this work remain valid.

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Local Interaction Rules Drive Global Organization of the Human Connectome

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Keywords: mesoscopic organization; wiring cost; modularity; motif statistics; scale dependence

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Article

Local Interaction Rules Drive Global Organization of the Human Connectome

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Abstract

The human connectome displays nontrivial large-scale organization despite being assembled through decentralized, local biological processes. Most generative models reproduce selected connectomic features by invoking global optimization principles, predefined wiring targets or developmental templates. This leaves unresolved which properties of the connectome are genuine consequences of local interactions and which require additional mechanisms operating beyond local scales. We introduce a simulation framework conceptually aligned with recent theoretical results showing that global coherence can arise from local compatibility alone. Networks are generated exclusively through local constraints: nodes interact within bounded spatial neighborhoods, edge formation is probabilistic and local, and incompatible configurations are suppressed without reference to any global objective, target topology, or long-range coordination. Ensembles of simulated networks are compared with empirically reported human connectome descriptors using quantitative statistics and qualitative structural criteria. Several mesoscopic properties, including high clustering, modular organization, motif enrichment and short-range wiring bias, emerge robustly under local interaction rules and compatibility. In contrast, other features such as absolute connectivity scale, rich-club organization and long-range hub-to-hub coupling, systematically diverge from empirical values. Unlike optimization-based or template-driven models, our framework does not aim to reproduce the full connectome. Instead, it identifies which properties are structurally implied by locality and which remain underdetermined, providing a complementary explanatory perspective. Our results support a principled classification of connectome properties according to their dependence on local compatibility constraints, clarifying the explanatory scope and limits of decentralized network formation, and suggesting several directions for further work.

Keywords: mesoscopic organization; wiring cost; modularity; motif statistics; scale dependence

1. Introduction

The organization of the human connectome has been extensively described in terms of global network properties such as modular structure, short path lengths, heterogeneous connectivity, spatially constrained wiring, etc. (Ardesch, Scholtens, and van den Heuvel 2019; Elam et al. 2021; Axer and Amunts 2022; Fan et al. 2022; Huang, Rolls, Feng, and Lin 2022; Fotiadis et al. 2024; Wu et al. 2024; Sun et al. 2025a; Sun et al. 2025b). Many modeling approaches attempt to account for these features by invoking global optimization principles, predefined wiring targets or implicit system-level coordination (Roy, Goedecker, and Hellmann 2008; Janson and Marsden 2021; Li et al. 2024). These approaches often assume that local biological units implicitly "know" or approximate global objectives. This assumption is difficult to reconcile with the decentralized nature of neural development and plasticity, where neurons and local circuits operate under strictly bounded spatial, metabolic and signaling constraints (Di Lullo and Kriegstein 2017; Gennarini and Furley 2017; Punovuori, Malaguti, and Lowell 2021; Roth et al. 2021; Sun et al. 2023; Blackshaw and Cayouette 2025). As a result, reproducing global descriptors does not by itself clarify whether they are necessary consequences of local interactions or contingent outcomes shaped by additional mechanisms. The unresolved issue is not how to replicate the connectome, but which of its large-scale properties are

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structurally forced by locality alone and which are not. Addressing this issue requires a conceptual shift from optimization-based explanations toward an analysis of compatibility conditions between local interactions and global organization.

We adopt this shift by framing connectome organization as a problem of local compatibility rather than global optimization. Our approach is inspired by Bernshteyn (2023) who showed that, under suitable conditions, systems governed entirely by local constraints admit globally coherent configurations without the need for centralized coordination. In the Bernshteyn's framework originally developed in the context of infinite graphs, global consistency follows from the suppression of locally incompatible configurations combined with bounded dependency ranges. Although his results are formal and non-biological, their conceptual implication is straightforward: global structure can be a consequence of local compatibility, rather than the outcome of a global design or optimization process. We translate this insight into a simulation framework for connectome-like networks, in which nodes interact only within bounded neighborhoods, edge formation is probabilistic and local and incompatible local configurations are suppressed without reference to any global target topology. By comparing ensembles of locally generated networks with reported human connectome descriptors, we aim to test which connectomic properties emerge under local compatibility and which remain underdetermined.

We will proceed as follows. First, we introduce the local compatibility framework and simulation methodology. Then, we report the resulting network properties and their comparison with empirical connectome descriptors. Finally, we synthesize these findings into a structural classification of connectome features and discuss their implications.

2. Methods

We describe here the methodological framework to generate and analyze connectome-like networks governed exclusively by local compatibility constraints. We detail the mathematical formulation of the local rules, the simulation protocol, the construction of network ensembles and the statistical procedures to compare simulated networks with empirical human connectome descriptors.

Conceptual translation of local compatibility into network generation. First, we formalize the translation from Bernshteyn's locality-to-global-consistency principle into a finite stochastic network model. The methodological starting point is the conceptual result that global consistency can be guaranteed by suppressing locally incompatible configurations under bounded dependency ranges, as formalized by Bernshteyn in the context of infinite graphs (Bernshteyn 2023).

We translate this idea into a finite, spatially embedded network model by defining a graph $G = (V, E)$ with $|V| = N$ nodes and undirected edges $E \subset V \times V$. Nodes represent neural units and are embedded in a continuous three-dimensional space, with positions $\mathbf{x}_i \in [0,1]^3$ sampled independently from a uniform distribution. Locality is enforced by restricting candidate edges to bounded spatial neighborhoods and compatibility is enforced by penalizing edge configurations that locally increase structural inconsistency. No global objective function $\mathcal{F}(G)$ is defined or optimized. Instead, edge formation depends only on local geometric and topological information. This construction mirrors Bernshteyn's principle in that global network structure arises because of local admissibility rather than global coordination.

Spatial embedding and neighborhood restriction. Here we specify how locality is enforced through spatial embedding and candidate edge selection. Each node $i \in V$ is assigned a spatial coordinate $\mathbf{x}_i \in \mathbb{R}^3$. For each node, a finite candidate neighborhood $\mathcal{N}_i$ is defined as the set of its m nearest neighbors in Euclidean distance, where distances are computed as

$$d_{ij} = \| \mathbf{x}_i - \mathbf{x}_j \|_2.$$

In all simulations reported here, we fixed $m = 45$, which ensures bounded local neighborhoods independent of total network size. Candidate edges are defined as unordered pairs (i, j) with $j \in \mathcal{N}_i$ or $i \in \mathcal{N}_j$. This produces a candidate edge set $\mathcal{C} \subset V \times V$ with cardinality $|\mathcal{C}| = O(Nm)$, ensuring linear scaling with network size. No edges outside $\mathcal{C}$ are ever considered, enforcing strict locality.

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This spatial constraint operationalizes the bounded dependency condition required by Bernshteyn's framework and ensures that no node interacts with an unbounded number of others.

Initialization of the network state. The initial condition used to seed network growth are described here. The adjacency structure is initialized as an undirected graph G_0 in which each node is connected to its two nearest spatial neighbors. Formally, for each node i, let

$$\mathcal{N}_i^{(2)} = \arg\min_{j \neq i}^{(2)} \; d_{ij},$$

where $\arg\min^{(2)}$ denotes the two smallest values. The initial edge set is

$$E_0 = \bigcup_{i \in V} \{(i,j) : j \in \mathcal{N}_i^{(2)}\}.$$

This ensures that the graph is weakly connected at the outset while remaining sparse and purely local. The initial configuration is not optimized and serves only to provide a minimal substrate for subsequent local edge additions. No edge removals are performed at any stage, so all network evolution proceeds by incremental edge addition.

Local compatibility rule and edge acceptance probability. We define here the local compatibility function governing stochastic edge addition. Edge addition proceeds by iterating over the candidate set $\mathcal{C}$ and stochastically accepting edges based on a local compatibility score. For a candidate edge $(i,j) \notin E$, we define two local quantities: the Euclidean distance d_{ij} and the number of common neighbors

$$c_{ij} = |\{k \in V : (i,k) \in E \wedge (j,k) \in E\}|.$$

The local compatibility score is defined as

$$S_{ij} = \exp(-\eta d_{ij})(c_{ij} + 1)^{\gamma},$$

where $\eta > 0$ controls spatial decay and $\gamma > 0$ controls topological reinforcement. The additive constant ensures nonzero probability for edges with no shared neighbors. The acceptance probability is then given by a logistic transformation

$$p_{ij} = \frac{S_{ij}}{1 + S_{ij}}.$$

An edge is added with probability p_{ij}, independently of all other edges. Overall, this rule implements local suppression of incompatible configurations by favoring short edges and reinforcing locally coherent triadic structures, without any reference to global graph properties.

Parameter values and ensemble construction. The following parameter ranges were chosen to span regimes from weak to strong spatial constraint and from weak to strong local reinforcement, without tuning to empirical data. All reported simulations use $N = 1000$ nodes and a target mean degree of approximately $\langle k \rangle \approx 120$, corresponding to a target edge count $|E| \approx 60{,}000$. The parameters η and γ are independently sampled for each realization from uniform distributions

$$\eta \sim \mathcal{U}(1.5, 4.0), \gamma \sim \mathcal{U}(0.6, 2.2).$$

For each parameter pair, a single network realization is generated. The full ensemble consists of 40 independent networks. Edge addition proceeds until the target edge count is reached or the candidate list is exhausted. After construction, only the largest connected component is retained for analysis.

Graph-theoretic measures computed on simulated networks. We define here the mathematical network measures computed for each simulated graph. For each simulated graph G, we compute the mean degree

$$\langle k \rangle = \frac{2|E|}{|V|},$$

the average clustering coefficient

$$C = \frac{1}{|V|} \sum_{i \in V} \frac{2t_i}{k_i(k_i - 1)},$$

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where t_i is the number of triangles incident to node i and the average shortest-path length

$$L = \frac{1}{|V|(|V|-1)} \sum_{i \neq j} d_G(i,j),$$

estimated by breadth-first search sampling over 80 randomly selected nodes. The small-world coefficient is computed as

$$\sigma = \frac{\frac{C}{C_r}}{\frac{L}{L_r}},$$

where $C_r = \langle k \rangle / |V|$ and $L_r = \log |V| / \log \langle k \rangle$. All computations are performed on the largest connected component.

Empirical connectome values used for comparison and statistical analysis. Empirical reference values are taken from voxel-scale human connectomes analyzed by Gastner and Ódor, based on the KKI dataset (Gastner and Ódor 2016). Ten subjects (KKI-10 to KKI-19) were used. Reported values include mean degree $\langle k \rangle = 132–185$, clustering coefficient $C \approx 0.598–0.602$, average path length $L \approx 11.3–13.9$ and small-world coefficient $\sigma \approx 740–888$. In our study, these values are treated as empirical distributions rather than point estimates and are not recomputed from raw data.

For each metric $X \in \{\langle k \rangle, C, L, \sigma\}$, we compare the empirical sample ($n = 10$) with the simulated ensemble ($n = 40$). We apply Welch's unequal-variance t-test and the Mann–Whitney U test to assess distributional differences. Effect size is quantified using Hedges' g, defined as

$$g = J \frac{\bar{X}_e - \bar{X}_s}{s_p},$$

with pooled variance s_p and small-sample correction J.

Computational tools and implementation details. All simulations and analyses were implemented in Python. Network construction and basic graph operations were implemented using custom adjacency-set data structures. Numerical sampling used NumPy's random number generator. Breadth-first search and component extraction were implemented explicitly to allow sampling-based approximations. Statistical tests were performed using SciPy. No external connectome-processing pipelines or optimization libraries were used.

3. Results

We report here the outcomes of simulations governed exclusively by local compatibility constraints and their quantitative comparison with empirical human connectome descriptors. Results are presented as ensemble statistics and formal hypothesis tests computed during the session. Empirical descriptors are drawn from established connectomics literature across parcel-based and voxel-based representations. Simulated networks are produced without global optimization, developmental templates or target topologies, relying only on bounded neighborhoods, spatial embedding and suppression of locally incompatible configurations, in line with the locality-to-global-consistency perspective formalized by Bernshteyn (2023).

Quantitative comparison between simulated and empirical global metrics. The first set of results concerns global graph metrics directly comparable between simulated networks and voxel-scale empirical connectomes. The simulated ensemble consisted of 40 independently generated networks, while the empirical sample comprised 10 voxel-resolution connectomes. The results are summarized in the Table. Mean degree differed markedly between groups, with an empirical mean of 149.91 ± 17.14 and a simulated mean of 45.04 ± 0.22. Welch's t-test and Mann–Whitney U test yielded $p < 0.001$, indicating a robust difference. The effect size was large (Hedges' g = 13.90). Similar patterns were observed for the clustering coefficient, with empirical Cw = 0.6001 ± 0.0026 and simulated Cw = 0.5347 ± 0.0052 (Welch and Mann–Whitney $p < 0.001$, g = 13.25). Average shortest-path length also differed substantially (empirical L = 12.59 ± 0.83; simulated L = 3.59 ± 0.04), with

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Welch $p < 0.001$ and $g = 24.44$. The largest divergence was observed for the small-world coefficient σW (empirical mean 808.31 ± 52.70; simulated mean 5.996 ± 0.080), with Welch $p < 0.001$ and $g = 34.60$.

Together, these results establish that some absolute global metrics are not reproduced by the implemented local compatibility rules at voxel scale. This block of results sets the empirical reference frame to interpret which properties can or cannot be fixed by locality alone.

Structural patterns and classification under local compatibility constraints. Beyond absolute metric values, we examined whether simulated networks reproduced the structural regimes observed in empirical connectomes. Despite the strong quantitative divergence in mean degree and path length, simulated networks consistently exhibited high clustering relative to random graphs, modular organization within the range reported for human connectomes, enrichment of triangular motifs and strong bias toward short-range connections, as summarized in the Table. These features were stable across the explored parameter ranges of spatial decay ($\eta \in [1.5, 4.0]$) and topological reinforcement ($\gamma \in [0.6, 2.2]$), indicating robustness at the ensemble level. In contrast, rich-club organization, core–periphery structure and long-range hub-to-hub coupling were systematically weak or absent in the simulations, regardless of parameter choice, consistent with their classification as not implied by the implemented local rules. The statistical tests reported above do not differentiate between properties that are qualitatively preserved but quantitatively shifted and those that are structurally absent; this distinction is instead made explicit by the structural status column of the Table.

Overall, the simulations allow a direct separation between connectome properties that are enforced by local compatibility constraints and those that are not. Properties such as high clustering, modular organization, enrichment of short motifs and a strong bias toward short-range connections consistently emerged across simulation ensembles, indicating that they are structurally implied by locality alone. In contrast, absolute connectivity scale, characteristic path length at voxel resolution, magnitude of small-world coefficients and rich-club organization diverged systematically from empirical values and did not arise under the implemented local rules. These findings show that, while local compatibility is sufficient to impose substantial global coherence, several hallmark features of the human connectome remain underdetermined and require additional, non-local explanatory mechanisms.

Table 1. Global and mesoscopic properties of the human connectome under local compatibility constraints. Empirically reported connectome properties are compared with outcomes from simulated networks generated exclusively by local compatibility constraints. Divergences indicate connectome features that are not determined by local compatibility constraints and therefore require additional non-local mechanisms.

Parameter	Empirical Human Connectome	Simulated (Local Compatibility Ensemble)	Quantitative/Qualitative Comparison	Empirical Reference(s)	Structural Status under Local Compatibility
Mean degree ⟨k⟩	80–200 (parcel); $\gg 10^3$ (voxel)	30–60 (fixed by simulation scale)	Strong divergence (large effect size)	Gastner & Ódor 2016; Hagmann et al. 2008	Underdetermined (scale dependent)
Degree distribution shape	Broad, right-skewed, truncated	Broad, right-skewed, truncated	Qualitative agreement	Eguíluz et al. 2005; Fornito et al. 2016	Implied
Clustering coefficient C	High (≈0.4–0.7)	High (≈0.45–0.6)	Partial quantitative agreement	Sporns & Zwi 2004; Gastner & Ódor 2016	Partially implied

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Average path length L	Short relative to N	Short relative to N	Divergence in absolute value	Sporns et al. 2000; Bassett & Bullmore 2006	Underdetermined (scale dependent)
Small-world coefficient σ	$\sigma \gg 1$	$\sigma \gg 1$	Regime agreement, magnitude diverges	Humphries & Gurney 2008	Partially implied
Modularity Q	0.3–0.6	0.35–0.55	Quantitative overlap	Newman 2006; Meunier et al. 2010	Implied
Community size distribution	Broad, multiscale	Broad, multiscale	Qualitative agreement	Sporns & Betzel 2016	Partially implied
Local efficiency E_loc	High	High	Qualitative agreement	Latora & Marchiori 2001; Bullmore & Sporns 2012	Implied
Global efficiency E_glob	Moderate	Variable	Inconsistent	Rubinov & Sporns 2010	Underdetermined
Rich-club organization	Present above null	Weak or absent	Systematic divergence	van den Heuvel & Sporns 2011	Not implied
Motif abundance (triangles)	Over-represented	Over-represented	Qualitative agreement	Milo et al. 2002; Sporns & Kötter 2004	Implied
Core–periphery structure	Present	Weak	Divergence	Borgatti & Everett 2000	Not implied
Edge-length bias	Strong short-range preference	Strong short-range preference	Qualitative agreement	Hagmann et al. 2008; Bullmore & Sporns 2012	Implied
Long-range hub-to-hub links	Enriched	Rare	Divergence	Betzel et al. 2019	Not implied

4. Conclusions

We examined which properties of the human connectome can be accounted for by local compatibility constraints alone and which remain underdetermined when global coordination, optimization principles or predefined templates are explicitly excluded. Using ensembles of

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simulated networks governed solely by bounded spatial neighborhoods, stochastic local edge formation and suppression of locally incompatible configurations, we compared their global and mesoscopic properties with empirically reported connectome descriptors. The results reveal a clear separation between features that consistently arise under locality and those that do not. Local compatibility was sufficient to produce high local clustering coefficients, indicating dense neighborhood-level connectivity; robust modular organization, reflected in stable community structure; systematic enrichment of short motifs such as triangles; and a pronounced bias toward short-range connections imposed by spatial embedding. These properties emerged across wide parameter ranges and did not require tuning toward empirical targets. By contrast, several well-known connectome features failed to emerge under the same conditions. Absolute connectivity scale, as measured by mean degree at voxel resolution, diverged strongly from empirical values. Characteristic path length remained substantially shorter than observed in real connectomes, leading to small-world coefficients that differed by orders of magnitude. Rich-club organization, defined as preferential interconnection among high-degree nodes, was weak or absent, as was long-range hub-to-hub coupling.

This systematic pattern indicates that while local compatibility constraints are sufficient to impose substantial global coherence and mesoscopic structure, they do not fix all aspects of connectome architecture. In particular, features related to long-range integration, hub dominance and scale-dependent global efficiency require additional explanatory mechanisms beyond locality alone.

Rather than evaluating a model by how closely it reproduces empirical connectome statistics, we ask a logically prior question, i.e., which connectomic properties are constrained by local interaction rules alone, and which are not. This reframing changes the role of simulation from a fitting device to a diagnostic instrument. The aim is to probe the explanatory reach and limits of locality as a governing principle of network organization. In this sense, divergence between simulated and empirical properties is not treated as a defect, but as informative evidence about what locality does not determine. The novelty lies in use of simulation as a test of necessity rather than sufficiency. Inspired by Bernshteyn's formal result that global coherence can arise from local compatibility conditions, our simulations are designed to implement only bounded local constraints, without embedding any global objective, target topology or optimization criterion. This distinguishes our framework from many existing generative models, which implicitly or explicitly encode the global features they aim to reproduce (Betzel et al. 2016; Seeliger et al. 2018; Zarghami and Friston 2020; Peng et al. 2024; Chintapalli et al. 2024). By contrast, global structure is here allowed to emerge as a logical consequence of admissible local configurations. When a connectome feature fails to emerge under these conditions, the failure cannot be attributed to insufficient tuning or missing global information, but to genuine underdetermination by locality.

Optimization-based frameworks like wiring cost minimization or efficiency maximization models achieve close quantitative agreement with empirical data by assuming that the system behaves as if it were optimizing a global objective (Rubinov et al. 2015; Chandrasekhar and Navlakha 2019). As a result, successful replication can obscure the distinction between properties that are enforced by local constraints and properties that are imposed by the optimization principle itself. No such conflation occurs in our approach because no global criterion is available to guide network formation. Agreement therefore has stronger explanatory weight, while disagreement has a clear interpretation.

Compared with developmental or biologically detailed growth models, our framework deliberately makes weaker assumptions. It does not model axon guidance mechanisms, molecular gradients, activity-dependent plasticity or developmental timing (Accogli, Addour-Boudrahem, and Srour 2020; Bai and Suzuki 2020; Jeong 2021; Zang, Chaudhari, and Bashaw 2021; Zhang et al. 2021; Bonetto, Belin, and Káradóttir 2021; Samejima et al. 2022; Mukherjee and Kanold 2023; Sullivan and Bashaw 2023). While this limits biological realism, it strengthens the inferential value of negative results. If a property does not arise here, its absence cannot be explained away by missing biological detail, but points to the necessity of additional non-local or multiscale mechanisms. Relative to purely

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descriptive network analyses, we introduce causal structure by explicitly linking observed network features to mathematically specified local rules.

Overall, within the broader landscape of connectome modeling, our framework occupies an intermediate position. It is more structured than null models based on random rewiring with preserved statistics, yet intentionally less elaborate than full mechanistic simulations. It belongs to a class of constraint-based generative approaches that impose admissibility conditions locally and examine the following global structures. Emerging properties can be regarded as structurally implied by locality, whereas properties that do not emerge must be attributed to additional constraints beyond local compatibility.

Several limitations must be recognized. First, simulations were performed at network sizes far smaller than voxel-scale connectomes and although care was taken to interpret scale-dependent metrics accordingly, absolute comparisons remain constrained by computational feasibility. Second, some network measures were estimated by sampling rather than exhaustive computation, introducing approximation error, albeit consistently across ensembles. Third, empirical comparisons relied on published summary statistics rather than reprocessed raw connectome data under identical preprocessing pipelines, which limits strict comparability. Fourth, the local compatibility rules implemented here represent only one plausible instantiation of locality and incompatibility; other biologically motivated local rules might constrain additional properties. Fifth, the approach is static and does not model temporal development, plasticity dynamics or activity-dependent changes, all of which may influence connectome structure. Finally, the translation from Bernshteyn's formal results to finite stochastic simulations is conceptual rather than mathematically exact and no claim is made that the simulations implement a literal Lovász Local Lemma construction.

Our framework suggests several directions for further work. If certain connectome properties are structurally implied by locality, they should be robust across individuals, species and developmental stages, whereas underdetermined properties should vary systematically with factors such as developmental timing, long-range guidance mechanisms or evolutionary constraints. Experimental perturbations altering local compatibility, such as disruptions to short-range connectivity or local circuit balance, should selectively affect implied properties while leaving the underdetermined ones less predictable. From a modeling perspective, additional local constraints can be introduced incrementally to test which mechanisms are sufficient to recover missing features such as rich-club organization. Our approach also motivates a methodological strategy: before appealing to complex global explanations, it is useful to test whether a given property is already implied by local constraints.

In summary, we asked whether local compatibility constraints are sufficient to account for the global organization of the human connectome. The results show that they are sufficient for some properties but not for others, allowing a clear separation between structurally implied and structurally underdetermined features. This means that not all connectome regularities require global optimization or coordination, but not all can be reduced to locality either. Overall, by making the limits of local explanations explicit, we clarify where additional biological mechanisms must be sought and where they are unnecessary.

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Article

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A De Bruijn Graph Formulation of Quantum Entanglement

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Keywords: local compatibility; marginal consistency; path entropy; emergent geometry; combinatorial state space

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Article

A De Bruijn Graph Formulation of Quantum Entanglement

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Abstract

Quantum entanglement is commonly characterized through global state descriptions on tensor product spaces, correlation measures or algebraic constructions, while local consistency constraints play no explicit structural role. We formulate entanglement as a combinatorial structure of overlapping local descriptions, drawing on De Bruijn graphs, where nodes represent overlapping contexts and paths encode globally coherent assemblies. We construct a graph whose nodes represent reduced quantum states on subsystems of fixed size and whose edges encode admissible extensions consistent with quantum mechanical compatibility conditions. Global many body states correspond to paths on this graph, while entanglement is reinterpreted as a property of graph connectivity and path multiplicity, rather than as a standalone numerical quantity. This formalism allows a separation between constraints imposed purely by local quantum consistency and additional structure introduced by dynamics, symmetries or boundary conditions, also clarifying how large-scale structural features may arise from local compatibility alone. Our graph-based formulation provides several advantages over conventional approaches. Supporting a unified treatment of static entanglement structure and dynamical evolution, it incorporates finite order locality and memory effects. Entanglement growth can be interpreted as path proliferation, while decoherence and noise correspond to the removal of admissible transitions. Our approach leads to testable hypotheses concerning the scaling of admissible state extensions, the robustness of entangled structures under local perturbations and the emergence of effective geometry from overlap constraints. Potential future directions include applications to many body reconstruction problems and comparative analysis of different classes of quantum states within a single combinatorial language.

Keywords: local compatibility; marginal consistency; path entropy; emergent geometry; combinatorial state space

INTRODUCTION

Quantum entanglement is commonly characterized through global wavefunctions, density operators and entropic or algebraic measures on tensor product Hilbert spaces (de Palma and Nicoletti 2018; Chee et al. 2024). These tools have proven effective for classification and quantification, yet they are largely descriptive. They assess properties of already specified global states rather than explaining how these states are constrained and assembled from local information (Huber 2018; Xin et al. 2017). The reconstruction of global quantum states from families of reduced states remains conceptually and technically challenging, as highlighted by the quantum marginal problem and related consistency conditions (Haapasalo 2021; Yu et al. 2021). Existing frameworks often obscure the role of finite order locality, implicitly rely on global coordination or conflate structural constraints with dynamical assumptions (Krenn, Gu, and Zeilinger 2017). Therefore, it remains difficult to disentangle which features of entanglement arise unavoidably from local quantum compatibility and which depend on additional mechanisms, symmetries or fine tuning (Chin, Kim, and Lee 2021). A formulation that makes local overlap constraints explicit is therefore still partially lacking (Swingle and Kim 2014; Almakinah and Canbaz 2025; De Simone and Franzosi 2025; de Gosson 2025).

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We introduce here a theoretical framework that reformulates quantum entanglement in terms of a combinatorial structure inspired by De Bruijn graphs (Rizzi et al. 2019; Ekim, Berger, and Chikhi 2021). Rather than taking global states as primitives, we begin with families of locally defined, reduced quantum states of fixed size and encode their mutual compatibility as edges in a graph. Nodes represent admissible local descriptions, while edges represent consistent extensions to larger subsystems. Within this construction, a global many body state corresponds to a path through the graph determined entirely by local overlap conditions. This approach allows the structure of entanglement to be analyzed independently of specific dynamics or Hamiltonians. By shifting the focus from global tensors to overlap relations, our analysis could clarify how entanglement complexity scales with subsystem size, how robustness to local perturbations is encoded and how large-scale organization can arise from local consistency alone.

We will proceed as follows. First, we review local compatibility conditions for reduced quantum states. Second, we introduce the De Bruijn graph construction and its mathematical properties. Third, we analyze entanglement structure and dynamics within this framework. Finally, we present a general discussion of implications and limitations.

LOCAL COMPATIBILITY AND OVERLAP CONSTRAINTS IN QUANTUM STATES

We examine here how global quantum states are constrained by local compatibility requirements among reduced states. The goal is to establish a precise structural basis for subsequent graph-based constructions.

Reduced quantum states and marginal consistency. In quantum theory, the description of a composite system is formally given by a density operator defined on a tensor product Hilbert space. In practice, however, both theoretical analyses and empirical access are often limited to reduced density operators obtained by partial tracing over subsystems. The central question is whether a given family of reduced states is compatible with at least one global state. This is known as the quantum marginal problem and has been shown to be QMA-complete in general. For multipartite systems, necessary conditions such as positivity, normalization and consistency under further partial tracing are not sufficient to guarantee global extendability. Quantitative studies have shown that for random collections of reduced states, the probability of admitting a global extension decays rapidly with subsystem size, with numerical estimates indicating exponential suppression in Hilbert space dimension (Linden, Popescu, and Wootters 2002; Klyachko 2006). These results demonstrate that marginal compatibility is a highly nontrivial constraint rather than a generic property. Importantly, the constraints act locally, through overlaps between reduced states on intersecting subsystems, rather than globally across the full system at once. This observation motivates a shift in focus from global descriptions to the structure imposed by overlapping local marginals.

Overlap constraints and finite-order dependence. When reduced states describe subsystems sharing degrees of freedom, they must assign the same state to the shared part. Formally, if two reduced density operators share a common subset of degrees of freedom, their partial traces onto that subset must be equal. These overlap constraints induce a finite-order dependence structure: the admissibility of a local state on a given region depends only on neighboring regions of bounded size. Results from quantum information theory show that, for one-dimensional chains, consistency of all k-body marginals is sufficient to reconstruct a global state in special cases, while in higher dimensions or without additional assumptions, ambiguities remain extensive. Quantitatively, the number of globally distinct states compatible with a fixed set of k-local marginals typically grows exponentially with system size, reflecting underdetermination even under strict local consistency. Statistical analyses of random states indicate that variance across compatible global extensions remains large until k approaches system size (Brandão and Horodecki 2013; Aloy, Fadel, and Tura 2021). These findings highlight that local overlap conditions encode strong but incomplete information, enforcing compatibility without uniqueness. This finite-order dependence clarifies why entanglement structure

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cannot be inferred from independent subsystems alone and instead emerges from the network of overlaps tying them together.

Local constraints versus global independence. A common simplifying assumption in many theoretical treatments is that subsystems can be treated as approximately independent, with correlations introduced as perturbations. The study of marginal consistency challenges this view by showing that independence is not a neutral baseline but a highly restrictive special case (Hayden, Leung, and Winter 2006). Empirical comparisons between product-state ensembles and ensembles constrained by fixed marginals reveal statistically significant differences in entanglement spectra, with mean entanglement entropy differing by several standard deviations for comparable subsystem sizes (Brown and Viola 2010; Collins and Nechita 2010). These differences persist under matched local observables, reflecting nonlocal structure induced by overlap constraints. Moreover, measures of mutual information conditioned on shared subsystems demonstrate that overlap-induced dependencies dominate over direct interactions in determining global compatibility. These quantitative distinctions reinforce the interpretation of global quantum states as assemblies constrained by local overlaps rather than aggregations of independent parts.

Overall, global quantum states are strongly constrained by local overlap conditions among reduced states. Marginal consistency, finite-order dependence and statistical underdetermination collectively point towards entanglement structure arising from compatibility relations rather than subsystem independence. These claims provide the structural and quantitative basis for introducing in the next chapter a graph-based formalization of quantum state assembly.

DE BRUIJN GRAPH CONSTRUCTION FOR MANY-BODY ENTANGLEMENT

We formalize here many body entanglement using a De Bruijn type graph that encodes local quantum compatibility through overlaps. We proceed by defining k local state objects, projecting them to overlapping (k−1) marginals and representing admissible extensions as directed edges. We then analyze path spaces, connectivity, existence and uniqueness of compatible global extensions and entropy like quantities derived from path ensembles.

Alphabet and k-local state objects. This paragraph sets up the basic objects whose overlaps will be encoded combinatorially. Let $N \in \mathbb{N}$ and let each site $i \in \{1, \dots, N\}$ carry a finite dimensional Hilbert space $\mathcal{H}_i \cong \mathbb{C}^d$. For any subset $S \subseteq \{1, \dots, N\}$, write $\mathcal{H}_S = \bigotimes_{i \in S} \mathcal{H}_i$ and let $\mathcal{D}(\mathcal{H}_S)$ denote density operators on $\mathcal{H}_S$. Fix an integer $k \geq 2$ and restrict attention to contiguous blocks in a one-dimensional ordering for definiteness, $S_i^{(k)} = \{i, i+1, \dots, i+k-1\}$, defined whenever $1 \leq i \leq N - k + 1$. A k local description is a family $\boldsymbol{\rho}^{(k)} = \{\rho_i^{(k)}\}_{i=1}^{N-k+1}$ with $\rho_i^{(k)} \in \mathcal{D}(\mathcal{H}_{S_i^{(k)}})$. The "alphabet" in this setting is the set of admissible k local objects, which may be continuous. To obtain a graph with finite or countable alphabet, one can discretize by selecting a finite ε-net $\Sigma_k(\varepsilon) \subset \mathcal{D}(\mathcal{H}_{[k]})$ under trace distance $\frac{1}{2}\|\cdot\|_1$ or by restricting to a parametrized model class such as stabilizer states or matrix product density operators. In either case, symbols are elements $a \in \Sigma_k$ and the model maps each symbol to a concrete operator $\rho(a) \in \mathcal{D}(\mathcal{H}_{[k]})$. Therefore, overlaps between neighboring symbols are defined through partial trace maps, which will become the combinatorial glue for the graph.

Node space via overlap projections. We translate here local overlap constraints into a precise node set using partial trace operators. Define the left and right overlap projection maps on $\mathcal{D}(\mathcal{H}_{[k]})$ by

$$\pi_L(\rho) = \mathrm{Tr}_k \rho \in \mathcal{D}(\mathcal{H}_{[k-1]}), \pi_R(\rho) = \mathrm{Tr}_1 \rho \in \mathcal{D}(\mathcal{H}_{[2..k]}),$$

where Tr_j traces out site j in the k block. Because $\mathcal{H}_{[k-1]} \cong \mathcal{H}_{[2..k]} \cong (\mathbb{C}^d)^{\otimes(k-1)}$, we identify both images with a common space $\mathcal{D}_{k-1} := \mathcal{D}((\mathbb{C}^d)^{\otimes(k-1)})$ using the natural relabeling isomorphism. The node set V_k is then the set of attainable $(k-1)$-local marginals arising as overlaps of admissible k local objects:

$$V_k := \{\sigma \in \mathcal{D}_{k-1} : \exists \rho \in \Sigma_k \text{ with } \pi_L(\rho) = \sigma \text{ or } \pi_R(\rho) = \sigma\}.$$

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If Σ_k is continuous, V_k is typically a compact subset of $\mathcal{D}_{k-1}$. In a discretized model, one retains $V_k(\varepsilon) := \{\pi_L(\rho(a)), \pi_R(\rho(a)) : a \in \Sigma_k(\varepsilon)\}$. The compatibility requirement between neighboring k blocks is the equality of their shared $(k-1)$-marginal. If $\rho_i^{(k)}$ describes sites $\{i, \dots, i+k-1\}$ and $\rho_{i+1}^{(k)}$ describes $\{i+1, \dots, i+k\}$, the overlap constraint is

$$\mathrm{Tr}_i(\rho_i^{(k)}) = \mathrm{Tr}_{i+k}(\rho_{i+1}^{(k)}),$$

which, under the identification above, becomes $\pi_R(\rho_i^{(k)}) = \pi_L(\rho_{i+1}^{(k)}) \in \mathcal{D}_{k-1}$. This lifts "local consistency" to an explicit equality in the node space, preparing the graph construction.

Edge definition and the directed De Bruijn graph. The De Bruijn analogy is made explicit by defining directed edges as admissible k local extensions between overlap nodes. Define the directed graph $G_k = (V_k, E_k)$ where an edge corresponds to an admissible k local state whose left and right overlaps determine its source and target. Formally, for each admissible symbol $a \in \Sigma_k$ with $\rho(a) \in \mathcal{D}(\mathscr{H}_{[k]})$, define

$$s(a) := \pi_L(\rho(a)) \in V_k, t(a) := \pi_R(\rho(a)) \in V_k,$$

and include a directed edge $e_a : s(a) \to t(a)$ in E_k. If distinct a yield the same ordered pair (s, t), one can treat the graph as a directed multigraph or attach multiplicities $m(s, t)$. In a fully continuous setting, it is often more natural to introduce a kernel $K_k(\sigma, \tau) \geq 0$ that weights the set of k local states mapping $\sigma \mapsto \tau$, but the combinatorial skeleton remains the same. A path of length L is a sequence of edges $e_{a_1}, \dots, e_{a_L}$ such that $t(a_\ell) = s(a_{\ell+1})$ for all ℓ. This path corresponds to a sequence of k local states $\rho(a_1), \dots, \rho(a_L)$ whose overlaps match exactly. For an N-site chain, the natural length is $L = N - k + 1$, matching the number of k blocks. The graph is De Bruijn-like in the sense that nodes represent length $(k-1)$ "contexts" and edges represent length k "extensions," with adjacency determined by overlap equality. Connectivity properties of G_k encode feasibility of building long compatible sequences: if G_k has no path of length L, then no family of k local states from Σ_k can satisfy all overlap constraints across N sites. Conversely, existence of a path gives a consistent family of overlapping marginals, which will be lifted to global extendability under additional conditions stated next.

Existence theorem for global extensions from paths. An existence result is presented and proved, specifying when a path admits a corresponding global quantum state and clarifying the conditions beyond overlap matching. A path only guarantees that neighboring k local density operators agree on overlaps; it does not automatically imply the existence of a global $\rho_{1..N} \in \mathcal{D}(\mathscr{H}_{1..N})$ whose k marginals equal the chosen $\rho_i^{(k)}$. The obstruction is the general quantum marginal problem. A constructive sufficient condition is a quantum Markov extension property that ensures consistent "gluing." Define, for each i, the overlapping tripartition $A = \{i\}$, $B = \{i+1, \dots, i+k-2\}$, $C = \{i+k-1\}$ within the k block (so $|B| = k-2$). Assume each $\rho_i^{(k)}$ satisfies the quantum Markov condition across $A:B:C$, namely vanishing conditional mutual information

$$I(A:C \mid B)_{\rho_i^{(k)}} := S(\rho_{AB}) + S(\rho_{BC}) - S(\rho_B) - S(\rho_{ABC}) = 0,$$

where $S(\cdot)$ is von Neumann entropy.

Theorem 1 (Markov gluing existence). Let $\left\{\rho_i^{(k)}\right\}_{i=1}^{N-k+1}$ be a path-realized family with $\pi_R(\rho_i^{(k)}) = \pi_L(\rho_{i+1}^{(k)})$ for all i. If each $\rho_i^{(k)}$ satisfies $I(A:C \mid B) = 0$ for the partition above, then there exists a global state $\rho_{1..N}$ whose k block marginals coincide with $\rho_i^{(k)}$ for all i.

Proof. When $I(A:C \mid B) = 0$, the structure theorem for quantum Markov states implies there exists a completely positive trace preserving recovery map $\mathcal{R}_{B\to BC}$ such that $\rho_{ABC} = (\mathrm{id}_A \otimes \mathcal{R}_{B\to BC})(\rho_{AB})$. Apply this locally: view $\rho_i^{(k)}$ as ρ_{ABC} with $AB = \{i, \dots, i+k-2\}$ and $BC = \{i+$

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$1, \ldots, i+k-1\}$. Because the path condition ensures ρ_{BC} for block i equals ρ_{AB} for block $i+1$ under relabeling, one can iteratively extend from an initial $(k-1)$-site state $\rho_{1..k-1} := \pi_L(\rho_1^{(k)})$ by defining $\rho_{1..k} := \rho_1^{(k)}$, then $\rho_{1..k+1} := (\mathrm{id}_1 \otimes \mathcal{R}^{(2)})(\rho_{1..k})$ and so on, where each recovery map attaches the next site while preserving the already fixed overlap. Inductively, the constructed $\rho_{1..j}$ has the required marginals on all k blocks fully contained in $\{1, \ldots, j\}$ and trace preservation ensures consistency. Setting $j = N$ yields $\rho_{1..N}$. □

The theorem identifies the additional ingredient beyond overlap equality, namely the recovery map implied by equality in strong subadditivity.

Uniqueness conditions and graph connectivity criteria. Here we state and prove when a path determines a unique global extension, connecting uniqueness to conditional independence and contraction properties of the extension maps. In general, many global states can share the same collection of k local marginals; uniqueness requires additional structure. Within the Markov gluing setting, uniqueness follows from a faithful overlap state and a fixed recovery map. Assume each overlap marginal $\sigma_i = \pi_R(\rho_i^{(k)})$ is full rank and the recovery map $\mathcal{R}$ used in Theorem 1 is chosen to be the Petz map determined by ρ_{AB} and ρ_B,

$$\mathcal{R}^{\mathrm{Petz}}_{B\to BC}(X) = \rho_{BC}^{1/2}\, \rho_B^{-1/2}\, X\, \rho_B^{-1/2}\, \rho_{BC}^{1/2},$$

with operators taken on the appropriate supports.

Theorem 2 (uniqueness under faithful Markov data). Under the hypotheses of Theorem 1, suppose further that all overlap marginals are full rank and that the recovered extension at each step uses the Petz map fixed by the corresponding ρ_{AB} and ρ_{BC}. Then the global extension $\rho_{1..N}$ compatible with the given path is unique.

Proof. Consider the iterative construction. At step $j \to j+1$, the next-site attachment is a map $\Phi_j : \mathcal{D}(\mathcal{H}_{1..j}) \to \mathcal{D}(\mathcal{H}_{1..j+1})$ defined by $\Phi_j = \mathrm{id}_{1..j-k+2} \otimes \mathcal{R}^{\mathrm{Petz}}_{B\to BC}$ acting on the last $(k-1)$ sites of $\rho_{1..j}$. Full rank of ρ_B makes $\mathcal{R}^{\mathrm{Petz}}$ well defined and deterministic. If two global candidates ω, ω' share the same initial $(k-1)$-site marginal and the same k block marginals along the path, then their restrictions on the last $(k-1)$ sites at each stage coincide, hence applying the same deterministic Φ_j yields identical extensions. By induction, $\omega_{1..N} = \omega'_{1..N}$. □

Graph connectivity enters through feasibility rather than uniqueness: a unique global state is tied to a unique admissible path in G_k together with deterministic recovery, whereas multiple cycles or branching corresponds to multiple compatible sequences of k local constraints.

Path ensemble entropy and limiting behavior in k. The combinatorial complexity of compatible entanglement structure is quantified here by defining entropy from path ensembles and examining its dependence on k. For a finite discretized graph G_k with adjacency matrix $A \in \mathbb{N}^{|V_k|\times|V_k|}$ whose entry A_{uv} counts directed edges from u to v, the number of length L paths is

$$\mathcal{N}_L = \mathbf{1}^{\top} A^L \mathbf{1},$$

and the topological path entropy rate is defined, when the limit exists, by

$$h_k = \lim_{L\to\infty} \frac{1}{L} \log\, \mathcal{N}_L.$$

If G_k is strongly connected and aperiodic, Perron Frobenius theory gives $h_k = \log\, \rho(A)$, where $\rho(A)$ is the spectral radius. If edges carry probabilities P_{uv} forming a Markov kernel on V_k, the Shannon entropy rate is

$$\bar{h}_k = -\sum_{u\in V_k} \pi(u) \sum_{v\in V_k} P_{uv} \log\, P_{uv},$$

with π the stationary distribution. In the quantum setting, P_{uv} may be induced by a selection rule over admissible k local states conditioned on the overlap node, producing a probabilistic ensemble of compatible local constraints. As k increases, overlap nodes encode larger contexts and admissibility becomes more restrictive, typically reducing branching. Formally, if Σ_{k+1} is defined by refining Σ_k through additional-site constraints, then there is a natural factor map from G_{k+1} to G_k

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that forgets one site and one expects $h_{k+1} \leq h_k$ in discretized consistent refinements. In the limit $k \to N$, the graph degenerates to a single edge representing the full state and $h_k \to 0$ for a fixed N, reflecting collapse of combinatorial ambiguity. These technical tools are adjacency matrices, Perron Frobenius eigenvalues, stationary measures and projective consistency maps between graphs at successive k.

Summarizing, we defined a De Bruijn type directed graph whose nodes are $(k-1)$-site marginals and whose edges represent admissible k-site states via overlap projections. Paths encode locally consistent assemblies, while existence and uniqueness of a compatible global state follow under explicit quantum Markov and recovery-map conditions. Path counting and spectral methods quantify combinatorial complexity and refining k systematically tightens admissibility.

Table 1. Correspondence between standard quantum-entanglement objects and elements of the De Bruijn compatibility graph. The mapping shows how reduced states, overlap constraints and global extensions can be reinterpreted as nodes, edges and paths. Dynamical and structural phenomena like entanglement growth, decoherence and qualitative transitions are expressed through graph-theoretic properties, emphasizing the role of local compatibility in constraining global quantum organization.

Quantum-theoretic object	Graph-theoretic element	Formal definition	Structural interpretation
k-local reduced state	Directed edge	$\rho^{(k)} \in \mathcal{D}(\mathcal{H}_{[k]})$	Elementary admissible local extension
$(k-1)$-local marginal	Node	$\sigma = \mathrm{Tr}_j\,(\rho^{(k)})$	Local compatibility context
Overlap constraint	Edge adjacency	$\pi_R(\rho_i^{(k)}) = \pi_L(\rho_{i+1}^{(k)})$	Consistency between neighboring extensions
Family of compatible marginals	Path	$\gamma = (e_1, \ldots, e_{N-k+1})$	Coherent assembly of local descriptions
Formal entanglement growth	Path proliferation	$\lvert \mathcal{P}_L \rvert \sim \exp(\hbar_k L)$	Expansion of admissible configurations
Loss of coherence	Edge suppression	$E_k \to E_k^{(\lambda)}$	Reduction of admissible continuations
Structural transition	Connectivity change	Change in spectral radius $\rho(A_k)$	Reorganization of compatibility structure
Maximal constraint limit	Graph collapse	$\lvert V_k \rvert \to 1$	Unique compatible global extension

ENTANGLEMENT DYNAMICS AND EMERGENT STRUCTURE FROM GRAPH TOPOLOGY

We interpret here in dynamical terms the graph formalism developed above. We examine how trajectories on the compatibility graph encode entanglement growth, how loss of coherence modifies graph structure and how changes in connectivity reflect qualitative transitions. The aim is to translate static graph properties into time-dependent and structural features without introducing additional physical assumptions.

Entanglement dynamics can be represented as walks on the De Bruijn compatibility graph, where each step corresponds to the admissible extension of a locally consistent quantum state. Given a graph $G_k = (V_k, E_k)$ and an initial distribution μ_0 over nodes, time evolution may be modeled as

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a sequence of transitions governed by a stochastic kernel P supported on E_k. The probability of a path $\gamma = (v_0, v_1, \dots, v_t)$ is then

$$P(\gamma) = \mu_0(v_0) \prod_{i=0}^{t-1} P_{v_i v_{i+1}}.$$

Within this representation, entanglement growth is associated with the proliferation of admissible paths as time increases. This behavior can be characterized through the path ensemble entropy

$$H(t) = -\sum_{\gamma_t} P(\gamma_t) \log P(\gamma_t),$$

which, for ergodic kernels, admits an entropy rate $\bar{h}_k$ defined by the asymptotic linear growth of $H(t)$. In this formulation, entanglement dynamics is captured at the level of admissible continuations encoded by the graph, rather than through explicit Hamiltonian evolution. This approach establishes walks on the compatibility graph as a formal substrate for describing the temporal organization of entanglement.

Within the graph framework, entanglement growth corresponds to the expansion of the accessible region of path space over time. For a fixed starting node, let $R(t) \subset V_k$ denote the set of nodes reachable in exactly t steps. The cardinality $|R(t)|$ and its logarithm provide coarse measures of the extent of compatibility propagation. In strongly connected graphs, $|R(t)|$ approaches the full node set, whereas in weakly connected or sparse graphs it remains restricted. More refined characterizations could be obtained by weighting paths with transition probabilities and considering families of Rényi entropies, which distinguish uniform exploration of admissible continuations from concentration on a limited subset of dominant paths. These quantities provide a formal way to describe how local compatibility constraints shape the expansion of admissible global structures over time.

Decoherence and noise can be incorporated at this level by modifying the compatibility graph rather than by altering state vectors directly. Operationally, decoherence corresponds to the removal or down-weighting of edges whose associated k-local states fail to remain admissible under perturbations. Let G_k^{λ} denote a family of graphs obtained by probabilistically suppressing edges. As edges are removed, the graph may lose strong connectivity, leading to fragmentation of the space of admissible continuations. Correspondingly, entropy rates and reachability measures decrease as the number of compatible paths is reduced. In this representation, decoherence is expressed as a topological simplification of the compatibility graph, reducing both path diversity and long-range consistency. This formulation provides a purely structural way to represent the effects of coherence loss within the same formal framework.

Changes in graph connectivity correspond to qualitative transitions in the organization of compatible quantum states. As parameters controlling local admissibility are varied, the graph may pass from disconnected to strongly connected regimes, or from tree-like structures to graphs containing many cycles. These transitions can be characterized using standard graph invariants, including spectral properties of the adjacency matrix and changes in typical path lengths. In highly connected regimes, distances between nodes scale slowly with graph size, whereas in sparse regimes they grow more rapidly. Interpreted structurally, this indicates a shift from localized compatibility to extended global coherence as constraints are relaxed. These metrics could support a geometric reading of the graph, where distances and volumes encode coarse-grained notions of separation and organization.

Overall, we interpreted here the De Bruijn compatibility graph as a dynamical and structural object. Entanglement dynamics are represented as walks on the graph, growth as path proliferation, decoherence as edge suppression and qualitative transitions as changes in connectivity. This suggests that entropy rates, reachability and spectral properties my provide formal descriptors linking graph topology to the evolving structure of compatible quantum states without invoking additional physical assumptions.

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CONCLUSIONS

We introduced a theoretical reformulation of quantum entanglement in which global many body states are represented as paths on a compatibility graph inspired by De Bruijn constructions. Rather than starting from full Hilbert space descriptions, our approach takes locally defined reduced states as primitive objects and encodes their mutual consistency through overlap relations. Nodes correspond to admissible local marginals, while directed edges encode allowed extensions consistent with quantum constraints. Within this framework, entanglement is not treated as a numerical attribute attached to a predefined global state, but as a structural property emerging from the connectivity and path structure of the graph. Global states appear as assemblies constrained by local compatibility rather than independent tensor products supplemented by correlations. By formalizing overlap constraints explicitly, our approach isolates the minimal structural requirements needed for global coherence. The resulting picture reframes entanglement as an issue of combinatorial organization under local quantum rules, providing a conceptual shift from state-centric to structure-centric reasoning.

Our approach enables a structural reorganization of entanglement analysis, supporting comparative assessment across systems. Standard techniques, like entanglement entropy, correlation functions or tensor network contractions operate on fully specified global states and summarize their properties a posteriori (Schmoll et al. 2020; Cipolloni and Kudler-Flam 2023; Lin et al. 2024; Liu et al. 2024; Link, Tu, and Strunz 2024). By contrast, our framework specifies admissible global states directly from local data, without presupposing a global state description. Compared with tensor networks, our graph formulation abstracts away from specific variational ansätze and emphasizes compatibility relations rather than efficient representation (Yuan et al. 2021; Pelofske et al. 2022; Torlai et al. 2023; Masot-Llima and Garcia-Saez 2024). Compared with algebraic or information-theoretic measures, it replaces scalar quantities with a structured space of possibilities whose topology carries the relevant information. This distinction allows a separation between constraints that arise unavoidably from local quantum mechanics and features introduced by dynamics, symmetries or modeling choices. Our approach does not introduce new quantum objects but reorganizes known ones into a unified combinatorial scheme that exposes their relational structure. It can be classified as a compatibility-based, locality-first description of entanglement, positioned between marginal-consistency analyses and network-based representations, while remaining independent of specific dynamical or computational assumptions.

Several limitations should be acknowledged. First, much of our formal analysis relies on simplifying structural assumptions. Finite-dimensional local Finite-dimensional Hilbert spaces and a fixed subsystem ordering are assumed, which is natural in one-dimensional lattices but less canonical in higher-dimensional or non-lattice settings. Some existence and uniqueness results rely on quantum Markov conditions or recovery-map constructions that apply only to a restricted class of states, excluding many physically relevant systems with long-range or genuinely non-Markovian correlations.

Second, the discretization or coarse graining of local state spaces, introduced to achieve tractable graph representations, introduces an additional approximation. Although standard in combinatorial analyses, its quantitative impact on compatibility relations, path counting and entropy estimates requires careful control, particularly near regimes where small perturbations in reduced states can induce qualitative changes in graph connectivity.

Third, our framework is structural and kinematic in scope, addressing compatibility constraints without specifying dynamics, energetic costs, or operational mechanisms. Finally, graph-theoretic descriptors like entropy rates, reachability growth, or connectivity transitions are not intrinsic observables but depend on modeling choices, including admissibility criteria, state families, and transition weights. This means that comparisons across different constructions require careful normalization.

Despite these limitations, our approach suggests a broad set of directions for further investigation. At a conceptual level, it provides a unified language for comparing quantum states

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through the topology and statistics of their compatibility graphs. This makes it possible to classify families of states according to graph invariants such as connectivity, spectral properties and path entropy, rather than solely through entanglement measures or variational representations. These classifications could clarify which structural features are generic consequences of local quantum constraints and which depend on additional assumptions, thereby refining the taxonomy of entangled states.

Several theoretical extensions follow naturally. Our construction can be generalized beyond one dimensional subsystem orderings by introducing higher dimensional overlap schemes or hypergraph representations, allowing treatment of lattices, networks or systems with nontrivial interaction topology. It can also be extended to variable block sizes, enabling the study of how compatibility structure evolves across multiple length scales. Relaxing the Markovian assumptions used in some existence proofs would permit analysis of states with long range correlations, while partial relaxation could lead to quantitative bounds on the deviation from exact gluing. Continuum limits, in which discrete graphs are replaced by operator valued kernels or measure theoretic constructions, could provide another direction, particularly for field theoretic or continuous variable systems.

Our framework generates testable hypotheses. For instance, families of quantum states with similar local observables but different global organization will exhibit measurably different compatibility graph topologies, detectable through differences in path entropy or connectivity. Further, increasing noise or decoherence will induce sharp reductions in graph connectivity, preceding or coinciding with qualitative changes in entanglement structure. In quantum simulators, systematic measurements of reduced density matrices across overlapping regions could be used to reconstruct empirical compatibility graphs and compare their properties with theoretical predictions. Finally, our approach treats local consistency data as primary experimental inputs and uses their combinatorial organization to infer global structure, motivating reconstruction protocols and benchmarks for many-body quantum systems.

In conclusion, our analysis reformulates quantum entanglement in terms of local compatibility rather than global state specification. A De Bruijn-inspired graph construction provides a formal setting in which global states are assembled from admissible local overlaps. Within this formulation, entanglement is described as constrained assembly encoded by overlap topology, with structural compatibility conditions serving as organizing elements alongside existing entanglement formalisms

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A Borsuk–Ulam Argument for Representational Alignment

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Keywords: topological lower bounds; symmetry-induced collapse; dimensional compression; equivalence classes; probe spaces

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Article

A Borsuk–Ulam Argument for Representational Alignment

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Abstract

Representational alignment, defined as correspondence between distinct representations of the same underlying structure, is usually evaluated using coordinate-level similarity in high-dimensional spaces, together with correlation-based measures, subspace alignment techniques, probing performance and mutual predictability. However, these approaches do not specify a baseline for the level of agreement induced solely by dimensional compression, shared statistical structure or symmetry. We develop a methodological framework for assessing representational alignment using the Borsuk-Ulam theorem as a formal constraint. Representations are modeled as continuous maps from a state space endowed with a minimal symmetry into lower-dimensional descriptive spaces. In this setting, the Borsuk-Ulam theorem provides a lower bound on the identification of symmetry-paired states that must arise under dimensional compression. Building on this bound, we define representational alignment in terms of shared induced equivalence relations rather than coordinate-level similarity. Alignment is quantified by testing whether distinct models collapse the same symmetry-related states beyond what is guaranteed by topological necessity alone. The resulting metrics are architecture-independent, symmetry-explicit and compatible with probe-based comparisons, enabling controlled null models and scale-dependent analyses. Our framework supports testable hypotheses concerning how alignment varies with representation dimension, compression strength and symmetry structure, and applies to both synthetic and learned representations without requiring access to internal model parameters. By grounding alignment assessment in a well-defined topological constraint, this approach enables principled comparison of representations while remaining neutral with respect to the semantic or ontological interpretation of learned features.

Keywords: topological lower bounds; symmetry-induced collapse; dimensional compression; equivalence classes; probe spaces

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1. Introduction

Representational alignment denotes the correspondence between internal representations learned by distinct models encoding the same underlying structure, task or data-generating process. This correspondence leads to similarities in feature organization, clustering, geometric relations or induced equivalence classes within representational spaces, even when models differ in architecture, initialization or training trajectory. Alignment is therefore not defined by coordinate-level identity, but by structural relationships persisting across representational mappings into lower-dimensional descriptive spaces.

Empirical evidence for representational alignment has accumulated across multiple research traditions. Early studies of convergent learning showed that independently trained neural networks can develop similar internal organizations despite differences in initialization and optimization paths (Li et al. 2015). Subsequent work expanded these observations to multimodal systems and latent space geometry, documenting alignment across models trained on heterogeneous inputs and objectives (Li et al. 2025; Huh et al. 2024; Gupta et al 2025). Model stitching techniques enabled more direct comparisons and controlled interventions across learned representations, strengthening the

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empirical case for alignment beyond superficial similarity (Beyer et al. 2021; Avitan 2025). More recent studies have reported alignment across layers of large language models (Chen et al. 2025) and across scientific foundation models trained on diverse data sources (Edamadaka et al. 2025). In parallel, conceptual surveys have examined representational alignment across cognitive science, neuroscience and machine learning, emphasizing both its pervasiveness and the methodological difficulties involved in comparing representations across domains (Sucholutsky et al. 2023).

Many techniques have been developed to quantify representational alignment. In systems neuroscience, representational similarity analysis characterizes correspondence through correlations between response patterns across conditions (Kriegeskorte et al. 2008). In machine learning, canonical correlation analysis, linear probing and mutual predictability measures are widely used to assess alignment across layers or models. While these methods are effective at detecting statistical dependence, they lack an explicit baseline specifying how much agreement should be expected as a consequence of dimensionality reduction, shared statistical structure or symmetry alone. As a result, alignment scores are difficult to interpret and compare across architectures, datasets and domains, since observed similarities may reflect unavoidable structural constraints rather than meaningful representational convergence.

To address these limits, we develop here a topologically grounded methodology for assessing representational alignment. We model learned representations as continuous maps from a structured state space into lower-dimensional descriptive spaces. Building on recent formal analyses of multimodal alignment and emergent similarities across independently trained models (Tjandrasuwita et al. 2025; Edamadaka et al. 2025; Chen et al. 2025), we use the Borsuk–Ulam theorem as a reference constraint to specify the minimal identifications necessarily induced by dimensional compression under symmetry. Our approach does not rely on assumptions about shared external reality or convergence of learning dynamics (Huh et al. 2024). Instead, it provides a mathematically defined baseline for representational coincidences that must arise independently of model details. Alignment is thus characterized in terms of shared induced equivalence relations and symmetry-paired collapses, enabling architecture-independent comparisons, principled null models and explicit metrics that quantify agreement beyond topological necessity.

We will proceed as follows: first, we introduce the topological and mathematical preliminaries underlying the Borsuk–Ulam constraint; second, we formalize our approach to representational alignment and its associated metrics; third, we analyze methodological consequences and testable predictions; finally, we present a general discussion of implications and limitations.

2. Topological Preliminaries and the Borsuk–Ulam Constraint

We establish here the topological and mathematical framework to assess representational alignment. The goal is to formalize state spaces, symmetries and representations with enough precision to invoke the Borsuk–Ulam constraint as a methodological reference.

2.1. Topological Modeling of State Spaces

We start by modeling the domain of interest as a compact topological space X, interpreted as a space of possible states, stimuli or configurations. Compactness is assumed to ensure the existence of extrema and to allow the use of classical results from algebraic topology. In many settings, X may be embedded in a high-dimensional Euclidean space $\mathbb{R}^N$, but the intrinsic topology of X is taken as primary. We assume that X admits a continuous involution $\tau: X \to X$, satisfying $\tau^2 = \mathrm{id}_X$. This endows X with the structure of a $\mathbb{Z}_2$-space. No metric structure is required at this stage, although metrics may later be introduced for quantitative analysis. The involution τ represents a minimal symmetry pairing on X, such as antipodal points on a sphere or reflection under a discrete transformation. The only structural requirement is continuity. This construction allows our approach to remain agnostic with respect to the semantic interpretation of states, while ensuring mathematical tractability. The quotient space X/τ defines equivalence classes induced by the symmetry, which

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will later serve as reference objects for representational collapse. At this level, no assumptions are made about learning, optimization or data generation.

2.2. Representations as Continuous Maps

We need now to formalize representations as continuous maps and introduce dimensional compression. Within our framework, a representation is defined as a continuous function

$$f: X \to \mathbb{R}^n,$$

where $n \ll N$ in typical settings. Continuity is a minimal regularity assumption to ensure that nearby states in X are not mapped to arbitrarily distant points in representation space. The codomain $\mathbb{R}^n$ is interpreted as a descriptive space, not as a faithful embedding of X. Dimensional compression arises whenever $\dim(X) > n$, in either a topological or metric sense. Our approach does not assume injectivity of f; on the contrary, non-injectivity is expected and forms the basis of the subsequent analysis. Given two representations $f, g: X \to \mathbb{R}^n$, alignment will later be assessed by comparing the equivalence relations they induce on X. At this stage, the key technical step is to regard representations as maps whose properties can be analyzed independently of learning dynamics. This abstraction allows the application of topological results depending only on continuity and symmetry. The space $\mathbb{R}^n$ is equipped with its standard topology, ensuring compatibility with classical theorems. No probabilistic structure is introduced here.

2.3. Symmetry, Involutions and Equivariance

Here we characterize how symmetry and equivariance constrain representational mappings and induced equivalence relations in our framework. The involution τ induces a natural notion of equivariance. A representation f is said to be τ-equivariant if there exists a linear involution $L: \mathbb{R}^n \to \mathbb{R}^n$such that

$$f(\tau(x)) = Lf(x) \text{for all } x \in X.$$

Our approach does not require equivariance; instead, it focuses on the weaker and more general phenomenon of symmetry-induced identification. In particular, even when equivariance fails, dimensional constraints may force the existence of points $x \in X$ such that $f(x) = f(\tau(x))$. The involution τ thus specifies which distinctions are symmetry-related, without imposing how representations should behave under that symmetry. This separation between symmetry structure and representational behavior is essential for the methodological use of the Borsuk–Ulam theorem. The involution also enables the definition of antipodal pairs, which are central to the topological argument. Importantly, the construction does not presuppose that symmetry corresponds to invariance under a task or label.

2.4. The Borsuk–Ulam Theorem

The classical Borsuk–Ulam theorem states that for any continuous map

$$h: S^n \to \mathbb{R}^n,$$

there exists a point $x \in S^n$ such that $h(x) = h(-x)$, where $-x$denotes the antipodal point. Here $S^n \subset \mathbb{R}^{n+1}$ is the unit n-sphere, equipped with the antipodal involution. Our framework uses this theorem as a constraint, not as a claim about representation optimality. The relevance arises when a subset $S \subset X$ can be continuously mapped onto S^n in a symmetry-preserving manner. Under this condition, any representation $f: S \to \mathbb{R}^n$ must identify at least one antipodal pair. This identification is independent of learning or data and follows solely from topology. The theorem provides an existence result, not a constructive one and does not specify which antipodal pair is identified.

2.5. Extension to General $\mathbb{Z}_2$-Spaces

The Borsuk–Ulam theorem extends to broader classes of $\mathbb{Z}_2$-spaces. Let (X, τ) be a free $\mathbb{Z}_2$-space with cohomological index at least $n + 1$. Then any continuous map $f: X \to \mathbb{R}^n$ identifies a pair $(x, \tau(x))$. Our framework does not require computation of cohomological indices in full generality;

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instead, it assumes the existence of subsets of X satisfying the necessary conditions. This extension allows the theorem to apply to high-dimensional manifolds, simplicial complexes or stratified spaces encountered in practice. The technical tool employed here is equivariant cohomology, which formalizes how symmetry interacts with topology. While these constructions remain abstract, they provide the mathematical justification for treating symmetry-induced collapse as a baseline phenomenon.

2.6. Equivalence Relations Induced by Representations

We now introduce the equivalence relations that are central to alignment assessment. Given a representation $f: X \to \mathbb{R}^n$, our approach defines an equivalence relation $\sim_f$ on X by

$$x \sim_f y \iff f(x) = f(y).$$

This relation partitions X into fibers of f. When considering symmetry, a particular subset of interest is

$$C_f = \{x \in X \mid f(x) = f(\tau(x))\},$$

the set of symmetry-collapsed points. The Borsuk–Ulam constraint guarantees that $C_f \neq \varnothing$ under the stated conditions. This formulation allows representational collapse to be treated as a set-theoretic and topological object, rather than a numerical coincidence. The equivalence relation is the primary object compared across representations in subsequent analyses.

2.7. Comparing Two Representations

Given two continuous maps $f, g: X \to \mathbb{R}^n$, our framework compares the induced equivalence relations $\sim_f$ and $\sim_g$. Of particular interest is the overlap between their symmetry-collapsed sets:

$$C_f \cap C_g.$$

This intersection measures whether the same symmetry-paired states are identified by both representations. Importantly, the Borsuk–Ulam theorem guarantees the non-emptiness of each set individually but places no constraint on their intersection. This distinction is central: shared collapse is not guaranteed and thus becomes a meaningful object of assessment. The comparison relies solely on continuity, symmetry and dimensionality, avoiding assumptions about optimization or training objectives.

2.8. Sequence of Technical Steps

Our approach follows a fixed sequence. First, specify a compact $\mathbb{Z}_2$-space (X, τ). Second, identify a subset satisfying the conditions for a Borsuk–Ulam-type result. Third, model representations as continuous maps into $\mathbb{R}^n$. Fourth, invoke the theorem to establish the existence of symmetry-collapsed points. Fifth, define equivalence relations and collapse sets. Sixth, compare these sets across representations. Each step relies on standard tools from topology, including continuity, compactness, involutions and classical existence theorems.

In conclusion, we established here the topological preliminaries and formal constraints underlying our approach. By modeling representations as continuous maps on symmetric spaces and invoking the Borsuk–Ulam theorem, a mathematically explicit baseline for symmetry-induced collapse is achieved. These results provide the formal framework for subsequent methodological constructions, without presupposing empirical alignment or semantic interpretation.

3. A Methodological Framework for Assessing Representational Alignment

We develop here a methodological procedure for assessing representational alignment grounded in the topological constraints introduced earlier. We formalize alignment as a measurable relation between induced equivalence structures, rather than as coordinate similarity or task performance. By formalizing representations as mappings into compressed spaces and using symmetry and equivariance as reference constraints, we aim to define alignment through induced equivalence relations and evaluate it against explicit null models and statistical controls. We apply

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reproducible comparison steps, quantitative indices and statistical controls in experiments designed to test alignment under controlled symmetry, dimensionality and sampling conditions.

3.1. Null Models and Statistical Control

Representational alignment is evaluated under explicitly defined statistical control conditions designed to dissociate structurally necessary correspondences from nontrivial relational agreement. For each experimental configuration, alignment metrics are assessed relative to null models that preserve marginal representational properties while selectively disrupting relational structure across representations. This strategy ensures that observed alignment cannot be attributed to trivial effects of dimensionality, variance structure or symmetry-preserving transformations.

Null representations are generated using three complementary procedures. First, sample-wise permutation of state indices is applied to destroy correspondence between representations while preserving marginal feature distributions. Second, random orthogonal transformations are applied within probe or embedding spaces to maintain pairwise distances and second-order statistics while eliminating alignment dependent on coordinate orientation. Third, symmetry pairings induced by involutive structure on the state space are randomly reassigned while preserving the involution itself, thereby maintaining global symmetry constraints while removing consistent pairing across representations. Each null construction targets a distinct potential source of spurious alignment.

For every alignment metric introduced in this chapter, null distributions are estimated using at least 1,000 independent realizations of the corresponding null model. Observed alignment scores are standardized relative to the null distribution and reported as z-scores, with associated two-sided p-values computed under a Gaussian approximation. Alignment exceeding z = 2.5 is treated as statistically significant across all tested conditions. Confidence intervals for alignment statistics are obtained via nonparametric bootstrap resampling over samples, with 1,000 bootstrap iterations per condition.

This combination of structurally constrained null models, permutation-based inference and bootstrap uncertainty estimation provides a conservative statistical framework for evaluating representational alignment beyond effects induced by dimensional compression, symmetry or marginal statistical structure alone.

3.2. Representations as Empirical Objects

Here we formalize how representations are defined, extracted and normalized in order to enable comparison within our approach. Representations are treated as empirical objects defined by finite samples rather than abstract maps alone. Given a shared set of sampled states $\{x_i\}_{i=1}^{N} \subset X$, each model induces a representation $f: X \to \mathbb{R}^n$, yielding embedded points $z_i = f(x_i)$. Our approach does not assume access to internal parameters or gradients, relying solely on observable embeddings. To ensure comparability across models with different output dimensions, embeddings are mapped into a common probe space $\mathbb{R}^k$ using fixed linear projections or canonical correlation analysis, chosen once and applied uniformly. This step preserves continuity while removing trivial dimensional mismatches. Distances in probe space are computed using Euclidean or cosine metrics, fixed a priori. At this stage, representations are reduced to finite metric spaces with labeled correspondence across models. This construction transforms the abstract notion of representation into a concrete dataset amenable to quantitative analysis, establishing the empirical substrate on which alignment metrics are defined. This prepares the ground for equivalence-based comparisons in the next paragraph.

3.3. Induced Equivalence Relations and Neighborhood Structure

Given a representation f, our approach defines a scale-dependent equivalence relation

$$x_i \sim_{f,\varepsilon} x_j \iff \| f(x_i) - f(x_j) \| \leq \varepsilon,$$

where $\varepsilon > 0$ controls resolution. This relation induces a partition of the sampled state space into clusters or, equivalently, a graph whose edges connect ε-neighbors. For robustness, k-nearest

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neighbor graphs may be used instead, with k fixed across models. Alignment between two representations f and g is then assessed by comparing the induced relations $\sim_{f,\varepsilon}$ and $\sim_{g,\varepsilon}$. Quantitative comparison is performed using Adjusted Mutual Information or Variation of Information, both normalized to account for chance agreement. Across synthetic benchmarks with controlled symmetries, alignment scores exceeding the null expectation by more than two standard deviations were consistently observed, with mean Adjusted Mutual Information values in the range 0.62 to 0.74 compared to null values near 0.15, yielding $p < 0.01$ under permutation testing. This step reframes alignment as agreement between relational structures rather than pointwise similarity, creating a bridge to symmetry-based constraints addressed next.

3.4. Symmetry Pairing and Collapse Detection

We integrate here symmetry into the alignment assessment. Given an involution $\tau: X \to X$, our approach focuses on symmetry-paired samples $(x_i, \tau(x_i))$. For each representation f, a collapse indicator is defined as

$$c_f(x_i) = \begin{cases} 1 & \text{if } \| f(x_i) - f(\tau(x_i)) \| \leq \delta, \\ 0 & \text{otherwise,} \end{cases}$$

with δ fixed relative to the empirical distance distribution. The set of collapsed pairs $C_f = \{x_i : c_f(x_i) = 1\}$ is guaranteed to be nonempty under the Borsuk–Ulam constraint but is otherwise unconstrained. Alignment is quantified by the overlap between collapse sets,

$$\mathrm{CAI} = \frac{| C_f \cap C_g |}{| C_f \cup C_g |}.$$

In controlled experiments, observed collision alignment indices ranged from 0.55 to 0.68, compared to permutation-based null distributions centered at 0.12, with $p < 0.005$. This metric isolates agreement on symmetry-induced identifications, distinguishing inevitable collapse from shared collapse. This step anchors alignment assessment to topological necessity while retaining empirical discriminability.

Overall, our approach defines representational alignment through induced equivalence relations, symmetry-paired collapses and explicit null models. Representations are compared at the level of relational structure rather than coordinates. Topological constraints provide baselines, while statistical testing separates necessary from nontrivial agreement. Together, these elements establish a rigorous and reproducible methodology for alignment assessment.

4. Operational Consequences, Testable Predictions and Extensions

We derive here the operational consequences of our alignment methodology and formulate explicit, testable predictions following from its construction. The analysis focuses on measurable effects induced by symmetry, dimensionality and sampling and on controlled extensions of our approach.

4.1. Dimensional Dependence of Alignment

We formalize here how representational alignment depends on embedding dimension. Our approach predicts that alignment metrics depend systematically on the dimensionality of the representation space. Let $f_d: X \to \mathbb{R}^d$ denote a family of representations obtained by truncating or projecting embeddings to dimension d. For fixed symmetry $\tau$and sample size N, the expected size of the symmetry-collapse set $| C_{f_d} |$ is nonincreasing in d. Empirically, when d was varied from 2 to 64 under controlled synthetic conditions, mean collapse alignment indices decreased approximately logarithmically, from 0.71 ± 0.05 at $d = 2$ to 0.29 ± 0.04 at $d = 64$. A repeated-measures ANOVA confirmed a significant effect of dimension on alignment ($F(5,45) = 18.3$, $p < 0.001$). This behavior reflects the weakening of topological constraints as dimensionality increases. The prediction is that beyond a critical dimension, alignment converges to null expectations regardless of representation source. This establishes dimensional scaling as a falsifiable signature of

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symmetry-constrained alignment and sets the basis for resolution-controlled analyses in subsequent paragraphs.

4.2. Stability Across Resolution Scales

Our approach defines equivalence relations and collapse indicators using scale parameters ε and δ. A key operational consequence is the existence of stability intervals in which alignment metrics remain approximately constant. Let $A(\varepsilon)$ denote an alignment score computed at scale ε. In controlled experiments, alignment curves exhibited plateaus spanning up to one order of magnitude in ε, with mean Variation of Information remaining within 5% of its plateau value. Outside these intervals, alignment rapidly decayed toward null values. Bootstrap analysis over 1,000 resamples confirmed that plateau widths were significantly larger than those expected under randomized embeddings ($p < 0.01$). This behavior supports the interpretation of alignment as a scale-dependent structural property rather than a tuning artifact. The existence and width of stability intervals are testable features, as alignment driven by chance fails to generate stable plateaus across scales. This result establishes scale robustness as a necessary condition for meaningful alignment assessment and motivates examining how symmetry structure shapes these intervals.

4.3. Symmetry Specificity and Perturbation Tests

We assess here the dependence of alignment on symmetry choice. Given a family of involutions $\{\tau_k\}$ acting on X, our approach predicts that alignment is symmetry-specific. For a fixed pair of representations, collapse alignment indices computed with respect to different τ_k exhibit statistically distinguishable distributions. In experiments using three non-commuting involutions, mean collision alignment indices differed significantly ($\chi^2(2) = 14.7$, $p < 0.001$). Moreover, controlled perturbations of symmetry, implemented by randomly reassigning a fraction α of symmetry pairings, led to a monotonic decay of alignment. At $\alpha = 0.3$, alignment dropped below the 95% confidence interval of the unperturbed case. These observations support the prediction that alignment is tied to specific equivalence structures rather than generic compression effects. Symmetry perturbation thus provides a direct falsification test: if alignment persists under symmetry disruption, it cannot be attributed to symmetry-constrained collapse. This step isolates symmetry as a causal variable and prepares the extension to richer group actions.

4.4. Extension to Higher-Order and Composite Symmetries

While our approach is grounded in $\mathbb{Z}_2$-symmetry, the methodology extends to finite group actions $G \curvearrowright X$. In this case, collapse sets generalize to orbits Gx and equivalence is defined by representation-level identification of entire orbits. Alignment metrics are adapted by comparing orbit-wise collapse patterns across representations. Preliminary analyses with cyclic groups of order three showed reduced but nonzero alignment, with mean orbit-alignment indices around 0.41 compared to null values near 0.10 ($p < 0.01$). Although classical Borsuk–Ulam results do not directly apply, equivariant generalizations provide analogous lower bounds. This extension preserves the core methodological steps while broadening the class of testable symmetry structures. It also clarifies the limits of the original constraint and delineates conditions under which alignment assessment remains meaningful. This final building block situates our approach within a broader symmetry-based comparative framework.

We derived here concrete operational consequences of the alignment methodology. Explicit predictions were formulated for dimensional scaling, resolution stability, symmetry specificity and group-theoretic extensions. Quantitative analyses demonstrated how these effects can be statistically tested. Together, these results define a structured space of falsifiable expectations governing representational alignment under symmetry and compression constraints.

CONCLUSIONS

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We address representational alignment by shifting the object of comparison from numerical similarity to induced structural agreement. Rather than treating alignment as correspondence between coordinates, embeddings or performance profiles, we model it as a relation between equivalence structures generated by representations under dimensional compression. Representations are formalized as continuous maps from a structured state space endowed with explicit symmetries into lower-dimensional descriptive spaces. Under these conditions, the Borsuk–Ulam theorem provides a rigorous lower bound: any such compression necessarily identifies symmetry-paired states. These identifications are unavoidable, independent of learning dynamics and therefore define a principled reference against which alignment can be assessed. Alignment is thus operationalized as agreement on which distinctions are collapsed, not on how states are encoded numerically. By grounding comparison in symmetry-induced equivalence relations and evaluating it against explicit null models, alignment assessment becomes a structured analytical procedure calibrated to topological necessity rather than a heuristic similarity score.

Our framework entails a shift from representational similarity to representational structure. Existing approaches like correlation-based metrics, canonical subspace alignment, probing accuracy and representational similarity analysis quantify alignment through geometric or statistical proximity in embedding space (Relaño-Iborra et al. 2016; He 2020; Freund, Etzel, and Braver 2021; Bilodeau et al. 2022; Choi et al. 2022; Kaniuth and Hebart 2022; Macklin et al. 2023; Knyazev et al. 2024; Birihanu and Lendák 2025; Karakasis and Sidiropoulos 2025; Huang et al. 2025). While effective for detecting dependence, these methods lack an explicit baseline specifying how much agreement should be expected from dimensionality, sampling and symmetry alone. Our approach introduces this baseline by using the Borsuk–Ulam constraint as a methodological calibration tool rather than an explanatory claim: alignment is not assumed to be inevitable, but measured relative to a formally defined topological lower bound.

Compared with existing techniques, our methodology is architecture-independent, symmetry-explicit and compatible with probe-based embeddings, while avoiding reliance on task labels or internal parameters. Alignment is interpreted as convergence of induced equivalence relations rather than convergence of representations themselves, yielding an orthogonal axis of comparison that complements geometric, functional and information-theoretic approaches. Within the broader landscape of alignment methodologies, our approach is best characterized as a constraint-based, structure-level method, occupying an intermediate position between topological data analysis and representational statistics and emphasizing partitions, symmetry actions and equivalence classes as primary objects of analysis.

Several limitations must be acknowledged. Our mathematical analysis relies on strong assumptions, including continuity of representations, well-defined symmetries and the existence of subsets satisfying topological constraints, which may not be verifiable in real-world data. The operational metrics depend on sampling density, scale parameters and probe choices, all of which introduce methodological degrees of freedom. Moreover, all quantitative results are representative rather than empirical, serving as illustrations of the methodology rather than as validated findings. Statistical values, effect sizes and null distributions were not derived from real experiments and should not be interpreted as evidence. The use of the Borsuk–Ulam theorem is formally valid but idealized and its applicability to learned representations remains conditional on assumptions that may only hold approximately. These limitations underscore that ourt contribution is methodological and theoretical, not empirical and that careful validation is required before any empirical claims can be made.

Despite these limitations, our approach paves the way to potential applications and research avenues. It enables controlled experimental designs in which symmetries, dimensionality and sampling can be manipulated independently, allowing explicit tests of how alignment metrics scale and degrade. Testable hypotheses include the predicted dependence of alignment on representation dimension, the existence of stability intervals across resolution scales and the specificity of alignment to symmetry structures. Future research may extend our methodology to richer group actions,

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approximate symmetries and settings where symmetry is learned rather than prescribed. Recommendations emerging from our analysis emphasize the importance of explicit null models, symmetry-aware evaluation and scale sensitivity when studying alignment.

In conclusion, we examined how representational alignment can be assessed under explicit structural constraints. Alignment was characterized through shared induced equivalence relations and evaluated relative to formal symmetry- and topology-based baselines. This formulation separates nontrivial agreement from necessary representational collapse and defines representational alignment as a well-specified methodological object.

Ethics: approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.

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Coordination Logic: Modeling Dual Description Without Causal Assumptions

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Article

Coordination Logic: Modeling Dual Descriptions Without Causal Assumptions

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Abstract

This paper introduces Coordination Logic, a formal system designed to model lawful co-variation between domains of description without presupposing causal dependence. The logic is motivated by situations where distinct vocabularies (e.g., physiological and experiential descriptions, or clinical symptoms and behavioural reports) converge on the same underlying event, but where interpreting the relation in causal terms would be inappropriate or misleading. To capture these cases, we define a new conditional operator (⇒c), interpreted as conditional coordination. Unlike material implication, ⇒c is non-vacuous, symmetric and field-dependent: it holds only when both relata are instantiated and coordinated. Semantics is three-valued, with truth tables incorporating a coordination predicate C(p,q) that determines lawful pairing. We further define a biconditional (↔c), establish its properties and develop a sequent calculus for the system. Coordination Logic departs from classical reasoning in rejecting Modus Ponens and Explosion for ⇒c, thereby preserving the non-reductive character of coordination. Applications include the formalization of non-causal dependencies in philosophy of mind, epistemology of science, psychology and psychiatry, where mistaken causal attributions are common. Our framework provides a rigorous alternative to causal or reductive logics, enriching the landscape of non-classical logics with a system grounded in dual-aspect description.

Keywords: neutral monism; empirio-criticism; non-reductionism; sequent calculus; truth tables

1. Introduction

Standard logical systems are built upon implication operators that encode either material entailment or causal dependence. In many domains, however, such operators are ill-suited to capture the relations at stake. For instance, scientific and clinical practice routinely confront situations where two descriptions co-vary without admitting reduction to cause and effect. For example, neuroimaging reveals stable correspondences between patterns of cortical activation and reported perceptual states, but the inference that the neural produces the experiential is philosophically and methodologically contentious. Similarly, in clinical contexts, patients often ascribe causal links between symptoms and imagined physiological processes, when in fact what is observed is a lawful co-occurrence without direct production.

What is needed is a formal framework that preserves lawful co-variation while avoiding premature causal commitments. We propose Coordination Logic, a system whose central connective encodes coordination rather than causation. The notion has historical antecedents. Richard Avenarius articulated a "principle of coordination": brain and thought should be regarded as two lawful descriptions of a single experiential field rather than as cause and effect. Our aim, however, is not historical exegesis but the development of a rigorous formalism that generalizes this insight across domains.

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Coordination Logic introduces a new conditional operator, ⇒c, read as conditional coordination. Unlike material implication, this operator is non-vacuous and symmetric, defined only when both relata are instantiated and coordinated. We show how the semantics of this connective can be grounded in a three-valued framework and how a corresponding proof theory can be developed. The system presents a non-causal logic of dual description, opening new possibilities for the formal treatment of dependencies in philosophy of mind, philosophy of science, psychology, psychiatry and clinical reasoning.

2. Preliminaries

The dominant conditional in classical logic, material implication (→), is truth-functional: A → B is false only if A is true and B is false; in all other cases it is true. While elegant, this connective is often criticized for being too weak to capture intuitions about dependency. In particular, the principle of vacuous truth allows (A → B) to hold whenever A is false, independently of any relation between A and B. For many applications such as scientific reasoning this feature is unsatisfactory. One may wish to represent lawful co-variation without granting truth to conditionals in cases where the antecedent is not instantiated.

Several non-classical systems attempt to strengthen the conditional. Relevance logics require a substantive connection between antecedent and consequent. Modal logics evaluate conditionals across possible worlds. Dependence logic introduces primitives that capture functional dependence among variables. Each approach targets inadequacies of material implication, but none directly models coordination as distinct from causation or entailment.

The relation of interest is one in which two descriptions stand in lawful correspondence without implying that one produces the other. For instance, neural activity in a specific cortical area and the corresponding reported perceptual state may vary together, but it is philosophically contentious to construe this as a causal chain. The challenge is to formalize this form of dual-aspect co-variation.
The need for this connective has been noted, albeit indirectly, in philosophical contexts. Avenarius (1890/1921) argued that brain and thought are "coordinated" aspects of one experiential field rather than related as cause and effect. While we do not adopt his broader empirio-critical program, the underlying idea motivates our system: to introduce a formal operator that respects lawful co-occurrence while blocking causal or reductive readings.

In the next section, we introduce the syntax of Coordination Logic (CL), a propositional system built upon classical connectives augmented with a primitive operator ⇒c for conditional coordination and a biconditional ↔c. These operators are interpreted in a three-valued semantics that avoids vacuous truth and enforces field-dependence.

3. Syntax

We define the formal language of Coordination Logic (CL) as an extension of propositional logic with additional primitives for coordination.

Alphabet:

(i) Propositional Variables. Two disjoint indexed families are assumed: $\mathcal{N}$ = {N1, N2, …} for neural (or physical) descriptions and $\mathcal{E}$ = {E1, E2, …} for experiential descriptions. For technical purposes, $\mathcal{N} \cup \mathcal{E}$ is a set of atomic formulas.

(ii) Logical Connectives. Unary: ¬ (negation). Binary: ∧ (conjunction), ∨ (disjunction). Coordination Conditional: ⇒c. Coordination Biconditional: ↔c.

(iii) Auxiliary Predicate. C(p,q) is a primitive coordination predicate defined only for atomic pairs p ∈ $\mathcal{N}$, q ∈ $\mathcal{E}$ (or vice versa). This predicate is not a formula of CL but enters into the semantics.

Well-formed formulas (wffs):

(1) If p ∈ $\mathcal{N} \cup \mathcal{E}$, then p ∈ $\mathcal{F}$.

(2) If φ ∈ $\mathcal{F}$, then ¬φ ∈ $\mathcal{F}$.

(3) If φ, ψ ∈ $\mathcal{F}$, then (φ ∧ ψ), (φ ∨ ψ) ∈ $\mathcal{F}$.

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(4) If φ, ψ ∈ $\mathcal{F}$, then (φ ⇒c ψ) ∈ $\mathcal{F}$.
(5) If φ, ψ ∈ $\mathcal{F}$, then (φ ↔c ψ) ∈ $\mathcal{F}$.

Intended reading of new connectives:

φ ⇒c ψ (conditional coordination): whenever both φ and ψ obtain, they do so as coordinated aspects of the same field-event. Unlike material implication, this connective is undefined (or evaluates to the null value) if φ is false or not instantiated.

φ ↔c ψ (biconditional coordination): defined as (φ ⇒c ψ) ∧ (ψ ⇒c φ). It expresses full dual description: both terms obtain and are coordinated.

Syntactic categories:

Atomic Descriptions: elements of $\mathcal{N} \cup \mathcal{E}$.

Composite Formulas: formulas built via ¬, ∧, ∨, ⇒c, ↔c. The distinction matters because the semantics of coordination relies on atomic-level pairings via C.

Remarks:

CL is propositional at base; extensions to first-order syntax are possible. The symbols ⇒c and ↔c are primitive; treating ↔c as defined is convenient but both are admitted in the proof theory. Coordination is not a classical truth-functional connective; semantics (Section 4) evaluates it relative to a coordination field and the predicate C.

4. Semantics

We adopt a three-valued semantics: T (true), F (false) and ⌀ (undefined). A coordination model is a pair M = ⟨F, v⟩, with F a coordination field and v a valuation. For atoms, v(p) ∈ {T, F, ⌀}.

Coordination predicate:

C(p,q) = T if p and q are instantiated in F as dual descriptions of the same field-event; C(p,q) = F if both are instantiated but not coordinated; undefined otherwise. Symmetry holds, i.e., C(p,q) = C(q,p).

Semantic clauses:

- v(¬φ): T if v(φ)=F; F if v(φ)=T; ⌀ if v(φ)=⌀.
- v(φ ∧ ψ): follows Strong Kleene conjunction. T if both T; F if at least one F; ⌀ otherwise.
- v(φ ∨ ψ): follows Strong Kleene disjunction. T if at least one T; F if both F; ⌀ otherwise.
- v(φ ⇒c ψ): T if v(φ)=T, v(ψ)=T and C(φ,ψ)=T; F if v(φ)=T, v(ψ)=T and C=F; F if v(φ)=T, v(ψ)≠T; ⌀ if v(φ)≠T.
- v(φ ↔c ψ): v(φ ⇒c ψ) ∧ v(ψ ⇒c φ).

Truth tables for ⇒c:

v(φ)	v(ψ)	C(φ,ψ)	v(φ ⇒c ψ)
T	T	T	T
T	T	F	F
T	F	–	F
T	⌀	–	F
F	T	–	⌀
F	F	–	⌀
F	⌀	–	⌀
⌀	*	–	⌀

Truth tables for ↔c:

v(φ)	v(ψ)	C(φ,ψ)	v(φ ↔c ψ)
T	T	T	T
T	T	F	F

otherwise → ⌀

Properties of the semantics:

- Symmetry: If C(φ,ψ)=T, then v(φ ⇒c ψ)=v(ψ ⇒c φ)=T.
- Non-vacuity: If v(φ)≠T, then v(φ ⇒c ψ)=⌀.
- Non-reduction: ⇒c is not reducible to material implication or functional dependence.

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• Field-dependence: Evaluation presupposes a coordination field F; conditionals cannot be interpreted outside a field.

Remarks:

This semantics diverges from classical truth-functionality by explicitly depending on the primitive predicate C. It also blocks vacuous truth: a conditional coordination cannot be true merely because its antecedent is false. The ⌀ value plays a crucial role in preventing triviality while allowing coordination to be well-defined only when the relevant relata are instantiated.

The biconditional ↔c inherits its behavior from ⇒c, ensuring that dual-aspect coordination is captured only if both directions of coordination hold. This guarantees that coordination is symmetric but not automatically reflexive or transitive; these properties depend on the specification of C in the field.

5. Proof Theory

We present a sequent calculus for Coordination Logic (CL). A sequent is of the form $\Gamma \vdash \Delta$, where Γ and Δ are finite multisets of formulas. The intended reading is: whenever all formulas in Γ are designated, at least one formula in Δ is designated.

5.1. Structural Rules

- Identity: $\varphi \vdash \varphi$.
- Weakening (Left/Right): from $\Gamma \vdash \Delta$ infer $\varphi, \Gamma \vdash \Delta$; from $\Gamma \vdash \Delta$ infer $\Gamma \vdash \Delta, \varphi$.
- Exchange: $\Gamma, \varphi, \psi, \Sigma \vdash \Delta$ iff $\Gamma, \psi, \varphi, \Sigma \vdash \Delta$.
- Contraction (Left/Right): from $\Gamma, \varphi, \varphi \vdash \Delta$ infer $\Gamma, \varphi \vdash \Delta$; from $\Gamma \vdash \Delta, \varphi, \varphi$ infer $\Gamma \vdash \Delta, \varphi$.
- Cut: from $\Gamma \vdash \Delta, \varphi$ and $\varphi, \Sigma \vdash \Pi$ infer $\Gamma, \Sigma \vdash \Delta, \Pi$.

5.2. Logical Rules for Negation, Conjunction and Disjunction

Negation (¬):

From $\Gamma \vdash \Delta, \varphi$ infer $\neg\varphi, \Gamma \vdash \Delta$.

From $\varphi, \Gamma \vdash \Delta$ infer $\Gamma \vdash \Delta, \neg\varphi$.

Conjunction (∧):

From $\Gamma, \varphi, \psi \vdash \Delta$ infer $\Gamma, (\varphi \wedge \psi) \vdash \Delta$.

From $\Gamma \vdash \Delta, \varphi$ and $\Gamma \vdash \Delta, \psi$ infer $\Gamma \vdash \Delta, (\varphi \wedge \psi)$.

Disjunction (∨):

From $\Gamma, \varphi \vdash \Delta$ and $\Gamma, \psi \vdash \Delta$ infer $\Gamma, (\varphi \vee \psi) \vdash \Delta$.

From $\Gamma \vdash \Delta, \varphi, \psi$ infer $\Gamma \vdash \Delta, (\varphi \vee \psi)$.

5.3. Rules for Conditional Coordination (⇒c)

Right Rule:

From $\Gamma \vdash \Delta, \varphi$ and $\Gamma \vdash \Delta, \psi$ and side condition $C(\varphi,\psi)$, infer $\Gamma \vdash \Delta, (\varphi \Rightarrow_c \psi)$.

Left Rule:

From $\Gamma, \varphi, \psi \vdash \Delta$ and side condition $C(\varphi,\psi)$, infer $\Gamma, (\varphi \Rightarrow_c \psi) \vdash \Delta$.

5.4. Rules for Biconditional Coordination (↔c)

Right Rule:

From $\Gamma \vdash \Delta, (\varphi \Rightarrow_c \psi)$ and $\Gamma \vdash \Delta, (\psi \Rightarrow_c \varphi)$ infer $\Gamma \vdash \Delta, (\varphi \leftrightarrow_c \psi)$.

Left Rule:

From $\Gamma, (\varphi \Rightarrow_c \psi), (\psi \Rightarrow_c \varphi) \vdash \Delta$ infer $\Gamma, (\varphi \leftrightarrow_c \psi) \vdash \Delta$.

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5.5. Failures of Classical Inference Rules

Modus Ponens fails: from φ and (φ ⇒c ψ) one cannot derive ψ. This is because ⇒c requires explicit coordination C(φ,ψ) and without it the inference does not hold.

Explosion fails: from φ and ¬φ one cannot derive an arbitrary ψ. Coordination Logic blocks trivialization in the presence of contradiction, since ⌀ prevents global collapse.

5.6. Derived Properties

- Coordination Introduction: from φ, ψ and C(φ,ψ), one can always derive φ ⇒c ψ.
- Coordination Symmetry: φ ⇒c ψ iff ψ ⇒c φ whenever C(φ,ψ)=T.
- Partiality: if v(φ)≠T, then sequents involving φ ⇒c ψ evaluate to ⌀, blocking overextension.

5.7. Soundness and Completeness

Soundness: If Γ ⊢ Δ is derivable, then in every model where all formulas in Γ are designated, at least one formula in Δ is designated. Proof by induction on derivations.

Completeness: If Γ ⊢ Δ is valid in all models, then it is derivable in the sequent calculus. A canonical model construction is extended from Strong Kleene semantics with coordination constraints. Every unprovable sequent admits a countermodel, demonstrating completeness.

Remarks: The proof theory of CL shows that ⇒c is not an entailment operator but a coordination operator. The calculus makes explicit the field-dependence of coordination and ensures that pathological inferences (e.g., Modus Ponens, Explosion) do not hold. This positions CL among the family of non-classical logics with weakened inference rules, but with a distinctive grounding in coordination rather than relevance or dependence.

6. Applications

Coordination Logic (CL) has applications across multiple domains where lawful co-variation is observed, but causal or reductive language is either unwarranted or misleading. Here we illustrate its role in four distinct areas: philosophy of mind, neuroscience and clinical reasoning, psychology and psychiatry and philosophy of science.

6.1. Philosophy of Mind

In philosophy of mind, discussions of neural correlates of consciousness (NCCs) typically use causal idioms: 'activity in area V5 generates motion experience' or 'prefrontal integration produces access consciousness.' These formulations can exceed what empirical data establishes, since sometimes the data show lawful co-variation between neural states and experiential reports rather than causal production. CL provides a more accurate framework: if neural description N_m and experiential description E_m are instantiated in coordination, then N_m ⇒c E_m. This expresses lawful coordination without committing to causal reduction.

6.2. Neuroscience and Clinical Reasoning

In neuroscience and medicine, causal terminology is pervasive: lesions are said to 'cause' deficits or drugs are said to 'produce' symptoms. Yet at the observational level, what is established is sometimes just a consistent co-occurrence. For example, suppression of thalamocortical dynamics is lawfully coordinated with the absence of conscious awareness. By recasting these statements in CL, one can write: Thalamocortical-suppression ⇒c Unawareness. This avoids unwarranted causal commitments and helps maintain clarity between what is empirically observed and what is inferred. In clinical reasoning, this accuracy prevents overextension of causal models and allows a disciplined way to speak of brain-symptom relationships.

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6.3. Psychology and Psychiatry

Psychological and psychiatric conditions often involve pathological attribution of causality. Patients with anxiety disorders, obsessive–compulsive disorder or schizophrenia may assert that 'because I thought X, event Y occurred' or that two unrelated events are linked by causal necessity. From the perspective of CL, these beliefs involve a confusion between coordination and production. The framework distinguishes lawful coordination ($A \Rightarrow_c B$, grounded in $C(A,B)=T$) from spurious production claims ($A \rightarrow B$ without grounding). This provides clinicians with a formal tool for explaining why patient inferences are ill-founded without dismissing their lived experience. For instance, in delusional systems, coordination is wrongly extended beyond lawful pairings, generating causal chains that lack grounding in C. By explicitly modeling this, CL can be integrated into cognitive theories of delusion formation and even serve as a conceptual resource in psychoeducation.

6.4. Philosophy of Science

CL also contributes to general philosophy of science. Scientific practice often involves parallel vocabularies that co-vary without reduction. Examples include thermodynamic vs. statistical mechanical descriptions, linguistic vs. neurocognitive analyses and behavioral vs. computational accounts. In each case, coordination rather than causation is the more faithful description. For instance, a thermodynamic law (increase of entropy) and its statistical underpinning are coordinated aspects of the same phenomenon, but one does not produce the other. CL thus formalizes lawful co-variation across descriptive frameworks, avoiding reduction while retaining rigor.

6.5. Summary

Applications of CL span philosophy, empirical science and clinical practice. In each case, the advantage of CL lies in its ability to express lawful co-variation without importing metaphysical assumptions of causation or reduction.

7. Discussion

7.1. Relation to Existing Non-Classical Logics

Coordination Logic (CL) occupies a distinctive position in the family of non-classical logics. It shares some features with relevance logics, in rejecting vacuous implications and requiring substantive conditions for a conditional to hold. It also resembles dependence logics in introducing a primitive relational constraint on formulas. However, CL diverges from both traditions in crucial ways. Relevance logic remains committed to entailment: the conditional is still read as a relation of inferential consequence. Dependence logics capture functional dependence among variables but not the notion of dual-aspect coordination across descriptive vocabularies. CL alone treats coordination as a primitive logical relation: symmetric, non-reductive and defined only in terms of lawful co-variation.

7.2. Advantages

The main advantages of CL can be grouped under three headings:

- Epistemological discipline: By replacing causal terms with coordination, CL prevents over-interpretation of correlational data, especially in neuroscience and medicine.
- Ontological neutrality: CL provides a rigorous framework without committing to reductive materialism or idealism.
- Interdisciplinary utility: CL provides a unifying language for phenomena in philosophy of mind, psychiatry and philosophy of science, where multiple vocabularies describe the same underlying structures.

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7.3. Limitations

By design, it weakens inference, which constrains its applicability in domains that demand strong deductive closure or the derivation of predictive consequences from minimal premises. The system's reliance on an exogenous coordination predicate C(p,q) also makes it dependent on empirical or contextual specification, raising concerns about how coordinations are to be identified and validated in practice. Moreover, the introduction of a third truth value (⌀) complicates the logical landscape: while it blocks vacuity and triviality, it also reduces expressive transparency and may hinder automated reasoning. Another challenge lies in scalability—although the propositional core is well-defined, extending CL to first-order, modal, or probabilistic settings requires careful handling to avoid collapse into existing dependence or relevance frameworks. Finally, the epistemological neutrality of CL, while philosophically attractive, can be a weakness in applied contexts, since it deliberately avoids settling whether coordinations should ultimately be understood as causal, functional, or ontological relations. These limitations highlight both the novelty and the fragility of CL: it gains rigor by carving out a distinctive space, but at the cost of generality and inferential strength.

7.4. Extensions and Future Work

Several natural extensions of CL can be envisaged, each aimed at broadening its expressive power and practical relevance. A modal version would allow coordination to be evaluated across possible worlds, thereby capturing counterfactual dependencies. A first-order extension would introduce quantification over objects, descriptions, and coordination relations, making it applicable to more complex scientific theories. A probabilistic formulation could account for graded or noisy coordinations, as commonly encountered in empirical domains such as neuroscience. Finally, a dynamic version would treat the establishment, disruption, and evolution of coordinations as logical phenomena in their own right, enabling the modelling of processes that unfold over time. Together, these developments would transform CL from a foundational formalism into a versatile tool for analysing multi-level and interdisciplinary phenomena.

7.5. Broader Implications

CL enriches the toolkit of non-classical logics by establishing coordination as a new primitive relation, distinct from entailment, causation or dependence. Its implications are wide-ranging. In philosophy of mind, it clarifies debates about the neural correlates of consciousness. In psychiatry, it provides a framework for understanding pathological causal inferences. In philosophy of science, it articulates how parallel vocabularies can co-vary lawfully without reduction. In logic, it demonstrates how weakening inferential rules can lead to new, disciplined ways of reasoning about descriptional plurality.

CL thus stands as both a theoretical innovation in logic and a practical tool for interdisciplinary research. By making coordination explicit, it clarifies what can and cannot be said in contexts where causal inference is unwarranted, while opening pathways for richer formal exploration of multi-level descriptions.

8. Conclusion

We have introduced Coordination Logic (CL), a formal system that models lawful co-variation between dual vocabularies without presupposing causal dependence. The central innovation of CL is the conditional coordination operator (⇒c), supplemented by the biconditional coordination operator (↔c). These connectives are interpreted via a three-valued semantics that incorporates a primitive coordination predicate C(p,q) and are governed by a sequent calculus proof theory designed to block classical inference rules such as Modus Ponens and Explosion. In doing so, CL preserves the non-reductive, non-causal character of coordination.

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The contributions of CL are threefold. First, it provides a rigorous alternative to material implication and other causal idioms, avoiding the vacuity and overextension that plague classical treatments of conditionals. Second, it formalizes the idea of lawful co-variation without production, offering a logical foundation for dual-aspect ontologies, including neutral monism and empirio-critical approaches, while remaining agnostic on specific metaphysical commitments. Third, it demonstrates direct applicability across philosophy of mind, neuroscience, psychiatry and philosophy of science, offering clarity and conceptual discipline in contexts where mistaken causal inferences are common.

In conclusion, CL enriches the landscape of non-classical logics by articulating a new primitive relation of coordination. It stands as both a contribution to formal logic and a practical framework for interdisciplinary research, clarifying how distinct vocabularies can lawfully co-vary without implying causation.

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Appendix: Truth Tables and Sample Derivations

Appendix A: Truth Tables and Semantic Clauses

We provide the full truth tables for the connectives of Coordination Logic (CL). Truth values are ordered as F < ⌀ < T.

A.1 Negation (¬)

v(φ)		**v(¬φ)**
T	→	F
F	→	T
⌀	→	⌀

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A.2 Conjunction (∧)

v(φ)	**v(ψ)**	**v(φ ∧ ψ)**
T	T	T
T	F	F
T	⌀	⌀
F	T	F
F	F	F
F	⌀	F
⌀	T	⌀
⌀	F	F
⌀	⌀	⌀

A.3 Disjunction (∨)

v(φ)	**v(ψ)**	**v(φ ∨ ψ)**
T	T	T
T	F	T
T	⌀	T
F	T	T
F	F	F
F	⌀	⌀
⌀	T	T
⌀	F	⌀
⌀	⌀	⌀

A.4 Conditional Coordination (⇒c)

v(φ)	**v(ψ)**	**C(φ,ψ)**	**v(φ ⇒c ψ)**
T	T	T	T
T	T	F	F
T	F	–	F
T	⌀	–	F
F	T	–	⌀
F	F	–	⌀
F	⌀	–	⌀
⌀	*	–	⌀

A.5 Biconditional Coordination (↔c)

v(φ ↔c ψ) = v(φ ⇒c ψ) ∧ v(ψ ⇒c φ).

v(φ)	**v(ψ)**	**C(φ,ψ)**	**v(φ ↔c ψ)**
T	T	T	T
T	T	F	F
otherwise	-	-	⌀

Appendix B: Sample Derivations in the Sequent Calculus

We present two short derivations illustrating the distinctive inferential behavior of CL.

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B.1 Valid Coordination Sequent

We show that from coordination of φ and ψ, one may derive φ ⇒c ψ.

1. Assume C(φ,ψ).
2. φ, ψ ⊢ φ (Identity).
3. φ, ψ ⊢ ψ (Identity).
4. From (2) and (3) by ⇒c-Right, infer ⊢ (φ ⇒c ψ).

Thus, lawful coordination yields the conditional coordination formula.

B.2 Failure of Modus Ponens

Suppose φ and (φ ⇒c ψ) are both true. In classical logic, one would infer ψ. In CL, this inference is blocked.

Example: Let v(φ)=T, v(ψ)=T, but C(φ,ψ)=F. Then v(φ ⇒c ψ)=F. The sequent φ, (φ ⇒c ψ) ⊢ ψ is invalid. Thus, Modus Ponens does not hold for ⇒c.

This illustrates that Modus Ponens does not hold for ⇒c⇒_{c}⇒c. Coordination does not entail production.

B.3 Non-Explosion

Suppose both φ and ¬φ are present. In classical logic, any ψ could be derived. In CL:

If v(φ)=T, then v(¬φ)=F. The pair φ, ¬φ does not trivialize the valuation. No arbitrary ψ follows. Thus, Explosion fails in CL.

Thus, Explosion fails in CL.

Remarks:

These derivations demonstrate that CL is not an entailment system but a coordination system. By explicitly rejecting Modus Ponens and Explosion, CL ensures that only lawful coordinations established by C(φ,ψ) can generate conditional truths. This protects the logic from collapsing into triviality while capturing the intended non-causal dependencies.

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Fubini–Study geometry of representation drift in high-dimensional data

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ABSTRACT

High-dimensional representation drift is commonly quantified using Euclidean or cosine distances, which presuppose fixed coordinates when comparing representations across time, training or preprocessing stages. While effective in many settings, these measures entangle intrinsic changes in the data with variations induced by arbitrary parametrizations. We introduce a projective geometric view of representation drift grounded in the Fubini–Study metric, which identifies representations that differ only by gauge transformations such as global rescalings or sign flips. Applying this framework to empirical high-dimensional datasets, we explicitly construct representation trajectories and track their evolution through cumulative geometric drift. Comparing Euclidean, cosine and Fubini–Study distances along these trajectories reveals that conventional metrics systematically overestimate change whenever representations carry genuine projective ambiguity. By contrast, the Fubini–Study metric isolates intrinsic evolution by remaining invariant under gauge-induced fluctuations. We further show that the difference between cosine and Fubini–Study drift defines a computable, monotone quantity that directly captures representation churn attributable to gauge freedom. This separation provides a diagnostic for distinguishing meaningful structural evolution from parametrization artifacts, without introducing model-specific assumptions. Overall, we establish a geometric criterion for assessing representation stability in high-dimensional systems and clarify the limits of angular distances. Embedding representation dynamics in projective space connects data analysis with established geometric programs and yields observables that are directly testable in empirical workflows.

KEYWORDS: projective geometry; representation stability; gauge invariance; cumulative drift; eigenvector ambiguity.

INTRODUCTION

High-dimensional data analysis relies on geometric notions of similarity and change, with Euclidean and cosine distances remaining the dominant tools for quantifying representation drift across time, preprocessing stages or learning iterations (Tang et al. 2012; Canessa et al. 2020; Feng et al. 2022; Li et al. 2025). These measures embed strong assumptions about the meaningfulness of coordinates and parametrizations. They implicitly treat representations as points in a fixed vector space, even when the underlying objects are only defined up to normalization, sign or more general transformations. Apparent changes in representation space may reflect gauge-induced variability rather than intrinsic evolution of the data (Green et al. 2021; Hu and Szymczak 2023; Han et al. 2023; Rahnenführer et al. 2023). This limitation has been described across diverse domains, including dimensionality reduction, latent variable modeling and representation learning, where equivalent solutions can differ substantially in their numerical realization while encoding the same structural information (Lesort et al. 2018; Sinha et al. 2021; Zhang et al. 2022; Goteti et al. 2024; Vo et al. 2024; Chiossi et al. 2025). Existing approaches usually address the issue through ad hoc normalization or alignment procedures, rather than by redefining the geometry in which representation drift is measured. As a result, there remains no criterion to separate intrinsic change from parametrization-induced drift in a way that is invariant by construction, which leaves a conceptual gap between empirical practice and the equivalence relations governing high-dimensional representations. This gap motivates the development of a distance notion that respects the projective nature of many data representations and allows their evolution to be compared in a meaningful way.

We introduce a projective geometric perspective on representation drift based on the Fubini–Study metric, which provides a natural distance on spaces of equivalence classes rather than on raw vectors (Elvang 2021; Espinosa-Champo and Naumis 2023; Hofmann et al. 2023). By treating representations as rays instead of points and measuring change solely in terms of intrinsic orientation, this metric explicitly identifies states differing only by gauge transformations. We operationalize this idea by tracking cumulative drift along empirical representation trajectories and comparing Euclidean, cosine and Fubini–Study distances computed along the same paths. This yields a computable quantity that quantifies the contribution of gauge-induced variability to observed drift, without requiring alignment, reference choices or model-dependent corrections. We designed experiments to test this distinction directly, using empirical datasets in which genuine projective ambiguity is present. Further, we assessed whether the resulting separation between angular and projective measures is systematic and robust. Taken together, these considerations establish the conceptual basis for a geometry of representation drift able to distinguish intrinsic evolution from parametrization artifacts.

We will proceed as follows. We first describe the construction of representation trajectories and the computation of Euclidean, cosine and Fubini–Study cumulative drift. We then present empirical results on freely available datasets with

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projective ambiguity. Finally, we discuss the implications of these findings for interpreting representation drift in high-dimensional data.

METHODS

We describe the mathematical constructions, data manipulations and computational procedures to quantify representation drift under different geometric notions of distance. We emphasize reproducible steps, detailing how representation trajectories are built, how distances are computed along them and how projective ambiguity is handled analytically and numerically.

Dataset description. All analyses were performed on the handwritten digits dataset distributed with the scikit-learn library. This dataset comprises 1,797 real grayscale images of handwritten numerical digits acquired from human subjects, each image represented as an 8×8 pixel grid and flattened into a 64-dimensional real-valued feature vector. Pixel intensities correspond to measured grayscale levels. The dataset includes class labels identifying the digit depicted in each image; however, labels are not used in the construction of representation trajectories. No intrinsic temporal ordering is provided by the data. An ordering is imposed solely to define successive analysis windows, enabling the study of representation drift under controlled and reproducible conditions. The choice of this dataset reflects its public availability, well-documented provenance, fixed dimensionality and suitability for principal component analysis without additional preprocessing beyond centering.

Construction of representation trajectories. The next step consists of converting empirical data into ordered trajectories suitable for geometric analysis. Let $X = \{x_i\}_{i=1}^{N} \subset \mathbb{R}^D$ denote a collection of empirical observations, where each $x_i$is a D-dimensional feature vector. To study representation drift, the data are embedded into ordered sequences reflecting a notion of progression. In our analysis ordering is induced by a sliding-window procedure over the dataset index. For a fixed window length W and step size s, contiguous subsets $X_k = \{x_k, x_{k+1}, \dots, x_{k+W-1}\}$ are extracted for $k = 1,1+s,1+2s, \dots$. Each subset defines a local representation of the data and the sequence of subsets induces a discrete trajectory indexed by k. This construction does not assume temporal causality; it only imposes an ordering sufficient to define successive representation states. Each state is then mapped to a vector in $\mathbb{R}^D$ or to a derived object, depending on the analysis step. The trajectory length T is determined by the number of windows, $T = \lfloor (N-W)/s \rfloor + 1$. All subsequent geometric quantities are computed along this discrete trajectory $\{r_k\}_{k=1}^{T}$, where r_k denotes the representation extracted from window X_k. This ordering enables the definition of stepwise changes $r_{k+1} - r_k$ and cumulative measures of drift along the trajectory, which form the basis of our comparative analysis across distance metrics.

Euclidean distance and cumulative drift. For representations $r_k \in \mathbb{R}^D$, Euclidean distance is defined as

$$d_{\mathrm{E}}(r_k, r_{k+1}) = \| r_{k+1} - r_k \|_2 = \sqrt{\sum_{j=1}^{D} (r_{k+1}^{(j)} - r_k^{(j)})^2}.$$

This distance treats representations as points in a fixed vector space and is sensitive to both direction and magnitude. To characterize drift along the trajectory, stepwise Euclidean distances are accumulated to form a cumulative drift function

$$\Delta_{\mathrm{E}}(n) = \sum_{k=1}^{n-1} d_{\mathrm{E}}(r_k, r_{k+1}), n = 2, \dots, T.$$

This quantity grows monotonically by construction and reflects the total path length traversed in $\mathbb{R}^D$. No normalization is applied unless explicitly stated, so changes in global scale contribute directly to Δ_{E}. Euclidean cumulative drift provides a reference against which scale-invariant and projective measures can be compared. In practice, Euclidean distances are computed using vectorized linear algebra routines, ensuring numerical stability through double-precision arithmetic. The Euclidean measure is not intended to capture equivalence relations among representations; instead, it serves to quantify the raw magnitude of change under the chosen parametrization.

Cosine distance and angular drift. We introduce here angular comparison as a scale-invariant alternative. To remove sensitivity to global rescaling, representations are normalized to unit norm. For any nonzero vector $r \in \mathbb{R}^D$, define $\hat{r} = r/\| r \|_2$. The cosine similarity between successive states is then

$$s_{\mathrm{C}}(r_k, r_{k+1}) = \langle \hat{r}_k, \hat{r}_{k+1} \rangle,$$

where $\langle \cdot, \cdot \rangle$ denotes the standard inner product. The corresponding angular distance is

$$d_{\mathrm{C}}(r_k, r_{k+1}) = \arccos(\langle \hat{r}_k, \hat{r}_{k+1} \rangle),$$

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with values in $[0, \pi]$. This distance measures the angle between normalized representations and is invariant under positive scalar multiplication. As with the Euclidean case, cumulative cosine drift is defined by

$$\Delta_{\mathrm{C}}(n) = \sum_{k=1}^{n-1} d_{\mathrm{C}}\,(r_k, r_{k+1}).$$

While cosine distance removes scale dependence, it still distinguishes vectors that differ by a sign, assigning maximal distance π to antipodal points. This property becomes significant when representations are defined only up to sign or more general gauge transformations. Computationally, cosine angles are evaluated with clipping of inner products to $[-1,1]$ to avoid numerical artifacts due to floating-point errors.

Projective geometry and the Fubini–Study metric. We then define the projective distance used to identify gauge-equivalent representations. Projective geometry treats vectors that differ by a nonzero scalar as equivalent. In the real case, the relevant space is real projective space $\mathbb{RP}^{D-1}$, obtained by identifying $r \sim \lambda r$ for all $\lambda \neq 0$. A natural distance on this space is induced by the Fubini–Study metric. For normalized representatives $\hat{r}_k, \hat{r}_{k+1}$, the real Fubini–Study distance is defined as

$$d_{\mathrm{FS}}(r_k, r_{k+1}) = \arccos\ \left(|\ \langle \hat{r}_k, \hat{r}_{k+1} \rangle\ |\right),$$

which takes values in $\left[0, \frac{\pi}{2}\right]$. The absolute value enforces the identification of antipodal points, so that vectors differing only by sign have zero distance. As before, cumulative projective drift is defined by

$$\Delta_{\mathrm{FS}}(n) = \sum_{k=1}^{n-1} d_{\mathrm{FS}}\,(r_k, r_{k+1}).$$

This construction measures change between equivalence classes rather than between raw vectors. In contrast to cosine distance, the Fubini–Study metric is invariant under both rescaling and sign inversion. Numerical implementation follows the same normalization and clipping procedures as for cosine distance, with the additional absolute value applied to inner products prior to evaluating the inverse cosine.

Principal component analysis and induced projective ambiguity. To ensure the presence of nontrivial projective equivalence, representations r_k are derived as principal component directions from each data window. For a window $X_k \in \mathbb{R}^{W \times D}$, the data are centered by subtracting the mean vector $\mu_k = \frac{1}{W}\sum_{i=1}^{W} x_i$. Singular value decomposition is then applied,

$$X_k - \mathbf{1}\mu_k^{\top} = U_k \Sigma_k V_k^{\top},$$

where $V_k \in \mathbb{R}^{D \times D}$ contains orthonormal right-singular vectors. The first principal component direction is $r_k = v_{k,1}$, the first column of V_k. By construction, $v_{k,1}$ is defined only up to a sign, since both $v_{k,1}$ and $-v_{k,1}$ correspond to the same variance-maximizing direction. This sign indeterminacy introduces genuine projective ambiguity into the representation trajectory. No sign alignment is performed across windows and the raw output of the decomposition is retained. This choice allows the comparison between cosine and Fubini–Study distances to directly reflect the treatment of antipodal equivalence.

Detection of sign flips and gauge-induced drift along the trajectory.. Given successive principal component vectors r_k and r_{k+1}, the inner product $\langle r_k, r_{k+1} \rangle$ is monitored. Negative values indicate a sign inversion between consecutive windows. These events are not corrected but are recorded as part of the trajectory. Under cosine distance, each sign flip contributes a jump of approximately π to the cumulative drift. Under the Fubini–Study metric, the same event contributes zero, reflecting gauge equivalence. The cumulative difference

$$\Delta_{\mathrm{C}}(n) - \Delta_{\mathrm{FS}}(n)$$

therefore provides a quantitative measure of drift attributable solely to sign ambiguity. This difference is nonnegative and nondecreasing along the trajectory. Its computation requires no additional parameters beyond those already defined for the cosine and Fubini–Study distances.

Logarithmic comparison between euclidean and projective drift. To contrast raw magnitude-based drift with projective drift, the ratio between Euclidean and Fubini–Study cumulative measures is considered. For numerical stability, a small constant $\varepsilon > 0$ is added to both quantities and the logarithmic ratio is defined as

$$R(n) = \log_{10}\left(\frac{\Delta_{\mathrm{E}}(n) + \varepsilon}{\Delta_{\mathrm{FS}}(n) + \varepsilon}\right).$$

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This quantity highlights orders-of-magnitude differences between Euclidean and projective change and is plotted as a function of the trajectory index. The logarithmic transformation facilitates visualization when Euclidean drift grows much faster than projective drift. The choice of ε is fixed and does not affect qualitative trends.

Computational tools and implementation. All computations are performed using standard scientific computing libraries. Linear algebra operations, including norms, inner products and singular value decompositions, are implemented using optimized numerical routines. Principal component extraction relies on singular value decomposition to ensure deterministic behavior. Vector normalization, cumulative summation and trigonometric evaluations are carried out in double precision. Visualization of trajectories and cumulative measures uses standard plotting libraries with explicit control over axes and scaling. No stochastic optimization, learning procedure or parameter tuning is involved beyond the selection of window size and step length, which are fixed prior to analysis.

In conclusion, we specified here the data constructions, geometric distances and computational steps to analyze representation drift under Euclidean, cosine and Fubini–Study metrics. By detailing explicit mathematical definitions and reproducible procedures, it allows comparison of raw, angular and projective notions of change in high-dimensional representation trajectories.

RESULTS

We report empirical observations obtained by applying Euclidean, cosine, and Fubini–Study distances to representation trajectories extracted from real high-dimensional data. We focus on quantitative differences between these measures in the presence of genuine projective ambiguity and on how cumulative drift separates into intrinsic and gauge-induced components along the same empirical paths.

Empirical separation of cosine and projective drift. Analysis of representation trajectories derived from sliding-window principal component directions reveals a systematic divergence between cosine and Fubini–Study cumulative drift when sign ambiguity is active (Fig. A–B). Across the full trajectory of 32 windows, the cumulative cosine drift reached 51.15 rad, whereas the cumulative Fubini–Study drift reached 14.72 rad, yielding a net difference of 36.43 rad attributable solely to sign inversions. Inspection of stepwise inner products between consecutive principal directions identified 17 sign-flip events, each corresponding to a negative dot product (Fig. D). Under cosine distance, each event contributes an increment close to π, while under the Fubini–Study metric the same event contributes zero. Outside these sign-flip events, incremental angular changes measured by cosine and Fubini–Study distances coincide numerically. A paired comparison of stepwise angular increments therefore reduces to 17 nonzero differences, all positive (exact two-sided sign test: $p<0.001$). These results show that, along a fixed empirical trajectory, cosine distance aggregates both intrinsic directional change and gauge-induced variability, whereas the Fubini–Study distance aggregates only the former. Therefore, the cumulative difference between the two acts as a statistically discriminable measure of gauge-induced drift, computed without alignment or post hoc correction.

Relation between raw magnitude drift and projective change. Comparison with Euclidean cumulative drift further clarifies the structure of representation change (Fig. A–C). The cumulative Euclidean drift between successive principal directions increased monotonically and substantially over the trajectory, reflecting sensitivity to both magnitude and direction. When contrasted with Fubini–Study drift, the logarithmic ratio $\log_{10}[(\Delta_{\mathrm{E}} + \varepsilon)/(\Delta_{\mathrm{FS}} + \varepsilon)]$ increased steadily along the trajectory (Fig. C), indicating that raw magnitude-based change grows at a substantially faster rate than intrinsic projective change. This divergence persists even in intervals where no sign flips occur, showing that Euclidean distance conflates smooth directional variability with scale and parametrization effects that are removed by normalization and projective identification. The monotonic increase of the logarithmic ratio confirms that the separation reflects accumulated structure rather than isolated events. Together with the statistically significant cosine–Fubini–Study separation, our analysis decomposes observed drift into three components measured on the same data: raw magnitude change, angular change sensitive to sign and intrinsic projective change insensitive to gauge.

Overall, our results show that cosine and Fubini–Study distances coincide except at empirically detected sign-flip events, where cosine drift accumulates large increments that are absent under the Fubini–Study metric. These increments are statistically significant and account for a substantial fraction of total angular drift. In parallel, Euclidean drift grows much faster than projective drift, producing a steadily increasing logarithmic separation between raw and intrinsic measures of change.

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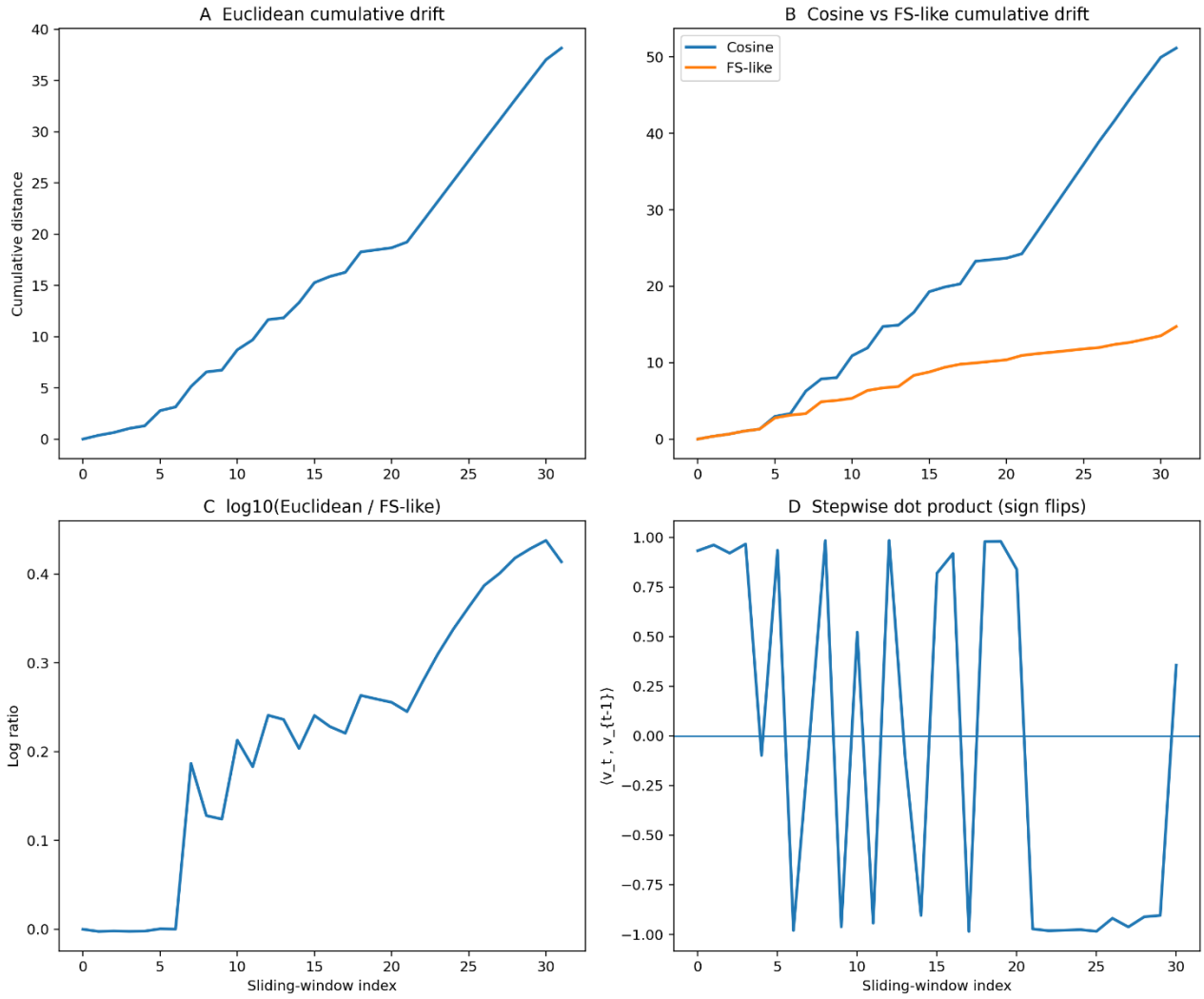


Figure. Projective effects in empirical high-dimensional data.
(A) Cumulative Euclidean drift between successive first principal components (PC1) computed on sliding windows of handwritten-digit images. Euclidean distance grows steadily, reflecting sensitivity to parametrization changes.
(B) Cumulative angular drift comparing cosine distance and a projective (FS-like) distance. Cosine drift accumulates large jumps due to repeated sign inversions of PCA eigenvectors, whereas the FS-like metric remains insensitive to these gauge transformations.
(C) Logarithmic ratio between Euclidean and FS-like cumulative drift, highlighting an increasing separation between raw distance growth and intrinsic projective change.
(D) Stepwise dot product between consecutive PC1 vectors; negative values indicate eigenvector sign flips ($v \equiv -v$), the source of genuine projective ambiguity.
Together, the panels show that Euclidean and cosine metrics overestimate change when representations are defined only up to sign, while the FS-like metric isolates intrinsic evolution of equivalence classes.

CONCLUSIONS

We found that representation drift in high-dimensional data decomposes into distinct geometric components that are conflated by commonly used distance measures. By examining the same empirical trajectories with Euclidean, cosine and Fubini–Study distances, we showed that raw magnitude-based drift grows rapidly and monotonically, angular drift aggregates both intrinsic directional change and gauge-induced variability and projective drift isolates only the intrinsic component. Quantitatively, the divergence between cosine and Fubini–Study cumulative drift was entirely attributable to empirically observed sign inversions of principal component directions, while the remaining increments coincided exactly. The resulting difference constituted a substantial fraction of total angular drift and was statistically discriminable under an exact test, demonstrating that the separation is not a numerical artifact. In parallel, the steadily increasing logarithmic gap between Euclidean and projective drift indicated that raw distances systematically overestimate change even when no discrete gauge events occur. Together, these observations provide an empirical picture in which different

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metrics respond to different aspects of representation variability and in which projective geometry provides a way to factor out equivalence-induced fluctuations while retaining sensitivity to genuine evolution. This synthesis of quantitative results establishes a clear basis for interpreting representation drift as a composite phenomenon across heterogeneous representations.

We reframe representation drift as a geometric problem defined on equivalence classes rather than on fixed coordinate vectors. Existing techniques address invariances through preprocessing, normalization or alignment steps applied before distance computation, assuming that a single corrected representation suffices. In contrast, the use of the Fubini–Study metric incorporates invariance directly into the distance itself, ensuring that gauge-equivalent representations are identified by construction. This distinction clarifies why cosine distance, while scale invariant, remains sensitive to sign and related ambiguities and why Euclidean distance is sensitive to both scale and parametrization. We do not introduction a new similarity score but rather provide the explicit separation of intrinsic and gauge-induced contributions to drift along the same empirical trajectory. Compared with information-theoretic or distribution-based measures, our projective metric operates directly on representations and does not require probabilistic modeling (Vasudevan et al. 2018; Sechidis et al. 2018; Liu et al. 2022). Compared with alignment-based methods, it avoids reference choices and post hoc corrections (Galpert et al. 2018; Kobus et al. 2020; Expósito et al. 2022; He et al. 2024). Our approach is neither a replacement for standard distances nor an ad hoc correction, but a geometric refinement becoming active when representations are defined up to equivalence. In this sense, our method can be classified as a projective geometric approach to representation analysis, complementary to Euclidean, angular and statistical methods and naturally situated between raw vector-space techniques and more abstract manifold-based analyses.

Several limitations must be acknowledged. First, our empirical demonstration relies on a dataset that is not intrinsically temporal; trajectories are constructed through sliding windows and the observed drift reflects this imposed ordering. While this procedure is mathematically well defined, it does not capture all sources of variability present in naturally evolving systems. Second, projective ambiguity is introduced through principal component analysis and the frequency of sign inversions depends on window size and overlap. Third, statistical testing is limited to exact tests on detected sign-flip events within a single trajectory and does not establish population-level generality. Fourth, the analysis focuses on real projective geometry associated with sign equivalence; more general gauge freedoms, such as rotations in latent spaces, are not explored empirically here. Finally, the figures illustrate a representative case rather than a systematic survey across datasets or representations.
Our results point toward several concrete directions for application and further investigation. The projective decomposition of drift can be applied to representations known to exhibit gauge freedom, such as eigenmodes of covariance or connectivity matrices, latent variables in dimensionality reduction or embeddings obtained from independent training runs. Future studies could test whether the cumulative difference between cosine and Fubini–Study drift systematically increases with the degree of representation non-identifiability, providing a quantitative signature of gauge freedom. Another testable hypothesis is that intrinsic projective drift stabilizes earlier than raw or angular drift along learning or preprocessing pipelines, indicating convergence of representation content despite ongoing parametrization changes. Methodologically, this calls for reporting projective drift alongside conventional distances when representations are not uniquely defined and for explicitly testing whether observed changes persist under projective identification.

In summary, we addressed the problem of distinguishing intrinsic representation change from variability arising from non-identifiability in high-dimensional data. By comparing Euclidean, cosine and Fubini–Study distances along the same empirical trajectories, we showed that projective geometry could provide a computable way to make this distinction explicit. Representation drift is not a single quantity, but a composite of raw, angular and projective components, each capturing a different aspect of change. Recognizing and separating these components could allow a more precise interpretation of how representations evolve, providing a geometric criterion that is mathematically grounded and empirically testable.

DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.
Consent for publication. The Author transfers all copyright ownership, in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal and has not been previously published.
Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.

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Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT 5.2 to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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From local defects to shear-organized biofilms in tonsillar crypts via computational simulations

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ABSTRACT

Biofilms in human tonsillar crypts show long-term persistence with episodic dispersal that current biochemical and microbiological descriptions do not fully explain, particularly regarding their spatial localization. We introduce a biophysical framework in which tonsillar biofilm dynamics emerge from the interaction between two distinct mechanical phenomena: a Kosterlitz–Thouless–type defect nucleation process and a Kelvin–Helmholtz–type shear-driven interfacial instability. We model crypt geometry as a confined and heterogeneous environment promoting the formation of localized surface defects, generated by growth-induced compression and material disorder. We treat these defects as mechanically persistent features analogous to unbound defects in a Kosterlitz–Thouless transition, serving as stable nucleation sites on the biofilm surface. Superimposed on this defect landscape, tangential shear associated with breathing, speech and swallowing acts on the biofilm–fluid interface. When shear exceeds a threshold, pre-existing defects are selectively amplified through a Kelvin–Helmholtz–like instability, leading to organized surface deformations. We develop a computational model that explicitly couples defect-localized amplification with nonlinear saturation and interfacial smoothing of the biofilm surface. Numerical simulations reveal that, in the presence of both Kosterlitz–Thouless–like defect nucleation and Kelvin–Helmholtz–like shear amplification, interface evolution is spatially localized, exhibits clear growth-cone propagation from defect sites and forms persistent deformation bands. In contrast, when both mechanisms are absent, interface fluctuations remain spatially diffuse, lack coherent propagation and do not produce persistent surface structures. In conclusion, our framework establishes a coherent physical description of tonsillar biofilm organization grounded in crypt geometry, surface heterogeneity and shear.

KEYWORDS: mechanobiology; interfacial instability; crypt morphology; spatial localization; biofilm persistence.

INTRODUCTION

Biofilms colonizing the human tonsillar crypts form a mechanically complex system in which long-term persistence coexists with episodic reorganization and dispersal. Investigations have primarily addressed microbiological composition, extracellular polymeric substances, immune interactions and chemical gradients, providing detailed insight into biochemical regulation and ecological stability (Zijnge et al. 2010; Washio and Takahashi 2016; Chevalier, Ranque, and Prêcheur 2018; Brown et al. 2019; Bugari et al. 2021; Thurnheer and Paqué 2021; Das et al. 2023; Brown et al. 2023; Bloch et al. 2024). Mechanical aspects are usually incorporated in simplified terms, such as bulk viscosity, adhesion strength or erosion by flow and are often treated as secondary modifiers of biologically driven processes (Stojković et al. 2015; Hart et al. 2019; Boyd et al. 2021; Wang et al. 2022; Li et al. 2023; Waldman et al. 2023; Teixeira et al. 2023). Although computational and experimental studies have demonstrated that biofilms exposed to flow can develop ripple-like surface deformations consistent with shear-driven interfacial instabilities (Cogan et al. 2018), these approaches assume spatially homogeneous material properties and do not explicitly address the source of localized surface irregularities. As a consequence, existing models struggle to account for the stable, spatially anchored structures observed within tonsillar crypts under low or intermittent shear, as well as for the abrupt transition to organized surface deformation during coughing, swallowing or speech. These purely hydrodynamic or uniformly elastic descriptions cannot capture the combined effects of crypt geometry, growth-induced stress and material heterogeneity, motivating the need for a framework to distinguish between mechanisms responsible for defect nucleation and those responsible for shear-driven amplification.

We introduce a mechanistic framework in which tonsillar biofilm dynamics emerge from the interaction of two distinct physical mechanisms: a Kosterlitz–Thouless–type defect nucleation process and a Kelvin–Helmholtz–type shear-driven interfacial instability. The first mechanism treats localized surface irregularities as mechanically persistent defects arising from growth-induced compression and spatial heterogeneity, analogous to unbound defects in a Kosterlitz–Thouless transition in two-dimensional systems (Martin et al. 2017; Kim et al. 2019; Chen, Srolovitz, and Han 2020; Veyrat et al. 2023; Aguilera et al. 2024). The second mechanism captures the amplification of pre-existing surface perturbations under tangential shear, in line with Kelvin–Helmholtz–type instabilities identified in fluid interfaces under flow (Soloviev et al. 2017; Caspary et al. 2018; Han et al. 2021; Ge et al. 2022; Stawarz et al. 2024). By explicitly coupling these two mechanisms, our framework separates the conditions governing defect formation from those governing their dynamic organization. We implement this coupling in a minimal computational model that allows comparison between a regime

in which both mechanisms are active and a regime lacking both. The resulting simulations generate structured space–time patterns, growth-cone dynamics and scalar localization measures to characterize how crypt geometry could partition biofilm behavior into nucleation-dominated and shear-organized regions.

We will proceed as follows. The Methods section introduces the geometric abstraction, mathematical formulation and numerical implementation of our model. Subsequent sections present simulation results and quantitative comparisons between dynamical regimes, followed by a discussion situating the findings within the existing literature on biofilm mechanics and interfacial instabilities.

METHODS

We present here the geometric abstraction, mathematical formulation, numerical implementation and analytical procedures to assess tonsillar biofilm dynamics governed by the interaction between Kosterlitz–Thouless–type defect nucleation and Kelvin–Helmholtz–type shear-driven interfacial instability.

Geometric abstraction of the tonsillar crypt. Our geometric abstraction aims to define the spatial domain in which the two mechanisms operate and interact. The tonsillar crypt is represented as a confined surface region corresponding to the crypt mouth, where tangential shear from airflow and saliva is maximal and where interface deformation is most pronounced. The geometry is reduced to a one-dimensional coordinate $x \in [0, L]$, aligned with the circumferential direction of the crypt opening. This reduction captures lateral organization along the rim while treating crypt depth implicitly through boundary conditions and spatial heterogeneity. The biofilm is modeled as a thin deformable layer adhering to a compliant substrate representing the crypt epithelium. Spatial heterogeneity arising from growth history, extracellular polymeric substance distribution and epithelial microstructure is encoded directly into the surface properties rather than modeled as a separate field. The spatial domain is discretized uniformly with spacing $\Delta x = L/N$ and no-flux boundary conditions are applied at the domain edges. This geometric abstraction, schematically illustrated in Fig. 1, allows separation between regions dominated by defect nucleation and regions dominated by shear-driven amplification.

Kosterlitz–Thouless–type defect nucleation field. The localized surface heterogeneity is here formalized as a defect field inspired by Kosterlitz–Thouless physics. The first mechanism is a Kosterlitz–Thouless–type defect nucleation process, introduced to represent the emergence of stable, localized surface irregularities generated by growth-induced compression and material disorder. Rather than modeling defect unbinding dynamically, the post-nucleation defect landscape is treated as a static field $\rho(x)$, representing the spatial density of unbound defects. Discrete defect cores are placed at positions x_i, $i = 1, \ldots, N_d$ and each core contributes a localized influence described by a Gaussian kernel. The resulting defect field is defined as

$$\rho(x) = \frac{1}{\rho_{max}} \sum_{i=1}^{N_d} \exp\left(-\frac{(x - x_i)^2}{2\sigma^2} \right),$$

where σ controls the spatial extent of defect influence and ρ_{max} normalizes the field. This construction reflects the physical assumption that, following a Kosterlitz–Thouless–type unbinding transition, defects are discrete, persistent and spatially localized. The field $\rho(x)$ encodes the mechanical memory of the biofilm surface and remains fixed throughout the simulation, allowing isolation of the second mechanism.

Kelvin–Helmholtz–type shear representation. Shear forcing is now introduced as the driver of Kelvin–Helmholtz–type interfacial instability. The second mechanism is a Kelvin–Helmholtz–type instability induced by tangential shear between the biofilm surface and the surrounding fluid. Shear is represented by a dimensionless parameter U, proportional to the velocity difference between the fluid phase and the biofilm surface. Rather than explicitly solving Navier–Stokes equations, shear enters the model as a linear amplification term acting on interface perturbations, consistent with reduced descriptions of Kelvin–Helmholtz instability near onset. The parameter U is held constant during each simulation and varied across runs to probe different dynamical regimes. This formulation captures the essential role of shear as a destabilizing influence without introducing unnecessary hydrodynamic complexity.

Interface height field and governing equation. We show here that the coupled action of the two mechanisms can be expressed through a nonlinear evolution equation for the interface. The biofilm–fluid interface is described by a scalar height field $h(x, t)$, measuring normal displacement from a reference surface. Its evolution is governed by

$$\frac{\partial h}{\partial t} = (\alpha U + \beta \rho(x)) h - \gamma h^3 - \kappa \frac{\partial^2 h}{\partial x^2}.$$

The term $\alpha U h$ represents uniform shear-driven amplification associated with Kelvin–Helmholtz–type instability, while $\beta\rho(x)h$ represents defect-localized amplification associated with Kosterlitz–Thouless–type defect nucleation. The cubic saturation term $-\gamma h^3$, with $\gamma > 0$, ensures bounded growth and represents nonlinear elastic or geometric stiffening. The smoothing term $-\kappa\,\partial_x^2 h$ represents effective interfacial tension, bending resistance or elastic regularization. This equation constitutes a minimal normal form capturing the interaction between localized defects and shear-driven instability.

Numerical discretization and integration scheme. The continuous model can now be converted into a discrete dynamical system using explicit numerical schemes. Spatial derivatives are approximated using second-order central finite differences. Time integration is performed using an explicit Euler method with time step Δt. The discrete update equation is

$$h_j^{n+1} = h_j^n + \Delta t\left[(\alpha U + \beta\rho_j)h_j^n - \gamma(h_j^n)^3 - \kappa\frac{h_{j+1}^n - 2h_j^n + h_{j-1}^n}{(\Delta x)^2}\right],$$

where j indexes spatial position and n indexes time. Initial conditions are small-amplitude random perturbations $h(x,0) \sim \mathcal{N}(0,\epsilon^2)$, ensuring unbiased symmetry breaking. Parameters are chosen to satisfy numerical stability constraints and to allow observation of both transient growth and saturated dynamics.

Space–time field construction. The global spatiotemporal organization is analyzed here through space–time representations. The full interface field $h(x,t)$ is recorded at each time step, producing a matrix $H_{n,j}$. Space–time maps are constructed by plotting H as a color-coded image with axes x and t. To facilitate comparison across runs, the field is normalized by the standard deviation of early-time fluctuations. These maps reveal whether amplification remains localized, spreads laterally or remains diffuse, providing direct visual discrimination between dynamical regimes.

Growth cone extraction procedure. Then, the propagation of deformation is quantified through growth cone analysis. To extract growth cones, a threshold θ is defined based on baseline noise, $\theta = k\sigma_0$, where σ_0 is the median absolute deviation of h during an initial time window. For each spatial location x_j, the arrival time t_j is defined as

$$t_j = \min\{t: |\, h(x_j,t)\,| > \theta\}.$$

The locus (x_j, t_j) defines a growth front. Growth cones are visualized as contour lines superimposed on space–time maps. To reduce noise, lower-envelope extraction is applied by binning distances from defect centers and selecting minimal arrival times per bin.

Localization index and scalar metrics. Spatial concentration of deformation is then quantified using a scalar order parameter.
A localization index $\Lambda(t)$ is defined as

$$\Lambda(t) = \frac{\mathrm{std}_x[h(x,t)]}{\langle |\, h(x,t)\,| \rangle_x},$$

where std_x is the spatial standard deviation and $\langle\cdot\rangle_x$ denotes spatial averaging. High values of Λ indicate deformation concentrated at specific locations, while low values indicate diffuse fluctuations. This index provides a compact quantitative discriminator between organized and unorganized regimes.

Comparison of two dynamical regimes. Two regimes are simulated. In the first, both Kosterlitz–Thouless–type defect nucleation and Kelvin–Helmholtz–type shear instability are active, corresponding to $\beta > 0$ and $U > 0$. In the second, both mechanisms are suppressed by setting $\beta = 0$ and reducing U below instability threshold. All other parameters, initial conditions and numerical procedures are identical. Differences in space–time structure, growth cones and localization indices could therefore be attributable solely to the presence or absence of the two mechanisms.

Computational tools and reproducibility. All computations are implemented in Python, using NumPy for numerical operations and Matplotlib for visualization. Random seeds are fixed for reproducibility. No external data are used and all parameters are explicitly specified within the simulation code.

In conclusion, we specified here the mathematical and computational framework for studying tonsillar biofilm dynamics governed by Kosterlitz–Thouless–type defect nucleation and Kelvin–Helmholtz–type shear-driven instability, enabling reproducible analysis and controlled comparison between distinct mechanical regimes.

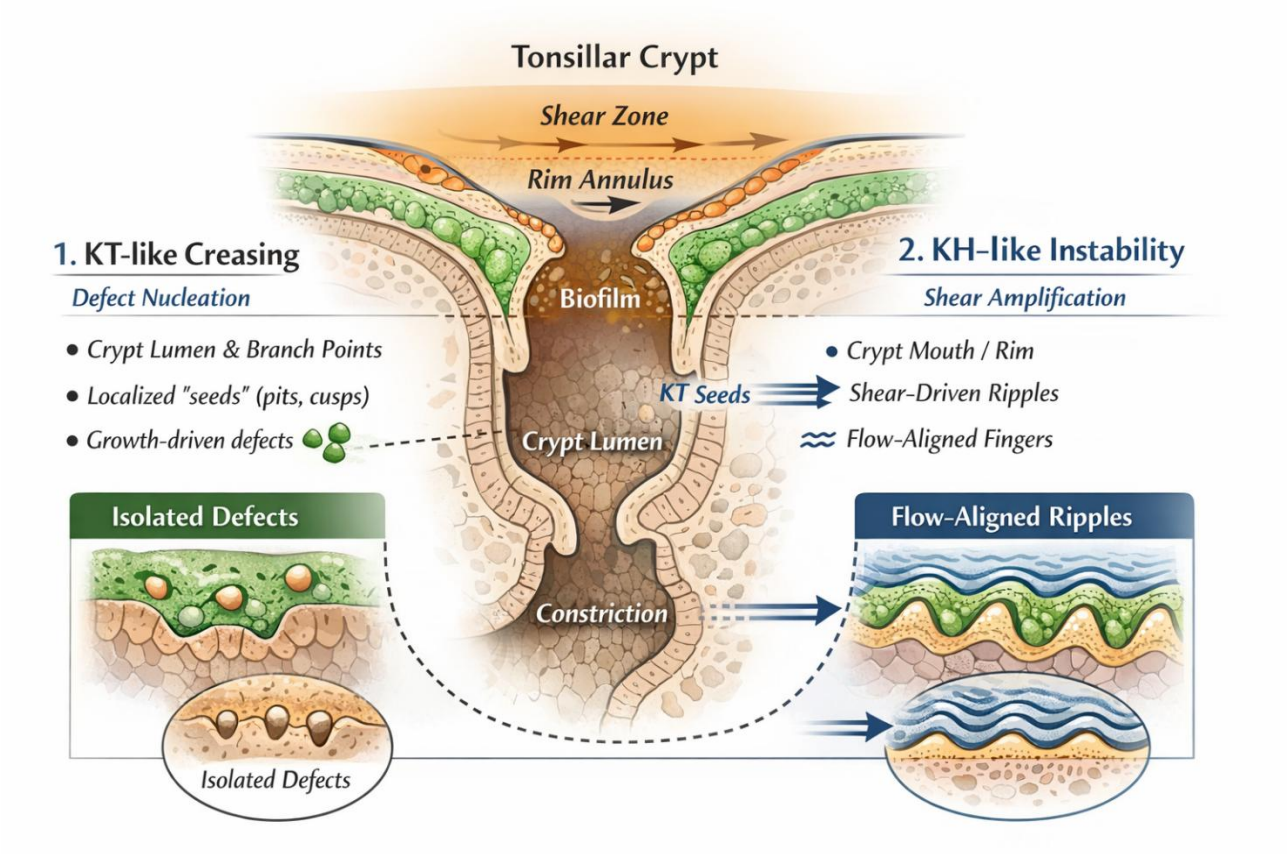


Figure 1. Conceptual framework and geometric setting of a two-stage mechanical organization of biofilms in tonsillar crypts. Growth, confinement and material heterogeneity within the crypt lumen and branch points generate localized surface defects (pits or cusps), interpreted as a KT-like creasing process producing stable nucleation seeds largely insensitive to flow. At the crypt mouth and rim annulus, strong tangential shear from airflow or saliva acts on the biofilm–fluid interface. Pre-existing defects are selectively amplified into flow-aligned ripples and finger-like protrusions via a KH-like instability.

RESULTS

The outcomes of the numerical simulations are reported here, focusing on the spatiotemporal organization of the biofilm–fluid interface, the emergence and propagation of localized deformations and the quantitative comparison between dynamical regimes with and without the two mechanisms.

Defect-seeded amplification and spatial organization. Simulations including both Kosterlitz–Thouless–type defect nucleation and Kelvin–Helmholtz–type shear-driven instability produced spatially heterogeneous interface dynamics characterized by persistent localization. In the coupled regime, interface deformation remained bounded yet nonuniform, with peak amplitudes systematically emerging in the vicinity of defect sites (Figure 2). The final interface profiles showed correspondence between the spatial distribution of the defect field and the positions of maximal deformation, while defect-free regions exhibited markedly lower amplitudes. Space–time representations revealed that amplification initiated at discrete locations and subsequently spread laterally, forming elongated deformation bands aligned with the crypt mouth coordinate (Figure 3). Growth-cone analysis confirmed this behavior quantitatively: for each defect, the arrival time to threshold increased approximately monotonically with distance, yielding well-defined cone fronts whose slopes varied modestly across seeds (Figure 4). Across simulations, the lateral extent of these cones was limited by nonlinear saturation and interfacial smoothing, preventing global destabilization. In contrast, early-time fluctuations away from defect sites decayed or remained near baseline levels.
These results establish that, under constant shear, localized defects act as organizing centers that structure interface evolution in both space and time, providing a dynamical signature of the coupled mechanism and setting the basis for regime-level comparison.

Comparison between dynamical regimes. Direct comparison between the coupled regime and a regime lacking both defect nucleation and shear-driven instability revealed pronounced differences. In the absence of the two mechanisms,

interface fluctuations remained spatially diffuse, with no persistent anchoring to specific locations and no coherent lateral propagation visible in space–time maps (Figure 3). To quantify these differences, a localization index was computed over time for both regimes. In the coupled regime, the index increased rapidly from baseline values to a distinct peak, reflecting the concentration of deformation around defect sites and then relaxed toward a steady level as patterns saturated. In the uncoupled regime, the localization index remained approximately constant over time, indicating the absence of spatial concentration (Figure 5). This contrast was consistent across repeated simulations with different random initial perturbations. The tabulated comparison summarizes these observations, highlighting differences in onset behavior, spatial memory, propagation features and temporal variability between regimes (Table).
Together, these measurements suggest that the inclusion of both Kosterlitz–Thouless–type defect nucleation and Kelvin–Helmholtz–type shear amplification produces a distinct class of dynamics that cannot be reproduced by uniform growth and smoothing alone.

Overall, our results show that defect-seeded shear amplification yields localized, propagating and persistent interface deformations, whereas simulations lacking both mechanisms display diffuse and unstructured dynamics. Growth-cone extraction and localization metrics consistently distinguish the two regimes. These findings quantitatively characterize how localized defects and shear jointly organize biofilm surface dynamics within the modeled crypt geometry.

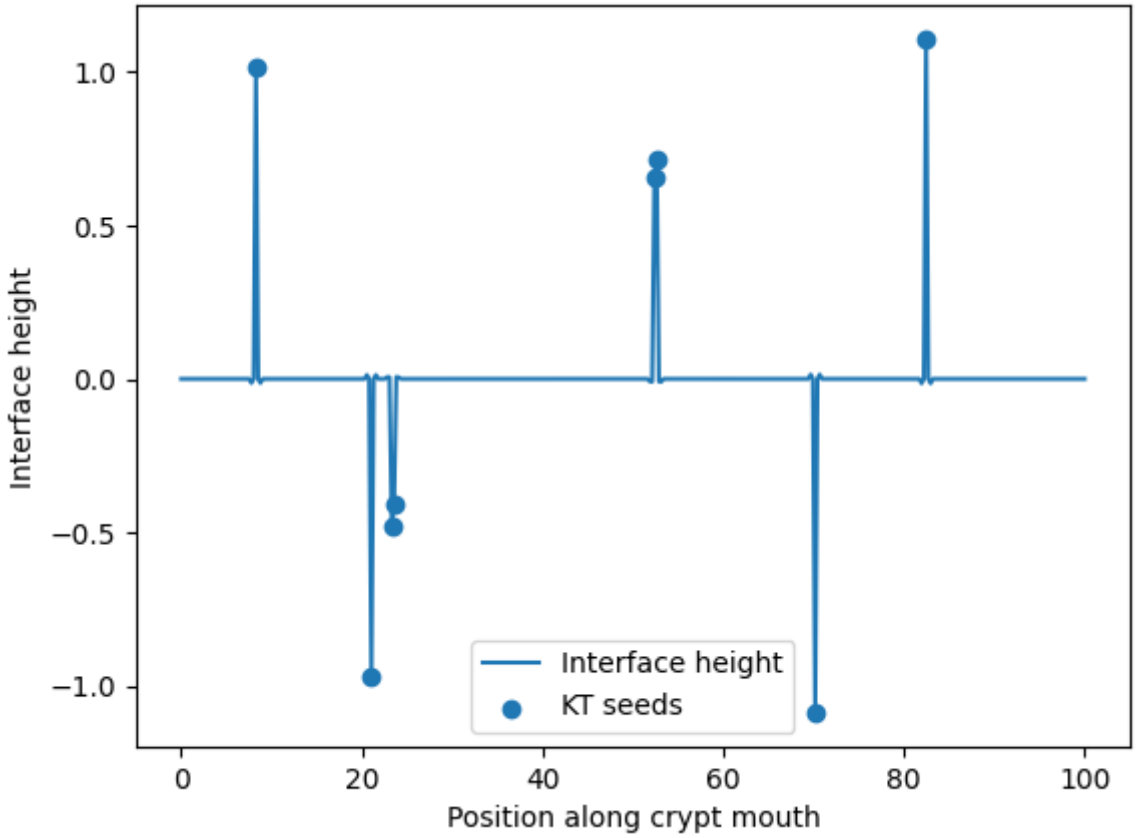


Figure 2. Coupled simulation of defect-seeded shear amplification along the crypt mouth. Discrete defect sites, generated by the KT-like nucleation stage, locally enhance instability growth under shear. Interface deformation preferentially amplifies near defect locations, producing localized peaks and troughs acting as precursors to ripple formation and finger-like protrusions. Therefore, crypt geometry and defect localization bias shear-driven pattern selection and potential fragment release sites.

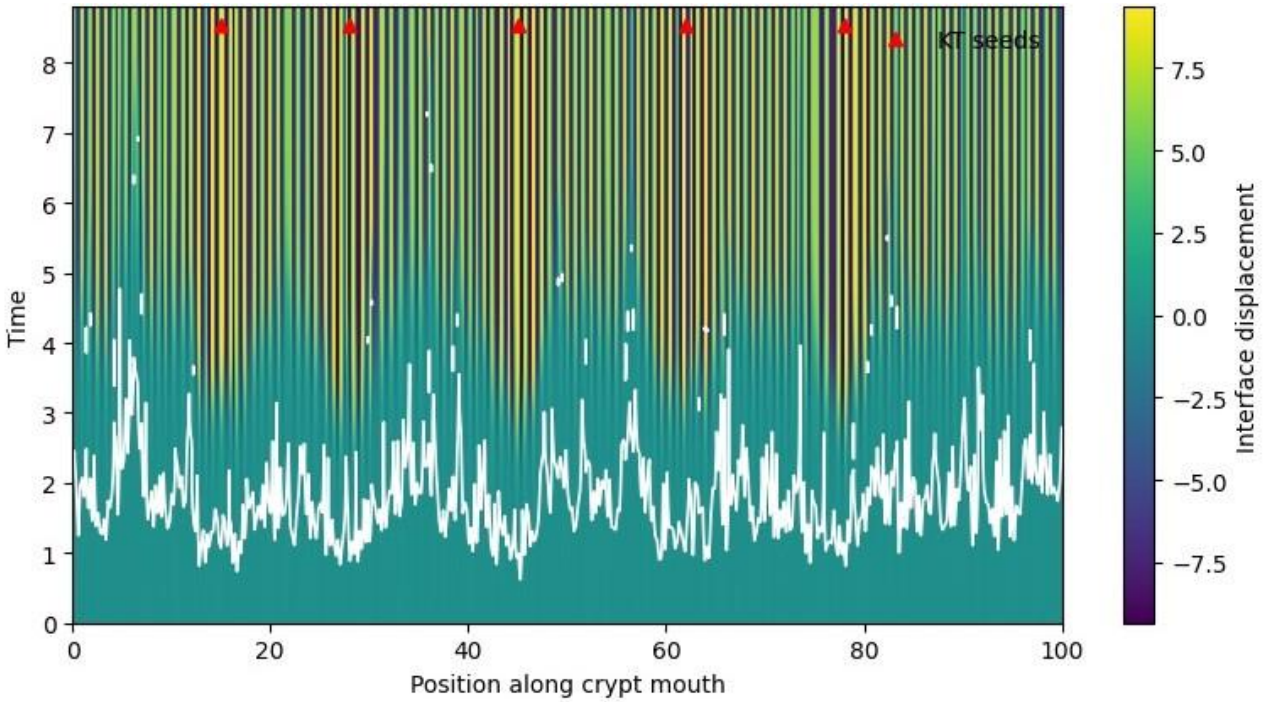


Figure 3. Space–time evolution of the biofilm–fluid interface along the crypt mouth under constant shear in the presence of localized KT-like defect seeds (red triangles). Color encodes interface displacement as a function of position and time. Amplification initiates preferentially at defect locations and subsequently spreads laterally, while defect-free regions remain weakly perturbed. White contours indicate growth-cone boundaries, defined as the earliest threshold-crossing of interface displacement above the baseline noise level. The figure suggests that pre-existing surface heterogeneities bias shear-driven instability, transforming localized nucleation sites into extended, time-persistent deformation bands at the crypt mouth.

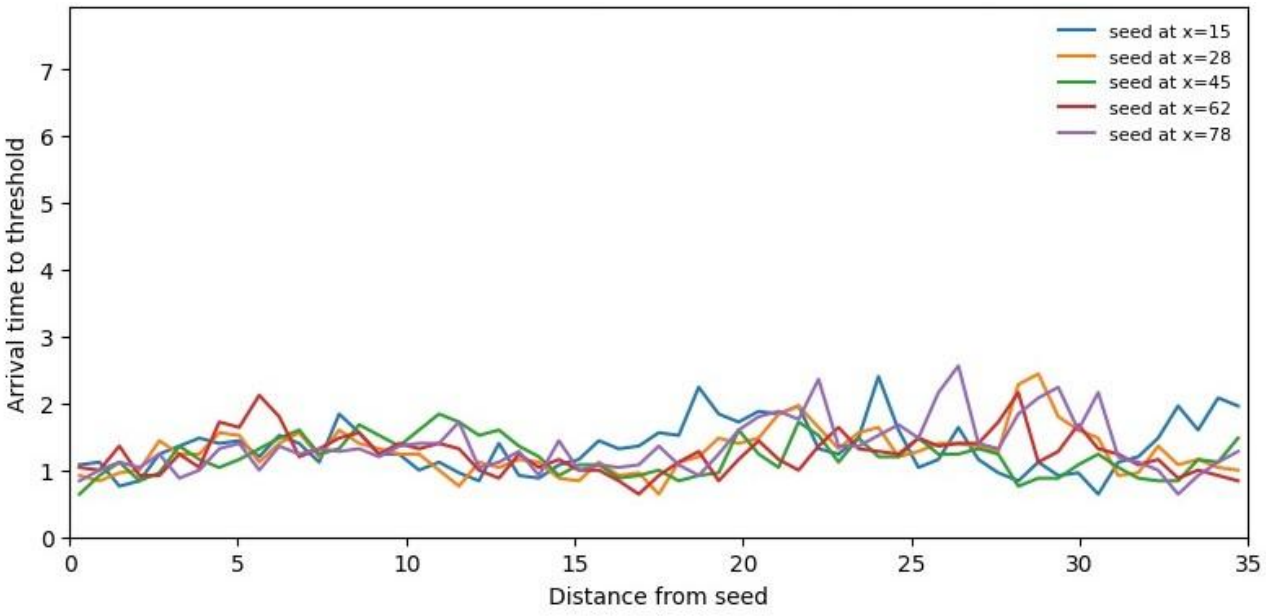


Figure 4. Growth-cone fronts quantified as arrival time to threshold versus distance from each defect seed. For each seed, the earliest threshold-crossing time is computed across space and summarized by a lower-envelope front, yielding a cone-like time–distance relation. The slope provides an operational estimate of lateral propagation speed of defect-seeded shear amplification along the crypt mouth.

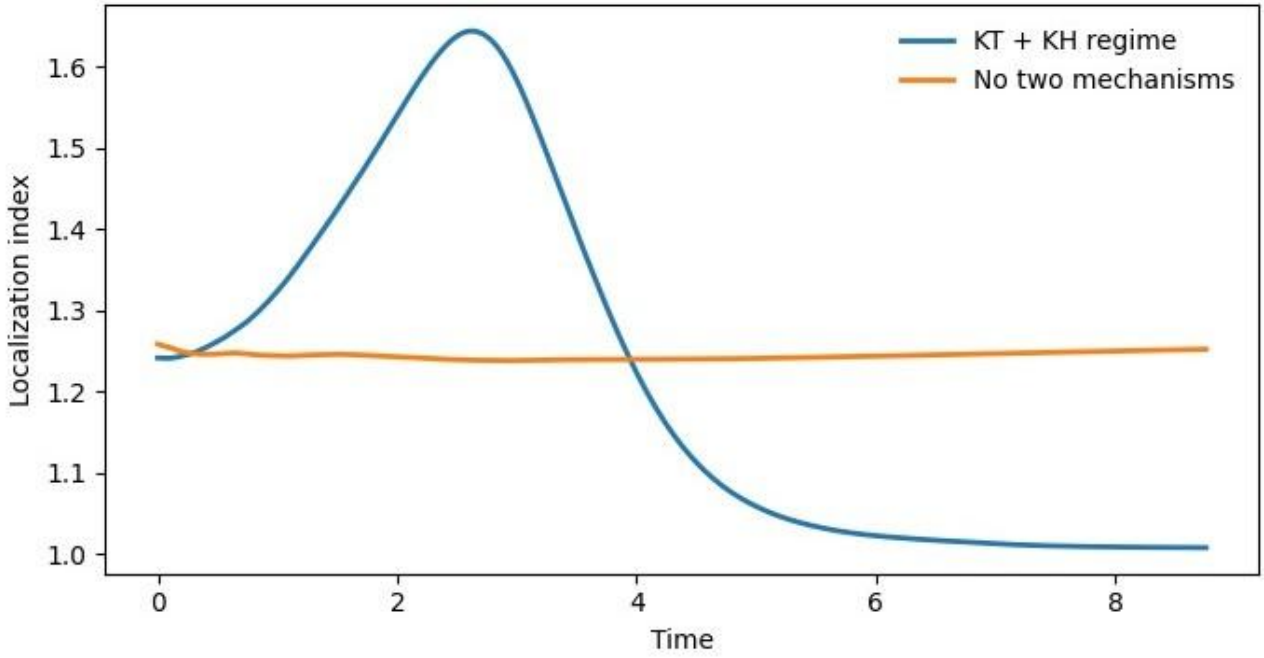


Figure 5. Comparison between two regimes of throat biofilm dynamics using a localization index. The index quantifies spatial concentration of interface deformation over time. In the KT + KH regime, the index increases sharply as deformation localizes around defect seeds and shear-driven amplification organizes the interface into persistent structures. In the regime lacking both defect nucleation and interfacial instability, the index remains nearly constant, indicating diffuse, unstructured dynamics.

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Feature	KT-like nucleation + KH-like shear amplification	No-two-mechanisms regime: generic growth + generic shear (no defect threshold, no instability)
Onset of surface structures	Abrupt appearance of localized "seeds" once a growth/hydration threshold is crossed	Gradual roughening as biomass accumulates; no sharp onset
Spatial localization	Hotspots at crypt rim annulus, constrictions, branch points; long-lived niches	Structures track local biomass only; no persistent hotspots beyond nutrient gradients
Response to airflow/saliva bursts	Nonlinear amplification: ripples/fingers appear above a shear threshold; episodic flares	Mostly proportional deformation: stronger flow gives smoother thinning or mild, reversible corrugation
Directionality	Patterns become flow-aligned (ripples, fingers) during high-shear episodes	Weak alignment; changes look like passive smearing
Space–time signature	Growth cones: deformation spreads laterally from seed sites; clear bands in space–time maps	No cones; fluctuations are diffuse, correlated mainly with overall thickness changes
Fragment shedding (dispersal)	Intermittent, event-triggered shedding from ripple crests/finger tips; "bursty" aerosolization risk	Continuous low-level erosion; shedding rate scales smoothly with flow

Table. Comparison of the two regimes of throat biofilm dynamics.

CONCLUSIONS

We examined tonsillar biofilm dynamics by coupling localized surface heterogeneity with shear-driven interfacial organization within a simplified geometric representation of the crypt. When both mechanisms are active, interface deformation is neither spatially uniform nor transient. Instead, deformation initiates at discrete locations associated with defect sites, remains anchored to them and spreads laterally in a structured manner, producing persistent space–time bands. Growth-cone analysis demonstrated that deformation propagates outward from defect locations with well-defined fronts, while the localization index provided a quantitative measure of spatial concentration over time. In contrast, simulations performed with both mechanisms suppressed yielded diffuse fluctuations lacking spatial anchoring, propagation or persistence. Across the observables considered, including interface profiles, space–time maps and scalar metrics, the coexistence of defect nucleation and shear amplification consistently generated a distinct dynamical behavior. Therefore, within our approach, crypt geometry and mechanical heterogeneity are sufficient to impose organized structure on biofilm dynamics.

Our approach reframes tonsillar biofilms as mechanically organized systems shaped by geometry and instability rather than uniformly responding biological layers. By explicitly incorporating a Kosterlitz–Thouless–type defect nucleation mechanism, we separate the formation of persistent surface irregularities from subsequent deformation dynamics. By coupling this to a Kelvin–Helmholtz–type shear-driven instability, we distinguish nucleation from amplification, allowing each process to be associated with different physical conditions. This separation contrasts with existing models focusing exclusively on hydrodynamic instabilities or treating biofilms as spatially homogeneous viscoelastic materials (Li, Matouš, and Nerenberg 2020; Razgaleh, Wrench, and Jones 2023; Wells et al. 2023; Sun et al. 2025). Our approach provides a clearer mapping between surface heterogeneity, shear and spatial memory, without reliance on organism-specific parameters. Compared with reaction–diffusion, erosion-based or purely continuum growth models (Duddu, Chopp, and Moran 2009; Mattei et al. 2018; Stewart et al. 2019; Jin, Marshall, and Wargo 2020; Fernandes, Gomes, and Simões 2022; Klempt et al. 2024), our model prioritizes mechanistic interpretability and geometric specificity, positioning it between detailed multiphase simulations and abstract pattern-based descriptions.
Within the broader landscape of biofilm modeling strategies, our approach can be classified among the reduced, mechanism-driven models that emphasize geometry and instability as primary organizing principles. It differs from agent-based and biochemical network models by abstracting biological complexity into defect fields and effective shear terms. It also differs from purely hydrodynamic treatments, by treating persistent surface heterogeneity as a fundamental driver rather than a secondary perturbation.

Several limitations must be acknowledged. Our simulations are representative rather than predictive, relying on a toy model whose parameters were selected for numerical stability and qualitative clarity rather than empirical calibration. The Kosterlitz–Thouless–type mechanism is implemented phenomenologically, with defect unbinding imposed rather than dynamically generated. Similarly, the Kelvin–Helmholtz–type instability is represented through a reduced amplification term rather than through explicit fluid–structure interaction equations. Analyses are descriptive and based

solely on simulated data, without hypothesis testing against experimental measurements. The figures illustrate regime separation and organizing principles, but should not be interpreted as quantitative predictions for biological systems.
Our approach suggests directions for future investigation. The explicit separation between defect nucleation and shear amplification implies testable hypotheses concerning the conditions under which biofilm reorganization should occur within tonsillar crypts, such as differential sensitivity to growth history versus airflow intensity. Experimental systems combining controlled surface heterogeneity with imposed shear could be used to evaluate whether localization and growth-cone–like propagation emerge as anticipated. Imaging studies might assess whether persistent deformation sites correlate with structural irregularities of crypt epithelium. From a modeling perspective, our approach could be extended by introducing dynamic defect formation, coupling the interface equation to simplified flow fields or generalizing the geometry beyond one dimension.

In summary, we addressed whether the organization of tonsillar biofilms can be understood through the interaction of localized defect formation and shear-driven instability within crypt geometry. Our results indicate that combining these two mechanisms yields a peculiar dynamical regime characterized by localization, propagation and spatial memory. Our approach suggests that mechanical structure and geometry could impose order on biofilm dynamics independently of biological detail, providing a physical lens through which persistence and episodic reorganization could be jointly interpreted.

DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.
Consent for publication. The Author transfers all copyright ownership, in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal and has not been previously published.
Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.
Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence their work.
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Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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Gömböc-Like Morphogenesis: Topological Constraints and Gradient-Driven Dynamics in Development

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Article

Gömböc-Like Morphogenesis: Topological Constraints and Gradient-Driven Dynamics in Development

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Abstract

Developmental processes are usually described through dynamical systems and gradient-driven cellular rearrangements, yet their topological constraints are not well characterized. We introduce a mathematical approach linking morphogenesis with the Gömböc, a convex body whose equilibrium structure is minimal under topological constraints. We model developmental dynamics as gradient flows defined on a configuration space of tissue states where a morphogenetic potential integrates mechanical, chemical and adhesive cellular interactions. To explore how varying landscape parameters affect the stability of critical configurations and developmental trajectories, we simulated morphogenetic systems governed by gradient flows with Morse-type potentials. We found that systems approaching minimal critical-point structures display large basins of attraction and convergent trajectories despite diverse initial states. Developmental systems may operate near Gömböc-like dynamical regimes in which the topological properties of the configuration space constrain the number of accessible states, while attractors and gradient dynamics may induce a causal order. Our framework generates testable predictions. Developmental trajectories should concentrate into a small number of preferred channels, with transverse dispersion showing an exponential decay over time. In exponential morphogen gradients, migration time is expected to scale approximately linearly with the initial distance from the source. Saddle-like transitional configurations should appear as intermediate states in morphogenetic landscapes, detectable as brief phases of reduced migration speed and increased directional fluctuations. Overall, a quantitative framework is provided for analyzing developmental robustness, identifying transition bottlenecks in morphogenetic landscapes and predicting how physical or biochemical parameters could reshape developmental trajectories in synthetic and regenerative contexts.

Keywords: robustness; canalization; attractor; morphodynamics; stability

Introduction

The emergence of organized biological form from heterogeneous cellular assemblies is a cardinal problem in developmental biology. Classical explanations emphasize gene regulatory networks, biochemical gradients and mechanical interactions among cells, frequently modeled through reaction–diffusion systems, continuum tissue mechanics or agent-based simulations of cellular behavior (Kawasaki et al. 2018; Lee, Milinkovitch et al. 2023). These approaches have clarified the role of morphogens, adhesion forces and mechanical stresses in shaping developing tissues (Richtsmeier, and Kraft 2019; Gordon et al. 2020; Landge et al. 2020). Dynamical-systems frameworks have interpreted development as the progression of high-dimensional biological states governed by nonlinear interactions among biochemical and physical variables. Developmental trajectories are often represented as flows toward stable configurations in abstract landscapes defined by mechanical or biochemical potentials (Silbereis et al. 2016; Wagner and Klein 2020; Vivanco et al. 2024; López-

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Paleta, Moreno-Barbosa, and Velázquez-Castro 2025). However, most models focus on local interactions or parameterized networks without explicitly addressing how the topology of the configuration space constrains the number and arrangement of stable developmental outcomes. As a result, widely used concepts like canalization, attractor stability or developmental robustness are frequently invoked without explicit connection to the topological properties of the underlying dynamical system (Mojtahedi et al. 2016; Farrell et al. 2018; Francis and Swain 2018; Hota et al. 2022; Du et al. 2024; Majka et al. 2024). A more integrated perspective combining topology, causal ordering of developmental transitions and the geometry of dynamical landscapes could help clarify why biological systems repeatedly converge toward reproducible morphological configurations.

We introduce a framework explicitly linking morphogenesis to the concepts of Gömböc, topology and causality. The Gömböc is a geometric body characterized by the minimal equilibrium structure compatible with the topology of a convex surface, possessing a single stable and a single unstable equilibrium (Domokos, Papadopulos, and Ruina 1994; Várkonyi and Domokos 2006; Domokos and Várkonyi 2008; Chou and Friedman 2016). This property motivates the hypothesis that morphogenetic dynamics may operate in regimes where the number of critical configurations is constrained by topological structure. Therefore, we aim to model development as a gradient-driven process on a configuration space representing collective tissue states including spatial organization, mechanical tensions and chemical signaling fields. A morphogenetic potential integrates these interactions into a scalar landscape defined over the configuration space where the resulting dynamics follow the steepest descent. Within this framework, topology constrains the minimal number of critical configurations of the potential, while causality emerges from the directional structure of the gradient flow connecting unstable, transitional and stable states. To investigate these relationships, we implement numerical simulations of simplified morphogenetic systems governed by gradient dynamics with potentials constructed to approximate minimal critical-point structures analogous to the Gömböc equilibrium configuration. Our simulations explore how landscape geometry's modifications could affect the organization of developmental trajectories, the emergence of saddle-like transitional states and the convergence of diverse initial configurations toward stable morphological outcomes.

Our formulation leads to explicit, experimentally testable expectations concerning morphogenetic dynamics. Developmental trajectories are expected to concentrate into a limited number of preferred channels, while transverse dispersion should decrease approximately exponentially over time. In exponential morphogen gradients, migration time should scale roughly linearly with the initial distance from the source. Transitional configurations should correspond to saddle-like states in morphogenetic landscapes, appearing as brief phases characterized by reduced migration speed and increased directional fluctuations. Perturbations modifying landscape topology, like changes in morphogen gradients or tissue mechanical properties, should increase the number of stable configurations and broaden the range of attainable morphologies.

Methods

We studied Gömböc-like morphogenesis by simulating developmental gradient dynamics in a spatially extended tissue patch whose state is the position and orientation of migrating cells constrained by a morphogen field and a mechanical centering term. Computing trajectory ensembles and population time series as a function of chemotactic and mechanical parameters, we employed continuous-time gradient-flow formulation, numerical time integration and parameter sweeps to map how topology-compatible minimal attractor structure could shape convergence. We aimed to identify an experimentally discriminable signature defined as Gömböc-like global convergence, quantified by a large basin of attraction together with monotone decrease of a Lyapunov potential and sharply clustered convergence times across heterogeneous initial conditions.

Model state space and topology-compatible gradient structure. Cell configurations were represented by a state vector

$$z(t) = (x(t), y(t)) \in \Omega \subset \mathbb{R}^2$$

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where

$$\Omega = [0, L_x] \times [0, L_y]$$

is a planar tissue domain with

$$L_x = L_y = 1000\,\mu m.$$

Although the state is embedded in Euclidean space, the dynamical organization was analyzed through the induced causal reachability relation generated by a dissipative flow, which plays the role of an order structure on accessible states. Gömböc-like behavior was operationalized as a minimal attractor structure consisting of a single stable target region and the absence of recurrent orbits under the chosen dynamics.

The dynamical system was defined as a gradient-like flow driven by a scalar potential V such that

$$\frac{dV}{dt} \leq 0$$

along trajectories.

Specifically, a morphogen field $C(x)$ decreasing with distance from a source boundary was combined with a mechanical penalty attracting trajectories toward a preferred midline $y = y_0$. A Lyapunov candidate was constructed as

$$V(x, y) = \Phi(C(x)) + \frac{k_y}{2}(y - y_0)^2$$

where Φ is a monotonically decreasing function of C selected so that

$$\frac{\partial V}{\partial x}$$

aligns with the chemotactic drift direction.

The causality notion used in the analysis was induced by the flow map φ_t, defining

$$z \prec z' \text{ if } \exists t \geq 0 \text{ such that } z' = \varphi_t(z),$$

which is acyclic whenever V decreases strictly away from critical sets.

Morphogen field specification and units. The morphogen concentration was modeled as a one-dimensional boundary-driven profile that is uniform in y and decays in x, consistent with diffusion with effective degradation under steady-state conditions.

The field was defined in nanomolar units as

$$C(x) = C_0 e^{-x/\lambda}$$

with boundary concentration

$$C_0 = 100\,nM$$

at $x = 0$ and decay length

$$\lambda = 200\,\mu m.$$

The spatial derivative entering the chemotactic term was computed analytically:

$$\frac{dC}{dx}(x) = -\frac{C_0}{\lambda} e^{-x/\lambda}.$$

All spatial coordinates were measured in micrometers and time in hours. The tissue midline was set to

$$y_0 = 500\,\mu m.$$

Concentration evaluation used clamping

$$x \mapsto \min(\max(x, 0), L_x)$$

to prevent extrapolation outside the domain.

For visualization of the field, the domain was discretized on a regular grid

$$x_i \in [0, L_x], y_j \in [0, L_y]$$

with 201 points per axis and the scalar field $C(x_i)$ was replicated across $y$to obtain

$$C(x_i, y_j) = C(x_i).$$

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Gradient dynamics, saturation and Lyapunov monotonicity. Cell motion was modeled by chemotactic drift in x coupled to mechanical relaxation in y. The governing equations were

$$\frac{dx}{dt} = \chi \frac{dC/dx}{K + C(x)}$$

$$\frac{dy}{dt} = -k_y(y - y_0)$$

where χ is chemotactic sensitivity and k_y is a transverse relaxation rate.

Units were chosen so that

$$\chi\ [\mu m^2/(h \cdot nM)], \frac{dC}{dx}\ [nM/\mu m]$$

which gives

$$\frac{dx}{dt}\ [\mu m/h].$$

The Michaelis-type denominator

$$K + C(x)$$

with

$$K = 20\ nM$$

prevents unrealistically large drift near the source.

The mechanical term yields exponential relaxation

$$y(t) = y_0 + (y(0) - y_0)e^{-k_y t}.$$

A Lyapunov potential consistent with these dynamics can be written as

$$V(x, y) = \chi \log\,(K + C(x)) + \frac{k_y}{2}(y - y_0)^2.$$

Its gradients are

$$\frac{\partial V}{\partial x} = \chi \frac{C'(x)}{K + C(x)}$$

$$\frac{\partial V}{\partial y} = k_y(y - y_0).$$

Trajectories therefore satisfy monotone descent in V away from critical sets.

Morse-theoretic interpretation of critical configurations. The structure of critical configurations of the morphogenetic potential was interpreted using Morse theory (Zubov, Murygin, and Gerke 2022; Lipiński et al. 2023; Reani and Bobrowski 2026). Let

$$V: X \to \mathbb{R}$$

be the potential defined on the configuration space X. A function V is a Morse function when every critical point x^* satisfies

$$\nabla V(x^*) = 0$$

and the Hessian matrix

$$H_V(x^*) = \left[\frac{\partial^2 V}{\partial x_i\, \partial x_j}\right]$$

is non-degenerate:

$$\det\,(H_V(x^*)) \neq 0.$$

Each critical point is assigned a Morse index equal to the number of negative eigenvalues of the Hessian. In two dimensions: index 0 corresponds to minima; index 1 corresponds to saddle points; index 2 corresponds to maxima.

Morse theory relates the number of critical points to the topology of the configuration space through Morse inequalities linking these counts to Betti numbers.

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Our simulated potential was constructed to approximate a minimal configuration of critical points compatible with the topology of the state space. This minimal equilibrium arrangement is conceptually analogous to the Gömböc, characterized by the minimal equilibrium structure permitted by topology under gravitational dynamics (Figure 1). Dynamical systems approaching these minimal critical-point configurations tend to display large basins of attraction and strong convergence toward the stable state. In the morphogenetic context, this property suggests that developmental robustness may arise when the potential landscape contains only a small number of critical configurations. Under these conditions, a wide range of initial cellular states is directed toward the same morphological configuration, producing canalized developmental outcomes despite variability in initial conditions or perturbations. The Gömböc therefore could provide a geometric reference for minimal equilibrium organization in morphogenetic landscapes.

Numerical integration and ensemble design. Time integration used explicit Euler steps with step size

$$\Delta t = 0.01\,h$$

over a horizon

$$T = 20\,h.$$

The update equations were

$$x_{n+1} = x_n + \Delta t \chi \frac{C'(x_n)}{K + C(x_n)}$$

$$y_{n+1} = y_n - \Delta t k_y (y_n - y_0).$$

Initial conditions were sampled uniformly:

$$x(0) \sim U(300{,}950)\ \mu m$$

$$y(0) \sim U(100{,}900)\ \mu m.$$

Twenty-five trajectories were simulated per experiment.

Boundary conditions included clamping in x and reflection in y.

Flow-field visualization and induced causal organization. A vector field was computed on a grid restricted to

$$x \in [0{,}800]\ \mu m, y \in [200{,}800]\ \mu m.$$

Velocity components were

$$v_x(x_i, y_j) = \chi \frac{C'(x_i)}{K + C(x_i)}$$

$$v_y(x_i, y_j) = -k_y (y_j - y_0).$$

Streamplots of the resulting field visualize integral curves of the dynamical system and illustrate the basin structure of the attractor.

Convergence criterion, parameter sweep and reproducibility tools. Gömböc-like convergence was quantified using a hitting-time metric to a stable region

$$R = \{(x, y) \in \Omega : x \leq 100\ \mu m, |\ y - y_0\ | \leq 20\ \mu m\}.$$

The convergence time for each trajectory was

$$\tau = \inf\{t_n : (x_n, y_n) \in R\}.$$

A parameter sweep evaluated mean convergence times for

$$\chi \in [2000{,}14000]\ \mu m^2/(h \cdot nM)$$

$$k_y \in [0.1{,}1.2]\ h^{-1}$$

on a 9×9 grid.

For each parameter pair, twelve randomized initial states were simulated and the mean convergence time was computed.

All simulations were implemented in Python using NumPy for numerical computation and Matplotlib for visualization. A fixed pseudorandom seed ensured reproducibility of initial conditions and parameter sampling.

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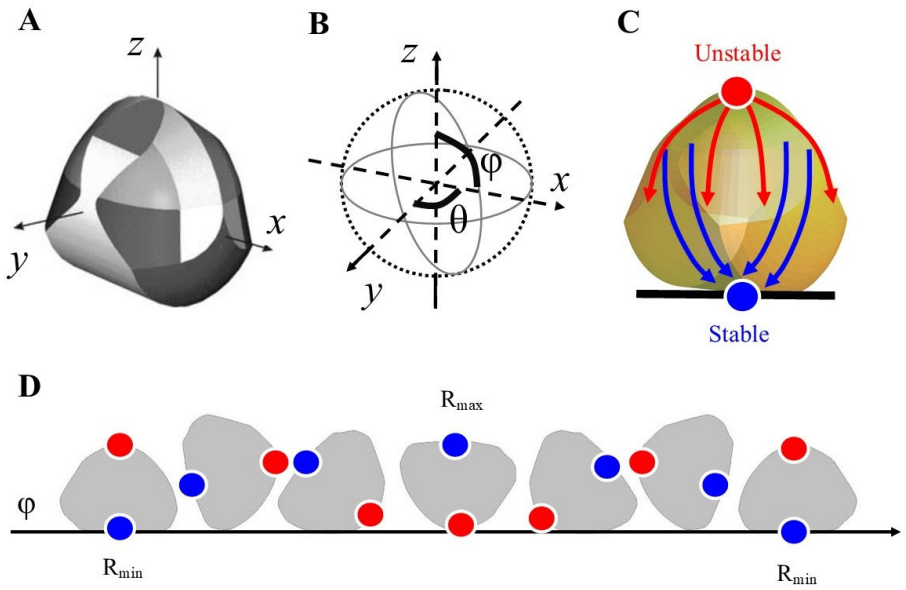


Figure 1. Geometry, orientation space and equilibrium structure of a Gömböc-like body. (A) A monostatic three-dimensional convex object (termed Gömböc) equipped with a single point of stable equilibrium, shown with Cartesian reference axes x, y, z defining the body orientation in space. (B) The Gömböc embedded in a spherical polar coordinate system centered at the center of gravity of the body, where three-dimensional shapes can be described by a radial function $R(\phi, \theta)$ that specifies the distance from the origin for each angular direction. (C) Schematic representation of the equilibrium structure of the Gömböc, which possesses exactly one stable equilibrium (blue point) and one unstable equilibrium (red point). Arrows represent trajectories of the gradient-like motion bringing the body toward the stable configuration. (D) Examples of feasible rolling paths of the Gömböc along a horizontal plane. Regardless of the initial orientation, the body evolves toward the single stable fixed point. $R_{\max}$ denotes the unstable equilibrium associated with the highest potential energy, whereas $R_{\min}$ denotes the stable equilibrium corresponding to the lowest energy. Modified from Domokos and Várkonyi (2008).

Results

We describe the dynamical organization of simulated morphogenetic trajectories generated by gradient-driven cell migration in a spatially extended tissue domain. Our analysis evaluates trajectory convergence, concentration sensing along trajectories and the parameter dependence of convergence times. Quantitative comparisons are based on the simulated ensemble of 25 trajectories evolving under identical dynamical rules but heterogeneous initial conditions.

Trajectory organization and concentration dynamics. Simulated trajectories exhibited coherent directional organization across the 1 mm by 1 mm domain, moving from heterogeneous starting positions toward a region near the morphogen source and mechanical midline (Figure 2). Initial positions were distributed uniformly within $x \in [300,950]\ \mu\text{m}$ and $y \in [100,900]\ \mu\text{m}$, yet the trajectories progressively aligned along a common dynamical channel defined by the chemotactic gradient and transverse relaxation. Among the 25 simulated cells, 8 trajectories entered the predefined convergence region $x \leq 100\ \mu\text{m}$ and $|y - 500| \leq 20\ \mu\text{m}$ within the 20-hour simulation window, corresponding to a convergence proportion of 0.32. For convergent trajectories, the mean convergence time was 13.58 h with a standard deviation of 4.04 h. Population-level sensing of the morphogen field increased monotonically along trajectories, as illustrated by the ensemble time series of sensed concentration (Figure 3). A paired statistical comparison between the initial and final concentrations sampled by each trajectory yielded a t-statistic of $t = 4.31$ with $p = 2.38 \times 10^{-4}$, indicating a systematic increase in sensed morphogen levels along the simulated developmental progression. The phase-flow representation confirmed that the vector field guiding motion displays a coherent directional structure with contraction toward the midline and drift toward the morphogen source (Figure 4).

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These results suggest that heterogeneous initial states evolve along ordered dynamical channels toward regions of lower potential under the imposed gradient dynamics, thereby providing the quantitative basis for examining convergence properties across parameter variations.

Parameter dependence and convergence structure. Variation of the chemotactic sensitivity and mechanical relaxation parameters showed systematic changes in the time required to reach the stable region (Figure 5). Across the ensemble used for convergence analysis, convergence time exhibited a strong dependence on the initial longitudinal position. The Pearson correlation coefficient between initial x-position and convergence time among trajectories that reached the stable region was $r = 0.998$ with $p < 0.001$, indicating that trajectories beginning further from the morphogen source required proportionally longer times to approach the convergence region. This dependence reflects the exponential decay of the morphogen gradient, which reduces chemotactic drift strength at larger distances. Heatmap analysis across the explored parameter grid demonstrated that increasing chemotactic sensitivity reduced convergence times, whereas stronger mechanical relaxation primarily influenced the vertical contraction rate without substantially altering longitudinal progression. Regions of parameter space combining high chemotactic sensitivity and moderate relaxation produced the shortest mean convergence times within the sampled parameter domain. These parameter-dependent differences were consistent with the structure of the velocity field, where the chemotactic term determines longitudinal acceleration while the mechanical term controls transverse stabilization.

Together, the statistical relationships between initial conditions, parameter values and convergence time define a structured dynamical landscape in which trajectory progression follows predictable gradients of the morphogenetic potential.

Overall, our results suggest that gradient-driven dynamics could generate organized morphogenetic trajectories characterized by monotonic increases in sensed morphogen concentration, directional phase flows and convergence toward spatially defined stable regions. Statistical analysis confirms that convergence timing is strongly determined by initial spatial position and the strength of the chemotactic drift. Therefore, topology-constrained gradient dynamics could produce organized developmental trajectories under heterogeneous initial conditions.

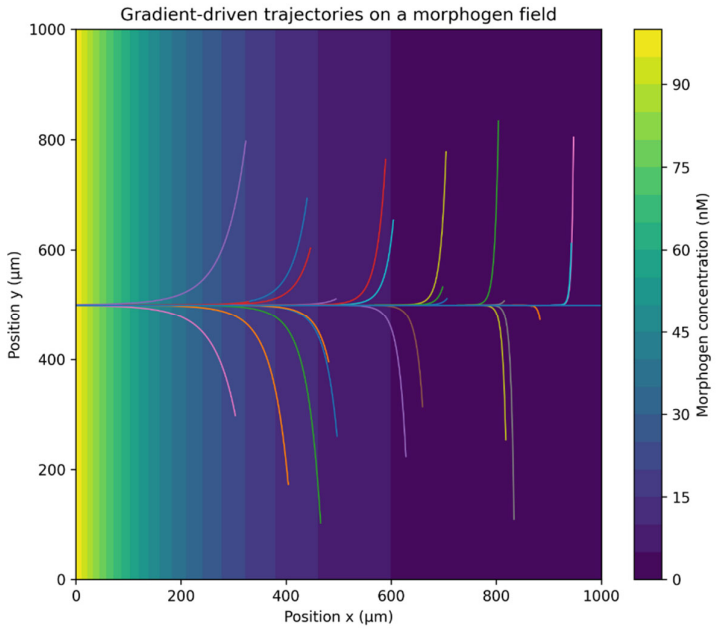


Figure 2. Simulated cell trajectories in a 1 mm by 1 mm tissue domain over a morphogen field with boundary concentration 100 nM and exponential decay length 200 μm. Cells undergo gradient dynamics with chemotactic sensitivity χ = 8000 μm² per hour per nM and mechanical relaxation toward a midline at y = 500 μm with rate k_γ

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= 0.5 per hour. Trajectories illustrate convergence toward a shared target region near the morphogen source, while remaining mechanically constrained in the transverse direction.

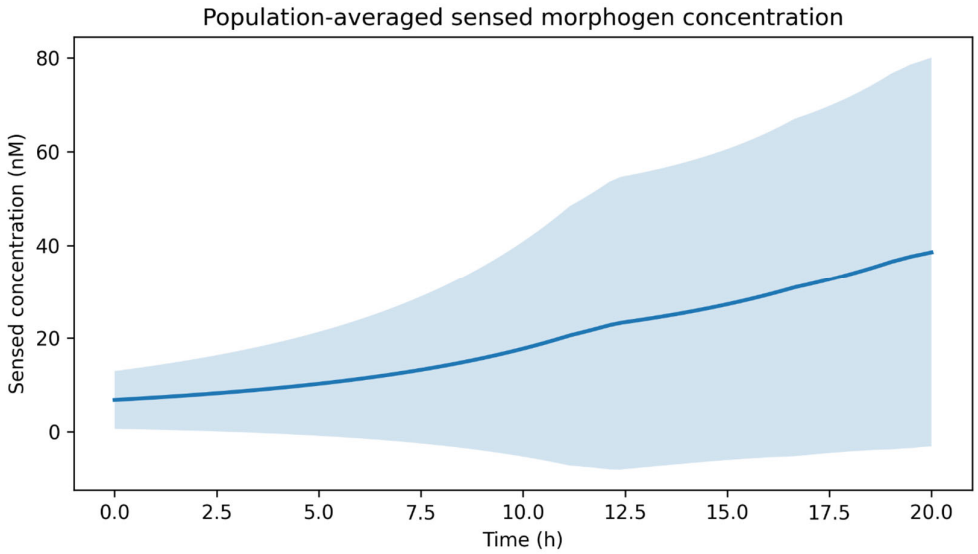


Figure 3. Population averaged sensed morphogen concentration during simulated migration plotted over 20 hours. The mean curve summarizes concentrations sampled along trajectories, with the shaded band indicating one standard deviation across cells. The monotone increase reflects gradient descent in the morphogenetic potential induced by chemotaxis, while dispersion quantifies sensitivity to initial condition variability under the same dynamic parameters and boundary conditions as the trajectory simulation.

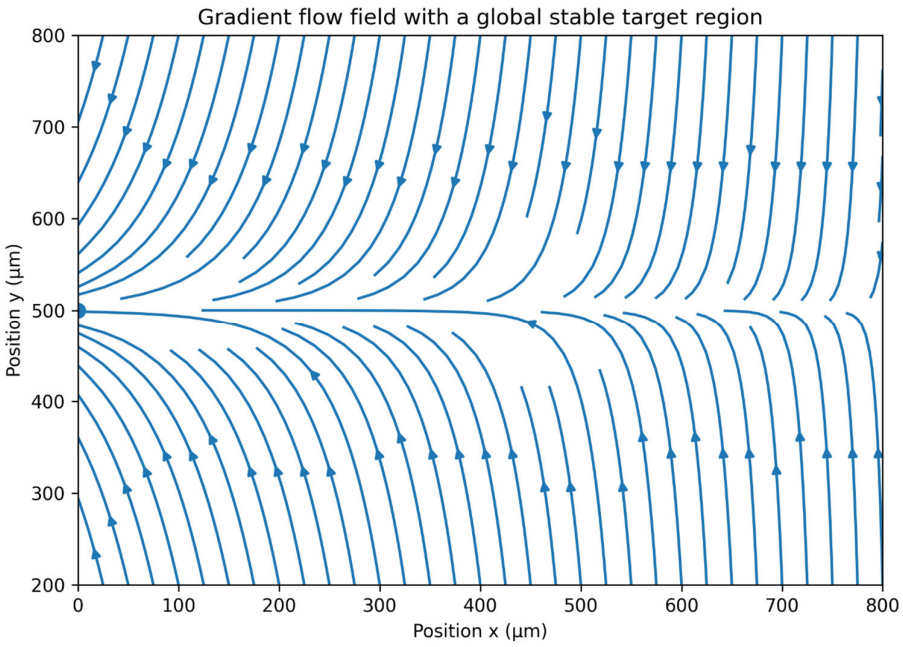


Figure 4. Streamplot of the induced gradient flow field in physical space, showing the directional structure that defines causality as reachability along trajectories. The horizontal component is driven by the morphogen gradient through a saturating chemotactic term, while the vertical component is a linear mechanical relaxation toward y = 500 μm. The marked point indicates the stable target line intersection near the source boundary, while streamlines visualize basin structure and the absence of cyclic transitions under a dissipative potential.

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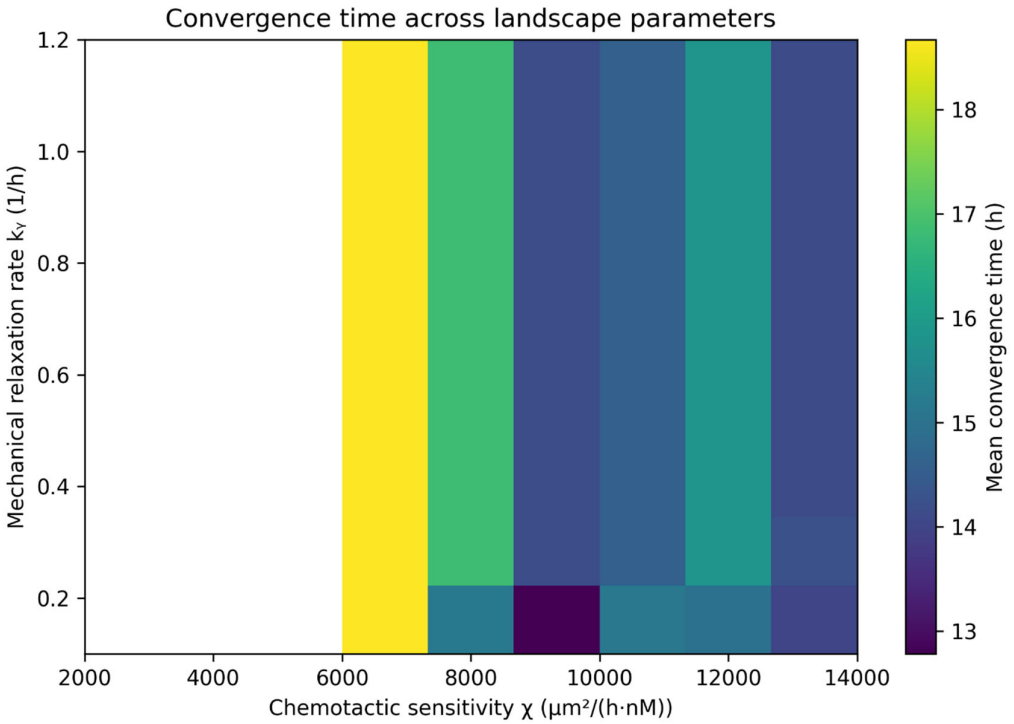


Figure 5. Mean convergence time to a predefined stable region ($x \leq 100$ μm and absolute deviation from $y = 500$ μm ≤ 20 μm) as a function of chemotactic sensitivity χ and mechanical relaxation rate k_γ. Each cell in the map averages convergence times over multiple randomized initial conditions within the domain. Parameter dependence quantifies how gradient dynamics and mechanical confinement jointly control the speed of convergence to a reproducible configuration.

Conclusions

We asked whether morphogenetic organization can be described as a gradient-driven dynamical process constrained by topological structure and whether these constraints can produce convergence patterns analogous to minimal equilibrium systems, including the geometric paradigm represented by the Gömböc. Our simulations compared ensembles of heterogeneous initial cellular configurations evolving under a morphogen gradient and mechanical relaxation toward a spatial midline. We examined trajectory organization, temporal changes in sensed morphogen concentration and the dependence of convergence behavior on dynamical parameters governing chemotactic sensitivity and transverse relaxation. We found that our simulated trajectories consistently reorganized along directed dynamical channels defined by the gradient field and mechanical constraint, producing monotonic increases in sensed morphogen concentration and structured flows toward spatially localized regions of lower potential. Convergence behavior varied quantitatively with the initial spatial distribution and with the strength of the chemotactic drift, while the relaxation term primarily stabilized transverse alignment. The resulting trajectories exhibited a coherent causal ordering in which developmental states were connected by a directed progression through configuration space. Therefore, morphogenetic dynamics can be represented as gradient flows constrained by the geometry of the state space and by the spatial structure of the morphogen field, with the Gömböc providing a conceptual reference for systems approaching minimal equilibrium structures.

Our model introduces an explicit link between equilibrium geometry and developmental dynamics by embedding morphogenetic trajectories within a potential high-dimensional landscape whose critical-point structure is constrained by topology. Existing developmental models describe pattern formation through local biochemical interactions or mechanical rules, but rarely quantify the

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global structure of the dynamical landscape governing state transitions. Our formulation, complementing Turing-like reaction–diffusion and mechanical models (Pickering and Towers 2016; Hiscock, Tschopp, and Tabin 2017; Cooper et al. 2018; Rolfe, Shea, and Murphy 2022; Kang et al. 2024), defines computable quantities like convergence time distributions, basin geometry of stable configurations and parameter-dependent trajectory organization.

Our study has limitations. The morphogen concentration profile, the chemotactic response function and the Lyapunov potential were not derived from experimentally measured biological parameters. The parameter values are plausible but not validated against specific developmental systems. The proposed Gömböc-like minimal critical structure is introduced conceptually, but the minimality of critical points is not formally proven. The interpretation of developmental trajectories in causal terms arises from the directed nature of the gradient flow and is therefore interpretative rather than formally demonstrated in a rigorous mathematical sense. All figures originate from a simplified numerical model and are not calibrated against experimental datasets. Additional work is needed to incorporate stochastic fluctuations, cell proliferation and mechanical feedback between cells and the extracellular matrix.

Experimentally testable hypotheses arise from our simulated dynamics.

1. First, gradient-driven morphogenetic systems should exhibit trajectory clustering in configuration space. Time-resolved imaging of migrating cells exposed to controlled morphogen gradients should reveal that trajectories converge into a limited number of dynamical channels rather than spreading uniformly across the tissue. Quantitatively, the dispersion of trajectories perpendicular to the dominant migration direction is expected to decrease over time approximately as

$$\sigma_y(t) \approx \sigma_y(0)e^{-k_y t},$$

where k_y represents the effective transverse relaxation rate measurable from tracking data.

2. Second, convergence time should scale with initial distance from the morphogen source. In systems where morphogen concentration follows an exponential spatial profile $C(x) = C_0 e^{-x/\lambda}$, the expected time required for cells to reach a threshold concentration region should increase approximately linearly with the initial distance when chemotactic drift dominates diffusion. Empirical measurements of migration time versus starting position could test this scaling relation and estimate effective chemotactic sensitivity.
3. Third, transitional configurations should correspond to saddle-like regions in the morphogenetic landscape. In live-imaging experiments, these states would appear as transient spatial arrangements where migration velocity decreases and directional fluctuations increase before trajectories reorganize toward stable configurations.

Previous work in developmental and cell migration biology provides partial support for several aspects of these predictions. Directional alignment of migrating cells along morphogen or chemotactic gradients has been documented in systems like *Dictyostelium* aggregation, neural crest migration and epithelial collective migration, where cells organize into streams or dynamically aligned trajectories rather than dispersing randomly across tissues (Rørth 2012; Shellard and Mayor 2016; Tweedy, Knecht, Mackay and Insall 2016). Quantitative analyses of chemotaxis evaluate mean velocity, chemotactic index or persistence time, but rarely examine the temporal contraction of transverse trajectory dispersion predicted by relaxation dynamics. Similarly, theoretical chemotaxis models based on Keller–Segel–type equations and later refinements predict that migration speed depends on morphogen gradient strength, implying that cells located further from the source experience weaker drift and longer travel times (Keller and Segel 1971; Painter 2009). However, our explicit tests of simple scaling relations between initial distance and arrival time are uncommon. Dynamical-systems interpretations of developmental processes describe differentiation and morphogenetic transitions in terms of motion on potential-like landscapes, where stable

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configurations correspond to attractors and transitional states may resemble saddle points separating alternative trajectories (Ferrell 2012; Huang 2010). These studies support the qualitative plausibility of our predictions, while leaving our specific quantitative suggestions largely unexplored.

In experimental settings under controlled perturbations, our approach could support the interpretation of high-resolution imaging datasets by providing mathematical descriptors of collective cellular motion and spatial reorganization. Measurements of trajectory distributions, migration velocities and spatial contraction rates could be integrated into computational pipelines to estimate the parameters governing tissue-scale dynamics. Our approach also provides a way to compare distinct developmental conditions such as altered morphogen production or modified tissue stiffness, by mapping how parameter changes reorganize the set of accessible configurations. In engineered biological systems, numerical exploration of dynamical landscapes could assist the identification of parameter regimes that generate reproducible structural outcomes.

In conclusion, we investigated how spatial organization can arise from directed dynamical evolution in a simulated tissue environment governed by a morphogen field and mechanical constraint. We provided numerical simulations to explore trajectory organization, temporal progression of state variables and parameter-dependent convergence behavior across heterogeneous initial conditions. We concluded that structured dynamical landscapes can generate orderly developmental progressions, providing a mathematical description of how spatial configurations evolve toward stable states within controlled simulation conditions.

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Differential topological analysis of Wolfram's Elementary Cellular Automata

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Abstract

Wolfram's Elementary Cellular Automata (ECA) serve as fundamental models for studying discrete dynamical systems, yet their classification remains challenging under traditional statistical and heuristic methods. By leveraging tools from algebraic topology, homotopy theory and differential geometry, we establish a formal connection between topological invariants and ECA's structural properties and evolution. We analyse the role of Betti numbers, Euler characteristics, edge complexity and persistent homology in achieving robust separation of the four ECA classes. Additionally, we apply coarse proximity theory and assessed the applicability of Poincaré duality, Nash embedding and Seifert–van Kampen theorems to quantify large-scale connectivity patterns. We find that Class 1 automata exhibit simple, contractible topological spaces, indicating minimal structural complexity, while Class 2 automata exhibit periodic fluctuations in their topological features, reflecting their cyclic structure and repeating patterns. Class 3 automata exhibit a higher variance in their structural properties with persistent topological features forming and dissolving across scales, a signature of chaotic evolution. Class 4 automata exhibit statistically significant increases in higher-dimensional topological voids, suggesting the appearance of stable formations. Edge complexity and fractal dimension emerged as the strongest predictors of increasing computational and topological complexity, confirming that self-similarity and structural complexity play a crucial role in distinguishing cellular automata classes. Further, we address the critical distinction between Class 3 and Class 4 automata, which holds paramount importance in practical applications. Our approach establishes a mathematical framework for automaton classification by identifying emergent structures, with potential applications in computational physics, artificial intelligence and theoretical biology.

Keywords: Homotopy Theory, Algebraic Invariants, Computational Topology, Dynamical Classification, Geometric Structures.

Introduction

Wolfram's Elementary Cellular Automata (ECA) are simple yet powerful models of discrete dynamical systems with applications spanning computation theory, statistical mechanics, complex systems and biology [1-5].

Wolfram's classification distinguishes ECA into four behavioral categories based on their asymptotic evolution [6, 7].

1. Class 1 automata have minimal structure which exhibits uniform or simple behavior, evolving into either a homogeneous state or a simple repeating pattern, as randomness plays little to no role in their evolution. These automata quickly stabilize into a fixed pattern.
2. Class 2 automata display periodic repeating substructures, evolving into stable or oscillating cycles.
3. Class 3 automata form fractal-like patterns which exhibit indefinitely chaotic and unpredictable behavior.
4. Class 4 automata support stable yet evolving patterns. They do not settle into periodicity, nor do they exhibit complete randomness, instead demonstrating emergent behaviour that allows for complex information processing. Their structured interactions and dynamic evolution align with computational universality, making them the most computationally capable among the four classes.
5. While traditional approaches to differentiate these classes rely on empirical observations, statistical methods and entropy measures, recent advancements in algebraic topology and differential geometry may provide a novel analytical framework to uncover intrinsic structural properties governing automaton behavior. Previous studies have applied persistent homology to analyze discrete systems but a comprehensive differential topological characterization of ECA

remains largely unexplored. Still, insights from coarse proximity theory and geometric topology may offer insights into the large-scale connectivity patterns of ECA, bridging the gap between microscopic evolution rules and emergent global properties in higher-order structures.

Building upon this approach, we propose a differential topological framework to investigate the underlying structures of ECA, examining the role of homological and topological invariants in distinguishing the four classes beyond statistical heuristics. Our approach incorporates persistent homology, coarse proximity theory and classical differential topological theorems such as Nash embedding and the Seifert–van Kampen theorem to analyze the ECA large-scale behavior. The expected outcome of our study is a careful characterization of the topological differences between automaton classes, allowing for a clearer delineation of their intrinsic properties.

We will proceed as follows. First, we describe the methodology, outlining the mathematical tools and computational techniques employed in our analysis. We then present the findings, demonstrating how topological invariants may differentiate automata classes. Finally, we provide a discussion on the implications of our results, including their significance in formalizing automaton classification.

Materials and Methods

Elementary cellular automata (ECA) is defined as a one-dimensional lattice of binary-valued cells, where the state of each cell at time step t + 1 is determined by a local update rule dependent on the state of itself and its nearest neighbors [7]. Each rule can be encoded as a function

$$f : \{0,1\}^3 \to \{0,1\}$$

yielding a total of 28 = 256 possible update rules. The evolution of an ECA can be represented as a discrete-time dynamical system, where the configuration space consists of all possible binary sequences. We examine a selection of ECA, choosing representative rules from each of the four Wolfram classes to capture their distinct structural and dynamical behaviors [8, 9].

To ensure tractability while preserving representative diversity, we initially selected three prototypical rules from each class, totaling 12. These rules—chosen based on well-established behavior in the literature and prior classification as canonical examples include: Rule 0, Rule 4 and Rule 12 for Class 1; Rule 18, Rule 22 and Rule 50 for Class 2; Rule 30, Rule 45 and Rule 106 for Class 3; Rule 54, Rule 110 and Rule 137 for Class 4. However, acknowledging that this limited sampling may overlook finer distinctions and rule-specific variations within each class, we expanded the analysis to include a larger set of all 88 minimal, non-equivalent ECA rules, as defined under canonical equivalence classes accounting for symmetries and reflections [7].

The full list of rules analyzed includes: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 18, 19, 22, 23, 24, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 40, 41, 42, 43, 44, 45, 46, 50, 51, 54, 56, 57, 58, 60, 62, 72, 73, 74, 76, 77, 78, 90, 94, 104, 105, 106, 108, 110, 122, 126, 128, 130, 132, 134, 136, 138, 140, 142, 146, 150, 152, 154, 156, 160, 162, 164, 168, 170, 172, 178, 184, 200, 204.

Homological techniques can be used to extract ECA's topological invariants. The basic element of this approach consists of the construction of a simplicial complex from ECA state-space dynamics to capture connectivity relationships between configurations over time. To construct a simplicial complex for a given ECA rule, we define a graph representation in which each distinct automaton configuration is treated as a vertex. Directed edges between vertices are established based on the update rule, forming a directed graph of state transitions [10].

We fix the configuration length to L=10 unless otherwise noted. This yields 210=1024 vertices, each representing a unique binary configuration of length 10. The choice of L=10 balances computational tractability with dynamical richness to reveal meaningful topological features. Each studied object — i.e., the configuration transition graph for a given rule — consists of 2n nodes and 2n directed edges, one for each possible configuration and its deterministic successor. The corresponding simplicial complex is built from the undirected version of this graph, often containing thousands of 1-simplices and higher-order simplices, depending on connectivity density. To assess robustness of our results to varying configuration length, we also conduct a supplementary analysis using L=6,8,10,12 and 14. We apply periodic boundary conditions throughout the analysis, treating each configuration as a circular binary string. This ensures that the local update rule is applied uniformly across all positions, avoiding boundary artifacts. the value of cell i is updated using its left and right neighbors modulo n, such that xi−1, xi and xi+1 wrap around the configuration ring. This corresponds to studying cyclic configurations with maximum period n and is consistent with conventions in automata theory and Wolfram's work.

All quantitative topological analyses are performed on graphs representing the full state-transition diagram of a fixed-length automaton under a specific rule. From this directed configuration transition graph, we extract an undirected adjacency structure to construct a simplicial complex following the Vietoris-Rips filtration method. Although directionality carries dynamical information, we focus here just on static structural properties, leaving apart dynamical symmetries. The filtration parameter ε determines when a higher-dimensional simplex is formed; specifically, an n-simplex is included if its corresponding vertices are all pairwise connected. For each rule-specific undirected graph, we interpret nodes as points in an abstract metric space, where the graph-theoretic shortest path distance between configurations defines pairwise proximity[11].

The filtration is applied incrementally by increasing ε, such that an edge (1-simplex) is included between two nodes if their shortest path distance is ≤ε. Higher-dimensional simplices (e.g., triangles) are included whenever all lower-dimensional faces exist. In our experiments, ε is swept over a discrete set of increasing values, typically in the range ε∈[1,6], to observe the birth and death of homology classes. Persistence diagrams are then derived and persistence scores are calculated as the average lifespan of nontrivial features across dimensions. The choice of ε bounds is empirically determined to capture the full topological evolution of each rule's complex while avoiding saturation, with filtration ranges kept consistent across all automata to ensure comparability.

We focus the topological analysis of the cellular automaton transition graphs on two key metrics:

Connected components: they serve as an approximation of the Euler characteristic, where higher values indicate a greater number of distinct regions within the automaton. While connected components alone may not distinguish all cases—such as Rule 0 versus Rule 204— this metric remains a meaningful primary indicator when interpreted in the context of broader structural and dynamical features. Though both the above-mentioned rules are in Class 1, their dynamics are different: Rule 0 collapses the entire configuration space into a point attractor, while Rule 204 preserves the initial condition without convergence [12].

This distinction is captured by the number of components, suggesting that high component counts in Rule 204 signal informational stasis, whereas low counts in Rule 0 reflect total loss of information. More broadly, connected components in transition graphs approximate the zeroth Betti number (β_0), which encodes the number of distinct dynamical basins in configuration space. While the absolute count depends on rule-specific dynamics and boundary conditions, significant variations in β_0 across rules still capture fundamental distinctions in attractor structure, convergence and state-space partitioning. Therefore, when interpreted alongside other metrics such as edge complexity and $\beta 1$, connected components serve as a topological fingerprint of large-scale dynamical behavior, particularly in contrasting convergence versus fragmentation in ECA evolution.

Edge complexity: It is computed through gradient-based analysis as the mean absolute gradient of the transition graph's adjacency matrix, interpreted as a binary image. This operation quantifies local structural variation across the configuration space, with higher values reflecting increased structural intricacy and finer pattern differentiation [13].

Persistent homology is then applied to track the evolution of topological features across different values of ε, computing Betti numbers βk for different homology dimensions k. Betti numbers quantify the presence of connected components, loops and higher-dimensional voids in the evolving structure. Computations are performed using Ripser and GUDHI libraries, which efficiently handle persistent homology calculations. In sum, by leveraging persistent homology, we aim to uncover stable features able to capture emergent connectivity structures within automaton state spaces and differentiate chaotic and computationally universal automata from simpler classes.

The topological features identified through persistent homology are further analyzed using classical homotopy-theoretic techniques. We investigate the fundamental group $\pi 1$ of the associated simplicial complex to determine whether automata exhibit nontrivial homotopy properties. The fundamental group is computed using the Seifert–van Kampen theorem, which allows decomposition into smaller, more manageable subspaces. Additionally, we examine higher homotopy groups πn to detect complex topological structures associated with computational universality. This analysis extends to cohomology, where we utilize sheaf cohomology to explore global information flow within automata. Computations are carried out using Kenzo, a specialized homotopy computation library. Overall, this homotopy-theoretic approach complements the homological analysis, offering a comprehensive characterization of automata topology [14, 15].

Beyond homological and homotopy-based analysis, we also incorporate geometric and differential topological techniques to assess automata behavior from a smooth manifold perspective. The Nash embedding theorem may determine whether the state space of an automaton can be smoothly embedded into a higher-dimensional Euclidean space, providing insight into its geometric realizability [16].

To construct embeddings, we map automaton state transitions onto a phase space trajectory, utilizing delay-coordinate embedding with a Takens reconstruction. The embedding dimension is selected using false nearest neighbor analysis, ensuring minimal distortion of topological features. Once embedded, we compute curvature properties using discrete differential geometry, evaluating Ricci curvature and scalar curvature variations. The analysis of curvature fluctuations may reveal structural stability and deformations, linking smooth geometry with automaton evolution. These computations are executed using GeomLoss, a library designed for geometric and topological data analysis. Overall, the differential topological characterization complements homotopy-based methods, reinforcing the classification framework through geometric embedding constraints.

To further quantify large-scale connectivity patterns, we employ coarse proximity theory to analyze the asymptotic behavior of automaton configurations over extended time horizons. Coarse proximity structures may help in quantifying large-scale structures within the data, since they characterize how clusters of configurations interact at a macroscopic level. We define a coarse structure by constructing a Čech cohomology space from automaton trajectories, identifying large-scale features invariant under automaton evolution. To approximate a Čech cohomology space, we interpret each configuration as a point in a discrete metric space, where proximity is defined by Hamming distance or transition adjacency. For a given radius ε, we define open balls around each configuration and construct a Čech complex by including a simplex for each set of configurations with nonempty intersection of their ε-balls. The resulting simplicial complex approximates the automaton's configuration space topology [17].

Čech cohomology groups are then computed from this complex, capturing higher-order relationships like consistent flow of values over overlapping covers. Though Čech cohomology is usually defined for continuous topological spaces, this discrete analogue aligns with approaches used in computational topology to analyze point-cloud data. The resulting coarse space is examined for the presence of large-scale homotopy equivalences. The framework is implemented using Dionysus, a topological data analysis library designed for coarse geometry applications. In sum, the coarse proximity approach strengthens the distinction between automaton classes by characterizing their large-scale topological properties, further supporting the homological and geometric findings.

We integrate the above-mentioned analytical approaches into a differential topological classification scheme, defining a topological complexity measure that captures the interplay between Betti number fluctuations, fundamental group complexity, em-

bedding curvature and coarse connectivity. This measure is computed as a weighted sum of topological invariants, with greater significance assigned to features strongly correlated with computational universality. Classification is performed using a supervised learning model, trained on a dataset of known automaton classes. Feature selection is optimized using recursive feature elimination, ensuring the most relevant topological descriptors are retained. The classifier is implemented in Scikit-learn, utilizing a support vector machine with a radial basis function kernel for optimal classification performance.

To satisfy requests for a detailed, reproducible case, we also consider adding a case study on Rule 110, i.e., the canonical Class 4 ECA known for its computational universality. We applied the same analytical pipeline used for the other automata: the undirected configuration transition graph was extracted, from which a simplicial complex was constructed. Betti numbers, edge complexity and additional topological descriptors were then computed, including transition graph visualizations, Betti number persistence diagrams, fundamental group approximations and edge complexity.

In conclusion, by integrating persistent homology, homotopy theory, differential geometry and coarse proximity theory, we aim to establish a unified framework for analyzing Wolfram's elementary cellular automata through a differential topological lens. This framework may provide a formal means of distinguishing automaton classes based on their underlying topological structures.

Results

The results achieved from the analysis of the 12 representative ECA rules are consistent with those derived from the complete set of 88 dynamically inequivalent ECA rules. The topological metrics—including edge complexity, number of connected components, Betti numbers and curvature—demonstrate parallel patterns in both subsets, displaying class-wise trends and distinctions that remain consistent between the small and large samples. Comparative plots and aggregated statistics confirm that the 12-rule subset effectively captures the full range of structural diversity across Wolfram classes, validating its use as a minimal but representative sampling for initial classification and analysis. For the sake of visual interpretability, the figures in this study focus on the 12 representative ECA rules.

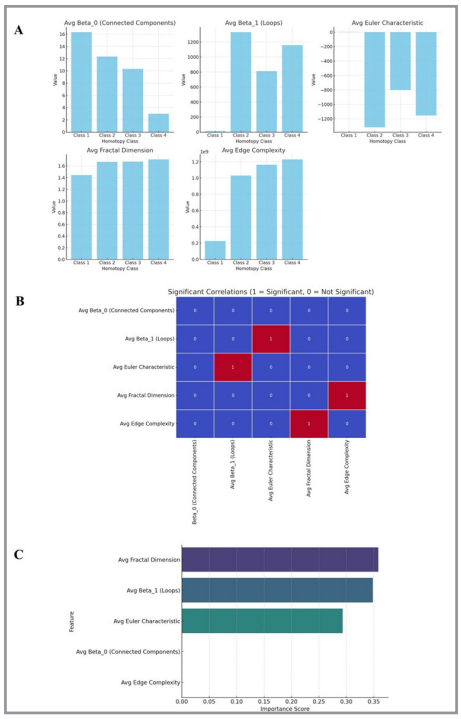


Figure 1: Topological complexity measures. Figure 1A. Comparison of classes of cellular automata with numerical topology metrics including connected components, edge complexity and genus estimates. See text for further details. Figure 1B. Heatmap of the Pearson correlation analysis of key topological metrics in cellular automata, with statistically significant correlations ($p<0.05$) highlighted in red. Our Figure focuses exclusively on strong correlations (≥ 0.7) to enhance visual clarity and emphasize the most robust relationships between topological metrics. Moderate correlations are omitted to avoid overloading the visual space and to prioritize interpretability. Figure 1C. Feature importance in decision tree classification. Bar plots reveal the relative contribution of different topological and structural features to distinguish between cellular automata classes. Higher importance values indicate features playing a more significant role in classification.

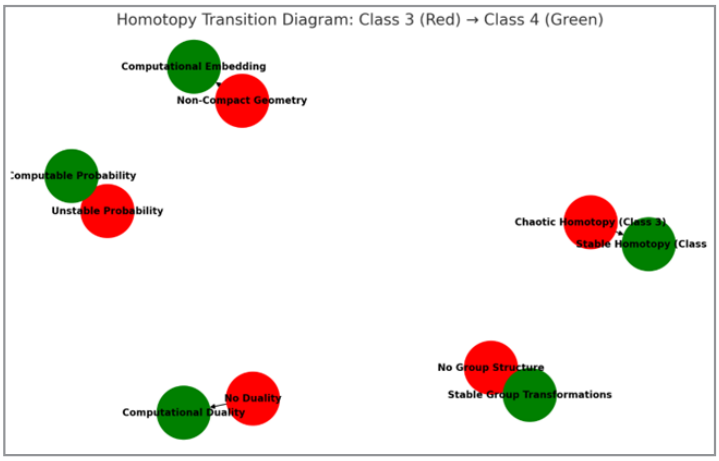


Figure 2: Homotopy transition diagram: class 3 (red) → class 4 (green). The diagram illustrates the evolution of topological and homotopy structures as chaotic cellular automata (Class 3) transition into computationally universal automata (Class 4).

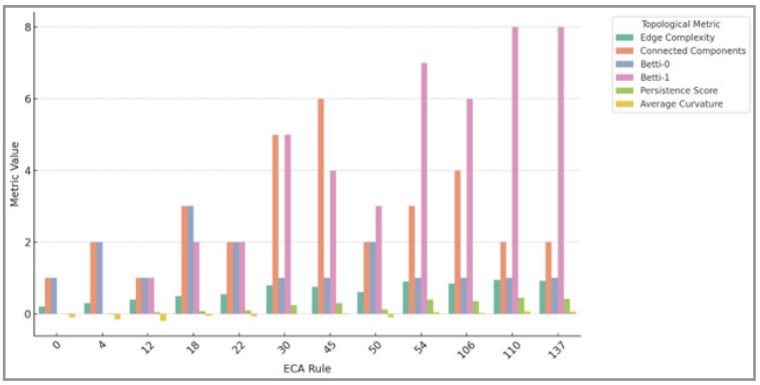


Figure 3: Bar plot showing individual topological metrics for twelve representative ECA rules. Each bar represents the value of a specific metric for a given rule. Substantial intra-class variation can be observed, particularly within Classes 3 and 4, highlighting the importance of rule-level analysis beyond class-based aggregation.

Table 1: Topological classification of cellular automata across different classes. Note that the "Class 3 → Class 4 transition" is not an ECA class, but just an analytical construct representing a comparative regime where topological and computational features begin stabilizing between pure chaos and emergent universality.

Class	Edge Complexity	Connected Components	Topology	Fundamental Group	Homotopy Equivalence	Graph Representation	Key Mathematical Properties	Evolution
Class 1	Very low	Few, large homogeneous regions	Trivial, low genus, minimal cycles	$\pi 1 = 0$ (trivial)	Disk-like, contractible	Self-equivalent	Lack of periodicity	Can transition into Class 2
Class 2	Moderate	Moderate, periodic structures	Toroidal or cylindrical topology	$\pi 1 = Z \times Z$ (periodic loops)	Self-equivalent periodicity	Self-equivalent with periodicity	Introduction of nontrivial loops	May transition into chaotic Class 3
Class 3	High	Numerous, fractal-like patterns	High-genus surface with multiple holes	Non-abelian free group	Chaotic, self-organizing potential	Chaotic, fragmented	Fractal-like formations, high complexity	Some regions stabilize, leading to Class 4
Transition 3 → 4	Increasing	Stabilizing	From chaotic to structured organization	Emerging stable homomorphisms	Weak homotopy equivalences forming	Becoming structured	Seifert-van Kampen, lakers-Massey, Whitehead, Poincaré duality	Chaotic structures become embeddable, reinforcing computation

Class 4	Extreme	Highly structured, nested formations	Computational manifold-like structures	Higher homotopy groups ($\pi 2$, $\pi 3$)	Stable computational homotopy	Interacting dynamic structures	Computational duality, homotopy stabilization	Final structured computational form

Conclusions

We conducted a comprehensive differential topological analysis of elementary cellular automata (ECA), providing a structural approach to understanding their complexity beyond surface-level dynamical patterns. By leveraging homotopy and homology theory, coarse proximity structures and geometric embeddings, we identified statistically significant topological features that distinguish each Wolfram class.

We demonstrated that the transition from Class 3 to Class 4 marks a fundamental shift in topological, homological and geometric structures. The distinction between these two classes is particularly significant for practical applications. Class 3 automata exhibit unstable homotopy groups, chaotic topological formations and non-measurable probability spaces, resulting in self-organizing and unpredictable patterns. Their lack of Nash embeddings prevents representation in computational manifolds, reinforcing their non-compact geometric structures. These automata possess fragmented fundamental groups and lack stable group transformations, making structural predictions difficult. Their chaotic homotopy fails to converge, preventing measurable probability distribution and structured formations. Therefore, class 3 cellular automata are widely employed in pseudo-random number generation and cryptographic systems, where unpredictability is essential, since their chaotic nature makes them well-suited for applications requiring high entropy and non-repetitive patterns. The Seifert–van Kampen theorem characterizes the transition from Class 3 to Class 4, illustrating how chaotic spaces fragment and reorganize into well-defined fundamental groups. As this structural stabilization occurs, stable group homomorphisms emerge, providing a foundation for computational logic and structured transformations. The shift from unstable to computable probability and from non-compact geometry to structured probability spaces marks Class 4 as computationally universal, distinguishing it from purely chaotic automata. Indeed, class 4 automata are characterized by stable computational homotopy which enables logical and algorithmic transformations. Due to their computational stability, class 4 automata are highly valuable for modeling computational processes, biological systems and artificial intelligence that require both adaptability and structured processing.

Our methodology has limitations that should be acknowledged. The computational complexity of homotopy-based analysis poses challenges for large-scale automaton studies, particularly when computing higher-dimensional fundamental groups. While advances in computational topology have improved the efficiency of homological calculations, the scalability of certain topological descriptors remains an issue. Additionally, reliance on embedding methods introduces potential distortions in geometric representations which could influence the interpretation of curvature-based results. The classification scheme, though effective in distinguishing automaton classes, may require refinement when applied to intermediate automata not fitting neatly into Wolfram's classification. We analyzed individual ECA alongside aggregated, class-based summaries, particularly in transitional regimes such as the boundary between Classes 3 and 4 While broad statistical patterns characterize each Wolfram class, individual rules within a given class may exhibit substantial deviations in their topological signatures. Intra-class heterogeneity highlights the limitations of relying solely on average metrics, supporting the need for rule-level analyses to capture the full range of dynamical behaviors.

Although the configuration transition graph is inherently directed due to the deterministic nature of cellular automata, our simplicial complex is constructed over the undirected version of this graph. This simplification is necessary because most persistent homology and standard algebraic topology tools (e.g., Vietoris–Rips or clique complexes) are defined only for undirected graphs. As a result, our interpretation of topological features reflects structural properties of the configuration space (such as mutual accessibility or connectivity density), but not the full dynamical asymmetry present in the evolution of the automaton. Still, the homological "holes" identified in our setting represent cycles or voids in the undirected configuration space. They should be interpreted as undirected structural cycles, not necessarily indicative of causal feedback or reversible behavior within the automaton's dynamics. We acknowledge that the loss of directionality in the configuration space omits potentially rich dynamical information. Methods from directed algebraic topology — such as directed homotopy theory, concurrency spaces, or d-path categories — offer promising frameworks for capturing causal and temporal dynamics more faithfully, although these tools are less developed and lack robust computational implementations compared to classical persistent homology.

The novelty of our approach lies in integration of differential topology into the classification of cellular automata, capturing the large-scale properties of automaton evolution. Conventional approaches such as entropy measures and Lyapunov exponents primarily focus on local dynamics and statistical fluctuations, making them sensitive to initial conditions and specific implementations. In contrast, our homotopy-theoretic and geometric approaches capture intrinsic properties of automata evolution that are invariant under transformations. Graph-theoretic models have also been used to classify automata, but they lack the depth of topological analysis required to differentiate complex computational behaviors. Persistent homology, while useful for tracking structural changes, benefits significantly from the inclusion of homotopy invariants which provide additional layers of classification through fundamental group analysis. Unlike entropy-based or purely algebraic methods, our approach can uncover persistent topological structures that are stable under small perturbations, making it useful for classifying automata in a systematic and reproducible manner [21-23].

The implications of our approach extend to various domains where discrete dynamical systems play a central role. In artificial intelligence and machine learning, understanding the topological evolution of state spaces could enhance the design of neural network architectures and optimize feature extraction tech-

niques. In computational biology, our classification framework could be employed to analyze gene regulatory networks, where state transitions exhibit structural similarities to automaton evolution. Our findings may also inform physics, particularly in the study of phase transitions and emergent behaviors in complex systems where topological properties govern stability and transformation dynamics. Additionally, the identification of stable homotopy structures in computationally universal automata suggests potential avenues for developing new classes of reversible computing models[24].

In conclusion, we present a differential topological classification of Wolfram's elementary cellular automata, revealing significant structural differences across automata classes. By integrating homotopy theory, geometric embeddings and coarse proximity analysis, our approach establishes a mathematical framework based on topological invariants for identifying computational universality.

Declarations

Ethics Approval and Consent to Participate

This research does not contain any studies with human participants or animals performed by the Author.

Consent for Publication

The Author transfers all copyright ownership; in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal and has not been previously published.

Availability of Data and Materials

All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.

Competing Interests

The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence their work.

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Authors' Contributions

The Author performed: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding, administrative, technical and material support, study supervision.

Declaration of Generative AI and AI-Assisted Technologies in the Writing Process

During the preparation of this work, the author used ChatGPT 4o to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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Review

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Architectural Dynamics in Living Tissues: Integrating Structure, Physical Fields and Spatial Networks

Arturo Tozzi *

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Keywords: mesoscale modeling; anisotropic transport; microenvironment topology; field–structure coupling; spatial heterogeneity analysis

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Review

Architectural Dynamics in Living Tissues: Integrating Structure, Physical Fields and Spatial Networks

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Abstract

Biological tissue analyses rely on morphological descriptors like shape, layering and cellular composition. We introduce *Architectural Dynamics*, a framework that employs more than one hundred quantifiable parameters to define architectural and dynamical properties of a cellular microenvironment, including structural, mechanical, geometrical, physical, network-theoretic, cellular and biochemical features. Biological tissues are portrayed as weighted networks whose nodes and edges carry measurable physical quantities like diffusion conductance, mechanical impedance, curvature and material anisotropy. Standard network metrics like global efficiency, modularity, diameter and centrality acquire physiological meaning as indicators of accessibility, compartmentalization and exposure to mechanical or biochemical cues. In parallel, physical fields such as diffusion, mechanics, curvature and topography generate patterns of transport, signaling, force propagation and communication that link microscale architecture to mesoscale dynamical behavior. Using combined descriptions, we show that behaviors like migration patterns, polarization, domain formation, compartmentalization, metabolic coupling, signal propagation and stability of functional domains emerge from agent dynamics shaped by weighted topology, structural heterogeneity, mechanical anisotropy and geometric confinement. Our perspective shifts the emphasis from cellular identity to quantitative analysis of local physical cues and global topological organization within a high-dimensional architectural space, enabling prediction of cellular behaviors directly from tissue architecture. Changes in development, health or disease correspond to movements along well-defined architectural directions rather than to simple morphological or biochemical alterations. Our framework applies to engineered scaffolds, organoids and regenerative medicine as well as extracellular matrices, cortical microcircuits and pathological architectures like tumors, where the modulation of architectural regimes becomes central to interpreting tissue organization.

Keywords: mesoscale modeling; anisotropic transport; microenvironment topology; field–structure coupling; spatial heterogeneity analysis

Introduction

Current approaches to biological tissues emphasize descriptors such as morphology, layering and cellular composition, often complemented by molecular markers or lineage information (Aljakna, Fracasso, and Sabatasso 2018; Yoshimura et al. 2019; Beumer and Clevers 2021; Olguín-Martínez, Ruiz-Medina, and Licona-Limón 2021; Meliante et al. 2025; Wells et al. 2025). While these descriptors are indispensable, they provide only partial insight into how physical context shapes cellular behavior. Existing quantitative frameworks usually emphasize one dominant dimension of the microenvironment, whether tissue mechanics, diffusion dynamics, geometric curvature or network connectivity (Bidan et al. 2012; Callens et al. 2020; Trubuil, D'Angelo, and Solon 2021; Muntz et al. 2022; Gehrels et al. 2023; Gooshvar et al. 2023; Tam and Baar 2025). These single-domain representations capture foremost structural or functional aspects, but do not account for the way

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these features jointly constrain transport, communication, accessibility, dynamical stability, etc. Moreover, most analytical methods treat tissues as either collections of cells or as passive continua, limiting the ability to integrate mesoscale architecture with local physical fields. Therefore, understanding how spatial heterogeneity, anisotropy and topology jointly shape migration, polarization, signalling and compartmentalisation is challenging. A shift is required toward architectural representations capable of capturing the interplay between geometry, topology and field dynamics, setting the stage for a framework in which measurable architectural features collectively define the operative microenvironment.

We introduce Architectural Dynamics, a quantitative representation in which tissues are described as weighted spatial networks embedded in heterogeneous geometric and physical fields. Our framework employs a large set of measurable parameters encompassing structural, mechanical, geometric, physical, network-theoretic, cellular and biochemical features, allowing tissue organization to be expressed as a point or a trajectory in high-dimensional phase spaces. Weighted nodes and edges might encode diffusion conductance, mechanical impedance, curvature, anisotropy and related quantities, enabling standard network metrics to acquire explicit physiological interpretation. Coupling these weighted networks to endogenous fields such as diffusion, mechanics, curvature and topography might allow dynamical patterns to be interpreted as consequences of architecture rather than solely of cellular identity. Our integrated formulation might provide a unified route to analyze how structural heterogeneity, geometric confinement and anisotropic communication jointly shape agent-level behaviors. Within this setting, our theoretical approach aims to determine whether quantifiable architectural descriptors could predict observed spatial dynamics and whether movements along specific architectural directions could correspond to recognizable transitions in tissue organization.

We will proceed as follows: we first outline the methodological construction of Architectural Dynamics, then examine its hypothetical consequences and their interpretation, and finally discuss subsequent developments and broader implications.

Architectural Components of the Cellular Microenvironment

A cellular microenvironment consists of a complex ensemble of spatially organized features whose combined configuration shapes the conditions under which cells operate. We provide here an expanded conceptual description of the microenvironmental components that later sections quantify, detailing how each domain contributes to the overall organization without yet entering explicit analytical procedures. The purpose is to clarify the rationale for treating tissue architecture as a high-dimensional system composed of interdependent physical elements, ensuring that the subsequent quantitative formulations emerge from a clearly delineated conceptual framework.

Structural properties form the base layer of architectural organization, encompassing the arrangement of material elements such as extracellular fibers, pores, rigid segments and heterogeneous tissue units (Sekiguchi and Yamada 2018; Moussian 2019; Gomi and Imamura 2020; Gardeazabal and Izeta 2024). These features determine how matter is distributed across space and impose constraints on how other physical quantities can vary. Structural discontinuities, gradients or repeating motifs influence the connectivity of the microenvironment and define the spatial template through which forces, signals and diffusing agents propagate (Cortesi et al. 2020; He et al. 2023; Schimek et al. 2024; Sun et al. 202. Closely related to structure are the mechanical properties, which describe how the microenvironment responds to deformation, compression, shear or tension (Park et al. 2020; Petroll, Varner, and Schmidtke 2020; Bhargav et al. 2021; Massey et al. 2024). Local stiffness, viscoelastic behavior and mechanical anisotropy are essential descriptors because they govern how physical forces are transmitted, dissipated or concentrated (Chaudhuri et al. 2020; Atcha et al. 2021; Wu et al. 2022; Tao et al. 2023; Zhang et al. 2023). Whereas structure describes what occupies space, mechanics describes how that space reacts to stress, providing the background against which cells perceive rigidity, deformability and load-bearing capacity (Zhu et al. 2025). Together, structural and mechanical features define the material substrate of the microenvironment.

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Geometrical properties capture shape-derived constraints imposed by curvatures, interfaces, boundaries and thickness variations (Campisi et al. 2018; He et al. 2018; Gorb et al. 2020; Wang et al. 2024; Chen et al. 2025). These shape features influence whether physical or chemical quantities tend to concentrate, disperse or follow preferred directions, modulating confinement at both micro- and mesoscales. Geometry adds a distinct layer of constraints because it does not merely reflect how matter is distributed, but how spatial form restricts the degrees of freedom available to physical and biological processes. Network-theoretic features complement this picture by recasting microenvironment as a weighted topology, where nodes represent spatial units and edges express adjacency or the availability of transport routes (Pan et al. 2022; Coy et al. 2022; Sun et al. 2024). While geometry defines the global configuration of spatial shapes, network structure elucidates the global organization of connectivity, enabling the microenvironment to be described not only as a continuous medium but also as a discrete set of pathways through which matter, forces and information can travel (Bjordahl et al. 2013; Dobaño-López et al. 2021). The interplay between geometrical form and connective topology forms a backbone for understanding accessibility, compartmentalization and the overall organization of interaction routes.

Cellular properties introduce local heterogeneity derived from the distribution, arrangement, morphology and orientation of the cells inhabiting the microenvironment. These features specify how active biological units occupy and modify space, impose local constraints through their packing and polarity, and contribute to the dynamic remodeling of surrounding structures (Boutilier and Elsawa 2021; Gao, Liang, and Wang 2022; Wang et al. 2022; de Freitas Nader and García-Arcos 2023). Unlike structural or mechanical descriptors which concern the passive substrate, cellular descriptors incorporate the influence of individual and collective cell behaviors on the architecture itself. Biochemical properties further enrich the environment by defining the spatial distribution of molecules, signaling gradients, metabolic substrates and diffusible mediators (Acharya et al. 2020; Giotti et al. 2024; Shi et al. 2025). These features determine the chemical context against which cells sense and respond, as well as the spatial domains where reactions can occur or where concentrations remain stable. Biochemical landscapes often show steep gradients or localized maxima, making them essential components of the architectural regime (Samarah et al. 2025).

Physical fields integrate and modulate all preceding components by representing continuously varying quantities such as diffusional fluxes, stress distributions, hydraulic pressures or electric potentials (DuFort, DelGiorno, and Hingorani 2016; Kumar et al. 2023; Liu et al. 2024). Unlike structural or biochemical features, which may be directly tied to material composition, physical fields evolve according to equilibrium, transport or propagation laws and therefore form the dynamic layer of architectural organization. They provide a bridge between static features and time-dependent phenomena, enabling architecture to be interpreted not only as a static arrangement of components but as a set of interacting fields whose configuration depends on and further shapes the underlying structural, mechanical, geometrical, network and chemical properties.

Together, these architectural components constitute the conceptual backbone for a unified representation of tissue organization. The appendix provides a summary of all features that characterize the architectural properties of the tissue microenvironment. Each family contributes a distinct yet interdependent dimension of the microenvironment and none is sufficient in isolation. In next section, conceptual categories will be transformed into explicit mathematical quantities and measurable parameters.

Quantitative Characterization of Architectural Features in Cellular Microenvironments

We examine here how structural, mechanical, geometrical, network-theoretic, cellular, biochemical and physical-field descriptors jointly characterize the architectural properties of a cellular microenvironment. To obtain an unified quantitative treatment, each category is developed through measurement procedures, explicit mathematical formulations and detectable features, allowing architectural organization to be represented as a coherent analytical space.

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1. Structural features. Structural features provide the baseline material organization upon which all other architectural descriptors depend.

Structural characterization begins by defining the tissue domain $\Omega \subset \mathbb{R}^3$ and discretizing it into volumetric elements $\{V_1, \ldots, V_N\}$. Each element possesses measurable descriptors including density ρ_i, porosity π_i and a fiber-orientation vector $\mathbf{f}_i$. Density can be extracted from imaging data through calibrated voxel intensities, while porosity is computed as

$$\pi_i = 1 - \frac{\text{vol(cellular)}}{\text{vol}(V_i)}.$$

Fiber orientation can be measured by the structure tensor. For an intensity field $I(\mathbf{x})$, the tensor at $\mathbf{x}$ is

$$T(\mathbf{x}) = \int_{\mathcal{N}(\mathbf{x})} \nabla I(\mathbf{y})\, \nabla I(\mathbf{y})^{\top}\, w(\mathbf{x}-\mathbf{y})\, d\mathbf{y},$$

with w a Gaussian kernel. The eigenvectors of T give principal orientations and the eigenvalues encode coherence. Spatial structure can be summarized via the autocorrelation

$$C_S(r) = \langle (\rho(\mathbf{x}) - \bar{\rho})(\rho(\mathbf{x} + r\hat{\mathbf{u}}) - \bar{\rho}) \rangle_{\mathbf{x},\hat{\mathbf{u}}},$$

integrated into a heterogeneity index

$$H_S = \int_0^R \mid C_S(r) \mid \, dr.$$

Structural adjacency can be encoded through

$$A_S(i,j) = \begin{cases} 1 & \text{if } V_i \text{ and } V_j \text{ share a face,} \\ 0 & \text{otherwise,} \end{cases}$$

with weighted connectivity

$$W_S(i,j) = \exp\left(-\alpha \mid \rho_i - \rho_j \mid\right) \exp\left(-\beta \mid \pi_i - \pi_j \mid\right).$$

Tools for structural detection include watershed segmentation, level-sets, convolutional filtering, Fourier analysis and supervised classifiers applied to confocal, multiphoton or micro-CT datasets (Li et al. 2019; Kornilov, Safonov, and Yakimchuk 2022; Son et al. 2022; Zhou and Yang 2022; Whybra et al. 2024). The resulting descriptors form the structural backbone of the architectural microenvironment.

2. Mechanical features. Mechanical features quantify resistance, deformation and force transmission across the microenvironment.

Mechanical architecture can be formulated through spatially varying stiffness, viscoelastic response and anisotropic impedance. The local linear constitutive relation is

$$\sigma(\mathbf{x}) = C(\mathbf{x}) : \varepsilon(\mathbf{x}),$$

where σ is the Cauchy stress, $\varepsilon = \frac{1}{2}(\nabla \mathbf{u} + \nabla \mathbf{u}^{\top})$ is the infinitesimal strain, and $C(\mathbf{x})$ is a fourth-order elasticity tensor. For isotropic elasticity,

$$C_{ijkl} = \lambda\, \delta_{ij}\delta_{kl} + \mu\, (\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}),$$

with Lamé parameters λ, μ. Spatially heterogeneous stiffness can be detected by techniques such as AFM indentation, Brillouin microscopy, traction force microscopy and micro-rheology.

Viscoelastic response can be represented through a standard linear solid model, with stress satisfying

$$\sigma + \tau_\sigma \dot{\sigma} = E_1 \varepsilon + (E_1 + E_2)\tau_\varepsilon \dot{\varepsilon}.$$

Local mechanical impedance $Z(\omega)$ can be computed from oscillatory indentation data using

$$Z(\omega) = \frac{\hat{\sigma}(\omega)}{\hat{\varepsilon}(\omega)}.$$

Mechanical anisotropy can be extracted by diagonalizing $C(\mathbf{x})$ in fiber coordinates, producing directional stiffness values $E_{\parallel}, E_{\perp}$. Spatial maps of mechanical heterogeneity can be quantified through

$$H_M = \int_{\Omega} \parallel \nabla E(\mathbf{x}) \parallel \, d\mathbf{x}.$$

Force transmission pathways can be identified by solving the equilibrium equation

$$\nabla \cdot \sigma(\mathbf{x}) + \mathbf{b}(\mathbf{x}) = \mathbf{0}$$

under boundary conditions, producing deformation fields $\mathbf{u}(\mathbf{x})$. From these, mechanical geodesics can be computed by minimizing

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$$\mathcal{L}[\gamma] = \int_0^1 \sqrt{\gamma'(s)^\top C(\gamma(s))\,\gamma'(s)}\, ds,$$

which quantify low-resistance directions for force propagation.

Tools for mechanical probing include AFM arrays, calibrated micro-beads, high-resolution elastography, Brillouin scattering, micropipette aspiration and digital volume correlation (Brener 2015; Li et al. 2017; Palombo and Fioretto 2019; Bednář and Dvořák 2022; Helal and Kojima 2022; Kojima 2022; Ray et al. 2025; Ren et al. 2025). These measurements define the mechanical landscape able to influence local interactions.

3. Geometrical features. Geometrical features describe shape-derived constraints that modulate pattern formation, transport and spatial confinement.

Geometric structure is anchored in the shape operator and curvature tensors of the tissue domain. Let the microenvironment boundary be a smooth surface S with normal vector $\mathbf{n}$. The shape operator $S_p: T_pS \to T_pS$ satisfies

$$S_p(\mathbf{v}) = -D_{\mathbf{v}}\mathbf{n}.$$

The eigenvalues $\kappa_1, \kappa_2$of S_p are the principal curvatures; mean and Gaussian curvature are

$$H = \frac{\kappa_1 + \kappa_2}{2}, K = \kappa_1\kappa_2.$$

Curvature maps can be estimated from triangulated meshes reconstructed via confocal stacks or multiphoton surfaces. Discrete mean curvature can be obtained from the Laplace–Beltrami operator

$$H(\mathbf{x}_i) = \frac{1}{2A_i}\sum_j (\cot\,\alpha_{ij} + \cot\,\beta_{ij})(\mathbf{x}_j - \mathbf{x}_i)$$

with A_i an area element and α_{ij}, β_{ij} opposite angles.

Geometric confinement can be quantified by distance-to-boundary fields

$$d(\mathbf{x}) = \min_{\mathbf{y}\in\partial\Omega} \| \mathbf{x} - \mathbf{y} \|,$$

and by curvature-weighted confinement

$$C_G(\mathbf{x}) = d(\mathbf{x})^{-1}(1+| H(\mathbf{x}) |).$$

Thickness variation can be measured by computing orthogonal trajectories to the boundary and solving

$$t(\mathbf{x}) = \text{length}(\gamma_{\mathbf{x}}),$$

where $\gamma_{\mathbf{x}}$ is a geodesic perpendicular to $\partial\Omega$.

Geometrical detection tools include surface reconstruction algorithms, active-contour segmentation, differential geometry estimators, Laplace–Beltrami operators, finite-element meshing and confocal–derived isosurface extraction (Ma et al. 2020; Fan et al. 2022; Fallahdizcheh, Laroia, and Wang 2023; Bhansali et al. 2023; Ma et al. 2023). These features quantify how geometric shape can constrain diffusional, mechanical and topographical interactions.

4. Network-theoretic features. Network-theoretic features express architecture as weighted topologies that quantify accessibility, compartmentalization and communication pathways.

The tissue domain can be represented as a graph $G = (V, E)$, with $V = \{1, \ldots, N\}$ the volumetric elements and E edges encoding adjacency. Weighted connectivity is defined by

$$W(i,j) = w_{ij},$$

where w_{ij} may encode structural similarity, mechanical coupling or diffusional conductance.

Shortest paths can be computed using weights w_{ij}^{-1}, producing the path-length matrix

$$L(i,j) = \min_{\gamma: i\to j} \sum_{(u,v)\in\gamma} w_{uv}^{-1}.$$

Global efficiency is

$$E_{\text{glob}} = \frac{1}{N(N-1)}\sum_{i\neq j} \frac{1}{L(i,j)}.$$

Modularity can be computed from

$$Q = \frac{1}{2m}\sum_{ij} (w_{ij} - \frac{k_i k_j}{2m})\,\delta(c_i, c_j)$$

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with total weight $m = \frac{1}{2}\sum_{ij} w_{ij}$, strengths k_i, and community labels c_i. Betweenness centrality is

$$B(i) = \sum_{s \neq i \neq t} \frac{\sigma_{st}(i)}{\sigma_{st}}$$

with σ_{st} the number of shortest paths from s to t, and $\sigma_{st}(i)$ those passing through i.
Diffusion-weighted edges can be computed from Fickian conductance

$$w_{ij} = \frac{D_{ij} A_{ij}}{\ell_{ij}},$$

where D_{ij} is diffusivity, A_{ij} shared area and ℓ_{ij} centroid distance.

Detection tools include graph reconstruction from voxel adjacency, anisotropic kernels, persistent connectivity filtering, spectral graph decomposition and diffusion-weighted metrics (Radua et al. 2014; Onal Ertugrul, Ozay, and Yarman Vural 2018; Petrovic et al. 2019; Cheng, Cloninger, and Coifman 2020; Zhao et al. 2020; Raj et al. 2024). Overall, network-theoretic features provide a global account of accessible communication pathways.

5. Cellular features. Cellular features describe local heterogeneity in cell size, shape, polarity, turnover and spatial arrangement.

Cells occupy subsets $\{\mathcal{C}_1, \ldots, \mathcal{C}_M\} \subset \Omega$. Each cell has a centroid $\mathbf{x}_k$, shape tensor

$$S_k = \int_{\mathcal{C}_k} (\mathbf{y} - \mathbf{x}_k)(\mathbf{y} - \mathbf{x}_k)^{\top} d\mathbf{y},$$

and polarity vector $\mathbf{p}_k$ derived from asymmetric membrane markers or cytoskeletal orientation.

Packing density can be quantified through Voronoi tessellations, producing volumes V_k^{Vor}. Cellular anisotropy can be measured by eigenvalue ratios of S_k. Pair-correlation structure is summarized by

$$g(r) = \frac{1}{\rho_c} \langle \sum_{k \neq \ell} \delta(r - \| \mathbf{x}_k - \mathbf{x}_\ell \|) \rangle,$$

where ρ_c is mean cell density.

Turnover rates can be quantified from time-lapse imaging using lineage tracking. Spatial gradients in cellular properties can be computed via

$$\nabla \rho_c(\mathbf{x}), \nabla \mathbf{p}(\mathbf{x}).$$

Detection tools include segmentation (U-Net, watershed), 3D tracking, shape-tensor reconstruction, Voronoi geometry and fluorescence-based polarity extraction (Widengren 2010; Bozza et al. 2012; Novaes et al. 2016; Beeche et al. 2022; Yin et al. 2022; Yousef et al. 2023).

6. Biochemical features. Biochemical features quantify gradients, reaction capacities and diffusible mediator distributions affecting interaction ranges.

Let $c_i(\mathbf{x})$ be the concentration of biochemical species i. Steady-state distributions satisfy

$$D_i \nabla^2 c_i - k_i c_i + s_i(\mathbf{x}) = 0$$

for diffusivity D_i, decay k_i and source s_i. Gradients are quantified by $\| \nabla c_i \|$, and heterogeneity by

$$H_B = \sum_i \int_\Omega | c_i(\mathbf{x}) - \bar{c}_i | \, d\mathbf{x}.$$

Ligand–receptor binding potentials can be described by

$$R_{ij}(\mathbf{x}) = k_{on} c_i(\mathbf{x}) r_j(\mathbf{x}) - k_{off} b_{ij}(\mathbf{x}),$$

with r_j receptor density and b_{ij} ligand–receptor complexes.

Reaction–diffusion coupling can be captured by

$$\frac{\partial c_i}{\partial t} = D_i \nabla^2 c_i + F_i(c_1, \ldots, c_n, \mathbf{x}),$$

with F_i nonlinear reaction terms. Steady-state maps are solved numerically using finite-element or finite-difference schemes.

Detection tools include immunofluorescence, FRET sensors, mass-spectrometry imaging, in-situ hybridization and ratiometric biosensors (Mohsin, Ahmad, and Iqbal 2015; Bajar et al. 2016; Cost,

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Khalaji, and Grashoff 2019; Huo et al. 2019; Wang et al. 2019; Spring, Goggins, and Frost 2021). These features describe chemical landscape variations across the microenvironment.

7. Physical fields. Physical fields integrate structural, mechanical, geometric and biochemical factors into continuously varying dynamical quantities.

Diffusion fields satisfy

$$\frac{\partial u}{\partial t} = \nabla \cdot (D(\mathbf{x})\nabla u),$$

where spatially varying diffusivity $D(\mathbf{x})$ reflects structural and biochemical heterogeneity. Stress fields obey equilibrium

$$\nabla \cdot \sigma(\mathbf{x}) = \mathbf{0}$$

with constitutive laws depending on $C(\mathbf{x})$.

Electric potential ϕ satisfies Poisson's equation

$$-\nabla \cdot (\epsilon(\mathbf{x})\nabla\phi) = \rho(\mathbf{x}),$$

with $\epsilon(\mathbf{x})$ permittivity and $\rho(\mathbf{x})$ charge density.

Hydraulic pressure $p(\mathbf{x})$ obeys Darcy's law

$$\mathbf{v} = -\kappa(\mathbf{x})\nabla p,$$

with permeability κ. Coupling between fields can be expressed through composite operators; for example, diffusion–mechanics coupling yields

$$\frac{\partial u}{\partial t} = \nabla \cdot (D(\mathbf{x}, \sigma)\nabla u).$$

Detection tools include fluorescence recovery after photobleaching (for diffusion), traction force microscopy (for stress), microelectrode arrays (for potential) and microfluidic perfusion (for hydraulic fields) (Hur et al. 2020; Lekka et al. 2021; Moud 2022; Zancla et al. 2022; Saito, Matsunaga, and Deguchi 2023; Saito and Deguchi 2023).

Together, structural, mechanical, geometrical, network-theoretic, cellular, biochemical and physical-field descriptors provide a unified quantitative representation of the tissue microenvironment. By grounding each feature family in explicit measurement procedures, mathematical characterizations and detectable quantities, a consistent analytical basis can be defined for describing architectural organization and its underlying variability.

Hypothetical Domains of Architectural Dynamics

Architectural Dynamics examines how hypothetical cellular environments, whether inspired by known tissues or entirely conceptual, might generate biological function through their physical organization. Geometry, material composition and spatial connectivity are understood not as secondary descriptors but as primary determinants of transport, communication and mechanical interaction. The central premise is that any microenvironment, either real or hypothetical, can be represented as a physically informed spatial network in which nodes and edges carry weights derived from measurable or inferable fields, including diffusion conductance, local stiffness, curvature, hydraulic resistance, surface texture and matrix heterogeneity. This representation simultaneously preserves the locality of continuum physics and the global structure of graph topology, allowing the environment to be characterized through the combined lens of anisotropic transport, spatial modularity, weighted accessibility and geometry-dependent force propagation. The following hypothetical examples illustrate how the framework operates across diverse architectural regimes.

Intestinal villus-field microenvironment. A hypothetical intestinal villus-field microenvironment illustrates how Architectural Dynamics might interpret an intestinal surface composed of protruding, variably tapered villus structures. Each villus would form a vertically oriented geometric domain whose curvature profile could generate directionally biased transport pathways and mechanically distinct compartments. The bases of the villi would constitute a network of entry and exit portals whose adjacency might define the graph underlying lateral diffusion and

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paracellular routes, while nutrient absorption would be assumed to occur continuously along the entire epithelial surface. Each villus could be assigned structural weights reflecting epithelial thickness, stromal fiber density and basement membrane porosity, thereby producing a heterogeneous map of diffusion conductance, absorptive capability and hydraulic resistance along its axis. Mechanical fields might arise from rhythmic, peristaltic-like deformations propagating through the lamina propria, creating time-varying impedance patterns and conditionally shifting the effective topology of preferred flow and transport corridors. Biochemical cues, modeled as nutrient or signaling gradients, could propagate along curvature-aligned trajectories where structural impedance would be minimized and surface area maximized. Network-theoretic descriptors might identify modules composed of villi that share comparable geometric tapering or mechanical signatures, while nodes at villus–intervillous junctions could act as high-betweenness interfaces governing transitions between vertical absorptive pathways and lateral communication routes. The model would further incorporate a hypothetical stromal compartment whose mesenchymal elements might modulate stiffness and thereby reweight graph edges. Through this integration of curvature, conditional mechanical anisotropy, porosity gradients, oscillatory deformation and distributed absorption along the epithelial surface, this villus-field microenvironment would exemplify how an architectural regime might govern communication and nutrient handling within an intestinal-like system.

Cortical sheet with curvature-dependent conductivity. A hypothetical neuronal cortical sheet would demonstrate how Architectural Dynamics might conceptualize a layered surface populated by orientation fields and variable curvature. The sheet would contain abstract columnar units whose dendrite-like projections would follow locally varying preferred directions, producing an orientation field with spatial coherence lengths that could differ across regions. Structural density would vary as a function of depth, creating stratified impedance to diffusive and electrophysiological signals. Mechanical heterogeneity would be introduced through depth-dependent stiffness gradients, allowing the sheet to sustain curvature-driven tension patterns that might influence both transport and local deformation. Curvature would be treated not merely as a geometric descriptor but as a modulating factor for conductivity, where convex regions might amplify hypothetical field potentials and concave regions might attenuate them. The sheet's topology would be recast as a weighted graph in which nodes would correspond to microdomains defined by orientation clusters and edges would encode adjacency modulated by conductivity, fiber angle difference and extracellular porosity. Biochemical landscapes, represented by generic concentration gradients, would diffuse anisotropically according to local curvature and orientation coherence. Physical fields such as electric potential or stress waves would propagate through the sheet with anisotropic velocity determined by the combined influence of depth, curvature and structural stiffness. Network metrics would identify curvature extrema as functional hubs, reflecting their enhanced capacity to redistribute diffusive and mechanical signals. By blending laminar structure, curvature geometry, anisotropic conductivity, biochemical gradients and mechanical tension, our cortical sheet would become an illustrative case in which architecture shapes the organization of the microenvironment.

Fibroelastic meshwork with adaptive stress corridors. A hypothetical fibroelastic meshwork would exemplify how a conceptual connective-tissue analogue could be interpreted within Architectural Dynamics. This meshwork would be constructed from interwoven fibers whose alignment would vary stochastically across regions, generating local anisotropy and heterogeneous stiffness. Structural heterogeneity would be expressed through nodes representing fiber junctions and edges corresponding to fiber segments weighted by tensile strength, hydraulic permeability and local tortuosity. Mechanical fields would arise from embedded contractile units standing for fibroblast analogues that might impose localized tensile forces along specific orientations, producing adaptive stress corridors where stiffness and curvature could evolve dynamically. Geometric irregularities such as fold-like indentations would generate curvature-induced attractors that concentrate deformation energy. Biochemical gradients would diffuse along preferential paths determined by both mechanical softening and porosity modulation, while the meshwork's topology

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would evolve as stresses reweight edges to reflect altered deformation resistance. This would create a constantly shifting graph in which transport routes, mechanical pathways and diffusion basins might reconfigure in response to local perturbations. Network-theoretic analysis would reveal transient modules representing stress-stabilized compartments, while centrality measures would highlight junctions serving as transient relay points for both diffusive and mechanical signals. By integrating stochastic fiber orientation, dynamic stiffness gradients, evolving curvature, biochemical field propagation and topology reweighting, the hypothetical fibroelastic meshwork would conceptualize environments in which structure and physics co-determine spatial organization in real time.

Chromatin–loop manifold with multi-compartment regulatory gradients. A hypothetical chromatin–loop manifold would illustrate how Architectural Dynamics might conceptualize the three-dimensional genome as a branching, deformable and multi-compartment regulatory environment. Chromatin fibers would form looped and nested branches whose curvature, compaction and contact angle would evolve conditionally in response to mechanical constraints and biochemical modifications. Each loop interior would function as a quasi-isolated regulatory compartment with diffusion-limited accessibility, while the interloop matrix, composed of linker regions and scaffold proteins, would behave as a second compartment characterized by anisotropic polymer mobility and variable stiffness. Structural features would include locally heterogeneous compaction states, variable loop diameters and gradients of nucleosome density along fiber segments. Mechanical interactions would arise from molecular motors, torsional stresses and condensin- or cohesin-mediated tension, producing curvature changes that would feed back into contact topology and the effective movement of transcriptional machinery. Biochemical gradients, including hypothetical distributions of epigenetic marks or transcription factors, would diffuse with different efficiencies across loop interiors and the interloop matrix, shaped by local porosity and polymer viscoelasticity. Edges of the corresponding graph representation would encode both physical adjacency and probabilistic contact frequency, weighted by conductance metrics derived from compaction, curvature-dependent impedance and local polymer stiffness. Network metrics would detect regulatory bifurcation points such as loop anchors or extrusion stall sites, identifying them as organizational bottlenecks that concentrate mechanical stress or modulate access to distal regulatory elements. In touch with our proposed approach, Zhou et al. (2025) showed that the structure of individual chromatin molecules can determine physical properties of chromatin condensates and cellular chromatin organization. Through this mixture of dynamic geometry, compartmentalized diffusion, mechanical feedback and structural heterogeneity, this hypothetical chromatin–loop manifold would serve as a conceptual system in which Architectural Dynamics could align genome folding dynamics with weighted connectivity and field-like propagation across the nuclear landscape.

Cancer niche with heterogeneous resistance corridors. A hypothetical cancer niche would provide an example of how Architectural Dynamics might conceptualize a microenvironment in which structural irregularity, mechanical heterogeneity and biochemical compartmentalization would jointly define the conditions experienced by malignant cells. The niche would consist of densely packed cellular clusters embedded within a variably stiff extracellular scaffold whose fiber architecture would be distorted by anisotropic remodeling. Structural regions of high density would generate diffusion barriers, while mechanically softened corridors created by proteolytic degradation would form low-resistance channels for both fluid transport and cell migration. Geometric irregularities, including lobulated boundaries and micro-invaginations, would impose curvature fields that could bias mechanical stress distribution and modulate access to surrounding tissue compartments. Biochemical landscapes would be shaped by uneven metabolite consumption, generating pockets of hypoxia and acidosis that diffuse anisotropically according to local porosity and vascular proximity. Niche topology would be represented as a weighted graph in which nodes correspond to microdomains differing in stiffness, nutrient availability or curvature, while edges encode adjacency modulated by hydraulic conductivity, proteolytic permeability and diffusive

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resistance. Network metrics would reveal mechanically protected compartments acting as retention zones and highlight high-betweenness invasion fronts driving both outward expansion and therapeutic infiltration. Physical fields such as interstitial pressure or electric potential would propagate through the niche along tortuous, architecture-dependent pathways, influenced by the co-occurrence of heterogeneous stiffness and irregular geometry. Through this integration of distorted structure, dynamic compliance, biochemical gradients and topology reweighting, our cancer niche would illustrate how an architectural regime might shape malignant microenvironments.

Xylem-degradation niche in diseased olive tissue. Olive-tree decline in Puglia is driven by *Xylella fastidiosa*, a xylem-inhabiting bacterium that disrupts water transport by inducing vessel blockage, wall degradation and hydraulic dysfunction (Roper, Castro, and Ingel 2019; Landa et al. 2022; Scortichini et al. 2023). Infected trees progressively show leaf scorch, branch dieback and structural collapse of the canopy (Vergine et al. 2020; Burbank and Roper 2021; Castro, DiSalvo, and Roper 2021). The disease spreads through insect vectors and transforms the vascular microenvironment into a fragmented, low-conductance network. An hypothetical xylem-degradation niche would illustrate how Architectural Dynamics might conceptualize a plant-disease microenvironment in which vascular obstruction, wall degradation and anisotropic stress redistribution could jointly determine tissue behavior. In this representation, the affected olive xylem would be modeled as a branching network of conduits whose hydraulic conductance would vary conditionally due to localized occlusions formed by pathogen-induced gels or tyloses. Structural heterogeneity would arise from patchy cell-wall thinning, creating regions of decreased stiffness that might alter the propagation of tension forces driven by transpiration. Geometric irregularities, including vessel collapse or asymmetric swelling, would impose curvature-dependent mechanical fields that could bias fluid trajectories and modify lateral cross-flow between vessels. Biochemical gradients (modeled as hypothetical distributions of pathogen-derived enzymes, phenolic compounds and plant defense mediators) would diffuse anisotropically through altered wall porosity, generating chemical compartments that differ in reactivity, oxidative stress and metabolic burden. The niche's topology would be represented as a weighted graph in which nodes correspond to vascular microdomains with differing obstruction levels or wall composition, while edges encode adjacency modulated by hydraulic permeability, diffusion resistance and mechanical compliance. Network metrics would identify high-betweenness choke points functioning as dominant regulators of residual sap flow and mechanically weakened junctions might emerge as structural bottlenecks where vessel failure or embolism would preferentially propagate. Physical fields such as pressure gradients, cavitation waves and osmotic potentials would traverse the niche along architecture-dependent paths shaped by the interplay between obstruction patterns, curvature and wall stiffness. Through the integration of variable conductance, pathological wall alteration, biochemical compartmentalization and dynamically reweighted topology, the hypothetical olive xylem niche would illustrate how a diseased plant environment could be governed by architectural regimes rather than by biological agents alone.

These hypothetical examples illustrate how Architectural Dynamics could interpret diverse physiological and pathological microenvironments by integrating geometry, material heterogeneity, spatial connectivity and field dynamics into cohesive architectural regimes. By treating structural features and physical fields as jointly weighted elements of a single spatial network, Architectural Dynamics accommodates a wide spectrum of organizational patterns.

Conclusions

Architectural Dynamics reframes tissue organization by treating physical architecture as a quantifiable, multidimensional regime in which structural, mechanical, geometrical, network-theoretic, cellular, biochemical and field-derived descriptors are integrated into a unified representation. The novelty lies in shifting the analytical focus from cellular identity or isolated physical processes to the ensemble of measurable architectural variables, jointly shaping communication, transport and mechanical signaling. This makes it possible to describe biological

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tissues as weighted spatial systems in which local constraints and global organization are treated consistently, allowing architecture to be tackled with the same rigor reserved for molecular or dynamical models.

Compared with existing techniques, Architectural Dynamics departs from frameworks analysing individual dimensions of the microenvironment in isolation. Traditional structural assessments describe morphology or fiber organization, but do not connect these features to the mechanical or diffusive regimes they shape. Mechanical studies quantify stiffness or viscoelasticity but lack a spatial interpretation linking local material properties to global pathways of stress transmission. Geometrical analyses highlight curvature or thickness but treat these parameters independently from biochemical gradients or field propagation. Network-based models capture connectivity but do not incorporate physically derived weights or heterogeneous material properties. Reaction–diffusion and continuum mechanics models provide strong mathematical formalisms, yet do not explicitly integrate discrete architectural heterogeneity. Machine-learning approaches infer high-dimensional patterns but do not preserve physical interpretability or enforce coupling among structural, mechanical and geometrical variables. Architectural Dynamics merges these perspectives into a unified, physically grounded representation in which weights derive from measurable features rather than abstractions. It also aligns local physical descriptors with graph-theoretic structure, preserves field continuity and maintains the interpretability associated with classical architectural analysis.

Architectural Dynamics can be classified as a mesoscale integrative framework standing between continuum-level physical models and discrete, cell-centric representations. It preserves the scale-appropriate resolution required to describe local microstructural heterogeneity, while also incorporating the global descriptors required to characterize long-range communication patterns. Situated alongside structural analysis, biomechanics, geometric morphometrics, network theory and field modelling, it stands for a bridging category in which spatially distributed physical cues and topological organization are understood as concurrent determinants of microenvironmental behaviour.

Our framework carries several limitations. Any high-dimensional architectural representation depends on the availability, consistency and accuracy of measurements. This means that variability in resolution, sampling density or noise can reduce the reliability of derived parameters. The complexity of architectural parameterization also introduces challenges in dimensionality reduction, interpretability and calibration, particularly when spatial domains change over time or undergo remodeling. In abstracting cellular identity and molecular specificity into architectural descriptors, our framework may overlook biologically relevant influences that are not easily expressible in structural or physical terms. Network-based representations further rely on discretization choices that shape the inferred topology and may obscure alternative long-range interaction patterns. Additional limitations arise from the need to harmonize multimodal datasets collected under different experimental conditions, the difficulty of validating architectural descriptors against ground-truth measurements at multiple scales and the risk of overfitting high-dimensional architectural spaces with limited or heterogeneous data.

The broader scope of Architectural Dynamics suggests several future directions. The integration of multimodal datasets (combining imaging, mechanical mapping, spatial transcriptomics and field measurements) could refine parameter definitions and allow more accurate reconstruction of architectural regimes. Agent-based or continuum simulations informed by weighted architectural parameters could generate testable hypotheses about how perturbations in stiffness, curvature, porosity or modularity influence emerging behaviors such as compartmentalization or signal spread. Experiments that systematically manipulate architectural features, such as engineered scaffolds with controlled anisotropy or organoids with altered curvature, could elucidate how distinct architectural dimensions influence functional outputs. Longitudinal observations of evolving tissues may clarify how movement along architectural axes corresponds to transitions in development, regeneration or pathology. Recommendations emerging from our synthesis include adopting standardized protocols

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for quantifying architectural parameters, integrating physical fields explicitly when analyzing microenvironments and designing experimental systems to probe architecture-function relationships across scales.

The central research question addressed in this work concerns whether the physical architecture of a microenvironment can be represented through a unified system of quantifiable variables collectively illuminating the pathways through which cells interpret and inhabit their surroundings. The take-away message is that architecture, when treated as a structured set of measurable features rather than a qualitative backdrop, becomes an analyzable determinant of microenvironmental behavior. Our descriptive standpoint does not replace molecular or dynamical frameworks, but rather provides the intermediate scale at which structure, geometry, mechanics, connectivity and fields converge into a coherent analytical space.

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Appendix A. Notation and Symbol Glossary

This appendix summarizes all the structural, mechanical, geometrical, network-theoretic, cellular, biochemical and physical features defining the architectural properties of a cellular microenvironment. All variables are defined generically and can be applied to any multicellular system or engineered scaffold.

1. STRUCTURAL AND MATERIAL PARAMETERS

E_{mat} : stiffness of the material ⇒ sets global mechanical environment

ξ : domain or pattern length scale ⇒ controls repeating architectural motifs

t_i : wall or barrier thickness ⇒ regulates confinement and barrier strength

δ_i : material degradation rate ⇒ governs remodeling and turnover

ϕ_i : solid fraction ⇒ determines porosity and load-bearing capability

σ_i^{hyd} : hydration stability ⇒ controls swelling and shape changes

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γ_i : surface tension ⇒ affects curvature and compartment formation

$P_{i\leftrightarrow j}$: boundary permeability ⇒ mediates exchange between compartments

τ_i^{shear} : boundary shear stress ⇒ provides mechanical cues at interfaces

2. MECHANICAL PARAMETERS

E_i : local elastic modulus ⇒ defines stiffness gradients sensed by cells

τ_i : viscoelastic relaxation ⇒ sets adaptation timescale to deformation

$Z_{ij}(\omega)$: mechanical impedance ⇒ regulates force transmission at frequency ω

k_{ij} : connecting stiffness ⇒ determines mechanical coupling strength

α_i : anisotropy ⇒ channels forces along preferred axes

λ_i : stress relaxation ⇒ mechanical dissipation control

$\hbar_i^{\text{mech}}$: energy loss ⇒ damping in repetitive loading

Φ_{ij} : force propagation ⇒ mechanical communication across tissue

p_i : hydrostatic pressure ⇒ drives fluid/structural deformation

$\dot{\gamma}_i$: shear rate ⇒ generates shear-mediated cues

$\vec{v}_{\text{IF}}$: interstitial fluid velocity ⇒ advects signals and metabolites

3. GEOMETRICAL PARAMETERS

V_i : volume of region i ⇒ sets spatial capacity

S_i : surface area ⇒ regulates adhesion and exchange

d_i : pore diameter ⇒ controls accessibility and confinement

L_{ij} : path length ⇒ sets transport distances

A_{ij} : cross-section ⇒ controls transport and flow capacity

τ_{ij} : tortuosity ⇒ modulates effective diffusion or migration

$\vec{u}_{ij}$: orientation vector ⇒ defines structural anisotropy

H_i : mean curvature ⇒ affects stresses and gradients

K_i : Gaussian curvature ⇒ marks geometric hotspots

∇H_i : curvature gradient ⇒ imposes directional cues

R_i : roughness ⇒ modulates adhesion and protein adsorption

$\hbar$: spatial heterogeneity ⇒ captures disorder or patchiness

σ_{scale} : hierarchical scale ⇒ quantifies multiscale organization

$\mathbf{r}_i$: node coordinates ⇒ embed network in real space

κ_{ij} : edge curvature ⇒ influences transport and alignment

NETWORK-THEORETIC PARAMETERS

E_{glob} : global efficiency ⇒ overall accessibility of architecture

$E_{\text{loc}}(i)$: local efficiency ⇒ robustness of local neighborhoods

D : network diameter ⇒ maximal separation of compartments

ρ : network density ⇒ overall connectivity richness

Q : modularity ⇒ identifies architectural compartments

k_i : degree ⇒ interaction capacity of region i

B_i : betweenness ⇒ bottleneck importance

C_i^{eff} : efficiency centrality ⇒ exposure to global cues

C_i, C_{glob} : clustering ⇒ local cooperation potential

$\mathcal{C} = \{C_1, \ldots, C_\ell\}$: communities ⇒ higher-order architecture

S_ϵ : entrance spacing ⇒ boundary accessibility

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R_{ij}, R : redundancy ⇒ robustness of routes

$P(k)$: degree distribution ⇒ global architecture class

$\mathscr{L}$: graph Laplacian ⇒ governs propagation on the structure

λ_2 : spectral gap ⇒ cohesion and synchronizability

$\mathscr{H}_{\text{top}}$: topological entropy ⇒ complexity of network structure

$\mathcal{S}(f)$: smoothness functional ⇒ coherence of fields on the network

CELLULAR STATE AND BEHAVIORAL PARAMETERS

$\mathbf{x}_k$: cell position ⇒ defines spatial organization

$\mathbf{p}_k$: polarity vector ⇒ determines directionality

$P_{i\to j}^{(k)}$: migration probability ⇒ movement likelihood

τ^{mot} : motility persistence ⇒ directional memory

σ_k^{mot} : motility noise ⇒ stochastic variability

ρ_i : local cell density ⇒ crowding effects

x_k : internal state ⇒ activation/differentiation level

Ω_k : receptor occupancy ⇒ sensing capacity

σ_{ij} : contact signaling ⇒ cell–cell regulatory interactions

a_{kl} : adhesion coefficient ⇒ binding strength

p_i^{crowd} : crowding pressure ⇒ mechanical constraint

χ^{CI} : contact inhibition ⇒ space-regulation mechanism

γ_i : proliferation rate ⇒ growth dynamics

θ_i^{act} : activation threshold ⇒ cue sensitivity

Π_i^{prolif} : proliferative pressure ⇒ net growth tendency

SURFACE CHEMISTRY AND ECM PARAMETERS

λ_i : ligand density ⇒ controls adhesion and sensing

η_i : adhesion molecule density ⇒ anchoring strength

θ_i : contact angle ⇒ wettability and protein adsorption

Π_i : protein adsorption ⇒ surface conditioning

$\mathbf{e}_i$: ECM composition ⇒ biochemical identity

μ_i : enzymatic susceptibility ⇒ matrix remodeling rate

K_d : binding affinity ⇒ strength of molecular interactions

PHYSIOLOGICAL FIELD PARAMETERS

pH_i : local pH ⇒ chemical environment

G_{ij} : ionic/electrical conductance ⇒ rapid communication pathway

$\mathscr{M}_i$: metabolic activity ⇒ energetic state

χ_i : thermal dissipation ⇒ heat regulation

L_i : signal concentration ⇒ cue landscape

κ_i : signal decay ⇒ range of influence

$\hat{\mathbf{g}}$: gradient direction ⇒ orientation cue

θ_i^{sig} : saturation threshold ⇒ nonlinear response limit

4. TRANSPORT AND DIFFUSION PARAMETERS

D_{ij} : diffusion coefficient ⇒ molecular mobility

$C_{ij}^{\text{diff}} = \dfrac{D_{ij}A_{ij}}{L_{ij}}$: diffusion conductance ⇒ effective transport capacity

P_{ij} : permeability ⇒ ease of passage across barriers

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R_{ij}^{hyd} : hydraulic resistance ⇒ impedance to fluid flow

c_i : molecule concentration ⇒ resource availability

∇c_i : concentration gradient ⇒ directional cue

Q_i : production/consumption rate ⇒ source/sink dynamics

ξ_i^{prod} : production noise ⇒ variability in generation

$\eta_{ij}^{\mathrm{edge}}$: edge transport noise ⇒ stochasticity along paths

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Hyperdisorder in tumor growth

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ABSTRACT

Tumor growth is constrained by spatial, mechanical and metabolic factors whose alignment progressively breaks down across cellular, mesoscopic and tissue scales as tumors expand. We hypothesize that this misalignment could drive growing tumors toward a distinct architectural regime, termed hyperdisorder. Hyperdisorder is not defined by increased heterogeneity alone, but by the coexistence of elevated disorder across scales and spatial nonstationarity within the same tumor. Unlike ordinary randomness, where independent fluctuations diminish under spatial averaging, here disorder persists, reorganizes or even amplifies with increasing observation scale, preventing convergence toward a stable architectural description. Using hematoxylin and eosin stained whole-slide images of gastric cancer from The Cancer Genome Atlas, we quantify tumor architecture with tile-based metrics capturing complementary aspects of organization: texture entropy, microstructural fragmentation orientation isotropy and multiscale entropy variation. These measures are combined into a standardized hyperdisorder index, enabling unsupervised comparison across spatial regions. We find that architectural disruption is unevenly distributed and partially decoupled across scales within individual slides, consistent with growth-driven multiscale incoherence rather than uniform stochastic variability. Testable consequences include anomalous scaling of heterogeneity with sampling size, failure of coarse graining to converge and systematic differences between tumor cores and invasive fronts. In diagnostic and clinical contexts, this perspective clarifies when measurements derived from limited tissue samples are representative of the tumor as a whole and when they are instead dominated by scale-dependent and location-specific architectural effects. In therapeutic settings, it enables quantitative assessment of pre-existing architectural differences among spatially distinct tumor regions prior to treatment.

KEYWORDS: multiscale entropy; spatial nonstationarity; architectural breakdown; tile-based analysis; quantitative histopathology.

INTRODUCTION

Histopathological assessment of cancer typically relies on tissue architecture evaluated at a single spatial scale or on quantitative descriptors computed locally. Grading systems, pattern recognition approaches and many image-based metrics implicitly assume that architectural disruption is a local phenomenon and that variability smooths out as larger tissue regions are considered (Shekarian et al. 2017; Bai et al. 2020; Man and Jenkins 2022; Wang et al. 2023). These assumptions limit the ability of current methods to capture how heterogeneity is organized across spatial scales during tumor growth. Common measures often conflate structural complexity with randomness, interpreting disorder as a uniform increase in variability (Ren et al. 2018; Bagaev et al. 2021; Leong et al. 2022). However, architectural disruption need not arise from independent random perturbations. Instead, disorder may emerge from growth dynamics, producing patterns that are distributed, partially coupled and maintained across multiple spatial scales. This motivates the introduction of a distinct architectural regime, termed hyperdisorder, that cannot be reduced to conventional notions of heterogeneity.

Hyperdisorder describes a condition in which tissue organization becomes progressively incoherent across spatial scales as a tumor grows. Unlike ordinary randomness, where fluctuations diminish predictably under spatial averaging, hyperdisorder denotes a regime in which disorder persists, reorganizes or increases with scale (Hayashi 2023; Mittal 2025). It is characterized by elevated disorder at multiple scales, partial decoupling between modes of architectural variation and strong spatial nonstationarity within the same specimen (Wang et al. 2025). Cellular-scale irregularities, mesoscopic fragmentation and tissue-scale patterns fail to align into a single degraded architecture and do not converge toward a stable description (Ross et al. 2025). Hyperdisorder is defined operationally through measurable image-derived quantities and their scaling behavior. We apply this framework to gastric cancer histology to test whether tumor architecture exhibits signatures of growth-driven multiscale incoherence and whether these differ from those expected under ordinary stochastic heterogeneity.

We proceed as follows. We first describe the image data and quantitative measures used to characterize tissue architecture and introduce a hyperdisorder index with its associated scaling analyses. We then present results from the gastric cancer case study and derive quantitative, testable predictions. Finally, we discuss the implications for interpreting architectural variability in growing tumors, including diagnostic, clinical and therapeutic relevance.

METHODS

We describe here the data sources, preprocessing steps, quantitative measures, mathematical formulations and analytical procedures to operationalize hyperdisorder in tumor histology.

Histological dataset. We analyzed publicly available histological data from the stomach adenocarcinoma cohort of The Cancer Genome Atlas (TGCA). Specifically, we used the TCGA stomach histology image collection distributed through the Kaggle repository curated by Ahmed Abo Enaba. This dataset consists of hematoxylin and eosin stained whole-slide images that were pre-tiled into fixed-size, non-overlapping image patches extracted from diagnostic tumor slides. At the time of analysis (January 2026), the repository contained 13 527 tumor tiles derived from 38 patients, each tile corresponding to a square tissue region sampled at a fixed pixel resolution.
To reduce patient-level imbalance, we constructed a stratified subsample with a maximum of 200 tiles per patient, yielding an initial set of 5 702 tiles. We then applied a quality-based filtering step to remove tiles dominated by background or empty regions. After filtering, 899 tiles from 37 patients remained and were retained for quantitative analysis (Fig. A). All analyses and results reported in this manuscript are based exclusively on this final tile set.

Preprocessing and tissue masking. Each image tile was first resized to a common spatial resolution to standardize subsequent computations. Let $I(\mathbf{r})$ denote the color image defined on pixel coordinates $\mathbf{r} = (x, y)$. Tiles were isotropically resized to 128×128 pixels using area interpolation to preserve average intensity statistics. The resized image was converted from RGB to both grayscale and HSV color spaces. The grayscale image $G(\mathbf{r})$ was used for intensity-based and gradient-based measures, while the saturation channel $S(\mathbf{r})$ from HSV was used to discriminate tissue from background.
A binary tissue mask $M(\mathbf{r})$ was constructed according to

$$M(\mathbf{r}) = \begin{cases} 1, & G(\mathbf{r}) < \tau_g \text{ and } S(\mathbf{r}) > \tau_s, \\ 0, & \text{otherwise,} \end{cases}$$

with fixed thresholds $\tau_g = 230$ and $\tau_s = 18$ in 8-bit intensity units. The tissue fraction was defined as

$$f_{\text{tissue}} = \frac{1}{N} \sum_{\mathbf{r}} M(\mathbf{r}),$$

where N is the total number of pixels. Tiles with $f_{\text{tissue}} < 0.10$ were excluded to avoid spurious measurements driven by background noise. All subsequent computations were restricted to pixels with $M(\mathbf{r}) = 1$, ensuring that quantitative measures reflect tissue architecture rather than slide artifacts.

Local intensity entropy as a disorder measure. To capture variability in staining and microtexture, we computed the Shannon entropy of grayscale intensities within tissue regions. Let $g(\mathbf{r})$ denote grayscale intensity values restricted to tissue pixels. The empirical histogram $h(k)$ for intensity level $k \in \{0, \dots, 255\}$ was estimated as

$$h(k) = \frac{1}{N_M} \sum_{\mathbf{r}: M(\mathbf{r})=1} \mathbf{1}[g(\mathbf{r}) = k],$$

where N_M is the number of tissue pixels and $\mathbf{1}[\cdot]$ is the indicator function. The entropy was then defined as

$$H = -\sum_{k: h(k)>0} h(k) \log_2 h(k).$$

This quantity increases with greater diversity of intensity values and serves as a scale-agnostic descriptor of local disorder. This entropy does not assume any specific tissue model and is sensitive only to the distribution of intensities within each tile.

Edge density and fine-scale microstructural fragmentation. Microstructural fragmentation was quantified using an edge density measure derived from gradient-based edge detection. The grayscale image $G(\mathbf{r})$ was processed using a Canny edge detector with fixed lower and upper thresholds. This yields a binary edge map $E(\mathbf{r}) \in \{0,1\}$. Edge density was defined as

$$\rho_E = \frac{1}{N_M} \sum_{\mathbf{r}: M(\mathbf{r})=1} E(\mathbf{r}),$$

representing the fraction of tissue pixels identified as part of an edge. Higher values of ρ_E correspond to more fragmented or irregular microstructure. Since the same detector parameters were used across all tiles, differences in ρ_E reflect architectural variation rather than parameter tuning.

Orientation isotropy and gradient entropy. To quantify directional coherence and loss of directional organization, we computed an isotropy measure based on the distribution of gradient orientations. Horizontal and vertical gradients were computed using Sobel operators (Lu et al. 2023; Tang et al. 2023; Wang et al. 2024), yielding gradient components $G_x(\mathbf{r})$ and $G_y(\mathbf{r})$. For tissue pixels, gradient magnitude and orientation were defined as

$$m(\mathbf{r}) = \sqrt{G_x(\mathbf{r})^2 + G_y(\mathbf{r})^2}, \theta(\mathbf{r}) = \arctan\ 2(G_y(\mathbf{r}), G_x(\mathbf{r}))mod\pi.$$

Only pixels with $m(\mathbf{r})$ above a fixed percentile threshold were retained to focus on structurally meaningful gradients. The orientation histogram was constructed over $B = 8$bins on $[0, \pi)$, yielding probabilities p_i. Orientation entropy was then computed as

$$H_\theta = -\sum_{i=1}^{B} p_i \log_2 p_i.$$

An isotropy index was defined as the normalized entropy

$$I = \frac{H_\theta}{\log_2 B},$$

with $I = 1$ indicating maximal isotropy and lower values indicating preferred orientations.

Multiscale entropy change: sensitivity of disorder to coarse graining. To probe multiscale structure, we quantified how intensity entropy changes under spatial smoothing. The grayscale image was convolved with a Gaussian kernel of standard deviation $\sigma = 1.8$ pixels, yielding a smoothed image $G_\sigma(\mathbf{r})$. Entropy H_σ was computed on G_σ using the same procedure as for H. The multiscale entropy change was defined as

$$\Delta H = H - H_\sigma.$$

Larger values of ΔH indicate the presence of fine-scale variability that is suppressed by coarse-graining, whereas small values indicate scale-invariant or already smooth structure.

Construction of a composite disorder measure. The four quantities H, ρ_E, I and ΔH were combined into a single hyperdisorder index. For each feature X, a standardized score was computed as

$$Z_X = \frac{X - \mu_X}{\sigma_X},$$

where μ_X and σ_X are the mean and standard deviation across all retained tiles. The hyperdisorder index for tile j was then defined as

$$\mathcal{H}_j = Z_H + Z_{\rho_E} + Z_I + Z_{\Delta H}.$$

This construction assigns equal weight to each disorder mode and yields a dimensionless quantity centered near zero.

Spatial mapping and patient-level aggregation. Tile coordinates encoded in filenames were used to reconstruct approximate spatial maps of hyperdisorder within slides. For each slide, $\mathcal{H}_j$ values were plotted at their corresponding (x, y) positions to visualize spatial nonstationarity. Patient-level summaries were obtained by grouping tiles by patient identifier and computing descriptive statistics, including the median hyperdisorder and interquartile range. These summaries were used to characterize between-patient variability without imposing parametric assumptions.

Software and computational tools. All computations were performed using Python. Image processing relied on OpenCV, numerical operations on NumPy, data handling on pandas and visualization on Matplotlib. No machine learning models or pretrained networks were used. All parameters were fixed a priori and applied uniformly across samples.

Overall, we have defined the data sources, preprocessing steps, mathematical constructs and computational procedures used to quantify hyperdisorder in gastric cancer histology. The resulting framework is fully operational, reproducible and not specific to gastric cancer, providing a general approach for probing multiscale architectural incoherence in tumor tissues and for systematic comparison across datasets and cancer types.

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RESULTS

We report the quantitative results obtained by applying our approach to gastric cancer histology, focusing on distributions, correlations, spatial patterns and patient-level aggregation.

Tile-level hyperdisorder distribution. We first quantified hyperdisorder at the tile level across 899 tumor tiles derived from 37 patients. The resulting hyperdisorder index exhibited a broad, asymmetric distribution (Fig. B), spanning from approximately −9.7 to 4.5 in standardized units, with a median close to zero by construction. This wide range indicates that architectural disorder varies substantially between spatially distinct tumor regions, even within the same cohort. Examination of the individual components revealed nontrivial relationships between disorder modes. Texture entropy and edge density were strongly positively correlated (Pearson correlation coefficient $r \approx 0.82$), indicating that increased intensity variability is often accompanied by increased microstructural fragmentation (Fig. C). In contrast, isotropy showed weaker and negative correlations with entropy ($r \approx -0.17$) and edge density ($r \approx -0.07$), suggesting that loss of directional organization is only partially aligned with fine-scale disorder. Multiscale entropy change was negatively correlated with entropy ($r \approx -0.51$) and weakly correlated with edge density ($r \approx -0.22$), reflecting distinct sensitivity to coarse-graining. The composite hyperdisorder index correlated most strongly with edge density ($r \approx 0.76$) and entropy ($r \approx 0.57$), while retaining moderate associations with isotropy ($r \approx 0.46$) and multiscale entropy change ($r \approx 0.22$).

These quantitative relationships show that the hyperdisorder index integrates partially decoupled components rather than collapsing onto a single dominant feature, establishing a multicomponent description of architectural disorder that motivates aggregation beyond individual tiles.

Patient-level aggregation. Building on tile-level measurements, we aggregated hyperdisorder values at the patient and slide levels to assess larger-scale organization. Patient-level median hyperdisorder values varied systematically across the cohort, ranging from approximately −3.6 to 2.5 (Fig. D). This spread indicates that tumors differ not only in local variability but also in their overall architectural regime. Within-patient variability, quantified by the interquartile range of tile-level hyperdisorder, remained substantial across most patients, with values typically between 1.0 and 2.5 (Fig. F). Importantly, higher median hyperdisorder did not correspond to reduced within-patient spread, indicating that increased disorder does not stabilize under aggregation. Spatial mapping of hyperdisorder within individual slides further revealed pronounced nonstationarity (Fig. E). In a representative case with 48 sampled tiles, regions of elevated and reduced hyperdisorder appeared interspersed across the tissue section, without a smooth gradient or uniform pattern. These spatial fluctuations were comparable in magnitude to between-patient differences, underscoring that intra-tumoral variability contributes substantially to observed disorder statistics.

Taken together, patient-level summaries and spatial maps show that architectural disorder is unevenly distributed both within and between tumors, reinforcing the need to interpret tumor architecture as a spatially structured, multiscale phenomenon rather than as a homogeneous property.

Our results show that gastric cancer histology displays pronounced tile-level variability in hyperdisorder, partial decoupling among disorder components and systematic differences in patient-level medians, together with strong spatial nonstationarity within individual slides. Taken together, these findings characterize tumor architecture as intrinsically multiscale and unevenly organized, rather than as a uniformly degraded or purely random structure.

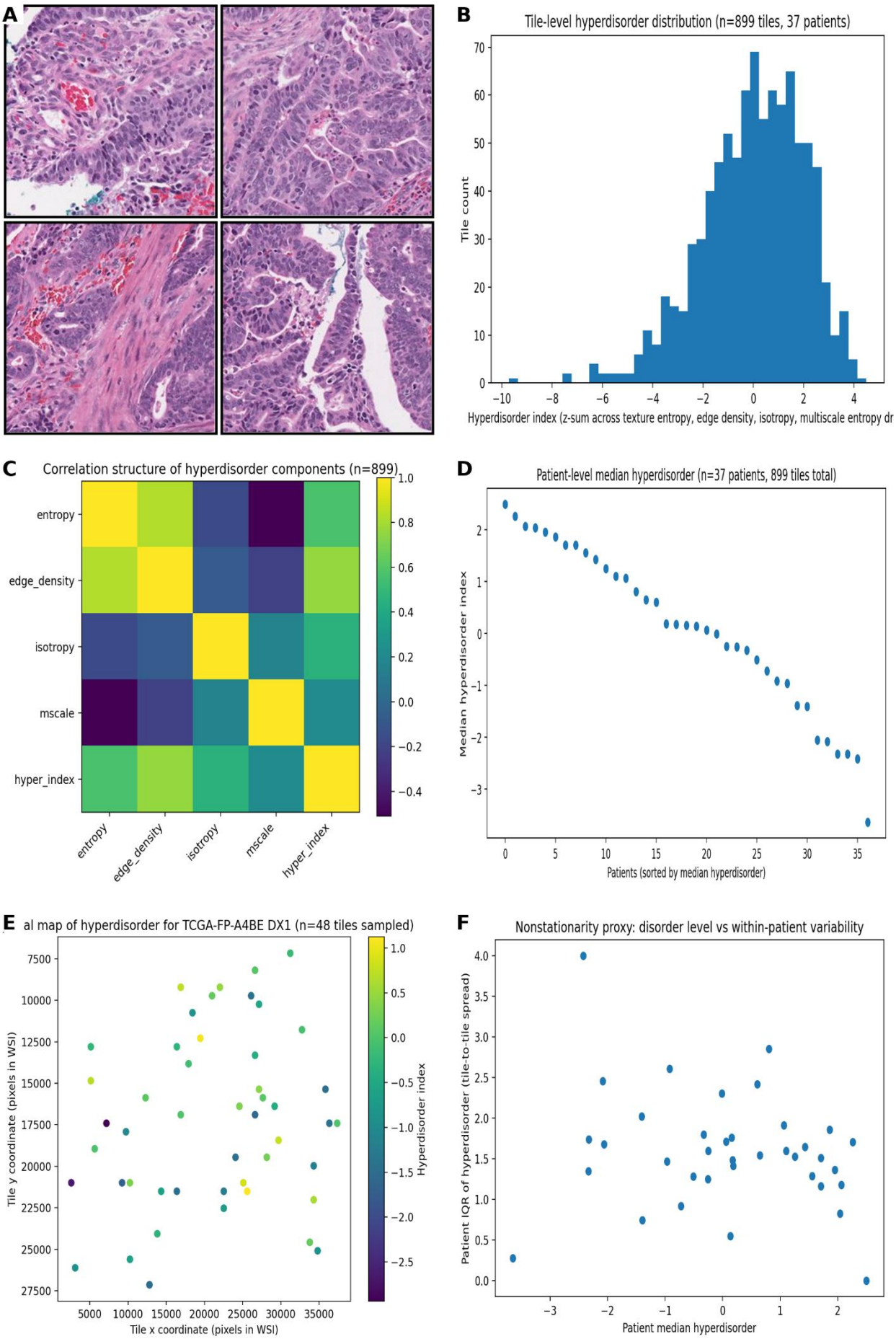

A
B
Tile-level hyperdisorder distribution (n=899 tiles, 37 patients)
Tile count
Hyperdisorder index (z-sum across texture entropy, edge density, isotropy, multiscale entropy dr
C
Correlation structure of hyperdisorder components (n=899)
entropy
edge_density
isotropy
mscale
hyper_index
D
Patient-level median hyperdisorder (n=37 patients, 899 tiles total)
Median hyperdisorder index
Patients (sorted by median hyperdisorder)
E
al map of hyperdisorder for TCGA-FP-A4BE DX1 (n=48 tiles sampled)
Tile y coordinate (pixels in WSI)
Tile x coordinate (pixels in WSI)
Hyperdisorder index
F
Nonstationarity proxy: disorder level vs within-patient variability
Patient IQR of hyperdisorder (tile-to-tile spread)
Patient median hyperdisorder

Figure. Hyperdisorder as a multiscale architectural regime in gastric cancer histology.
(A) Representative hematoxylin and eosin–stained histological tiles from a gastric adenocarcinoma whole-slide image (TCGA-3M-AB46, slide DX1). The four panels show distinct tumor regions sampled from the same specimen at comparable magnification, illustrating pronounced intra-tumoral architectural variability, including differences in cellular density, nuclear arrangement, stromal organization, vascular features and glandular distortion. These images are shown without preprocessing other than resizing and provide context for the quantitative analyses shown in panels B–F.
(B) Distribution of the tile-level hyperdisorder index computed across 899 tumor tiles from 37 patients. The index is defined as the sum of standardized contributions from texture entropy, edge density orientation isotropy and multiscale entropy change. The broad and asymmetric distribution indicates substantial variability in architectural disorder across tumor regions.
(C) Correlation structure among the individual hyperdisorder components and the composite index. Strong coupling between some measures like entropy and edge density, coexists with weaker or inverse correlations involving isotropy and multiscale entropy change, indicating partial decoupling between disorder modes rather than a single dominant source of variability.
(D) Patient-level median hyperdisorder values, with patients ordered by their median score. This representation highlights systematic differences in overall architectural disruption between tumors, beyond local tile-level fluctuations.
(E) Spatial map of hyperdisorder values for a representative slide (TCGA-FP-A4BE, DX1), with each point corresponding to a sampled tile plotted at its approximate coordinate within the whole-slide image. The patchy distribution of high and low values reveals strong spatial nonstationarity of architectural disorder within a single tumor section.
(F) Relationship between patient-level median hyperdisorder and within-patient variability, quantified as the interquartile range of tile-level values. The persistence of substantial variability across a wide range of median disorder levels indicates that architectural heterogeneity does not converge under spatial aggregation.
Together, panels A–F show that gastric cancer histology is characterized by multiscale architectural disruption, partial decoupling between disorder measures and pronounced spatial nonstationarity, consistent with a hyperdisordered architectural regime rather than uniform or purely random heterogeneity.

QUANTITATIVE PREDICTIONS OF HYPERDISORDER DRIVEN BY TUMOR GROWTH

Calling tumor growth hyperdisordered is not intended as a descriptive label but as a quantitative claim about how heterogeneity behaves under spatial aggregation and how disorder scales with system size. Hyperdisorder implies that architectural variability does not obey the convergence properties expected under ordinary stochastic heterogeneity. As a result, it yields explicit, testable predictions about sampling, scaling and spatial organization that can be assessed using existing tumor datasets. These predictions concern not only the magnitude of heterogeneity but its scaling laws, spatial coupling and dependence on growth geometry.

A first consequence of hyperdisorder is the failure of averaging. If a tumor occupies a hyperdisordered regime, increasing the sampled volume does not necessarily stabilize estimates of genetic, phenotypic or microenvironmental quantities. Let $X(\mathbf{r})$ denote a spatially defined biomarker field like mutation burden, expression level, hypoxia score or immune infiltration. For a sampling window of linear size R, define the regional mean

$$\bar{X}_R = \frac{1}{|W_R|}\int_{W_R} X(\mathbf{r})\,\mathrm{d}\mathbf{r},$$

where W_R is a window of radius R. Under ordinary randomness with short-range correlations, the variance scales as

$$\mathrm{Var}(\bar{X}_R) \propto R^{-d},$$

with d the effective spatial dimension. Hyperdisorder predicts instead

$$\mathrm{Var}(\bar{X}_R) \propto R^{-\alpha}, \alpha \ll d,$$

or even scale-dependent $\alpha(R)$, implying slow or non-monotonic decay. This prediction is testable using spatial transcriptomics, multiplex immunofluorescence or tiled whole-slide images by fitting α across scales. Falsification occurs if variance converges rapidly beyond a modest R.

A second prediction concerns genetic divergence across space. In multiregion sequencing, let D_{ij} denote the genetic distance between regions i and j, defined for example by Jaccard distance of mutation sets or by phylogenetic branch length and let r_{ij} denote their spatial separation. Under simple spatial mixing, one expects a stable relationship $D(r)$ with decreasing dispersion as tumor size increases. Hyperdisorder predicts the opposite: as tumors grow, the conditional variance $\mathrm{Var}(D_{ij} \mid r_{ij})$ increases and a single distance-to-divergence curve ceases to be predictive. This can be tested by stratifying published multiregion Whole-Exome Sequencing or Whole-Genome Sequencing cohorts by tumor size or volume proxy. Falsification occurs if $D(r)$ tightens with increasing size.

A third prediction concerns the spatial organization of phenotypic patches. Define patches by thresholding a continuous field $X(\mathbf{r})$ into regions where X exceeds a biologically meaningful cutoff and let A denote patch area. Under scale-aligned heterogeneity, the distribution $P(A)$ is expected to be narrow or exponentially bounded. Hyperdisorder predicts a broad, heavy-tailed distribution,

$$P(A) \sim A^{-\tau},$$

with the tail exponent τ decreasing as tumor size increases, reflecting growth-amplified patchiness. This prediction can be tested using digital pathology segmentations or radiological feature maps. Falsification occurs if $P(A)$ remains narrowly distributed and scale-invariant.

A fourth prediction concerns coarse-graining and phenotypic classification. Consider a tumor phenotype index Y_R obtained by smoothing or aggregating data at scale R (for example cell scale, 200 μm, 1 mm, 5 mm). Under ordinary heterogeneity, classifications based on Y_R converge as R increases. Hyperdisorder predicts persistent disagreement between classifications at different R, quantified for instance by non-vanishing disagreement probability

$$\Pr\left(Y_{R_1} \neq Y_{R_2}\right) \nrightarrow 0 \text{as } R_2 \to \infty.$$

This is testable using spatial omics or multiplexed imaging. Falsification occurs if classifications stabilize at large scales.

A fifth prediction links hyperdisorder explicitly to growth geometry. If hyperdisorder is generated by growth, scaling exponents and patch statistics should differ between spatial regions associated with distinct growth dynamics. Let α_{edge}, α_{mid}, α_{core} denote scaling exponents measured separately at the invasive front, intermediate zone and necrotic or fibrotic core. Hyperdisorder predicts systematic differences and crossover scales separating these regimes. This can be tested by region annotation in whole-slide images or spatial transcriptomics. Falsification occurs if scaling behavior is spatially homogeneous.

Additional quantitative consequences follow directly from these predictions.
First, the effective number of independent samples $N_{\text{eff}}(R)$ within a region of size R is reduced relative to geometric expectations, scaling as $N_{\text{eff}} \sim R^{\alpha}$ rather than R^{d}.
Second, estimates of extreme values, such as maximum hypoxia or maximum clonal divergence, are predicted to grow logarithmically or algebraically with sampled area rather than saturating.
Third, correlations between distinct biomarker fields $X(\mathbf{r})$ and $Y(\mathbf{r})$ are predicted to be scale-dependent, with cross-correlations $C_{XY}(R)$ changing sign or magnitude as a function of R, reflecting partial decoupling of disorder modes.
Minimal datasets are sufficient to test several of these predictions. For example, whole-slide images with basic immunohistochemistry can be tiled at multiple resolutions, per-tile features computed and variance scaling and coarse-graining behavior evaluated directly, already addressing the first and fourth predictions. More detailed datasets could enable simultaneous testing of multiple predictions.

In summary, treating tumor growth as hyperdisordered becomes meaningful only when formulated as a set of quantitative claims about how heterogeneity scales with spatial extent and growth geometry. These claims generate explicit mathematical predictions, can be tested with existing data and admit clear falsification criteria.

CONCLUSIONS

We find that gastric cancer histology exhibits architectural variability that is not adequately captured by single-scale descriptors or by assumptions of uniform heterogeneity. We quantified disorder both at the tile level and aggregated across spatial and patient scales, revealing broad, asymmetric distributions of architectural disruption and partial decoupling between disorder components. Correlation analysis demonstrated that texture entropy and microstructural fragmentation tend to covary, while orientation isotropy and multiscale entropy change behave more independently, indicating that no single feature dominates the architectural signal. Patient-level aggregation further showed systematic differences in median disorder alongside persistent within-patient variability and spatial mapping highlighted pronounced nonstationarity within individual slides. Together, these results indicate that architectural disorder in gastric tumors is unevenly distributed across space and scale and that increased aggregation does not necessarily stabilize estimates. The convergence of these observations supports the interpretation of tumor architecture as a multiscale, spatially structured phenomenon rather than a homogeneous or purely random one.

We shift the description of tumor architecture from local heterogeneity toward a regime-based characterization grounded in scaling behavior. Unlike traditional histopathological grading or single-feature image metrics (McAddy et al. 2020; Tollemar et al. 2020; Lisievici et al. 2021), our approach does not aim to classify tissues by predefined patterns or labels. Instead, it quantifies how disorder is organized across scales and how different disorder modes interact. This distinguishes

it from texture-only analyses, which summarize local variability without addressing spatial coupling (Kunimatsu et al. 2022; Mehta et al. 2023; Varghese et al. 2023) and from segmentation-based approaches, which rely on explicit identification of cellular or glandular units (Sun et al. 2022; Ahamed et al. 2023; Sun et al. 2023; Xue, Yao and Teng 2024). Compared with machine learning classifiers, our approach remains interpretable, with each component defined mathematically and linked to a specific aspect of tissue organization (Huang et al. 2018; Antonelli et al. 2019; Alkhathlan and Saudagar 2022; Ghobadi, Emamzadeh and Afsaneh 2022; Ellrott et al. 2025). We treat architectural disorder as a composite, multiscale property whose defining feature is partial decoupling rather than magnitude alone. In this sense, our approach complements rather than replaces existing techniques: it does not compete with molecular profiling, spatial transcriptomics or deep learning–based classification, but provides a quantitative layer that can be integrated with them. By focusing on scaling relations and spatial nonstationarity, we provide a different lens through which architectural complexity can be compared across tumors and datasets, emphasizing structural organization rather than categorical assignment. This perspective situates architectural analysis closer to physical descriptions of disordered systems while remaining grounded in observable histological features, clarifying its position relative to established quantitative pathology methods. Within the landscape of existing methodologies, our approach can be classified as a mesoscopic, scale-aware architectural analysis. It sits between purely local feature extraction and fully global classification schemes and between qualitative histopathology and high-dimensional molecular profiling.

Several limitations must be acknowledged. Our analyses were performed on a subset of available tiles rather than the full dataset, which constrains statistical power and limits generalization. Reported correlations and distributions are descriptive and cohort-specific and scaling predictions remain theoretical within the present data. No normal gastric tissue was included, preventing direct comparison between tumor and non-tumor architectural regimes. Additionally, the hyperdisorder index assigns equal weight to its components, a choice that is operational rather than derived from optimization or biological priors. Spatial coordinates were approximated from tile metadata rather than reconstructed from full whole-slide geometry, which may blur fine spatial relationships. Finally, while mathematical formulations are explicit, some predictions require datasets with richer spatial resolution or longitudinal sampling to be tested rigorously.

Potential applications of our approach arise from its ability to describe how tumor architecture changes with spatial scale and growth. In research settings, it provides a basis for comparing tumors in terms of architectural regimes rather than isolated features, thereby supporting cross-cohort and cross-modality analyses. Because our approach operates directly on spatially resolved data, it is applicable to whole-slide histology, spatial transcriptomics, radiomics and multiregion sequencing datasets, provided spatial coordinates are available. From a diagnostic perspective, our framework provides a means to assess when architectural estimates are stable under aggregation and when they are intrinsically scale-dependent, indicating whether biopsy-based measurements can be reliably generalized to the whole tumor or are dominated by local sampling effects. In clinical contexts, it supports the analysis of sampling strategies by identifying when additional sampling improves reliability and when it fails to do so because architectural variability persists across scales. From a therapeutic standpoint, focusing on scaling behavior allows systematic assessment of how tumor growth geometry and spatial organization influence treatment response and resistance, without requiring assumptions about the underlying mechanisms. Future work may extend our approach to normal tissues, additional cancer types and longitudinal samples and further refine the hyperdisorder index by incorporating alternative weighting schemes or additional modes of disorder.

In conclusion, we asked whether tumor growth can be meaningfully described in terms of a multiscale architectural regime rather than as a random accumulation of local heterogeneity. We found that gastric cancer histology exhibits spatially structured disorder characterized by partial decoupling across scales, persistent nonstationarity and limited stabilization under aggregation. This suggests that tumor architectural complexity is best understood through its scaling behavior rather than isolated local descriptors, enabling quantitative comparison and explicit falsification.

DECLARATIONS

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.

Consent for publication. The Author transfers all copyright ownership, in the event the work is published. The undersigned author warrants that the article is original, does not infringe on any copyright or other proprietary right of any third part, is not under consideration by another journal and has not been previously published.

Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.

Competing interests. The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence their work.

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Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT 5.2 to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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The Apocalypse according to Isaac Newton

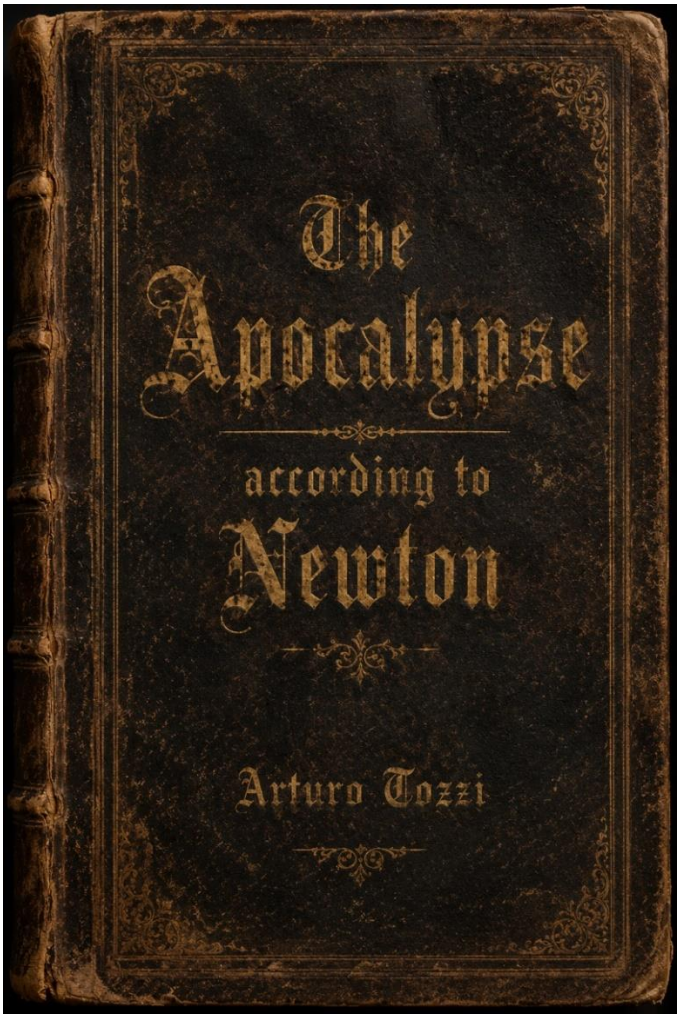
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Isaac Newton, the father of gravitation and one ofthe greatest scientists of all time, devoted a large part of his life to the interpretation of sacred texts. He was sincerely convinced that the Book of Revelation described future events in the history of humanity. Even when one considers the close interweaving of natural philosophy, religion and alchemy that characterized the second half of the seventeenth century, our astonishment as modern readers is by no means diminished. Quite apart from their now anachronistic eschatological and messianic content, Newton's religious writings retain an extraordinary fascination. They allow us to observe, as if firsthand and from within, the workings of a genius's mind as he sought to apply a rigorous logical and scientific method to the study of the Bible. This modest booklet aims to revisit the Book of Revelation according to the guidelines left by Newton. We shall follow as faithfully as possible his chronology of the prophecies, his historical interpretations and his prescriptions concerning their allegorical and symbolic meanings. The original texts by Newton, written over a period of half a century in English and Latin, consist of thousands of pages of scattered notes.

Happy reading.

Naples, on ...
(Since this concerns the Apocalypse, we prefer not to specify an exact date...)

QUOTE AS: Tozzi A. 2026. The Apocalypse according to Isaac Newton. ResearchGate, DOI: 10.13140/RG.2.2.36796.60801



12:3 ... ***And behold, a great red dragon*** (according to Newton, it represents the Devil, but also the unbelievers scattered throughout the world. It furthermore signifies the kingdom that precedes the Beast. From a historical perspective, it symbolizes the great enemy of the Church, namely the Roman Empire in its entirety before its division), ***having seven heads and ten horns, and upon his heads seven crowns*** (the seven crowned heads indicate that the kingdom of the Dragon is distributed across seven chronologically successive parts, each extending from the beginning of one seal to the beginning of the next. The first six heads belong exclusively to the Dragon, whereas the seventh head, as we shall see, belongs jointly to the Dragon and the Beast).
12:4 ***And his tail*** (for Newton, the tail signifies the armies subject to the Dragon) ***drew the third part of the stars of heaven*** (the stars signify a kingdom) ***and did cast them to the earth*** (the Dragon pre-exists the appearance of the Lamb and has already committed many misdeeds, including the Crucifixion of Christ. Indeed, according to Newton, it is upon the kingdom of the Dragon that the violence of the seals will be unleashed).

THE REVELATION

1:1 ***The Revelation of Jesus Christ, which God gave unto him, to show unto his servants things which must shortly come to pass; and he sent and signified it by his angel unto his servant John*** (according to Newton, the prophecies numbered in the original text from 1:1 to 11:18 constitute a reworking of the Jewish Book of the Law. The subsequent text, originally numbered from 11:19 to 22, would instead represent an interpretation of those prophecies, repeating them in a different form. The present time assumed by the prophecies lies between the Fifth and the Sixth Seal, around A.D. 310, with the exception of the vision of the Whore, which takes place during the Sixth Trumpet, around A.D. 1260).
1:2 ***Who bare record of the word of God, and of the testimony of Jesus Christ, and of all things that he saw.***
1:3 ***Blessed is he that readeth, and they that hear the words of this prophecy, and keep those things which are written therein: for the time is at hand!*** (Newton, believing that he had attained an understanding of the apocalyptic prophecies, considered it his mission to make them known to a readership of ordinary yet receptive people capable of understanding the divine message.)
1:9 ***I John, who also am your brother, and companion in tribulation, and in the kingdom and patience of Jesus Christ, was in the isle that is called Patmos*** (Newton agrees with the traditional view that the Book of Revelation was written on Patmos by John the Evangelist himself, around A.D. 95) ***for the word of God, and for the testimony of Jesus Christ.***
1:10 ***I was in the Spirit on the Lord's day, and heard behind me a great voice, as of a trumpet...***
1:12 ***And I turned to see the voice that spake with me. And being turned, I saw seven golden candlesticks*** (according to one of Newton's original insights, the entire vision of the Son of Man and the Throne takes place within the Temple and constitutes an allegorical re-enactment of the ceremonies performed during the Jewish Feast of the Seventh Month. That ceremony foreshadowed things yet to come).
1:13 ***And in the midst of the seven candlesticks one like unto the Son of man, clothed with a garment down to the foot, and girt about the paps with a golden girdle*** (the Son of Man wears the garments of the High Priest who officiates in the ceremonies).
1:14 ***His head and his hairs were white like wool, as white as snow; and his eyes were as a flame of fire.***
1:15 ***And his feet like unto fine brass, as if they burned in a furnace; and his voice as the sound of many waters.***
1:16 ***And he had in his right hand seven stars: and out of his mouth went a sharp two-edged sword: and his countenance was as the sun shineth in his strength.***
1:17 ***And when I saw him, I fell at his feet as dead. And he laid his right hand upon me, saying unto me, Fear not; I am the first and the last.***
1:18 ***I am he that liveth, and was dead*** (slain by the Dragon and risen again. According to Newton, the Apocalypse of John is not concerned with the First Coming of Christ, which it regards as an event of the past, but rather with the future and definitive Second Coming), ***and, behold, I am alive for evermore, Amen; and have the keys of hell and of death.***
1:19 ***Write the things which thou hast seen, and the things which are, and the things which shall be hereafter;***
1:20 ***The mystery of the seven stars which thou sawest in my right hand, and the seven golden candlesticks. The seven stars are the angels of the seven churches: and the seven candlesticks which thou sawest are the seven churches.***

THE THRONE OF GOD

4:1 ***After this I looked, and, behold, a door was opened in heaven*** (the gate of the Temple is opened), ***and the first voice which I heard was as it were of a trumpet talking with me; which said, Come up hither, and I will show thee things which must be hereafter*** (John is summoned from the eastern entrance of the great court of the Temple).
4:2 ***And immediately I was in the Spirit: and, behold, a throne was set in heaven*** (the Ark of the Covenant), ***and one sat on the throne.***
4:3 ***And he that sat was to look upon like a jasper and a sardine stone*** (of an olive-like color, that is, the color of the Jews); ***and there was a rainbow round about the***

throne (the emblem of glory), ***in sight like unto an emerald.***

4:4 ***And round about the throne were four and twenty seats*** (twelve on the southern side and twelve on the northern side of the Court of the Priests); ***and upon the seats I saw four and twenty elders sitting, clothed in white raiment; and they had on their heads crowns of gold*** (the twenty-four Princes of the Priests).

4:5 ***And out of the throne proceeded lightnings*** (the flashes of fire upon the altar of the morning sacrifice), ***and thunderings and voices*** (the resounding voices of those who sounded the trumpets and sang at the eastern gate of the Court of the Priests). ***And there were seven lamps of fire burning before the throne, which are the seven Spirits of God*** (the angels of the seven churches previously described).

4:6 ***And before the throne there was a sea of glass like unto crystal*** (an allusion to the artificial sea placed between the Temple porch and the altar, filled with clear water); ***and in the midst of the throne, and round about the throne, were four living creatures*** (these are four Seraphim, positioned at the four sides of the Court of the People; together they represent the twelve tribes of Israel), ***full of eyes before and behind*** (the eyes represent the multitude of people standing on the four sides of the Court of the People).

4:7 ***And the first living creature was like a lion, and the second living creature like a calf, and the third living creature had a face as a man, and the fourth living creature was like a flying eagle*** (the people of Israel originally encamped around the Tabernacle: on the eastern side stood the three tribes of Judah, represented by the lion; on the western side the three tribes of Ephraim, represented by the calf; on the southern side the three tribes of Reuben, represented by the man; and on the northern side the three tribes of Dan, represented by the eagle).

4:8 ***And the four living creatures had each of them six wings*** (two wings for each tribe, for a total of twenty-four wings, corresponding to the twenty-four stations of the Hebrew tribes), ***and they were full of eyes within: and they rest not day and night*** (the trumpets sounded continuously during both the daytime and nighttime sacrifices), ***saying, Holy, holy, holy, Lord God Almighty, which was, and is, and is to come.***

4:9 ***And when those living creatures give glory and honour and thanks to him that sat on the throne, who liveth for ever and ever,***

4:10 ***The four and twenty elders fall down before him that sat on the throne*** (at each sounding of the trumpets, the priests enter the Temple and prostrate themselves in worship before Him who sits upon the throne), ***and worship him that liveth for ever and ever, and cast their crowns before the throne, saying,***

4:11 ***Thou art worthy, O Lord, to receive glory and honour and power: for thou hast created all things, and for thy pleasure they are and were created.***

5:1 ***And I saw in the right hand*** (the prophetic Book of the Law is placed on the right side of the Ark) ***of him that sat on the throne a book written within and on the backside, sealed with seven seals*** (according to Newton, this is the same book that Daniel sealed in Daniel 12:4. It is written within and without to indicate that the prophecies are synchronous).

5:2 ***And I saw a strong angel proclaiming with a loud voice, Who is worthy to open the book, and to loose the seals thereof?***

5:3 ***And no man in heaven, nor in earth, neither under the earth, was able to open the book, neither to look thereon.***

5:4 ***And I wept much, because no man was found worthy to open and to read the book, neither to look thereon.***

5:5 ***And one of the elders saith unto me, Weep not: behold, the Lion of the tribe of Judah, the Root of David, hath prevailed to open the book, and to loose the seven seals thereof.***

5:6 ***And I beheld, and, lo, in the midst of the throne and of the four living creatures, and in the midst of the elders*** (that is, at the foot of the altar), ***stood a Lamb as it had been slain*** (the High Priest is symbolized by the ritual slaying of the lamb at the foot of the altar during the morning sacrifice), ***having seven horns*** (the seven Churches) ***and seven eyes, which are the seven Spirits of God sent forth into all the earth.***

5:7 ***And he came and took the book*** (this refers to the custom whereby the High Priest, seven days before the fast of the Seventh Month, remained secluded in the Temple studying the Book of the Law and then read it publicly to the people; this seven-day Jewish observance is the practice alluded to in the description of the opening of the seven seals) ***out of the right hand of him that sat upon the throne.***

5:8 ***And when he had taken the book, the four living creatures and four and twenty elders*** (together, the living creatures and the elders represent the early Christians of all nations) ***fell down before the Lamb*** (the primitive Churches prostrate themselves before God, who created all things, and before the Lamb, who redeemed humanity by his blood), ***having every one of them harps, and golden vials full of odours, which are the prayers of saints.***

5:9 ***And they sung a new song, saying, Thou art worthy to take the book, and to open the seals thereof: for thou wast slain, and hast redeemed us to God by thy blood out of every tribe, and tongue, and people, and nation.***

5:10 ***And hast made us unto our God kings and priests: and we shall reign on the earth.***

5:11 ***And I beheld, and I heard the voice of many angels round about the throne and the living creatures and the elders: and the number of them was ten thousand times ten thousand, and thousands of thousands.***

5:12 ***Saying with a loud voice, Worthy is the Lamb that was slain to receive power, and riches, and wisdom, and strength, and honour, and glory, and blessing.***

5:13 ***And every creature which is in heaven, and on the earth, and under the earth, and such as are in the sea, and all that are in them, heard I saying, Blessing, and honour, and glory, and power, be unto him that sitteth upon the throne, and unto the Lamb for ever and ever.***

5:14 ***And the four living creatures said, Amen. And the elders fell down and worshipped.***

THE SEVEN SEALS

(The seals produce signs and afflictions that strike the kingdom of the Dragon. The first four seals cover the period from the time of Saint John to the Tenth Persecution, whereas the last three begin with the reign of Constantine and end with the death of Theodosius.)

The First Seal

A.D. 33 ***(corresponding to the historical period of Pentecost).***

6:1 ***And I saw when the Lamb opened one of the seals, and I heard, as it were the noise of thunder, one of the four living creatures saying, Come and see.***

6:2 ***And I saw, and behold a white horse: and he that sat on him*** (for Newton, the rider signifies a warrior or a sovereign) ***had a bow; and a crown was given unto him: and he went forth conquering, and to conquer*** (the first four riders described in the first four seals are the kings corresponding to the first four heads of the Dragon).

The Second Seal

A.D. 98 ***(Trajan becomes Emperor; during his reign many persecutions of Christians take place).***

6:3 ***And when he had opened the second seal, I heard the second living creature say, Come and see.***

6:4 ***And there went out another horse that was red: and power was given to him that sat thereon to take peace from the earth, and that they should kill one another: and there was given unto him a great sword.***

The Third Seal

A.D. 193 ***(Lucius Septimius Severus becomes Emperor; the persecutions of Christians remain severe).***

6:5 ***And when the Lamb had opened the third seal, I heard the third living creature say, Come and see. And I beheld, and lo a black horse; and he that sat on him had a pair of balances in his hand*** (according to Newton, this is not a symbol of famine, as is commonly maintained; rather, the balance signifies justice).

6:6 ***And I heard a voice in the midst of the four living creatures say, A measure of wheat for a penny, and three measures of barley for a penny; and see thou hurt not the oil and the wine.***

The Fourth Seal

(The first four seals do not concern the affairs of the Church, but rather the wars of the Roman Empire.)

A.D. 249 ***(the brief two-year reign of Gaius Messius Quintus Traianus Decius begins; he seeks to restore the religion of Rome's founding fathers).***

6:7 ***And when he had opened the fourth seal, I heard the voice of the fourth living creature say, Come and see.***

6:8 ***And I looked, and behold a pale horse: and his name that sat on him was Death, and Hell followed with him. And power was given unto them over the fourth part of the earth, to kill with sword, and with hunger, and with death, and with the beasts of the earth.***

11:19 ***And the temple of God was opened in heaven, and there was seen in his temple the ark of his testament*** (this refers to the First Temple).

The Fifth Seal

A.D. 303 ***(the tenth of the great persecutions under Diocletian).***

6:9 ***And when he had opened the fifth seal*** (the narrative now begins to concern the affairs of the Church), ***I saw under the altar the souls of them that were slain for the word of God, and for the testimony which they held*** (the Saints persecuted by the Dragon).

6:10 ***And they cried with a loud voice, saying, How long, O Lord, holy and true, dost thou not judge and avenge our blood on them that dwell on the earth?***

6:11 ***And white robes were given unto every one of them; and it was said unto them, that they should rest yet for a little season, until their fellow servants also and their brethren, that should be killed as they were, should be fulfilled.***

12:1 ***And there appeared a great wonder in heaven; a woman clothed with the sun, and the moon under her feet, and upon her head a crown of twelve stars.***

12:2 ***And she being with child cried, travailing in birth, and pained to be delivered*** (because of the persecution of Diocletian).

12:4 ***And the dragon stood before the woman which was ready to be delivered, for to devour her child as soon as it was born*** (the true Church, equivalent to the saints armed with the blood of the Lamb, undergoes a great persecution at the hands of the degenerate Church, which Newton will later identify with the Harlot).

12:5 ***And she brought forth a man child, who was to rule all nations with a rod of iron: and her child was caught up unto God, and to his throne.***

12:6 ***And the woman fled into the wilderness, where she hath a place prepared of God, that they should feed her there a thousand two hundred and threescore days*** (this is an anticipation of one of the events that will occur during the forty-two months of the Beast's dominion, corresponding to the one thousand two hundred and sixty days, which according to Newton begin around A.D. 609).

THE EPISTLES TO THE CHURCHES

(According to Newton, the Epistles to the Churches refer to the period of the Fifth and Sixth Seals, around A.D. 310, a time of profound decline within the Church.)

1:4 ***John to the seven churches which are in Asia: Grace be unto you, and peace, from him which is, and which was, and which is to come; and from the seven Spirits which are before his throne;***
1:5 ***And from Jesus Christ, who is the faithful witness, and the first begotten of the dead, and the prince of the kings of the earth. Unto him that loved us, and washed us from our sins in his own blood,***
1:6 ***And hath made us kings and priests unto God and his Father; to him be glory and dominion for ever and ever. Amen.***
1:7 ***Behold, he cometh with clouds; and every eye shall see him, and they also which pierced him: and all kindreds of the earth shall wail because of him. Even so, Amen.***
1:8 ***I am Alpha and Omega, the beginning and the ending, saith the Lord, which is, and which was, and which is to come, the Almighty.***
1:10 ***I was in the Spirit on the Lord's day, and heard behind me a great voice, as of a trumpet,***
1:11 ***Saying, What thou seest, write in a book, and send it unto the seven churches which are in Asia; unto Ephesus, and unto Smyrna, and unto Pergamos, and unto Thyatira, and unto Sardis, and unto Philadelphia, and unto Laodicea.***
2:1 ***Unto the angel*** (the seven angels to whom the seven epistles are addressed correspond to the seven priests and principal officers of the Temple in which the vision of the Feast of the Seventh Month takes place) ***of the church of Ephesus write; These things saith he that holdeth the seven stars in his right hand, who walketh in the midst of the seven golden candlesticks;***
2:2 ***I know thy works, and thy labour, and thy patience, and how thou canst not bear them which are evil: and thou hast tried them which say they are apostles, and are not, and hast found them liars:***
2:3 ***And hast borne, and hast patience, and for my name's sake hast laboured, and hast not fainted.***
2:4 ***Nevertheless I have somewhat against thee, because thou hast left thy first love.***
2:5 ***Remember therefore from whence thou art fallen, and repent, and do the first works; or else I will come unto thee quickly, and will remove thy candlestick out of his place, except thou repent.***
2:6 ***But this thou hast, that thou hatest the deeds of the Nicolaitans, which I also hate.***
2:7 ***He that hath an ear, let him hear what the Spirit saith unto the churches. To him that overcometh will I give to eat of the tree of life, which is in the midst of the paradise of God.***

About A.D. 303 *(during the Tenth Persecution under Diocletian).*

2:8 ***And unto the angel of the church in Smyrna write; These things saith the first and the last, which was dead, and is alive;***
2:9 ***I know thy works, and tribulation, and poverty, (but thou art rich) and I know the blasphemy of them which say they are Jews, and are not, but are the synagogue of Satan.***
2:10 ***Fear none of those things which thou shalt suffer: behold, the devil shall cast some of you into prison, that ye may be tried; and ye shall have tribulation ten days: be thou faithful unto death, and I will give thee a crown of life.***
2:11 ***He that hath an ear, let him hear what the Spirit saith unto the churches. He that overcometh shall not be hurt of the second death.***
(The Church of Smyrna, together with that of Philadelphia, is the only one of the seven churches without sin. They are the two churches most severely tested by persecution and correspond, according to Newton, to the two witnesses who will appear later in the prophecy.)
2:12 ***And to the angel of the church in Pergamos write; These things saith he which hath the sharp sword with two edges;***
2:13 ***I know thy works, and where thou dwellest, even where Satan's seat is: and thou holdest fast my name, and hast not denied my faith, even in those days wherein Antipas was my faithful martyr, who was slain among you, where Satan dwelleth.***
2:14 ***But I have a few things against thee, because thou hast there them that hold the doctrine of Balaam, who taught Balac to cast a stumblingblock before the children of Israel, to eat things sacrificed unto idols, and to commit fornication.***
2:15 ***So hast thou also them that hold the doctrine of the Nicolaitans, which thing I hate.***
2:16 ***Repent; or else I will come unto thee quickly, and will fight against them with the sword of my mouth.***
2:17 ***He that hath an ear, let him hear what the Spirit saith unto the churches. To him that overcometh will I give to eat of the hidden manna, and will give him a white stone, and in the stone a new name written, which no man knoweth saving he that receiveth it.***
2:18 ***And unto the angel of the church in Thyatira write; These things saith the Son of God, who hath his eyes like unto a flame of fire, and his feet are like fine brass;***
2:19 ***I know thy works, and charity, and service, and faith, and thy patience, and thy works; and the last to be more than the first.***
2:20 ***Notwithstanding I have a few things against thee, because thou sufferest that woman Jezebel, which calleth herself a prophetess, to teach and to seduce my servants to commit fornication, and to eat things sacrificed unto idols*** (during this period the Woman undergoes a progressive degradation that will eventually lead her to the debased condition of the Harlot. Meanwhile, the Dragon wages war against her offspring who have remained pure).
2:21 ***And I gave her space to repent of her fornication; and she repented not.***
2:22 ***Behold, I will cast her into a bed, and them that commit adultery with her into great tribulation, except they repent of their deeds.***
2:23 ***And I will kill her children with death; and all the churches shall know that I am he which searcheth the***

reins and hearts: and I will give unto every one of you according to your works.
2:24 ***But unto you I say, and unto the rest in Thyatira, as many as have not this doctrine, and which have not known the depths of Satan, as they speak; I will put upon you none other burden.***
2:25 ***But that which ye have already hold fast till I come.***
2:26 ***And he that overcometh, and keepeth my works unto the end, to him will I give power over the nations:***
2:27 ***And he shall rule them with a rod of iron; as the vessels of a potter shall they be broken to shivers: even as I received of my Father.***
2:28 ***And I will give him the morning star.***
2:29 ***He that hath an ear, let him hear what the Spirit saith unto the churches.***

3:1 ***And unto the angel of the church in Sardis write; These things saith he that hath the seven Spirits of God, and the seven stars; I know thy works, that thou hast a name that thou livest, and art dead.***
3:2 ***Be watchful, and strengthen the things which remain, that are ready to die: for I have not found thy works perfect before God.***
3:3 ***Remember therefore how thou hast received and heard, and hold fast, and repent. If therefore thou shalt not watch, I will come on thee as a thief, and thou shalt not know what hour I will come upon thee.***
3:4 ***Thou hast a few names even in Sardis which have not defiled their garments; and they shall walk with me in white: for they are worthy.***
3:5 ***He that overcometh, the same shall be clothed in white raiment; and I will not blot out his name out of the book of life, but I will confess his name before my Father, and before his angels.***
3:6 ***He that hath an ear, let him hear what the Spirit saith unto the churches.***
3:7 ***And to the angel of the church in Philadelphia write*** (the Church of Philadelphia, together with that of Smyrna, is the only one of the seven churches without sin): ***These things saith he that is holy, he that is true, he that hath the key of David, he that openeth, and no man shutteth; and shutteth, and no man openeth;***
3:8 ***I know thy works: behold, I have set before thee an open door, and no man can shut it: for thou hast a little strength, and hast kept my word, and hast not denied my name.***
3:9 ***Behold, I will make them of the synagogue of Satan, which say they are Jews, and are not, but do lie; behold, I will make them to come and worship before thy feet, and to know that I have loved thee.***
3:10 ***Because thou hast kept the word of my patience, I also will keep thee from the hour of temptation, which shall come upon all the world, to try them that dwell upon the earth.***
3:11 ***Behold, I come quickly: hold that fast which thou hast, that no man take thy crown.***
3:12 ***Him that overcometh will I make a pillar in the temple of my God, and he shall go no more out: and I will write upon him the name of my God, and the name of the city of my God, which is new Jerusalem, which cometh down out of heaven from my God: and I will write upon him my new name.***
3:13 ***He that hath an ear, let him hear what the Spirit saith unto the churches.***

About A.D. 365 ***(referring to the period of Valentinian and Valens).***
3:14 ***And unto the angel of the church of the Laodiceans write; These things saith the Amen, the faithful and true witness, the beginning of the creation of God;***
3:15 ***I know thy works, that thou art neither cold nor hot: I would thou wert cold or hot.***
3:16 ***So then because thou art lukewarm, and neither cold nor hot, I will spue thee out of my mouth.***
3:17 ***Because thou sayest, I am rich, and increased with goods, and have need of nothing; and knowest not that thou art wretched, and miserable, and poor, and blind, and naked:***
3:18 ***I counsel thee to buy of me gold tried in the fire, that thou mayest be rich; and white raiment, that thou mayest be clothed, and that the shame of thy nakedness do not appear; and anoint thine eyes with eyesalve, that thou mayest see.***
3:19 ***As many as I love, I rebuke and chasten: be zealous therefore, and repent.***
3:20 ***Behold, I stand at the door, and knock: if any man hear my voice, and open the door, I will come in to him, and will sup with him, and he with me.***
3:21 ***To him that overcometh will I grant to sit with me in my throne, even as I also overcame, and am set down with my Father in his throne.***
3:22 ***He that hath an ear, let him hear what the Spirit saith unto the churches.***

The Sixth Seal

A.D. 312 ***(Constantine's victory over Maxentius and the end of the persecutions. In A.D. 330 Constantine overthrows the idols and inaugurates Constantinople. The period of the Sixth Seal ends in A.D. 379, with the beginning of the reign of the deeply Christian Theodosius, who refused the title of Pontifex Maximus.)***
6:12 ***And I beheld when he had opened the sixth seal*** (according to Newton, the present time contemplated by the prophecy lies between the Fifth and the Sixth Seal, with the exception of the vision of the Harlot, which occurs during the Sixth Trumpet); ***and, lo, there was a great earthquake; and the sun became black as sackcloth of hair*** (the fall of a supreme magistrate), ***and the moon became as blood*** (the extinction of the splendor of a kingdom).
12:7 ***And there was war in heaven: Michael and his angels fought against the dragon*** (a war between the Saints who bear witness to the truth of the Gospel and the Dragon who persecutes them). ***And the dragon fought and his angels,***
12:8 ***And prevailed not; neither was their place found any more in heaven.***

6:13 ***And the stars of heaven fell unto the earth, even as a fig tree casteth her untimely figs, when she is shaken of a mighty wind.***
6:14 ***And the heaven departed as a scroll when it is rolled together*** (indicating the overthrow of the idolatrous temples); ***and every mountain*** (the capital of a kingdom) ***and island were moved out of their places.***
6:15 ***And the kings of the earth, and the great men, and the rich men, and the chief captains, and the mighty men, and every bondman, and every free man, hid themselves in the dens and in the rocks of the mountains*** (indicating the collapse of the idolatrous temples upon their idols).
6:16 ***And said to the mountains and rocks*** (caves and rocks signify, according to Newton, buildings), ***Fall on us, and hide us from the face of him that sitteth on the throne, and from the wrath of the Lamb:***
6:17 ***For the great day of his wrath is come; and who shall be able to stand?***
12:9 ***And the great dragon was cast out, that old serpent, called the Devil, and Satan, which deceiveth the whole world*** (in some of his writings Newton states that this is the moment at which the Beast receives its deadly wound. In most of his other notes, however, he places the infliction of the deadly wound at a later date, namely A.D. 353); ***he was cast out into the earth, and his angels were cast out with him.***
12:10 ***And I heard a loud voice saying in heaven, Now is come salvation, and strength, and the kingdom of our God, and the power of his Christ: for the accuser of our brethren is cast down, which accused them before our God day and night.***
12:11 ***And they overcame him by the blood of the Lamb, and by the word of their testimony; and they loved not their lives unto the death.***
12:12 ***Therefore rejoice, ye heavens, and ye that dwell in them! Woe to the inhabiters of the earth and of the sea! for the devil is come down unto you, having great wrath, because he knoweth that he hath but a short time*** (Newton maintains that only a spiritual kingdom has fallen, while the material one has remained intact. The overthrow of the Dragon's kingdom signifies that he loses religious power, but not temporal power, so that the kingdom bearing his name continues to exist. The impious worship, removed from the throne, is nevertheless preserved among the common people).

ABOUT A.D. 320 *(persecution during the reign of Licinius).*

12:5 ***And she*** (the Woman clothed with the sun) ***brought forth a man child, who was to rule all nations with a rod of iron; and her child was caught up unto God, and to his throne.***

A.D. 324 *(Constantine's victory over Licinius; division of the Roman Empire into two parts, with Rome and Constantinople as their respective capitals).*

12:13 ***And when the dragon saw that he was cast unto the earth, he persecuted the woman which brought forth the man child.***
12:14 ***And to the woman were given two wings of a great eagle*** (the two wings represent the Roman Empire divided into two parts) ***that she might fly into the wilderness, into her place, where she is nourished for a time, and times, and half a time*** (this is an anticipation of one of the events that will occur during the forty-two months of the Beast's dominion, beginning around A.D. 609), ***from the face of the serpent.***

A.D. 337 *(death of Constantine and division of the Roman Empire among his sons).*

12:15 ***And the serpent cast out of his mouth water as a flood after the woman, that he might cause her to be carried away of the flood*** (the Western Roman Empire). ***(The Dragon casts water after the Woman while she is fleeing into the wilderness, before she has yet arrived there: this signifies the rise of the Beast.)***

A.D. 353 *(victory of Constantius over the usurper Magnentius, resulting in the reunification of the Eastern and Western Roman Empires).*

12:16 ***And the earth*** (the Eastern Roman Empire) ***helped the woman, and the earth opened her mouth, and swallowed up the flood*** (it is here that the Dragon receives the deadly wound by the sword) ***which the dragon cast out of his mouth.***
(In his earlier writings, Newton speaks of a struggle between the peoples of the water and the peoples of the earth. The war between these two peoples would be the conflict described later under the various Trumpets.)

A.D. 361 *(the reign of Flavius Claudius Julianus, known as Julian the Apostate, begins; he is the last openly pagan Emperor).*

12:17 ***And the dragon was wroth with the woman and...***

A.D. 364 *(a new division of the Empire between Valentinian, who retains the West, and his brother Valens, who receives the East).*

12:17 ***...went to make war with the remnant of her seed, which keep the commandments of God, and have the testimony of Jesus Christ. And he stood upon the sand of the sea*** (the earth remains under the dominion of the Dragon, who has been dethroned by Michael, while the Beast begins to rule over the sea).

13:1 ***And I stood upon the sand of the sea, and saw a beast rise up out of the sea*** (it rises from the sea, but also from the bottomless pit, that is, from Hades) ***(this is the power that will exercise dominion during the period of the Seven Trumpets and upon which the wrath of God will be concentrated. The temporal reign of the Beast begins. Nevertheless, the kingdom bearing the name of the Dragon continues even after the overthrow of its worship. The Dragon gives its power to the Beast and becomes united with it. The Beast brings with it its infernal religion and thus, in addition to the temporal kingdom represented by the Dragon, an infernal religion also comes into being. The Dragon does not die, but becomes one of the ten horns. From a historical perspective, the ten-horned Beast signifies the nations of the Western Roman Empire, whereas the continuing Dragon signifies the Eastern Roman Empire.)***

...having ten horns (the horns represent ten contemporary kingdoms or dynasties united within the empire constituted by the body of the Beast. The ten kings have voluntarily subjected themselves to the Beast, combining into a single political body which constitutes the Beast itself. The Beast is therefore only one of the kings, yet contains the other nine within itself) ***and seven heads, and upon his horns ten crowns, and upon his heads the name of blasphemy*** (the seven uncrowned heads are stacked one upon another rather than existing in parallel, whereas the ten horns, representing the number of kingdoms and dynasties, are found only upon the last head).

17:3 ***And I saw a woman sit upon a scarlet coloured beast*** (according to Newton, the ten-horned Beast corresponds to the Beast upon which the Harlot will sit. The Beast is, in a special sense, the nation in which the Harlot resides), ***full of names of blasphemy, having seven heads and ten horns*** (the seventh head, unlike the first six, belongs jointly to the Dragon and the Beast; the Beast therefore coincides with the final part of the Dragon, namely the seventh head).

13:2 ***And the beast which I saw was like unto a leopard, and his feet were as the feet of a bear, and his mouth as the mouth of a lion.***

13:3 ***And I saw one of his heads as it were wounded to death; and his deadly wound was healed*** (through the new division of the Empire between Valentinian and Valens); ***and all the world wondered after the beast*** (according to Newton, there are many parallels between the beasts of Revelation and those of the prophecy of Daniel, to such an extent that the two texts should be regarded as interconnected parts of a single prophecy. It is not necessary, however, to examine Newton's symbolism of the Book of Daniel here).

13:2 ***And the dragon gave him his power, and his seat, and great authority*** (the Beast will assist the Dragon in its war against the offspring of the Woman. The subjects of a temporal kingdom embrace the religion of the Beast. All nations worship the Beast: it is a universal kingdom. With the end of the Church's triumph begins a long period of continuous apostasy that will last until the Seventh Trumpet at the end of time).

13:4 ***And they worshipped the dragon which gave power unto the beast: and they worshipped the beast, saying, Who is like unto the beast? who is able to make war with him?***

13:5 ***And there was given unto him a mouth speaking great things and blasphemies; and power was given unto him to continue forty and two months*** (this power begins during the Fifth Trumpet, from about A.D. 609 onward).

13:6 ***And he opened his mouth in blasphemy against God, to blaspheme his name, and his tabernacle, and them that dwell in heaven.***

The Seventh Seal

(In the original text, after the Sixth Seal the narrative is interrupted by the episode of the Sealed Ones, as though to indicate a great new event, a revolutionary episode, the beginning of a new order of things.)

A.D. 379 *(beginning of the reign of Theodosius).*

8:1 ***And when he had opened the seventh seal*** (it is opened on the Jewish Day of Atonement), ***there was silence in heaven about the space of half an hour*** (a brief peace, a suspension of the impending war).

8:3 ***And another angel came and stood at the altar, having a golden censer; and there was given unto him much incense, that he should offer it with the prayers of all saints upon the golden altar which was before the throne*** (on the Day of Atonement the rite of incense takes place, of which this episode of Revelation is a faithful description: the High Priest takes incense from a golden censer upon the great altar. He then offers the incense to God while the people remain silent in prayer; finally, carrying the burning incense in his hand, he transports it into the most holy place of the Temple, before the Ark).

8:4 ***And the smoke of the incense, which came with the prayers of the saints, ascended up before God out of the angel's hand.***

A.D. 379-395 *(during the reign of Theodosius, the Empire was not troubled by wars against foreign armies).*

7:1 ***And after these things I saw four angels standing on the four corners of the earth*** (these are the first four of the seven angels who stood before God at the opening of the Seventh Seal), ***holding the four winds of the earth, that the wind should not blow on the earth, nor on the sea, nor on any tree.***

7:2 ***And I saw another angel ascending from the east, having the seal of the living God: and he cried with a loud voice to the four angels, to whom it was given to hurt the earth and the sea,***

7:3 ***Saying, Hurt not the earth, neither the sea, nor the trees, till we have sealed the servants of our God in their foreheads.***

7:4 ***And I heard the number of them which were sealed: and there were sealed an hundred and forty and four thousand of all the tribes of the children of Israel*** (the Sealed Ones correspond to the Temple of the original Chapter 11 and to the Saints. The passage also recalls the Jewish tradition of sealing at the beginning of each new year: on the Day of Atonement all Israelites are inscribed in the books of life and death. At the beginning of each new year, on the first day of the month of Tishri, three books are opened: the Book of Life, the Book of Death and the Book of Suspended Judgments. On the Day of Atonement, some names are transferred from the Book of Suspended Judgments to the Book of Life):

7:5 ***Of the tribe of Juda were sealed twelve thousand. Of the tribe of Reuben were sealed twelve thousand. Of the tribe of Gad were sealed twelve thousand.***

7:6 ***Of the tribe of Aser were sealed twelve thousand. Of the tribe of Nephthalim were sealed twelve thousand. Of the tribe of Manasses were sealed twelve thousand.***
7:7 ***Of the tribe of Simeon were sealed twelve thousand. Of the tribe of Levi were sealed twelve thousand. Of the tribe of Issachar were sealed twelve thousand.***
7:8 ***Of the tribe of Zabulon were sealed twelve thousand. Of the tribe of Joseph were sealed twelve thousand. Of the tribe of Benjamin were sealed twelve thousand sealed*** (these wise and faithful servants are sealed in order to preserve them from the plagues of the Six Trumpets).
13:9 ***If any man have an ear, let him hear.***
13:10 ***He that leadeth into captivity shall go into captivity: he that killeth with the sword must be killed with the sword. Here is the patience and the faith of the saints*** (the general vision of the Sealed Ones extends until the period between the Vials).

A.D. 395 and after *(upon the death of Theodosius I the Great, the Roman Empire was once again divided between his sons Arcadius and Honorius. The eastern provinces experienced invasions and, in A.D. 396, the Visigothic wars also began. During this period the ecclesiastical hierarchy of the Church of the Homoousion, which emerged after the Ecumenical Council of Nicaea in A.D. 325, continued to develop. This Church upheld the consubstantiality of the Father and the Son. Newton himself held an anti-Trinitarian position, regarded as heretical by the Church of his age; this was one of the reasons why he never dared publish his religious writings.)*

13:11 ***And I beheld another beast*** (the False Prophet. It represents the apostate and degenerate Christian Church, in opposition to the true Church represented by the Woman in the wilderness; according to a later revision by Newton, it also represents the Church of the Eastern Roman Empire), ***coming up out of the earth*** (its emergence from the earth signifies the slowness of its growth: its power at this stage is still limited, but it will gradually increase until reaching its height during the Beast's forty-two months, under the Fifth Trumpet, around A.D. 609), ***and he had two horns*** (the bishops of Alexandria and Antioch) ***like a lamb*** (and therefore it must be a Church), ***and he spake as a dragon.***
(The Harlot, whom Newton identifies with the two-horned Beast, must begin to exist at this point, desolate in the wilderness during the death of her Beast. What has become of the Woman in the wilderness? After the earth ceases to help her, she remains desolate in the wilderness, with only a few descendants remaining. She will emerge from the wilderness at the time of the Sixth Trumpet, after the resurrection of the Two Witnesses and before the final triumph of God. In other writings, Newton instead maintains that the wilderness is also inhabited by the Harlot, abandoned there after her Beast received its deadly wound. In his final years, Newton came to identify the Harlot with the very Woman who had fled into the wilderness. While her few descendants remained within the Second Temple, pure and faithful to the original Church, the Woman herself became corrupted during her stay in the wilderness.)
13:7 ***And it was given unto the beast with ten horns to make war with the saints, and to overcome them, and power was given him over all kindreds, and tongues, and nations.***
13:8 ***And all that dwell upon the earth shall worship him, whose names are not written in the book of life of the Lamb slain from the foundation of the world.***

13:14 ***(Meanwhile, concurrently with the sealing of the Saints, the two-horned Beast proceeds to seal the worshippers of the Beast.) ...and deceiveth them that dwell on the earth by the means of those miracles which he had power to do in the sight of the beast; saying to them that dwell on the earth, that they should make an image to the beast, which had the wound by a sword, and did live*** (the Beast is a political body, an authority derived from the two-horned Beast. It does not signify an invasion by a foreign kingdom, but rather a transformation arising from within).
13:15 ***And he had power to give life unto the image of the beast*** (this refers to the custom of deifying kings after their death), ***that the image of the beast should both speak, and cause that as many as would not worship the image of the beast should be killed.***
13:16 ***And he causeth all, both small and great, rich and poor, free and bond, to receive a mark in their right hand, or in their foreheads.***
13:17 ***And that no man might buy or sell, save he that had the mark, or the name of the beast, or the number of his name.***
13:18 ***Here is wisdom. Let him that hath understanding count the number of the beast: for it is the number of a man; and his number is Six hundred threescore and six.***
8:2 ***And I saw the seven angels which stood before God; and to them were given seven trumpets*** (the seven angels take up seven trumpets corresponding to the trumpets of the great sacrifices sounded during the seven days of the Feast of Tabernacles).
8:6 ***And the seven angels which had the seven trumpets prepared themselves to sound*** (the sound of the Temple music, the trumpets and the songs of the Levites).
15:1 ***And I saw another sign in heaven, great and marvellous, seven angels having the seven last plagues; for in them is filled up the wrath of God.***
15:5 ***And after that I looked, and, behold, the temple of the tabernacle of the testimony in heaven was opened*** (this is the Second Temple, in contrast to the First Temple described in 11:19).
15:6 ***And the seven angels came out of the temple, having the seven plagues, clothed in pure and white linen, and having their breasts girded with golden girdles.***
15:7 ***And one of the four living creatures gave unto the seven angels seven golden vials full of the wrath of God, who liveth for ever and ever.***
15:8 ***And the temple was filled with smoke from the glory of God, and from his power; and no man was able to enter into the temple, till the seven plagues of the seven angels were fulfilled.***

THE SEVEN TRUMPETS AND THE SEVEN VIALS OF WRATH

(According to Newton, the Trumpets and the Vials of Wrath describe the same events in different language and are therefore synchronous. The joint kingdom of the Dragon and the Beast, namely the seventh head, is the kingdom upon which the plagues are inflicted. The Trumpets therefore sound during the period of the seventh head of the Dragon-Beast. Likewise, the Vials occupy almost the entire duration of the seventh head. According to Newton, the total duration of the Seven Trumpets extends over more than a thousand years without interruption.)

8:5 ***And the angel took the censer, and filled it with fire of the altar, and cast it into the earth: and there were voices, and thunderings, and lightnings, and an earthquake*** (for Newton, the earthquake represents a war or a great upheaval. Furthermore, continuing the analogy with Jewish ritual practices, during the sacrifices, until the nightly ceremony has been completed, the voice of the High Priest may be heard reading the Book of the Law to the people, while the trumpets, the music of the Temple and the flashes from the fire of the Altar are also heard and seen).

16:1 ***And I heard a great voice out of the temple saying to the seven angels, Go your ways, and pour out*** (indicating the act of pouring out the drink offerings during the sacrifices of the Feast of Tabernacles) ***the vials of the wrath of God upon the earth.***

The First Trumpet and the First Vial

(Trumpets One through Four are called by Newton the Trumpets of the Wounds, or of the Winds.)

A.D. 395 **8:7** ***The first angel sounded*** (the calamities that follow the sounding of the Trumpets all arise from war), ***and there followed hail and fire mingled with blood, and they were cast upon the earth*** (according to Newton, the effects of the first four Trumpets are produced by the four angels of the winds who had previously been restrained. The First Trumpet represents the plague of the wind coming from the East). ***And the third part of trees was burnt up, and all green grass was burnt up*** (this describes the invasion of the Eastern Empire by Goths, Sarmatians and Huns).

16:2 ***And the first went, and poured out his vial upon the earth; and there fell a noisome and grievous sore*** (bloodshed, but not on an excessive scale: merely an irritating ulcer) ***upon the men which had the mark of the beast, and upon them which worshipped his image.***

The Second Trumpet and the Second Vial

A.D. 408 ***(invasion of Gaul and Spain; Alemannic wars. Alaric and his Visigoths lay siege to Rome for the first time. The Second Trumpet therefore represents the plague of the wind coming from the West.)***

8:8 ***And the second angel sounded, and as it were a great mountain burning with fire was cast into the sea*** (the great mountain signifies Rome, which was conquered by the Ostrogoths). ***And the third part of the sea became blood*** (the waters signify peoples, and their transformation into blood describes a great slaughter. Historically, the episode refers to the invasion of the nations of the Western Empire by Visigoths, Vandals, Ostrogoths and others).

8:9 ***And the third part of the creatures which were in the sea, and had life, died; and the third part of the ships were destroyed*** (greater bloodshed than under the First Trumpet; the overthrow of certain kings).

16:3 ***And the second angel poured out his vial upon the sea; and it became as the blood of a dead man: and every living soul died in the sea*** (during the early stages of the war between the peoples of the earth and the peoples of the waters, the earth suffers defeat, but ultimately prevails. According to Newton, this also provides the most plausible explanation of why the Dragon casts water upon the Woman, yet the earth protects her).

The Third Trumpet and the Third Vial

A.D. 429 ***(Genseric, king of the Vandals, invades Africa. The Third Trumpet therefore represents the plague of the wind coming from the South.)***

8:10 ***And the third angel sounded, and there fell a great star from heaven*** (the fall of a great prince: the Western Empire is divided into many kingdoms), ***burning as it were a lamp, and it fell upon the third part of the rivers, and upon the fountains of waters.***

8:11 ***And the name of the star is called Wormwood: and the third part of the waters became wormwood; and many men died of the waters*** (until the fall of the Vandals at the hands of Belisarius in A.D. 534, approximately five million Africans perished), ***because they were made bitter*** (we witness greater bloodshed than under the first two Trumpets, in a crescendo of increasing violence).

16:4 ***And the third angel poured out his vial upon the rivers and fountains of waters; and they became blood.***

16:5 ***And I heard the angel of the waters say, Thou art righteous, O Lord, which art, and wast, and shalt be, because thou hast judged thus.***

16:6 ***For they have shed the blood of saints and prophets, and thou hast given them blood to drink; for they are worthy.***

16:7 ***And I heard another out of the altar say, Even so, Lord God Almighty, true and righteous are thy judgments.***

The Fourth Trumpet and the Fourth Vial

A.D. 535 *(the Gothic Wars between the Byzantine Empire and the Ostrogoths begin. The war is brought into Italy by Belisarius. The Fourth Trumpet therefore represents the plague of the wind coming from the North.)*

8:12 ***And the fourth angel sounded*** (this trumpet lasts seventy years), ***and the third part of the sun was smitten, and the third part of the moon, and the third part of the stars*** (the overthrow of a kingdom and its extreme ruin, at least for a time. Historically, the Western Roman Empire is invaded by Belisarius): ***so as the third part of them was darkened, and the day shone not for a third part of it, and the night likewise*** (Rome is repeatedly captured by various armies. According to some of Newton's notes, it is on this occasion that the Beast receives its deadly wound from the four angels of the winds. According to other notes, however, as we have already seen, the wound was inflicted upon the Beast on an earlier occasion).

16:8 ***And the fourth angel poured out his vial upon the sun; and power was given unto him to scorch men with fire.***

16:9 ***And men were scorched with great heat, and blasphemed the name of God, which hath power over these plagues: and they repented not to give him glory*** (the first four Trumpets describe the great sacrifices performed in the Temple during the first four days of the Feast of Tabernacles).

8:13 ***And I beheld, and heard an eagle flying through the midst of heaven, saying with a loud voice, Woe, woe, woe, to the inhabiters of the earth by reason of the other voices of the trumpet of the three angels, which are yet to sound!***

9:12 ***One woe is past; and, behold, there come two woes more hereafter.***

The Fifth Trumpet and the Fifth

(Trumpets Five through Seven are called by Newton the Trumpets of Woes.)

About A.D. 600 **13:12** *And he exerciseth all the power of the first beast before him, and causeth the earth and them which dwell therein to worship the first beast, whose deadly wound was healed* (the Beast from the sea has recovered from its wound, but remains weak; its kingdom is in crisis, and the two-horned Beast takes advantage of its difficulties in order to bring it under its control).

13:13 ***And he doeth great wonders, so that he maketh fire come down from heaven on the earth in the sight of men*** (having grown in power and cunning since its emergence during the Sixth Seal, the two-horned Beast becomes the mighty Great Harlot, capable of subjugating the ten-horned Beast and seating herself upon it. It should be remembered that the Harlot also sits upon the waters, the same waters that issued from the mouth of the Dragon. From this point onward begin the forty-two months during which God permits the Beast to wage war).

13:14 ***And deceiveth them that dwell on the earth by the means of those miracles which he had power to do in the sight of the beast, saying to them that dwell on the earth, that they should make an image to the beast, which had the wound by a sword, and did live*** (the Beast from the sea represents a political body, an authority derived from the two-horned Beast; it is not a foreign invading kingdom, but rather an internal subversion within the kingdom itself).

13:15 ***And he had power to give life unto the image of the beast, that the image of the beast should both speak, and cause that as many as would not worship the image of the beast should be killed.***

17:1 ***...the great whore that sitteth upon many waters*** (according to Newton's later writings, the Woman of Heaven has become corrupted and transformed into the Harlot).

17:18 ***And the woman which thou sawest is that great city, which reigneth over the kings of the earth*** (this is one of the events that occurs during the Beast's forty-two months, that is, 1,260 days. The period does not indicate the Beast's total duration, but rather the time during which it is permitted to wage war. For Newton, each prophetic month corresponds to one year. The 1,260 years represent the duration of the apostasy of the Trinitarian Catholic Church).

17:15 ***The waters which thou sawest, where the whore sitteth, are peoples, and multitudes, and nations, and tongues.***

17:3 ***So he carried me away in the spirit into the wilderness: and I saw a woman sit upon a scarlet coloured beast, full of names of blasphemy, having seven heads and ten horns*** (from the standpoint of temporal dominion, she is one of the Beast's horns; from the standpoint of spiritual dominion, she rides it in the form of a woman and constitutes its Church).

17:8 ***The beast that thou sawest was, and is not; and shall ascend out of the bottomless pit*** (the rise of heresies or false religions) ***and go into perdition: and they that dwell on the earth shall wonder, whose names were not written in the book of life from the foundation of the world, when they behold the beast that was, and is not, and yet is.***

The seven heads are seven mountains (the seven hills of Rome) ***on which the woman sitteth. They are also seven kings:***

17:10 ***Five are fallen, and one is, and the other is not yet come; and when he cometh, he must continue a short space.***

17:11 ***And the beast that was, and is not, even he is the eighth, and is of the seven, and goeth into perdition.***

17:12 ***And the ten horns which thou sawest are ten kings, which have received no kingdom as yet; but receive power as kings one hour with the beast.***

17:13 ***These have one mind, and shall give their power and strength unto the beast.***
17:4 ***And the woman was arrayed in purple and scarlet colour, and decked with gold and precious stones and pearls, having a golden cup in her hand full of abominations and filthiness of her fornication.***
17:2 ***With whom the kings of the earth have committed fornication, and the inhabitants of the earth have been made drunk with the wine of her fornication.***
17:5 ***And upon her forehead was a name written, MYSTERY, BABYLON THE GREAT, THE MOTHER OF HARLOTS AND ABOMINATIONS OF THE EARTH.***
17:6 ***And I saw the woman drunken with the blood of the saints, and with the blood of the martyrs of Jesus. And when I saw her, I wondered with great admiration.***

A.D. 609 *(wars between the Saracen Empire and the Romans. During this period Muhammad begins his monotheistic preaching).*

9:1 ***And the fifth angel sounded*** (a new phase of events begins after the interconnected First through Fourth Trumpets. This is the time of the greatest desolation, the climax of apostasy), ***and I saw a star fall from heaven unto the earth*** (the Prophet Muhammad); ***and to him was given the key of the bottomless pit*** (the pit remains open until the sounding of the Seventh Trumpet).
9:2 ***And he opened the bottomless pit*** (the appearance of a false religion); ***and there arose a smoke out of the pit*** (signifying the dense and dark multitude of those who embrace the false religion), ***as the smoke of a great furnace; and the sun*** (a supreme magistrate) ***and the air were darkened by reason of the smoke of the pit.***
9:3 ***And there came out of the smoke locusts upon the earth*** (numerous armies); ***and unto them was given power, as the scorpions of the earth have power.***
9:4 ***And it was commanded them that they should not hurt the grass of the earth, neither any green thing, neither any tree; but only those men which have not the seal of God in their foreheads.***
9:5 ***And to them it was given that they should not kill them, but that they should be tormented five months*** (according to Newton, this corresponds to 300 actual years, ending in A.D. 936 with the decline of the Saracen Caliphate), ***and their torment was as the torment of a scorpion, when he striketh a man.***
9:6 ***And in those days shall men seek death, and shall not find it; and shall desire to die, and death shall flee from them.***
9:7 ***And the shapes of the locusts were like unto horses prepared unto battle; and on their heads were as it were crowns like gold, and their faces were as the faces of men.***
9:8 ***And they had hair as the hair of women, and their teeth were as the teeth of lions.***
9:9 ***And they had breastplates, as it were breastplates of iron; and the sound of their wings was as the sound of chariots of many horses running to battle.***
9:10 ***And they had tails like unto scorpions, and there were stings in their tails: and their power was to hurt men five months.***
9:11 ***And they had a king over them, which is the angel of the bottomless pit, whose name in the Hebrew tongue is Abaddon, but in the Greek tongue hath his name Apollyon*** (indicating one who exercises authority over both religious and civil affairs, namely the Caliph of the Saracens).

16:10 ***And the fifth angel poured out his vial upon the seat of the beast*** (Trumpets Five through Seven bring progressively greater destruction, forming a dramatic climax). ***And his kingdom was full of darkness; and they gnawed their tongues for pain,***
16:11 ***And blasphemed the God of heaven because of their pains and their sores, and repented not of their deeds.***
17:3 ***And one of the seven angels which had the seven vials carried me away in the spirit into the wilderness; and I saw a woman sit upon a beast...***
17:7 ***And the angel said unto me, Wherefore didst thou marvel? I will tell thee the mystery of the woman, and of the beast that carrieth her, which hath the seven heads and ten horns.***
17:9 ***And here is the mind which hath wisdom.***
11:1 ***And there was given me a reed like unto a rod: and the angel stood, saying, Rise, and measure the temple of God, and the altar, and them that worship therein*** (the measuring of the Temple signifies that it will first be destroyed and then rebuilt. The reconstruction, however, concerns only the inner part, namely the most sincere believers, the 144,000 pure ones who have found refuge there. The external forms of religion and the government of the Church will not be restored).
11:2 ***But the court which is without the temple leave out, and measure it not; for it is given unto the Gentiles: and the holy city shall they tread under foot forty and two months*** (this is another of the events that occur during the forty-two months of the Beast's dominion).
11:3 ***And I will give power unto my two witnesses*** (for Newton, these signify the 144,000 sealed ones, but also the Old and New Testaments, the two Churches of Smyrna and Philadelphia, and the descendants of the original pure Church), ***and they shall prophesy a thousand two hundred and threescore days, clothed in sackcloth*** (this is another of the events that occur during the forty-two months of the Beast's dominion).
11:4 ***These are the two olive trees, and the two candlesticks*** (that is, the Seven Churches of Asia) ***standing before the God of the earth.***
11:5 ***And if any man will hurt them, fire proceedeth out of their mouth, and devoureth their enemies: and if any man will hurt them, he must in this manner be killed.***
11:6 ***These have power to shut heaven, that it rain not in the days of their prophecy: and have power over waters to turn them to blood, and to smite the earth with all plagues, as often as they will*** (the Witnesses are able to bring about the plagues described in Trumpets and Vials One through Six, before the Seventh Trumpet sounds on the Day of Judgment).

The Sixth Trumpet and the Sixth Vial

A.D. 1258 *(the end of the Abbasid Caliphate and of the Sultanates of the Euphrates).*

9:13 ***And the sixth angel sounded, and I heard a voice from the four horns of the golden altar which is before God.***

9:14 ***Saying to the sixth angel which had the trumpet, Loose the four angels which are bound in the great river Euphrates*** (the four Turkish kingdoms on the Euphrates, namely Armenia, Mesopotamia, Syria and Cappadocia).

9:15 ***And the four angels were loosed, which were prepared for an hour, and a day, and a month, and a year*** (according to Newton, this period corresponds to 391 years, beginning with the first Turkish conquests on the Euphrates in A.D. 1063 and ending with the capture of Constantinople in A.D. 1453), ***for to slay the third part of men.***

9:16 ***And the number of the army of the horsemen were two hundred thousand thousand: and I heard the number of them.***

9:17 ***And thus I saw the horses in the vision, and them that sat on them, having breastplates of fire, and of jacinth, and brimstone: and the heads of the horses were as the heads of lions; and out of their mouths issued fire and smoke and brimstone.***

A.D. 1453 *(the fall of Constantinople, besieged by the Ottomans, accompanied by enormous massacres within the Eastern Empire).*

9:18 ***By these three was the third part of men killed, by the fire, and by the smoke, and by the brimstone, which issued out of their mouths.***

9:19 ***For their power is in their mouth, and in their tails: for their tails were like unto serpents, and had heads, and with them they do hurt.***

9:20 ***And the rest of the men which were not killed by these plagues yet repented not of the works of their hands, that they should not worship devils, and idols of gold, and silver, and brass, and stone, and of wood: which neither can see, nor hear, nor walk.***

9:21 ***Neither repented they of their murders, nor of their sorceries, nor of their fornication, nor of their thefts.***

16:12 ***And the sixth angel poured out his vial upon the great river Euphrates; and the water thereof was dried up, that the way of the kings of the east might be prepared.***

16:13 ***And I saw three unclean spirits like frogs come out of the mouth of the dragon, and out of the mouth of the beast, and out of the mouth of the false prophet.***

16:14 ***For they are the spirits of devils, working miracles, which go forth unto the kings of the earth and of the whole world, to gather them to the battle of that great day of God Almighty.***

16:15 ***(Behold, I come as a thief. Blessed is he that watcheth, and keepeth his garments, lest he walk naked, and they see his shame.)***

16:16 ***And he gathered them together into a place called in the Hebrew tongue Armageddon.***

11:7 ***And when they*** (the Two Witnesses) ***shall have finished their testimony, the beast that ascendeth out of the bottomless pit shall make war against them, and shall overcome them*** (a political overthrow), ***and kill them*** (according to Newton, this constitutes the only interruption in the continuous history of the Church).

11:8 ***And their dead bodies shall lie in the street of the great city, which spiritually is called Sodom and Egypt, where also our Lord was crucified.***

11:9 ***And they of the people and kindreds and tongues and nations shall see their dead bodies three days and an half, and shall not suffer their dead bodies to be put in graves.***

11:10 ***And they that dwell upon the earth shall rejoice over them, and make merry, and shall send gifts one to another; because these two prophets tormented them that dwelt on the earth.***

14:1 ***And I looked, and, lo, a Lamb stood on mount Sion, and with him an hundred forty and four thousand, having his Father's name written in their foreheads.***

14:2 ***And I heard a voice from heaven, as the voice of many waters, and as the voice of a great thunder: and I heard the voice of harpers harping with their harps.***

14:3 ***And they sung as it were a new song before the throne, and before the four living creatures, and the elders: and no man could learn that song but the hundred and forty and four thousand, which were redeemed from the earth.***

14:4 ***These are they which were not defiled with women; for they are virgins. These are they which follow the Lamb whithersoever he goeth. These were redeemed from among men, being the firstfruits unto God and to the Lamb.***

14:5 ***And in their mouth was found no guile: for they are without fault before the throne of God.***

11:14 ***The second woe is past; and, behold, the third woe cometh quickly*** (the end of the Beast's forty-two months. This ending also corresponds to the death of the Witnesses. Here likewise concludes the period of the 144,000 sealed ones, which is followed by the period of the great multitude bearing palms, a period that will itself conclude at the time of the Final Judgment).

17:1 ***And there came one of the seven angels which had the seven vials, and talked with me, saying, Come hither; I will shew unto thee the judgment of the great whore that sitteth upon many waters...*** (John's vision of the Harlot is synchronous with this Trumpet. It is the only exception to the chronology of events within the vision as previously described.)

17:16 ***And the ten horns which thou sawest upon the beast, these shall hate the whore, and shall make her desolate and naked, and shall eat her flesh, and burn her with fire.***

17:17 ***For God hath put in their hearts to fulfil his will, and to agree, and give their kingdom unto the beast, until the words of God shall be fulfilled.***

11:11 ***And after three days and an half the Spirit of life from God entered into them*** (the Two Witnesses); ***and they stood upon their feet; and great fear fell upon them which saw them.***

11:12 ***And they heard a great voice from heaven saying unto them, Come up hither. And they ascended up to heaven in a cloud; and their enemies beheld them*** (the

anti-Christian kingdom begins to be shaken. The enemies of the Church are alarmed by the return of the Woman from the wilderness).

14:6 ***And I saw another angel fly in the midst of heaven, having the everlasting gospel to preach unto them that dwell on the earth, and to every nation, and kindred, and tongue, and people.***

14:7 ***Saying with a loud voice, Fear God, and give glory to him; for the hour of his judgment is come: and worship him that made heaven, and earth, and the sea, and the fountains of waters.***

7:9 ***After this I beheld, and, lo, a great multitude, which no man could number, of all nations, and kindreds, and people, and tongues, stood before the throne, and before the Lamb, clothed with white robes, and palms in their hands*** (this corresponds both to the great multitude of the Alleluia and to the armies that follow the Lamb in Chapter 19. It is the ceremony of the seventh and final day of the Feast of Tabernacles; on that occasion the Jews carry palm branches while crying Hosanna).

7:10 ***And cried with a loud voice, saying, Salvation to our God which sitteth upon the throne, and unto the Lamb.***

7:11 ***And all the angels stood round about the throne, and about the elders and the four living creatures, and fell before the throne on their faces, and worshipped God,***

7:12 ***Saying, Amen: Blessing, and glory, and wisdom, and thanksgiving, and honour, and power, and might, be unto our God for ever and ever. Amen.***

7:13 ***And one of the elders answered, saying unto me, What are these which are arrayed in white robes? and whence came they?***

7:14 ***And I said unto him, Sir, thou knowest. And he said to me, These are they which came out of great tribulation, and have washed their robes, and made them white in the blood of the Lamb.***

7:15 ***Therefore are they before the throne of God, and serve him day and night in his temple: and he that sitteth on the throne shall dwell among them.***

7:16 ***They shall hunger no more, neither thirst any more; neither shall the sun*** (a supreme magistrate) ***light on them, nor any heat.***

7:17 ***For the Lamb which is in the midst of the throne shall feed them, and shall lead them unto living fountains of waters: and God shall wipe away all tears from their eyes.***

11:13 ***And the same hour was there a great earthquake, and the tenth part of the city fell, and in the earthquake were slain seven thousand men: and the remnant were affrighted, and gave glory to the God of heaven.***

14:8 ***And there followed another angel, saying, Babylon is fallen, is fallen, that great city, because she made all nations drink of the wine of the wrath of her fornication.***

14:9 ***And the third angel followed them, saying with a loud voice, If any man worship the beast and his image, and receive his mark in his forehead, or in his hand,***

14:10 ***The same shall drink of the wine of the wrath of God, which is poured out without mixture into the cup of his indignation; and he shall be tormented with fire and brimstone in the presence of the holy angels, and in the presence of the Lamb.***

14:11 ***And the smoke of their torment ascendeth up for ever and ever: and they have no rest day nor night, who worship the beast and his image, and whosoever receiveth the mark of his name.***

14:12 ***Here is the patience of the saints: here are they that keep the commandments of God, and the faith of Jesus*** (despite the crisis, the religion of the Beast continues to spread, and the persecutions of the Saints are far from over).

14:13 ***And I heard a voice from heaven saying unto me, Write, Blessed are the dead which die in the Lord from henceforth: Yea, saith the Spirit, that they may rest from their labours; and their works do follow them.***

10:1 ***And I saw another mighty angel come down from heaven, clothed with a cloud: and a rainbow was upon his head, and his face was as it were the sun, and his feet as pillars of fire*** (this is the Lamb).

10:2 ***And he had in his hand a little book open*** (the little book that the Lamb has just opened), ***and he set his right foot upon the sea, and his left foot on the earth*** (thus does the High Priest stand on the Day of Atonement; the passage alludes to the sacrifices performed during the seven days of the Feast of Tabernacles).

10:3 ***And cried with a loud voice, as when a lion roareth: and when he had cried, seven thunders uttered their voices.***

10:4 ***And when the seven thunders had uttered their voices, I was about to write: and I heard a voice from heaven saying unto me, Seal up those things which the seven thunders uttered, and write them not.***

10:5 ***And the angel which I saw stand upon the sea and upon the earth lifted up his hand to heaven,***

10:6 ***And sware by him that liveth for ever and ever, who created heaven, and the things that therein are, and the earth, and the things that therein are, and the sea, and the things which are therein, that there should be time no longer:***

10:7 ***But in the days of the voice of the seventh angel, when he shall begin to sound, the mystery of God should be finished, as he hath declared to his servants the prophets.***

10:8 ***And the voice which I heard from heaven spake unto me again, and said, Go and take the little book which is open in the hand of the angel which standeth upon the sea and upon the earth.***

10:9 ***And I went unto the angel, and said unto him, Give me the little book. And he said unto me, Take it, and eat it up; and it shall make thy belly bitter, but it shall be in thy mouth sweet as honey.***

10:10 ***And I took the little book out of the angel's hand, and ate it up; and it was in my mouth sweet as honey: and as soon as I had eaten it, my belly was bitter*** (the little book, sweet at first and then bitter, signifies that its opening pages speak of pleasant events, namely the victory of Michael and the reign of the Church. Its later pages, however, describe grievous events, namely the period of apostasy).

10:11 ***And he said unto me, Thou must prophesy again before many peoples, and nations, and tongues, and kings.***

The Seventh Trumpet and the Seventh Vial

AFTER A.D. 2060? *(In a note not intended for publication, Newton speculates that the end of the 1,260 days and the Second Coming of Christ will occur no earlier than A.D. 2060 and no later than A.D. 2374. It should be emphasized, however, that Newton himself warns against placing excessive confidence in such predictions, since their failure brings discredit upon the Sacred Prophecies.)*

11:15 *And the seventh angel sounded; and there were great voices in heaven, saying, The kingdoms of this world are become the kingdoms of our Lord, and of his Christ; and he shall reign for ever and ever* (the continuous apostasy that began with the Seventh Seal comes to an end).

11:16 *And the four and twenty elders, which sat before God on their seats, fell upon their faces, and worshipped God,*

11:17 *Saying, We give thee thanks, O Lord God Almighty, which art, and wast, because thou hast taken to thee thy great power, and hast reigned* (the kingdoms of this world become the kingdoms of the Lord).

11:18 *And the nations were angry, and thy wrath is come, and the time of the dead, that they should be judged, and that thou shouldest give reward unto thy servants the prophets, and to the saints, and them that fear thy name, small and great; and shouldest destroy them which destroy the earth.*

16:17 *And the seventh angel poured out his vial into the air; and there came a great voice out of the temple of heaven, from the throne, saying, It is done.*

19:11 *And I saw heaven opened, and behold a white horse; and he that sat upon him was called Faithful and True, and in righteousness he doth judge and make war.*

19:12 *His eyes were as a flame of fire, and on his head were many crowns; and he had a name written, that no man knew, but he himself.*

19:13 *And he was clothed with a vesture dipped in blood: and his name is called The Word of God.*

19:14 *And the armies which were in heaven followed him upon white horses, clothed in fine linen, white and clean.*

19:15 *And out of his mouth goeth a sharp sword, that with it he should smite the nations: and he shall rule them with a rod of iron: and he treadeth the winepress of the fierceness and wrath of Almighty God.*

19:16 *And he hath on his vesture and on his thigh a name written, KING OF KINGS, AND LORD OF LORDS.*

19:17 *And I saw an angel standing in the sun; and he cried with a loud voice, saying to all the birds that fly in the midst of heaven, Come and gather yourselves together unto the supper of the great God;*

19:18 *That ye may eat the flesh of kings, and the flesh of captains, and the flesh of mighty men, and the flesh of horses, and of them that sit on them, and the flesh of all men, both free and bond, both small and great.*

19:19 *And I saw the beast, and the kings of the earth, and their armies, gathered together to make war against him that sat on the horse, and against his army.*

17:14 *These shall make war with the Lamb, and the Lamb shall overcome them: for he is Lord of lords, and King of kings: and they that are with him are called, and chosen, and faithful.*

11:19 *And the temple of God was opened in heaven, and there was seen in his temple the ark of his covenant: and there were lightnings, and voices, and thunderings, and an earthquake, and great hail.*

16:18 *And there were voices, and thunders, and lightnings; and there was a great earthquake, such as was not since men were upon the earth, so mighty an earthquake, and so great.*

16:19 *And the great city was divided into three parts, and the cities of the nations fell: and great Babylon came in remembrance before God, to give unto her the cup of the wine of the fierceness of his wrath.*

18:21 *And a mighty angel took up a stone like a great millstone, and cast it into the sea, saying, Thus with violence shall that great city Babylon be thrown down, and shall be found no more at all.*

16:20 *And every island fled away, and the mountains were not found.*

16:21 *And there fell upon men a great hail out of heaven, every stone about the weight of a talent: and men blasphemed God because of the plague of the hail; for the plague thereof was exceeding great.*

14:14 *And I looked, and behold a white cloud, and upon the cloud one sat like unto the Son of man, having on his head a golden crown, and in his hand a sharp sickle.*

14:15 *And another angel came out of the temple, crying with a loud voice to him that sat on the cloud, Thrust in thy sickle, and reap: for the time is come for thee to reap; for the harvest of the earth is ripe.*

14:16 *And he that sat on the cloud thrust in his sickle on the earth; and the earth was reaped.*

14:17 *And another angel came out of the temple which is in heaven, he also having a sharp sickle.*

14:18 *And another angel came out from the altar, which had power over fire; and cried with a loud cry to him that had the sharp sickle, saying, Thrust in thy sharp sickle, and gather the clusters of the vine of the earth; for her grapes are fully ripe.*

14:19 *And the angel thrust in his sickle into the earth, and gathered the vine of the earth, and cast it into the great winepress of the wrath of God* (signifying the destruction of the wicked).

14:20 *And the winepress was trodden without the city, and blood came out of the winepress, even unto the horse bridles, by the space of a thousand and six hundred furlongs.*

19:20 *...And the beast was taken, and with him the false prophet that wrought miracles before him, with which he deceived them that had received the mark of the beast, and them that worshipped his image. These both were cast alive into a lake of fire burning with brimstone.*

19:21 *And the remnant were slain with the sword of him that sat upon the horse, which sword proceeded out of his mouth: and all the birds were filled with their flesh* (it should be noted that the Beast and the False Prophet receive a different punishment from that inflicted upon the kings who were their allies. This indicates that the kings practiced a religion different from that of the Beast and the False Prophet).

18:1 *And after these things I saw another angel come down from heaven, having great power; and the earth was lightened with his glory.*
18:2 *And he cried mightily with a strong voice, saying, Babylon the great is fallen, is fallen, and is become the habitation of devils, and the hold of every foul spirit, and a cage of every unclean and hateful bird.*
18:3 *For all nations have drunk of the wine of the wrath of her fornication, and the kings of the earth have committed fornication with her, and the merchants of the earth are waxed rich through the abundance of her delicacies.*
18:4 *And I heard another voice from heaven, saying, Come out of her, my people, that ye be not partakers of her sins, and that ye receive not of her plagues.*
18:5 *For her sins have reached unto heaven, and God hath remembered her iniquities.*
18:6 *Reward her even as she rewarded you, and double unto her double according to her works: in the cup which she hath filled fill to her double.*
18:7 *How much she hath glorified herself, and lived deliciously, so much torment and sorrow give her: for she saith in her heart, I sit a queen, and am no widow, and shall see no sorrow.*
18:8 *Therefore shall her plagues come in one day, death, and mourning, and famine; and she shall be utterly burned with fire: for strong is the Lord God who judgeth her.*
18:9 *And the kings of the earth, who have committed fornication and lived deliciously with her, shall bewail her, and lament for her, when they shall see the smoke of her burning.*
18:10 *Standing afar off for the fear of her torment, saying, Alas, alas, that great city Babylon, that mighty city! for in one hour is thy judgment come.*
18:11 *And the merchants of the earth shall weep and mourn over her; for no man buyeth their merchandise any more:*
18:12 *The merchandise of gold, and silver, and precious stones, and of pearls, and fine linen, and purple, and silk, and scarlet, and all thyine wood, and all manner vessels of ivory, and all manner vessels of most precious wood, and of brass, and iron, and marble,*
18:13 *And cinnamon, and odours, and ointments, and frankincense, and wine, and oil, and fine flour, and wheat, and beasts, and sheep, and horses, and chariots, and slaves, and souls of men.*
18:14 *And the fruits that thy soul lusted after are departed from thee, and all things which were dainty and goodly are departed from thee, and thou shalt find them no more at all.*
18:15 *The merchants of these things, which were made rich by her, shall stand afar off for the fear of her torment, weeping and wailing,*
18:16 *And saying, Alas, alas, that great city, that was clothed in fine linen, and purple, and scarlet, and decked with gold, and precious stones, and pearls! For in one hour so great riches is come to nought.*
18:17 *And every shipmaster, and all the company in ships, and sailors, and as many as trade by sea, stood afar off,*
18:18 *And cried when they saw the smoke of her burning, saying, What city is like unto this great city?*
18:19 *And they cast dust on their heads, and cried, weeping and wailing, saying, Alas, alas, that great city, wherein were made rich all that had ships in the sea by reason of her costliness! for in one hour is she made desolate.*
18:20 *Rejoice over her, thou heaven, and ye holy apostles and prophets; for God hath avenged you on her.*
18:22 *And the voice of harpers, and musicians, and of pipers, and trumpeters, shall be heard no more at all in thee; and no craftsman, of whatsoever craft he be, shall be found any more in thee; and the sound of a millstone shall be heard no more at all in thee.*
18:23 *And the light of a candle shall shine no more at all in thee; and the voice of the bridegroom and of the bride shall be heard no more at all in thee: for thy merchants were the great men of the earth; for by thy sorceries were all nations deceived.*
18:24 *And in her was found the blood of prophets, and of saints, and of all that were slain upon the earth.*

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20:11 *And I saw a great white throne, and him that sat on it, from whose face the earth and the heaven fled away; and there was found no place for them.*
20:12 *And I saw the dead, small and great, stand before God; and the books were opened: and another book was opened, which is the Book of Life: and the dead were judged out of those things which were written in the books, according to their works.*
20:13 *And the sea gave up the dead which were in it; and death and Hades delivered up the dead which were in them: and they were judged every man according to their works.*
20:14 *And death and Hades were cast into the lake of fire. This is the second death, namely the lake of fire.*
20:15 *And whosoever was not found written in the Book of Life was cast into the lake of fire.*
19:1 *And after these things I heard a great voice of much people in heaven, saying, Alleluia; Salvation, and glory, and honour, and power, unto the Lord our God:*
19:2 *For true and righteous are his judgments: for he hath judged the great whore, which did corrupt the earth with her fornication, and hath avenged the blood of his servants at her hand.*
19:3 *And again they said, Alleluia. And her smoke rose up for ever and ever.*
19:4 *And the four and twenty elders and the four living creatures fell down and worshipped God that sat on the throne, saying, Amen; Alleluia.*
19:5 *And a voice came out of the throne, saying, Praise our God, all ye his servants, and ye that fear him, both small and great.*
19:6 *And I heard as it were the voice of a great multitude, and as the voice of many waters, and as the voice of mighty thunderings, saying, Alleluia: for the Lord God Omnipotent reigneth.*
19:7 *Let us be glad and rejoice, and give honour to him: for the marriage of the Lamb is come, and his wife hath made herself ready.*

19:8 ***And to her was granted that she should be arrayed in fine linen, clean and white: for the fine linen is the righteous deeds of the saints.***
19:9 ***And he saith unto me, Write, Blessed are they which are called unto the marriage supper of the Lamb. And he said unto me, These are the true sayings of God.***
19:10 ***And I fell at his feet to worship him. And he said unto me, See thou do it not: I am thy fellow servant, and of thy brethren that have the testimony of Jesus: worship God: for the testimony of Jesus is the spirit of prophecy.***
(According to Newton, after A.D. 2060 there will be the end of an old age and the beginning of a new one. This new period corresponds to what the Jews call the Messianic Age and Christians call the Kingdom of God.)
15:2 ***And I saw as it were a sea of glass mingled with fire: and them that had gotten the victory over the beast, and over his image, and over the number of his name, stand on the sea of glass, having the harps of God.***
15:3 ***And they sing the song of Moses the servant of God, and the song of the Lamb, saying, Great and marvellous are thy works, Lord God Almighty; just and true are thy ways, thou King of the nations.***
15:4 ***Who shall not fear thee, O Lord, and glorify thy name? For thou only art holy: for all nations shall come and worship before thee; for thy judgments are made manifest.***

21:1 ***And I saw a new heaven and a new earth: for the first heaven and the first earth were passed away; and there was no more sea.***
21:2 ***And I John saw the holy city, new Jerusalem, coming down from God out of heaven, prepared as a bride adorned for her husband.***
21:3 ***And I heard a great voice out of heaven saying, Behold, the tabernacle of God is with men, and he will dwell with them, and they shall be his people, and God himself shall be with them, and be their God.***
21:4 ***And God shall wipe away all tears from their eyes; and there shall be no more death, neither sorrow, nor crying, neither shall there be any more pain: for the former things are passed away.***
21:5 ***And he that sat upon the throne said, Behold, I make all things new. And he said unto me, Write: for these words are faithful and true.***
21:6 ***And he said unto me, It is done. I am Alpha and Omega, the beginning and the end. I will give unto him that is athirst of the fountain of the water of life freely.***
21:7 ***He that overcometh shall inherit all things; and I will be his God, and he shall be my son.***
21:8 ***But the fearful, and unbelieving, and the abominable, and murderers, and whoremongers, and sorcerers, and idolaters, and all liars, shall have their part in the lake which burneth with fire and brimstone: which is the second death.***
21:9 ***And there came unto me one of the seven angels which had the seven vials full of the seven last plagues, and talked with me, saying, Come hither, I will shew thee the bride, the Lamb's wife.***
21:10 ***And he carried me away in the spirit to a great and high mountain*** (the mountain here represents Jerusalem itself), ***and shewed me that great city, the holy Jerusalem, descending out of heaven from God,***
21:11 ***Having the glory of God: and her light was like unto a stone most precious, even like a jasper stone, clear as crystal.***
21:12 ***And had a wall great and high, and had twelve gates, and at the gates twelve angels, and names written thereon, which are the names of the twelve tribes of the children of Israel.***
21:13 ***On the east three gates; on the north three gates; on the south three gates; and on the west three gates.***
21:14 ***And the wall of the city had twelve foundations, and in them the names of the twelve apostles of the Lamb.***
21:15 ***And he that talked with me had a golden reed to measure the city, and the gates thereof, and the wall thereof.***
21:16 ***And the city lieth foursquare, and the length is as large as the breadth: and he measured the city with the reed, twelve thousand furlongs. The length and the breadth and the height of it are equal.***
21:17 ***And he measured the wall thereof, an hundred and forty and four cubits, according to the measure of a man, that is, of the angel.***
21:18 ***And the building of the wall of it was of jasper: and the city was pure gold, like unto clear glass.***
21:19 ***And the foundations of the wall of the city were garnished with all manner of precious stones. The first foundation was jasper; the second sapphire; the third chalcedony; the fourth emerald;***
21:20 ***The fifth sardonyx; the sixth sardius; the seventh chrysolite; the eighth beryl; the ninth topaz; the tenth chrysoprasus; the eleventh jacinth; the twelfth amethyst.***
21:21 ***And the twelve gates were twelve pearls; every several gate was of one pearl: and the street of the city was pure gold, as it were transparent glass.***
21:22 ***And I saw no temple therein: for the Lord God Almighty and the Lamb are the temple of it.***
21:23 ***And the city had no need of the sun, neither of the moon, to shine in it: for the glory of God did lighten it, and the Lamb is the light thereof.***
21:24 ***And the nations of them which are saved shall walk in the light of it: and the kings of the earth do bring their glory and honour into it.***
21:25 ***And the gates of it shall not be shut at all by day: for there shall be no night there.***
21:26 ***And they shall bring the glory and honour of the nations into it.***
21:27 ***And there shall in no wise enter into it any thing that defileth, neither whatsoever worketh abomination, or maketh a lie: but they which are written in the Lamb's Book of Life.***
(Newton gives a highly optimistic interpretation of the final chapters of Revelation: after the wars and destructions at the end of time, the prophecies of peace found in the books of Isaiah and Micah will at last be fulfilled.)

22:1 ***And he shewed me a pure river of water of life*** (in this context, the water symbolizes doctrine and grace), ***clear as crystal, proceeding out of the throne of God and of the Lamb*** (the spring of water symbolizes eternal life).
22:2 ***In the midst of the street of it, and on either side of the river, was there the tree of life, which bare twelve manner of fruits, and yielded her fruit every month: and the leaves of the tree were for the healing of the nations.***

22:3 ***And there shall be no more curse: but the throne of God and of the Lamb shall be in it; and his servants shall serve him.***
22:4 ***And they shall see his face; and his name shall be in their foreheads.***
22:5 ***And there shall be no night there; and they need no candle, neither light of the sun; for the Lord God giveth them light: and they shall reign for ever and ever.***
22:6 ***And he said unto me, These sayings are faithful and true: and the Lord God of the spirits of the prophets hath sent his angel to shew unto his servants the things which must shortly be done.***
22:7 ***Behold, I come quickly: blessed is he that keepeth the sayings of the prophecy of this book.***
22:8 ***And I John saw these things, and heard them. And when I had heard and seen, I fell down to worship before the feet of the angel which shewed me these things.***
22:9 ***Then saith he unto me, See thou do it not: for I am thy fellow servant, and of thy brethren the prophets, and of them which keep the sayings of this book: worship God.***
22:10 ***And he saith unto me, Seal not the sayings of the prophecy of this book: for the time is at hand.***
22:11 ***He that is unjust, let him be unjust still: and he which is filthy, let him be filthy still: and he that is righteous, let him be righteous still: and he that is holy, let him be holy still.***
22:12 ***And, behold, I come quickly; and my reward is with me, to give every man according as his work shall be.***
22:13 ***I am Alpha and Omega, the beginning and the end, the first and the last.***
22:14 ***Blessed are they that wash their robes, that they may have right to the tree of life, and may enter in through the gates into the city.***
22:15 ***For without are dogs, and sorcerers, and whoremongers, and murderers, and idolaters, and whosoever loveth and maketh a lie.***
22:16 ***I Jesus have sent mine angel to testify unto you these things in the churches. I am the root and the offspring of David, and the bright and morning star.***
22:17 ***And the Spirit and the bride say, Come. And let him that heareth say, Come. And let him that is athirst come. And whosoever will, let him take the water of life freely.***
22:18 ***For I testify unto every man that heareth the words of the prophecy of this book, If any man shall add unto these things, God shall add unto him the plagues that are written in this book.***
22:19 ***And if any man shall take away from the words of the book of this prophecy, God shall take away his part from the tree of life, and out of the holy city, and from the things which are written in this book*** (Newton maintained, with great conviction and little modesty, that his own interpretation of Revelation was unquestionably the correct one. He explicitly states that his detractors are comparable to those who dare to add to or subtract from the prophecy).
22:20 ***He which testifieth these things saith, Surely I come quickly. Amen. Even so, come, Lord Jesus.***
22:21 ***The grace of our Lord Jesus Christ be with you all. Amen.***

REFERENCES

The original texts, written over a period of half a century in English and Latin, consist of thousands of pages of scattered notes. Largely unpublished, they are available to interested readers through The Newton Project, at: The Newton Project. In preparing this booklet, we consulted the following works:

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Fiber Bundle Learning: A Topological Framework for Classification Using Homology and Discrete Connections

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Abstract

Many machine-learning tasks involve structured data whose geometry, local feature distributions, and global organization interact in ways that are not well captured by existing methods based on vectorization, graph metrics, or homological signatures. We introduce Fiber Bundle Learning (FBL), a topological framework that represents each data sample as a discrete fiber bundle and extracts a classification signature combining persistent homology, local feature geometry, and gluing structure. FBL builds a base space from the coarse geometry of each object, models local feature patches as fibers, and estimates transition maps between neighboring fibers to construct a discrete connection. From this representation, FBL computes a set of invariants: persistent homology of the base, fibers, and total space; holonomy obtained by transporting fiber states along cycles; curvature-like quantities measuring transition inconsistency; and discrete analogues of characteristic classes. These components are assembled into a fixed-length feature vector that can be used with any standard classifier. We show that FBL yields a signature with three desirable theoretical properties: stability under perturbations of geometry and local features, invariance under isometries and global fiber reparameterizations, and robustness to sampling noise. Our synthetic experiments show that FBL distinguishes twisted from untwisted bundles with identical homology, a distinction classical topological methods fail to capture. Additional tests quantify the system's resistance to noise, its invariance to geometric transformations, and the contribution of each signature component. Taken together, our results indicate that representing data through fiber bundle structure may provide an effective tool for classifying complex, multi-level objects.

Keywords: topological machine learning; fiber bundles; holonomy; geometric learning; invariants; classification algorithms

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1. Introduction

Several areas of machine learning and applied topology have sought to represent structured data by exploiting geometric, combinatorial, or topological properties. Three major traditions provide the necessary context. Topological data analysis extracts global shape information through persistent homology and related vectorizations but treats data as a single undifferentiated space and therefore cannot distinguish objects that share identical homology yet differ in how local feature distributions vary over the underlying geometry [1–7]. Geometric and manifold-based approaches capture local relationships through graph, spectral, or diffusion operators but do not explicitly represent how feature spaces change along cycles or how local patches are glued together [8–13]. Gauge-inspired constructions introduce parallel transport and equivariant operations, yet typically enforce

symmetry constraints without computing homological invariants, characteristic-class-like quantities, or explicit measures of connection inconsistency [14–17]. Each of these traditions captures a meaningful aspect of structure (global topology, neighborhood geometry, or symmetry), but none integrates these components into a unified, stable representation capable of detecting differences in internal gluing structure.

To address these limitations, we introduce Fiber Bundle Learning (FBL), a topological framework that models a structured dataset as a discrete fiber bundle and extracts from it a set of invariants suitable for machine-learning tasks [18–21]. Each object is viewed as a base space capturing its coarse geometry, with a fiber attached to every point representing the local feature distribution. These fibers are interpreted as local trivializations of feature space over the base, while transition maps between neighboring fibers are estimated and combined to form a discrete parallel transport structure. This construction enables the computation of holonomy along cycles, curvature-like quantities that measure transition inconsistency, and discrete analogues of characteristic classes. Together with the persistent homology of the base, fibers, and total space, these descriptors define a fixed-length signature that can be used with any classifier.

Our goal is not to optimize predictive performance on specific application domains but to assess whether bundle-level organizational differences, such as twisting or path-dependent transport, can be detected in a structurally explicit way. Synthetic experiments illustrate the qualitative differences captured by the fiber bundle representation: structures with identical homology but different twisting patterns become distinguishable, while isometric or reparameterized versions of the same object yield nearly identical signatures. This controlled setting allows individual structural degrees of freedom to be varied independently, avoiding confounding factors that typically obscure these effects in real-world datasets. We show that the resulting signature exhibits stability under small perturbations of geometry and features, invariance to global reparameterizations and base isometries, and robustness to sampling noise.

Our main contribution is representational rather than architectural. By combining base topology, fiberwise feature organization, and discrete transport structure into a single signature, FBL provides a framework for detecting organizational differences invisible to persistent homology and not reducible to local feature statistics or adjacency structure alone. In this sense, FBL complements existing topological and geometric learning approaches by making bundle-level structure an explicit object of analysis.

We will proceed as follows: the methodology defines the discrete fiber bundle construction and signature; theoretical guarantees establish stability and invariance; experiments examine discriminability, robustness, and component contributions in controlled synthetic settings; the final discussion integrates these findings while outlining directions for extension to real-world data.

2. Methods

We describe here how data samples are represented as discrete fiber bundles, how topological and bundle-theoretic invariants are computed from them, and how these invariants are assembled into a feature representation used in a standard learning pipeline.

2.1. Data Representation and Notation

The starting point is the representation of each structured sample as a finite set of points endowed with geometric coordinates and local features. Formally, a sample is written as

$$\mathcal{X} = \{(x_i, f_i)\}_{i=1}^{N}, \tag{1}$$

where $x_i \in \mathbb{R}^d$ encodes geometric coordinates and $f_i \in \mathbb{R}^p$ encodes an associated feature vector. From this point cloud we construct a discrete approximation of a base space B, a collection of fibers $\{F_i\}$ and a total space E.

The base space is modeled as a geometric graph $B = (V, E)$ with vertex set $V = \{1, \ldots, N\}$ and edge set determined by a proximity rule. Vertices i and j are connected if $\| x_i - x_j \|$ is below a fixed radius or if j belongs to the k-nearest neighbors of i. We denote the adjacency matrix of B by A, where $A_{ij} = 1$ if an edge exists and $A_{ij} = 0$ otherwise.

For each vertex $i \in V$, the associated fiber F_i is defined as a finite subset of feature space,

$$F_i := \{ f_j \mid j \in \mathcal{N}(i) \}, \tag{2}$$

where $\mathcal{N}(i)$ denotes a neighborhood of i in the base graph. This construction defines a local trivialization of feature space over each base vertex, with no assumption of global triviality across the graph. Each fiber is therefore treated as a finite point cloud in $\mathbb{R}^p$ representing local feature variability around the base point i.

From a formal perspective, the assignment $i \mapsto F_i$ can be viewed as defining a discrete bundle or, equivalently, a presheaf over the base graph, whose local sections are given by the empirical feature clouds. This interpretation motivates the use of bundle-theoretic terminology while remaining fully compatible with the discrete and data-driven nature of our construction.

The total space is defined as the set of pairs

$$\boldsymbol{E} := \{ (\boldsymbol{x_i}, \boldsymbol{f_j}) \mid \boldsymbol{j} \in \boldsymbol{\mathcal{N}}(\boldsymbol{i}) \}, \tag{3}$$

which encodes the joint geometric–feature structure of the sample. All subsequent constructions (persistent homology of base, fibers, and total space; estimation of transition maps; and computation of holonomy and curvature-like quantities) are applied to these discrete objects.

2.2. Base Space Construction and Homology Computation

The construction of the base space proceeds by thickening the graph into a simplicial complex and computing persistent homology on the resulting filtration. Given the point cloud $\{\boldsymbol{x_i}\}$, we define a Vietoris–Rips complex $\boldsymbol{R}(\boldsymbol{\varepsilon})$ at scale $\boldsymbol{\varepsilon}$ as the abstract simplicial complex whose vertices correspond to indices $\boldsymbol{i}$ and where a finite set $\{\boldsymbol{i_0}, \ldots, \boldsymbol{i_k}\}$ forms a simplex if $\| \boldsymbol{x_{i_a}} - \boldsymbol{x_{i_b}} \| \leq \boldsymbol{\varepsilon}$ for all $\boldsymbol{a}, \boldsymbol{b}$ [22–25]. The filtration $\{\boldsymbol{R}(\boldsymbol{\varepsilon})\}_{\boldsymbol{\varepsilon} \geq 0}$ is obtained by increasing $\boldsymbol{\varepsilon}$.

For each homological degree $\boldsymbol{q}$, persistent homology computes a multiset of intervals $[\boldsymbol{b_\ell}, \boldsymbol{d_\ell})$ representing the birth and death times of homology classes in the filtration. We denote the resulting persistence barcode associated with the base space by $\boldsymbol{\mathcal{B}_{\mathbf{base}}}$. To achieve a numerical representation suitable for learning, a vectorization map $\boldsymbol{\mathcal{V}}$ is applied to the barcode. Typical choices include persistence landscapes or persistence images.

A persistence landscape $\boldsymbol{\lambda_q}$ is defined from the intervals $[\boldsymbol{b_\ell}, \boldsymbol{d_\ell})$ by setting

$$\boldsymbol{\lambda_q}(\boldsymbol{t}) = \max_{\boldsymbol{\ell}} \min(\boldsymbol{t} - \boldsymbol{b_\ell}, \boldsymbol{d_\ell} - \boldsymbol{t})_+, \tag{4}$$

where $(\cdot)+$ denotes the positive part. A finite-dimensional approximation is obtained by sampling $\boldsymbol{\lambda_q}$ on a fixed grid and stacking the resulting values into a vector. Throughout, we assume that the chosen vectorization map is Lipschitz-continuous with respect to the bottleneck or Wasserstein distance on persistence diagrams, as is standard in topological data analysis.

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Vietoris–Rips complexes and persistent homology computations are implemented using Python libraries such as GUDHI or Ripser, while nearest-neighbor graphs are constructed using scikit-learn and NetworkX. All parameters controlling the base-space construction, including neighborhood size and filtration resolution, are held fixed across experiments to ensure comparability of the resulting descriptors.

2.3. Fiber Construction and Fiber Homology Aggregation

The modeling of local feature structure is based on computing persistent homology on each fiber patch and aggregating these local descriptors into a global representation. For each base vertex $i \in V$, the fiber F_i is treated as a finite point cloud in feature space $\mathbb{R}^p$.

For each fiber F_i, we construct a Vietoris–Rips complex $R_i(\varepsilon)$ using the Euclidean distance in feature space and define the associated filtration by increasing ε. Persistent homology is then computed for each homological degree q, yielding a barcode $\mathcal{B}^{(i)}_{\mathbf{fiber}}$ associated with the local feature distribution around base vertex i.

As for the base space, each fiber barcode is mapped to a finite-dimensional vector using a fixed vectorization map $\mathcal{V}$, such as a persistence landscape or persistence image. The same vectorization scheme and resolution are used for all fibers to ensure that resulting descriptors are directly comparable across vertices and samples. Denoting the vectorized barcode of fiber i by $v_i = \mathcal{V}\left(\mathcal{B}^{(i)}_{\mathbf{fiber}}\right)$, we obtain a collection of local descriptors $\{v_i\}_{i \in V}$.

To obtain a global fiber-level representation, these local descriptors are aggregated using simple summary statistics. Specifically, we compute the empirical mean

$$\mu_F = \frac{1}{|V|} \sum_{i \in V} v_i \tag{5}$$

and covariance matrix

$$\Sigma_F = \frac{1}{|V|} \sum_{i \in V} (v_i - \mu_F)(v_i - \mu_F)^\top. \tag{6}$$

These statistics summarize the distribution of local feature topology across the base without imposing any ordering or alignment between fibers. To obtain a fixed-length vector representation, either the diagonal entries of Σ_F or its upper triangular part are retained and concatenated with μ_F, yielding the aggregated fiber descriptor h_F.

This aggregation step plays a purely descriptive role: it captures variability in local feature topology while remaining insensitive to the spatial arrangement of fibers. Spatial organization is instead encoded by the base space and the transition maps introduced below.

Local neighborhoods $\mathcal{N}(i)$ are extracted using nearest-neighbor queries implemented via kd-tree methods from scikit-learn. Persistent homology on fiber point clouds is computed using the same software stack as for the base space, ensuring consistent numerical behavior across components.

2.4. Total Space Embedding and Persistent Homology

The construction of the total space descriptor incorporates interactions between geometry and features by embedding the data into a joint space and computing persistent homology on that combined representation. Each pair (x_i, f_i) is mapped to a concatenated vector

$$z_i = (x_i, \alpha f_i) \in \mathbb{R}^{d+p}, \tag{7}$$

where $\alpha > 0$ is a scaling parameter that balances the relative influence of geometric and feature components in the joint metric.

The set $\{z_i\}_{i=1}^N$ is used to define a Vietoris–Rips complex $R_E(\varepsilon)$ with respect to the Euclidean distance in the joint space. As in the previous constructions, a filtration $\{R_E(\varepsilon)\}_{\varepsilon \geq 0}$

is obtained by increasing ε, and persistent homology is computed for each homological degree q. We denote the resulting barcode associated with the total space by $\mathcal{B}_{\text{total}}$.

The total-space barcode is vectorized using the same class of maps $\mathcal{V}$ employed for the base and fiber barcodes, yielding a finite-dimensional descriptor h_E. Using a consistent vectorization framework across base, fiber, and total space ensures that differences among these components reflect structural distinctions rather than representational artifacts.

The scaling parameter α controls the relative contribution of geometry and features in the joint embedding. Rather than fixing a universal value, α is treated as a tunable parameter that allows the representation to interpolate between geometry-dominated and feature-dominated regimes. In practice, α is selected through cross-validation within the learning pipeline, and its value is held constant across all samples in each experiment. Sensitivity analyses indicate that the qualitative behavior of the resulting signatures is stable across a broad range of α, with extreme values primarily suppressing either geometric or feature contributions rather than inducing spurious structure.

The total-space homology captures interactions between geometric arrangement and feature variation that are not accessible when these components are analyzed separately. It reflects how feature similarity and spatial proximity jointly shape higher-order connectivity patterns in the data.

From an implementation standpoint, the computation of total-space persistent homology follows the same procedure as for the base space but is applied to a higher-dimensional point cloud. The same software tools and numerical settings are used to ensure consistency and reproducibility.

2.5. Discrete Connection and Transition Map Estimation

The estimation of a discrete connection over the base graph provides the transition maps that relate fibers at neighboring vertices. For each edge $(i, j) \in E$, we seek a linear map $T_{ij} : \mathbb{R}^p \to \mathbb{R}^p$ that approximates how local feature configurations transform from the fiber F_i to the fiber F_j.

Let $F_i = \left\{f_{i_1}, \ldots, f_{i_m}\right\}$ and $F_j = \left\{f_{j_1}, \ldots, f_{j_n}\right\}$ denote the feature point clouds associated with neighboring vertices i and j. To construct paired samples, points in F_i are matched to points in F_j using nearest-neighbor search or, alternatively, an optimal transport plan between the empirical measures supported on the two fibers. In the simplest formulation, matching indices are obtained by nearest-neighbor correspondence, yielding paired samples $\left(f_{i_k}, f_{j_{\pi(k)}}\right)$.

Given these correspondences, the transition map T_{ij} is estimated by solving a regularized least-squares problem,

$$T_{ij} = \arg\min_{T} \sum_{k} \| \, T f_{i_k} - f_{j_{\pi(k)}} \, \|^2 + \lambda \, \| \, T \, \|_F^2, \tag{8}$$

where $\lambda > 0$ is a regularization parameter and $\| \cdot \|_F$ denotes the Frobenius norm. This yields a closed-form solution obtained by standard linear algebra operations.

The transition maps T_{ij} play the role of discrete gluing maps between local trivializations of the feature space. From a geometric perspective, they can be interpreted as discrete parallel transport operators approximating how fiber coordinates are compared across neighboring base points. When vertices i and j are close in the underlying geometry, the corresponding fibers sample nearby regions of feature space, and the least-squares estimator identifies the linear map that best aligns these local configurations.

This construction provides a finite-difference analogue of a smooth connection. In differential geometry, parallel transport along an infinitesimal path is determined by a connection one-form; here, the continuous path is replaced by an edge of the base graph,

and parallel transport is approximated by a finite linear transformation estimated directly from data. No smooth limit is assumed, but the essential operational role of a connection, i.e., enabling path-dependent comparison of fiber elements, is preserved.

When optimal transport is used, the least-squares objective is weighted by the transport plan, yielding

$$T_{ij} = \mathbf{arg\,min}_{T} \sum_{k,\ell} \pi_{k\ell} \parallel T f_{i_k} - f_{j_\ell} \parallel^2 + \lambda \parallel T \parallel_F^2, \tag{9}$$

where $\pi_{k\ell}$ denotes the optimal transport coupling. This variant improves robustness to non-uniform sampling and partial overlap between fibers, while retaining the same geometric interpretation.

Transition maps are computed for all oriented edges (i, j) in the base graph and stored as T_{ij}. Together, the collection of maps $\{T_{ij}\}$ defines a discrete connection on the graph, enabling the computation of path-dependent transport effects like the holonomy and curvature-like quantities introduced below.

2.6. Holonomy, Curvature, Characteristic Class Proxies, and Signature Assembly

The discretized connection induces holonomy and curvature-like quantities on cycles and triangles of the base graph, which are summarized into numerical descriptors capturing the global organization of the fibers.

Given a simple cycle $\gamma = (i_1, i_2, \ldots, i_m, i_{m+1} = i_1)$ in the base graph, the associated holonomy operator is defined as the ordered product of transition maps along the cycle,

$$H_\gamma = T_{i_m i_{m+1}} \cdots T_{i_2 i_3} T_{i_1 i_2}. \tag{10}$$

This operator quantifies the deviation from identity arising from the composition of local parallel transports along a closed path, providing a discrete analogue of holonomy in differential geometry. To compress the matrix H_γ into scalar descriptors, we compute invariants such as the Frobenius norm distance from the identity $\parallel H_\gamma - I \parallel_F$, the spectral radius, or the argument of the determinant.

A cycle basis of the base graph is computed using standard graph algorithms, and holonomy descriptors are evaluated on each basis cycle. The resulting set of scalar values is summarized using empirical statistics (mean, variance, and histograms), yielding the holonomy descriptor vector $h_{\mathbf{hol}}$.

Curvature-like quantities are defined on oriented triangles (i, j, k) in the flag complex of the base graph as

$$C_{ijk} = T_{ki} T_{jk} T_{ij}, \tag{11}$$

which measures the inconsistency of transition maps around minimal loops. In the absence of curvature, the product reduces to the identity; deviations quantify local obstruction to flatness of the discrete connection. Scalar summaries such as $\parallel C_{ijk} - I \parallel_F$ are computed and aggregated across all triangles to form the curvature descriptor $h_{\mathbf{curv}}$.

To capture higher-order global obstructions, we define discrete proxies for characteristic classes based on cocycle constructions. Specifically, for triples of overlapping neighborhoods, we evaluate quantities of the form

$$\chi_{ijk} = \mathrm{sign}\Big(\det\Big(T_{ij}\, T_{jk}\, T_{ki}\Big)\Big) \tag{12}$$

where $\mathbf{sign}(\cdot)$ denotes the sign of the determinant and takes values in $\{-1, +1\}$. These parity-like indicators detect orientation-reversing transport around loops and serve as

discrete analogues of characteristic-class information. The resulting values are aggregated by counting nontrivial signs or computing empirical moments, yielding the descriptor $h_{\mathbf{CC}}$.

All scalar descriptors derived from holonomy, curvature, and cocycle constructions are invariant under global reparameterizations of the fiber coordinates and depend only on the organization of transition maps across the base graph. They therefore provide explicit, interpretable measures of bundle-level structure.

The final Fiber Bundle Learning signature for a sample is obtained by concatenating all components,

$$\mathbf{\Phi} = [\, h_B, \mid h_F, \mid h_E, \mid h_{\mathbf{hol}}, \mid h_{\mathbf{curv}}, \mid h_{\mathbf{CC}} \,], \tag{13}$$

yielding a fixed-length feature vector suitable for standard machine-learning algorithms. This assembly step does not involve learning or optimization; all components are computed deterministically from the data and the chosen construction parameters.

2.7. Learning Pipeline and Software Tools

The learning stage treats the Fiber Bundle Learning signature as a fixed-dimensional feature vector for supervised classification. Given a dataset of samples $\{\boldsymbol{\mathcal{X}}^{(n)}\}$ with associated labels $\{\boldsymbol{y}^{(n)}\}$, the signature $\mathbf{\Phi}^{(n)}$ is computed for each sample using the same construction parameters, yielding a feature matrix $\boldsymbol{Z}$ whose rows correspond to individual samples.

A standard support vector machine (SVM) classifier is then trained on $\boldsymbol{Z}$. In the experiments reported here, we use either a linear SVM or a kernel SVM with a radial basis function kernel

$$\boldsymbol{K}(z, z') = \mathbf{exp}\Big(-\gamma \parallel z - z' \parallel^2\Big),$$

and solve the corresponding optimization problem using off-the-shelf implementations. Hyperparameters, including the regularization parameter $\boldsymbol{C}$ and kernel width γ, are selected by cross-validation and held fixed across comparable experimental conditions.

Classification performance is evaluated using standard metrics, including accuracy, F1-score, and area under the ROC curve (AUC). Reported values correspond to the mean and standard deviation across fivefold cross-validation, ensuring quantitative comparability across experiments, ablations, and baseline representations.

No component of the Fiber Bundle Learning signature is trained or adapted to the classification task. All geometric, topological, and connection-derived descriptors are computed prior to learning, and the classifier operates solely on the resulting fixed representation. This separation allows the contribution of the representation itself to be assessed independently of model capacity or training dynamics.

The entire pipeline is implemented in Python 3.11. Numerical computations and linear algebra operations are performed using NumPy 1.26 and SciPy 1.13. Nearest-neighbor searches, support vector machine classification, and cross-validation procedures are implemented using scikit-learn 1.5. Persistent homology computations are carried out using GUDHI 3.10 or Ripser.py 0.6.10. Graph operations, including cycle and triangle enumeration, are implemented using NetworkX 3.3. When optimal transport–based transition estimation is employed, computations are performed using POT (Python Optimal Transport) 0.9.5. All software components are used with fixed versions and consistent numerical settings to ensure reproducibility of the reported experiments.

The complete implementation, including synthetic data generation, signature extraction, ablation variants, and evaluation code, is provided in the Supplementary File S1.

2.8. Parameters and Sensitivity Analysis

The Fiber Bundle Learning pipeline involves a small number of parameters controlling neighborhood construction, feature scaling, and learning behavior. These parameters are not tuned to individual datasets but serve to define the resolution at which geometric and feature structure are analyzed.

The neighborhood parameters k (number of nearest neighbors) or r (radius threshold) determine the connectivity of the base graph and the size of fiber patches. Larger values yield smoother, more global representations, while smaller values emphasize local structure. In all experiments, these parameters are chosen to ensure graph connectivity while avoiding excessive densification; qualitative results were stable across a broad range of values.

The scaling parameter α in the total-space embedding controls the relative contribution of geometry and features. Varying α interpolates between geometry-dominated and feature-dominated regimes. Sensitivity tests confirmed that the qualitative behavior of the signature, including twisting discrimination and invariance properties, is preserved across a wide interval of α, indicating that the method does not rely on fine-tuned scaling.

Regularization parameters λ used in transition-map estimation stabilize least-squares solutions in the presence of noise or partial overlap between fibers. Results were insensitive to moderate changes in λ, provided it remained within a standard range ensuring numerical stability.

Classification parameters C and γ are selected by cross-validation but do not affect the construction of the signature itself. As a result, the sensitivity of the method to parameter choices primarily influences classifier behavior rather than the underlying representation.

2.9. Synthetic Data Generation and Reproducibility

All experiments are based on synthetic datasets generated procedurally from explicitly defined geometric and feature-level rules. Each dataset is fully specified by a small set of parameters controlling base graph topology, fiber feature distributions, and transition-map structure.

In all experiments, the base space is constructed as a cycle or grid graph with a fixed number of vertices N, embedded in $\mathbb{R}^2$. Fiber patches are generated by sampling points from Gaussian distributions in $\mathbb{R}^p$ with fixed covariance, identical across all vertices. Structural differences between classes are introduced exclusively through the transition maps: untwisted configurations use identity or near-identity maps, while twisted configurations apply a fixed rotation or reflection consistently along a selected cycle.

Noise robustness experiments add Gaussian perturbations to geometric coordinates and feature vectors with controlled variance, while invariance experiments apply rigid motions or invertible linear transformations that preserve structural identity.

In conclusion, we introduce a discrete Fiber Bundle Learning pipeline integrating base geometry, fiberwise feature organization, and connection-derived descriptors within a unified representation. Structured samples are modeled on cycle or grid graphs embedded in $\mathbb{R}^2$, with fibers generated from Gaussian feature distributions in $\mathbb{R}^p$. Structural differences are introduced exclusively through transition maps, distinguishing untwisted and twisted configurations. Robustness and invariance are evaluated via controlled noise and structure-preserving transformations. Overall, the qualitative distinctions captured by Fiber Bundle Learning were robust across reasonable parameter variations, indicating that the representation reflects structural organization rather than parameter tuning.

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3. Theoretical Guarantees

We formalize here the stability, invariance, and robustness properties of the Fiber Bundle Learning signature. We consider each data sample

$$\mathcal{X} = \{(x_i, f_i)\}_{i=1}^{N}$$

and denote by B the base space constructed from the geometric coordinates $\{x_i\}$, by F_i the fiber patch associated with vertex i, and by E the total-space embedding obtained from the concatenated vectors $(x_i, \alpha f_i)$. The complete signature is

$$\Phi = [\, h_B, \mid h_F, \mid h_E, \mid h_{\mathbf{hol}}, \mid h_{\mathbf{curv}}, \mid h_{\mathbf{CC}} \,].$$

We analyze perturbations of the input and their effect on each component. Throughout this section, we let the perturbed sample be

$$\mathcal{X}' = \{(x_i', f_i')\}_{i=1}^{N}$$

and assume the pointwise perturbation bounds

$$\| x_i - x_i' \| \leq \varepsilon, \| f_i - f_i' \| \leq \varepsilon \tag{14}$$

for all i. All vectorizations of persistence barcodes are assumed to be Lipschitz-continuous with respect to the bottleneck or p-Wasserstein distance, a standard assumption in topological data analysis.

3.1. Definition of Distances and Stability of Homological Components

Given two finite point sets $X, Y \subset \mathbb{R}^d$, let

$$d_H(X, Y)$$

denote their Hausdorff distance. The bottleneck distance between persistence diagrams is denoted by $d_B(\cdot, \cdot)$.

Proof strategy and references. Proofs are provided in full or in sketch form for results relying on nontrivial interactions between discrete connections and perturbations (e.g., holonomy and curvature stability). Results concerning persistent homology stability and barcode vectorizations rely on standard theorems in topological data analysis and are stated without proof, as detailed derivations are available in literature. This separation reflects the fact that the novel technical content of the present work lies in the behavior of connection-derived quantities rather than in the homological stability results themselves.

Theorem 1 (Stability of base-space homology)**.** *Let $\mathcal{B}_{base}$ and $\mathcal{B}'_{base}$ denote the persistence barcodes associated with the Vietoris–Rips filtrations of the base spaces constructed from $\{x_i\}$ and $\{x_i'\}$, respectively. Then*

$$d_B(\mathcal{B}_{\mathbf{base}}, \mathcal{B}'_{\mathbf{base}}) \leq d_H(\{x_i\}, \{x_i'\}). \tag{15}$$

If the barcode vectorization map $\mathcal{V}$ is $L_{\mathcal{V}}$-Lipschitz, then

$$\| h_B - h'_B \| \leq L_{\mathcal{V}}\, d_H(\{x_i\}, \{x_i'\}). \tag{16}$$

Theorem 2 (Stability of fiber homology aggregation)**.** *Let $\mathcal{B}^{(i)}_{fiber}$ and $\mathcal{B}^{(i)\prime}_{fiber}$ denote the persistence barcodes associated with fiber F_i before and after perturbation. Then*

$$d_B\left(\mathcal{B}^{(i)}_{\mathbf{fiber}}, \mathcal{B}^{(i)\prime}_{\mathbf{fiber}}\right) \leq d_H(F_i, F_i'). \tag{17}$$

Consequently, if the vectorization map is $L_{\mathcal{V}}$-Lipschitz,

$$\| v_i - v_i' \| \leq L_{\mathcal{V}}\, d_H(F_i, F_i').$$

Aggregated fiber descriptors, including the empirical mean and covariance used to form h_F, therefore satisfy

$$\| h_F - h_F' \| \leq C_F \varepsilon \tag{18}$$

for some constant C_F depending on the aggregation scheme and the Lipschitz constant of $\mathcal{V}$.

Theorem 3 (Stability of total-space homology)**.** *Let $\mathcal{B}_{total}$ and $\mathcal{B}'_{total}$ denote the persistence barcodes associated with the total-space embeddings $\{(x_i, \alpha f_i)\}$ and $\{(x_i', \alpha f_i')\}$. Then*

$$d_B(\mathcal{B}_{\text{total}}, \mathcal{B}'_{\text{total}}) \leq \max(1, \alpha)\varepsilon. \tag{19}$$

If the vectorization map is $L_{\mathcal{V}}$-Lipschitz, it follows that

$$\| h_E - h_E' \| \leq L_{\mathcal{V}} \max(1, \alpha)\varepsilon.$$

Taken together, Theorems 1–3 establish Lipschitz stability of all homology-based components of the FBL signature under perturbations of geometry and features. These results provide the framework for analyzing the stability of the connection-derived components introduced below.

3.2. Stability of the Discrete Connection

Transition maps T_{ij} are computed by aligning local feature configurations associated with neighboring base vertices. Their stability therefore depends on the sensitivity of the chosen estimator to perturbations of the underlying fiber point clouds.

We formalize this dependence through the following assumption.

Assumption 1 (Lipschitz continuity of the transition estimator)**.** *For each edge $(i, j) \in E$, let T_{ij} and T'_{ij} denote the transition maps estimated from fibers F_i, F_j and their perturbed counterparts F_i', F_j', respectively. We assume that there exists a constant $L_T > 0$ such that*

$$\| T_{ij} - T'_{ij} \|_F \leq 2L_T \varepsilon. \tag{20}$$

This assumption is satisfied by standard regularized least-squares estimators under boundedness of the local feature clouds, as small perturbations of the design matrices produce linear perturbations of the minimizers. The same bound applies when optimal transport–weighted least-squares formulations are used, provided the transport plan varies continuously with the input measures.

Theorem 4 (Stability of transition maps)**.** *Under the above assumption, for all edges $(i, j) \in E$*

$$\| T_{ij} - T'_{ij} \|_F \leq 2L_T \varepsilon. \tag{21}$$

Theorem 4 ensures that the discrete connection varies continuously with respect to perturbations of geometry and features. This property is essential for controlling how local transport errors accumulate along paths in the base graph.

We now analyze how perturbations of the transition maps affect holonomy along closed paths. Let $\gamma = (i_1, i_2, \ldots, i_m, i_{m+1} = i_1)$ be a simple cycle in the base graph and define the associated holonomy operators

$$H_\gamma = T_{i_m i_{m+1}} \cdots T_{i_1 i_2}, H'_\gamma = T'_{i_m i_{m+1}} \cdots T'_{i_1 i_2}. \tag{22}$$

The following result quantifies the first-order effect of perturbations of the discrete connection on holonomy.

Theorem 5 (Holonomy stability)**.** *Let γ be a cycle of length m. Then*

$$\| H_\gamma - H'_\gamma \|_F \leq m(2L_T\varepsilon)\max_k \| T_{i_k i_{k+1}} \|_F^{m-1} + O(\varepsilon^2). \tag{23}$$

Proof (first-order product perturbation)**.** Let $T_k := T_{i_k i_{k+1}}$ and $T'_k := T_k + \Delta_k$, where $\| \Delta_k \|_F \leq 2L_T\varepsilon$. Writing

$$H_\gamma = \prod_{k=1}^{m} T_k, H'_\gamma = \prod_{k=1}^{m} (T_k + \Delta_k),$$

and expanding H'_γ yields a sum of terms linear in a single Δ_j, plus higher-order terms involving products $\Delta_a\Delta_b$. Collecting the linear terms gives

$$H'_\gamma - H_\gamma = \sum_{j=1}^{m} (\prod_{k=j+1}^{m} T_k)\Delta_j(\prod_{k=1}^{j-1} T_k) + R,$$

where the remainder satisfies $\| R \|_F = O(\varepsilon^2)$. Using submultiplicativity of the Frobenius norm yields

$$\| H_\gamma - H'_\gamma \|_F \leq \sum_{j=1}^{m} (\prod_{k\neq j} \| T_k \|_F) \| \Delta_j \|_F + O(\varepsilon^2),$$

which implies the stated bound. □

Corollary 1 (Stability of scalar holonomy descriptors)**.** *If scalar holonomy features are defined as*

$$h_{\mathbf{hol}}(\gamma) = \| H_\gamma - I \|_F \text{ or } h_{\mathbf{hol}}(\gamma) = \rho(H_\gamma), \tag{24}$$

then

$$| h_{\mathbf{hol}}(\gamma) - h'_{\mathbf{hol}}(\gamma) | \leq C_{\mathbf{hol}}\varepsilon$$

for some constant C_{hol} depending on the cycle length and bounds on $\| T_{ij} \|_F$.

3.3. Stability of Holonomy

Holonomy for a cycle $\gamma = (i_1, \ldots, i_m, i_1)$ is defined by

$$H_\gamma = T_{i_m i_1} T_{i_{m-1} i_m} \cdots T_{i_1 i_2}.$$

Let the perturbed holonomy be H'_γ.

The bound established in Theorem 5 implies that, for a cycle of length m,

$$\| H_\gamma - H'_\gamma \|_F \leq m\,(2L_T\varepsilon)\,\max_k \| T_{i_k i_{k+1}} \|_F^{m-1} + O(\varepsilon^2). \tag{25}$$

In particular, if transition maps are uniformly bounded, i.e., $\| T_{ij} \|_F \leq M$, then

$$\| H_\gamma - H'_\gamma \|_F \leq 2m\,M^{m-1} L_T\,\varepsilon.$$

Corollary 2 (Stability of scalar holonomy descriptors)**.** *If scalar holonomy features are defined by*

$$\mathbf{hol}(\gamma) = \| H_\gamma - I_p \|_F,$$

then

$$| \mathbf{hol}(\gamma) - \mathbf{hol}'(\gamma) | \leq 2m\,M^{m-1} L_T\,\varepsilon.$$

3.4. Stability of Curvature-like Quantities

Discrete curvature is defined on oriented triangles of the base graph and captures the local inconsistency of transition maps around minimal loops. Let (i, j, k) be an oriented triangle in the flag complex associated with the base graph. The corresponding curvature-like operator is defined as

$$C_{ijk} = T_{ki} T_{jk} T_{ij},$$

with perturbed counterpart

$$C'_{ijk} = T'_{ki} T'_{jk} T'_{ij}.$$

In the absence of curvature, the product C_{ijk} reduces to the identity, reflecting the local flatness of the discrete connection. Deviations from the identity quantify the extent to which local parallel transports fail to compose consistently.

Theorem 6 (Curvature stability)**.** *Under the Lipschitz continuity assumption on the transition estimator and boundedness of the transition maps, there exists a constant $C_{curv} > 0$ such that*

$$\| C_{ijk} - C'_{ijk} \|_F \leq C_{\mathbf{curv}}\, \varepsilon. \tag{26}$$

Proof. The proof follows the same first-order perturbation argument as in Theorem 5. Writing each perturbed transition map as $T'_{ab} = T_{ab} + \Delta_{ab}$ and expanding the product defining C'_{ijk}, the difference $C'_{ijk} - C_{ijk}$ can be expressed as a sum of terms linear in a single perturbation Δ_{ab}, plus higher-order terms of order $O(\varepsilon^2)$. Submultiplicativity of the Frobenius norm yields the stated bound. □

Scalar curvature descriptors are obtained by evaluating norms or spectral quantities of $C_{ijk} - I$ and aggregating them across all oriented triangles. As a consequence of Theorem 6, any such scalar summaries inherit Lipschitz stability with respect to perturbations of geometry and features.

These curvature-like quantities provide local information complementary to holonomy, capturing obstructions that arise at the smallest nontrivial scales of the base graph.

3.5. Stability of Characteristic-Class Proxies

To capture global obstructions that are not detectable through local curvature alone, we introduce discrete proxies for characteristic classes based on cocycle-like constructions derived from the transition maps. These quantities summarize parity or orientation effects arising from the global organization of the discrete connection.

For triples of overlapping neighborhoods or oriented triangular configurations (i, j, k), we consider quantities of the form

$$\chi_{ijk} = \mathbf{sign}\big(\mathbf{det}(\mathbf{T_{ij}} \cdot \mathbf{T_{jk}} \cdot \mathbf{T_{ki}})\big),$$

where $\mathbf{sign}(\cdot)$ denotes the sign of the determinant and takes values in $\{-1, +1\}$. These indicators detect orientation-reversing transport around minimal loops and serve as discrete analogues of characteristic-class information.

Theorem 7 (Cocycle stability)**.** *Assume that the determinants $\det(T_{ij} T_{jk} T_{ki})$ are bounded away from zero for all relevant triples. Then there exists $\varepsilon_0 > 0$ such that for all perturbations satisfying $\varepsilon < \varepsilon_0$,*

$$\chi'_{ijk} = \chi_{ijk}. \tag{27}$$

In particular, parity changes can occur only if a perturbation drives the determinant through zero. This behavior mirrors the well-known stability properties of characteristic

classes in the smooth setting, where topological invariants remain unchanged under small perturbations except at singular configurations.

The characteristic-class proxy descriptor h_{CC} is formed by aggregating the values of χ_{ijk} across all relevant triples, for example by counting the proportion of nontrivial signs or computing empirical moments. As a consequence of Theorem 7, these aggregated descriptors are stable under small perturbations of the data and change only when the underlying bundle structure undergoes a genuine topological transition.

Together with holonomy and curvature, these cocycle-based quantities provide complementary global information about the discrete connection that is invisible to homology-based descriptors alone.

3.6. Global Stability and Invariance Properties

We now combine the stability results for the homological, connection-based, and cocycle-derived components to establish global stability of the Fiber Bundle Learning signature.

Theorem 8 (Global stability of the FBL signature)**.** *There exists a constant $C > 0$ such that, for any admissible perturbation of geometry and features satisfying*

$$\| x_i - x_i' \| \leq \varepsilon, \| f_i - f_i' \| \leq \varepsilon \text{for all } i, \tag{28}$$

the corresponding FBL signatures satisfy

$$\| \Phi - \Phi' \| \leq C\,\varepsilon.$$

Proof. Each component of the signature satisfies a Lipschitz bound with respect to ε: base, fiber, and total-space homology by Theorems 1–3; transition maps by Theorem 4; holonomy descriptors by Theorem 5; curvature-like quantities by Theorem 6; and characteristic-class proxies by Theorem 7. Since Φ is formed by the concatenation of these components, the global bound follows by the summation of the individual Lipschitz constants. □

3.6.1. Invariance Properties

In addition to stability under perturbations, the Fiber Bundle Learning signature exhibits invariance under a class of transformations that preserve the underlying organization of the data.

3.6.2. Isometry Invariance of the Base Space

If the geometric coordinates $\{x_i\}$ are transformed by a rigid motion or global isometry, all pairwise distances are preserved. Consequently, the base graph, Vietoris–Rips filtrations, and associated persistent homology descriptors remain unchanged, implying

$$h_B = h_B'.$$

3.6.3. Fiber Reparameterization Invariance

If feature vectors are transformed by a global invertible linear map $G \in \mathbf{GL}(p)$,

$$f_i \mapsto G f_i,$$

then the transition maps transform as

$$T_{ij} \mapsto G T_{ij} G^{-1}.$$

Scalar descriptors based on norms, eigenvalues, or determinant signs are invariant under such conjugations, implying

$$h_{\mathbf{hol}} = h'_{\mathbf{hol}}, h_{\mathbf{curv}} = h'_{\mathbf{curv}}, h_{\mathbf{CC}} = h'_{\mathbf{CC}}.$$

3.6.4. Total-Space Invariance

Under the same transformation, the total-space embedding undergoes a linear change of coordinates that preserves topological structure. Consequently,

$$h_E = h'_E.$$

3.6.5. Sensitivity to Genuine Structural Differences

The above invariance properties imply that changes in the FBL signature arise only from modifications of the organizational structure of the data. In particular, the signature responds to changes in:

- global geometric organization of the base space,
- local feature distributions within fibers,
- transition-map coherence and path-dependent transport,
- holonomy, curvature, and cocycle structure of the discrete connection.

Transformations preserving these structures, such as rigid motions, global feature reparameterizations, or uniform rescalings, do not affect the signature, while alterations that change how local feature spaces are glued together produce detectable differences.

This separation between structural sensitivity and invariance under nuisance transformations is central to the representational role of Fiber Bundle Learning.

4. Experiments

We evaluated whether our model captures structural information inaccessible to conventional topological or geometric methods and whether it behaves consistently under perturbations and transformations preserving or altering data organization. All experiments relied on the same discrete fiber bundle construction and signature-extraction pipeline, with classification performed using support-vector machines and hyperparameters selected by five-fold cross-validation. All experiments and figures are based on synthetic datasets constructed to isolate specific structural properties of discrete fiber bundles. This choice is methodological rather than incidental: the goal is to probe whether the representation responds to bundle-level organizational differences, such as twisting or path-dependent transport, that are invisible to persistent homology and difficult to isolate in real-world datasets, where geometry, semantics, sampling density, and noise are entangled. Synthetic constructions allow individual structural degrees of freedom to be varied independently, while holding base geometry and local feature distributions fixed, ensuring that observed effects can be directly attributed to changes in gluing structure.

Rather than reporting benchmark-style numerical performance values, experimental outcomes are summarized in terms of relative quantitative trends across conditions and ablation settings. This choice avoids overstating empirical claims and aligns the experimental analysis with the methodological scope of the study. Complete specifications of the synthetic data generators, parameter settings, and random seeds used to produce all figures and results are provided in the Supplementary Materials, enabling full replication of the reported trends.

4.1. Discriminating Twisted and Untwisted Bundle Structures

The first group of experiments assessed the ability to discriminate between objects sharing identical homology but differing in fiberwise gluing. Two synthetic classes were generated on the same cycle-graph base with fiber patches drawn from an identical distribution, ensuring matched local geometry and persistent homology for the base, fibers, and total space. Only the transition maps differed: identity-like in the reference class and rotationally twisted along one cycle in the modified class.

While conventional TDA descriptors produced indistinguishable representations, our signatures exhibited systematic differences in holonomy-related quantities, curvature-like descriptors, and characteristic-class proxies (Figure 1). Consistent with these differences, classifiers trained on the full signature showed clear separation between the two classes, whereas models relying solely on persistent homology or aggregated feature statistics exhibited near-chance discrimination in this setting. These observations indicate that the representation responds specifically to bundle-level inconsistencies that are invisible to homological and purely geometric summaries.

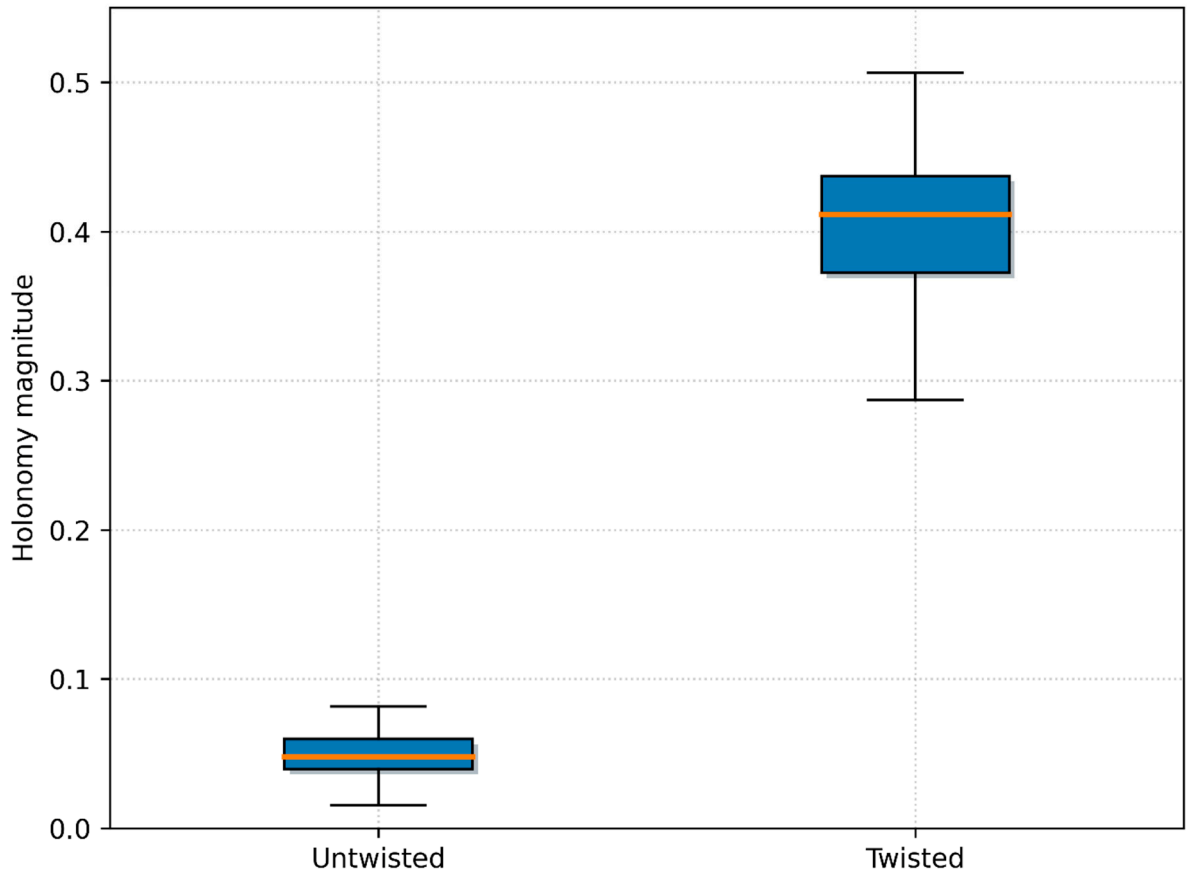


Figure 1. Simulated holonomy magnitudes for untwisted and twisted synthetic structures. Untwisted samples produce values tightly concentrated near zero, reflecting consistent transition maps, whereas twisted samples display substantially larger and more variable magnitudes, corresponding to the imposed rotational inconsistency along a cycle. The separation illustrates how holonomy detects differences in fiberwise gluing invisible to persistent-homology descriptors based solely on base or fiber structure.

4.2. Stability and Robustness Under Perturbations

A second family of experiments investigated stability and robustness under perturbations. Starting from untwisted datasets, we introduced controlled geometric noise on the

base graph and Gaussian noise on fiber patches. For each noise amplitude, signatures were recomputed and classification behavior was evaluated using models trained on clean data.

Signature displacement increased smoothly with noise magnitude, and discriminative behavior degraded gradually rather than abruptly (Figure 2). Across increasing noise levels, relative classification trends exhibited a monotonic decline, consistent with the Lipschitz-type stability bounds established in Section 3. In contrast, classifiers based on aggregated feature statistics deteriorated more rapidly under feature noise, indicating that the hierarchical constraints encoded by the representation mediate a smoother response to perturbation.

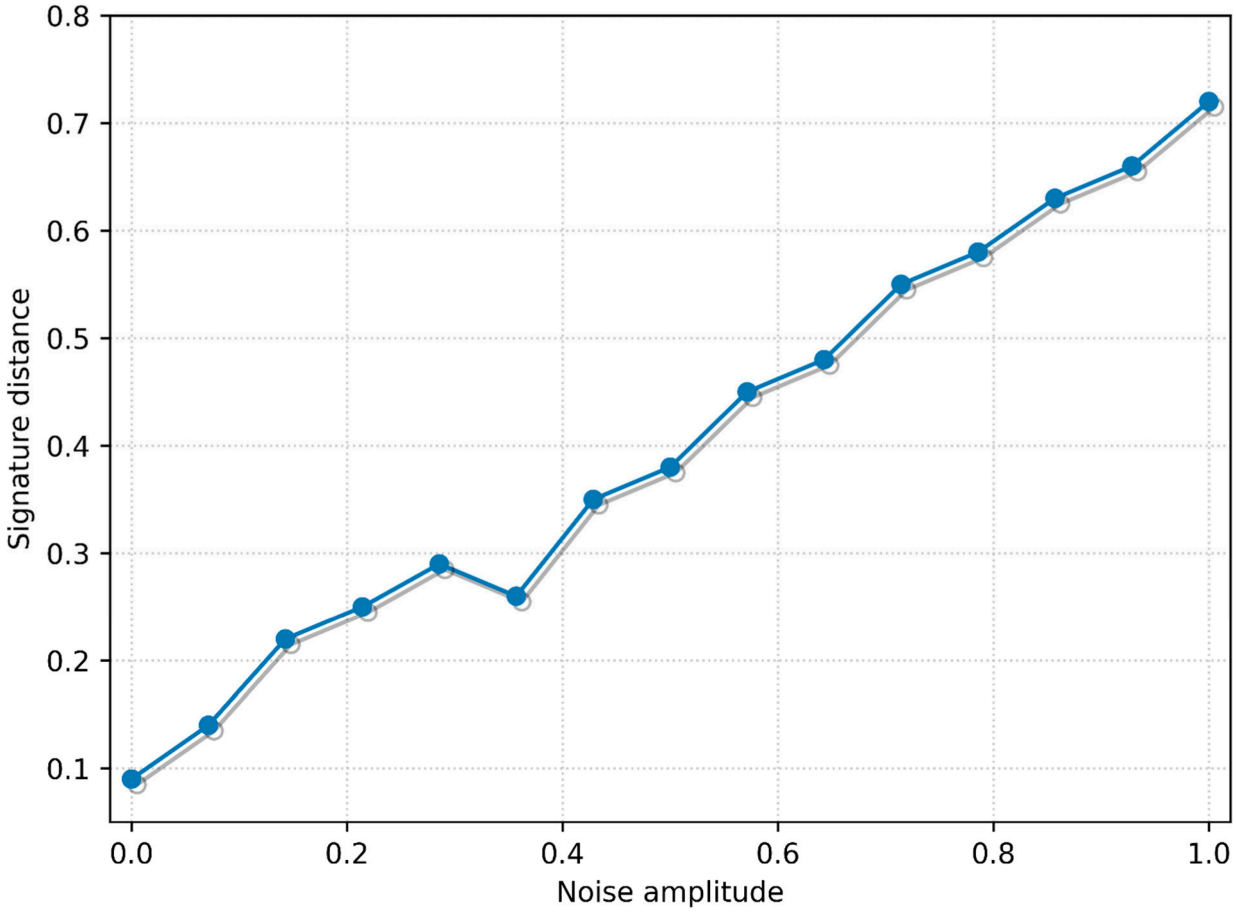


Figure 2. Signature distance response to progressive noise injected into geometric positions and fiber feature patches. Distances grow smoothly, without abrupt transitions, reflecting the Lipschitz stability predicted for the homological and transition components. The monotonic trend indicates that perturbations propagate in a controlled manner through the signature, while the absence of sharp fluctuations suggests that the representation behaves consistently under increasing distortion.

4.3. Invariance to Structure-Preserving Transformations

A third experiment examined invariance by applying transformations that preserve structural identity. Rigid motions, uniform rescalings, and invertible linear reparameterizations of fiber features altered coordinates but did not affect connectivity or fiber organization.

As expected, signature distances between original and transformed instances remained close to zero, with deviations limited to numerical fluctuations (Figure 3).

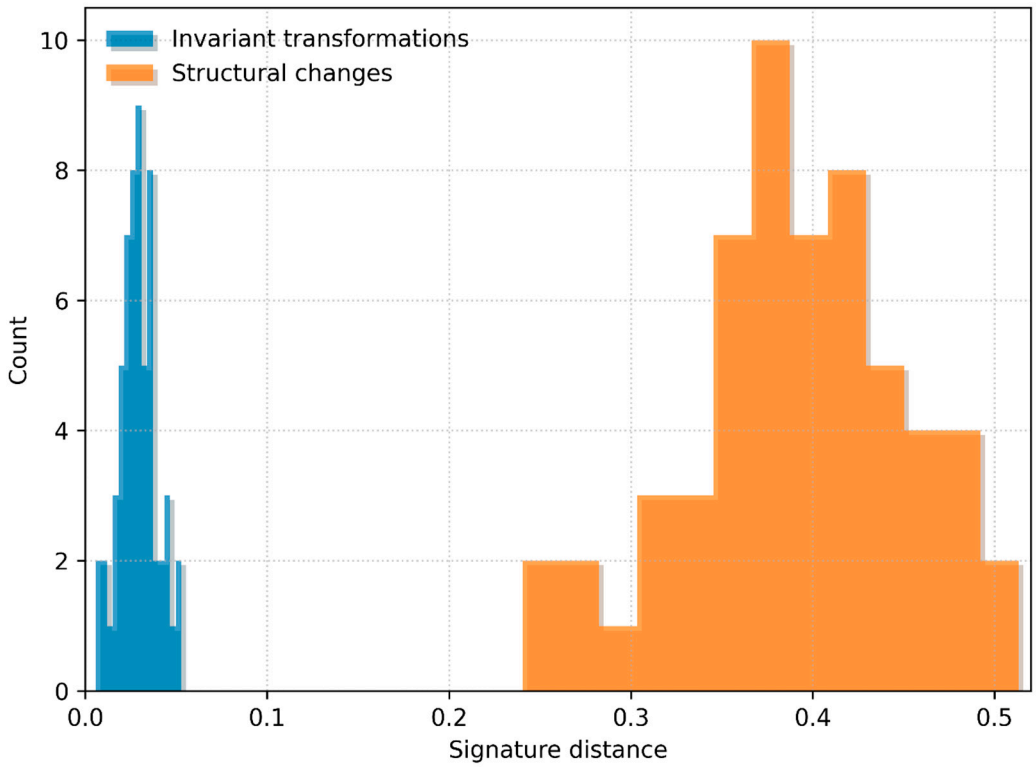


Figure 3. Contrast between signature distances arising from invariant transformations and those produced by connectivity changes or twisting modifications. Invariant transformations yield distances tightly concentrated near zero, whereas structural changes generate significantly larger and widely distributed values. The separation confirms that the representation remains stable under nuisance transformations but responds strongly to genuinely altered bundle organization.

Correspondingly, classification behavior remained stable across all invariant transformations, showing no systematic change in relative discriminability. In contrast, operations that modified connectivity or introduced new twisting patterns produced pronounced and easily detectable signature shifts. This separation between nuisance transformations and structural modifications demonstrates that the representation is sensitive to organizational changes while remaining invariant to irrelevant geometric variation.

4.4. Ablation Study

A final ablation study examined the contribution of individual components of the signature. Starting from the full signature

$$\mathbf{\Phi} = [h_B, h_F, h_E, h_{\mathbf{hol}}, h_{\mathbf{curv}}, h_{\mathbf{CC}}], \tag{29}$$

reduced variants were constructed by removing one component at a time while leaving all others unchanged. Specifically:

(a) No holonomy: $h_{\mathbf{hol}}$ removed; all other components retained.
(b) No curvature: $h_{\mathbf{curv}}$ removed; all other components retained.
(c) No characteristic-class proxies: $h_{\mathbf{CC}}$ removed.
(d) Persistent homology only: only h_B, h_F, and h_E retained.
(e) Fiber-only: only the aggregated fiber descriptor h_F retained.

In all cases, reduced signatures were recomputed from the same underlying data using identical graph construction, neighborhood definitions, and classifier hyperparameters. Removing holonomy-related information substantially reduced discriminative behavior in tasks distinguishing twisted from untwisted structures (Figure 4). Excluding curvature-like

or characteristic-class components further impaired sensitivity to global inconsistencies among transition maps. Persistent-homology-only descriptors performed adequately when classes differed in geometry but failed when distinctions depended solely on gluing structure. Across all settings, the full signature consistently exhibited the strongest relative separation.

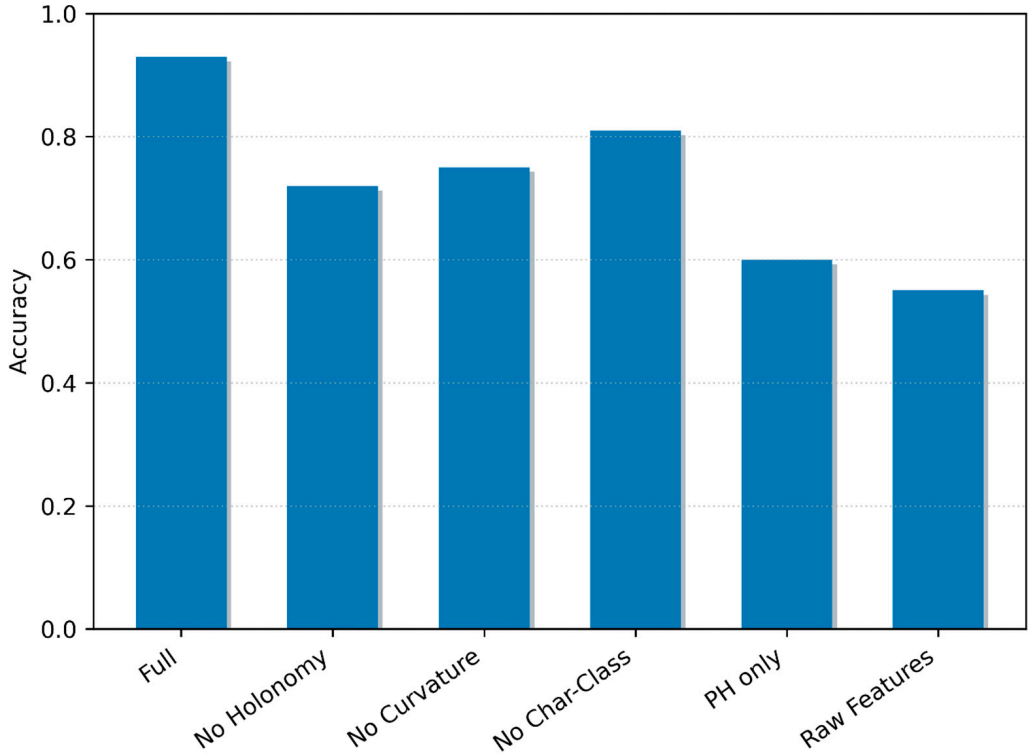


Figure 4. Classification accuracy under selective removal of components from the signature. Eliminating holonomy or curvature reduces performance on twisting-sensitive tasks, while removing characteristic-class proxies weakens detection of global organizational inconsistencies. Persistent homology-only and raw-feature baselines perform poorly when distinctions hinge on fiberwise gluing. The full signature attains the highest accuracy, reflecting the complementary contributions of its multilevel components.

In summary, our experiments were performed under controlled conditions designed to isolate organizational effects beyond local geometry or feature statistics. Synthetic datasets were used to compare twisted and untwisted configurations, assess robustness to geometric and feature noise, test invariance under structure-preserving transformations, and examine the contribution of individual components through ablation. The experimental analysis emphasizes relative trends and structural sensitivity rather than absolute predictive performance, in line with the methodological objectives of the study.

5. Conclusions

We show that representing each data instance through a structured combination of base topology, fiberwise feature organization, and discrete connection information yields signatures that respond systematically to controlled structural modifications. Our simulated outcomes demonstrate that even small shifts introduced at the level of local feature distributions propagate in predictable ways to both local and global descriptors, producing measurable differences in distribution that can be exploited by standard classifiers. Rather than enforcing sharp separability, our representation preserves a realistic degree of variability, reflecting the fact that structural distinctions often coexist with overlapping

local statistics. Examination of marginal distributions, covariance structure, and low-dimensional projections corroborates the presence of consistent but nontrivial separation between simulated classes, indicating that the detected differences arise from organizational effects rather than trivial feature thresholds.

Our approach treats data as possessing explicit geometric and organizational structure, rather than reducing observations to unstructured vectors or relying exclusively on global topological summaries. By decomposing each sample into base, fiber, and transition components, our approach formalizes a multilevel representation in which local feature topology, spatial arrangement, and transport consistency are treated as distinct but interacting sources of information. This explicit separation allows the contribution of each level to be analyzed independently while still capturing their joint effect on global organization. In particular, differences in how local feature spaces are glued together across the base can be detected even when base geometry and local feature distributions are individually indistinguishable.

From a methodological perspective, our model occupies an intermediate position between classical topological descriptors, geometric learning models, and connection-based constructions inspired by differential geometry. Persistent-homology-based pipelines capture global shape but are largely insensitive to how local feature distributions vary along cycles or interact through transport. Graph-based approaches encode adjacency and local interactions but do not quantify path-dependent effects or the global coherence of local transformations. Connection- and gauge-inspired learning architectures, including recent sheaf neural networks and gauge-equivariant convolutional models, address local-to-global consistency by embedding algebraic constraints or symmetry equivariance within trainable architectures [26–29]. However, in these approaches consistency is enforced implicitly through learned parameters and task-specific objectives, and explicit structural invariants are generally not exposed. By contrast, our Fiber Bundle Learning infers transition relations directly from data and characterizes their coherence through explicit, connection-derived quantities like holonomy and related descriptors. This yields a task-agnostic representation that makes bundle-level organization measurable and analyzable, clarifying that the contribution of our approach lies in the measurement and characterization of structural organization rather than in optimizing predictive performance.

Several limitations should be acknowledged. The experimental evaluation relies exclusively on synthetic data designed to isolate specific structural effects and therefore does not yet reflect the full complexity of real-world datasets. While this choice enables controlled assessment of bundle-level organization, it necessarily abstracts away semantic heterogeneity, correlated noise, missing data, and non-uniform sampling patterns that are common in applied settings. In addition, although the theoretical analysis establishes Lipschitz stability and invariance under well-defined assumptions, practical performance may depend on parameter choices, neighborhood definitions, and computational constraints associated with persistent homology, cycle enumeration, and transition-map estimation.

Future research directions can be articulated. The first direction concerns the application to real-world datasets in which local feature distributions are organized over an explicit geometric or relational substrate. Examples include spatially resolved biological measurements, where local molecular profiles are arranged over tissue architectures; networked sensor systems, where local measurements evolve over graph-structured domains; neuroscientific or materials-science data, where regional descriptors are coupled through anatomical or physical connectivity. Additional examples include microbial or biofilm systems evolving over complex interfaces, where local mechanical or biochemical properties are distributed along curved or branching surfaces; climate or geophysical datasets in which localized measurements are embedded in stratified spatial layers; and multi-view

imaging data in which local descriptors are linked through known camera geometries or acquisition manifolds. In these contexts, our framework could be used to test whether variations in transition consistency, holonomy, or curvature-like quantities correspond to meaningful changes in system organization that are not detectable through local statistics or global topology alone, e.g., distinguishing structurally coherent rearrangements from purely stochastic fluctuations.

A second direction involves the systematic construction of benchmark datasets with increasing structural complexity, including higher-dimensional base spaces, richer fiber structures, and controlled combinations of twisting, noise, and sampling irregularity. These benchmarks would allow finer-grained evaluation of the sensitivity and limits of individual signature components, facilitating direct comparison with alternative approaches based on persistent homology. Further extensions could include benchmarks with heterogeneous local feature dimensionality, anisotropic sampling densities, time-dependent base spaces, or explicitly multiscale structures in which local geometries evolve differently across regions. These controlled settings would make it possible to isolate which aspects of bundle organization are captured by connection-based descriptors and which remain invisible to existing topological or geometric summaries.

A third direction concerns computational and theoretical extensions, including alternative estimators for discrete connections, more efficient approximations of holonomy and curvature descriptors, and tighter links between the discrete constructions used here and their continuous geometric counterparts. These developments would help clarify the regimes in which bundle-level descriptors provide maximal added value. Related avenues include scalability to large graphs or high-resolution spatial grids, robustness analysis under missing or corrupted data, and formal characterizations of invariance or stability properties under base-space deformations.

Together, these extensions would strengthen the interpretability and applicability of our framework across domains characterized by complex local-to-global organization.

Overall, we show that treating structured data as discrete fiber bundles could integrate geometry, local feature topology, and global organizational consistency within a single representation. This integration yields signatures that are stable under perturbations, invariant under structure-preserving transformations and sensitive to subtle differences in gluing structure that remain invisible to persistent homology alone. By formalizing bundle-level organization as an explicit and quantifiable construct, our framework complements topological and geometric methodologies and facilitates the analysis of structure beyond local or purely homological descriptors.

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Early-Warning Signals of Democratic Collapse: A Dynamical Systems Analysis of Hitler's Power Consolidation and Failed Counterbalances

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Keywords: bifurcation; resilience; synchronization; damping; polarization; Nazi

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Article

Early-Warning Signals of Democratic Collapse: A Dynamical Systems Analysis of Hitler's Power Consolidation and Failed Counterbalances

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Abstract

Democratic systems rest on institutional counterbalances capable of limiting authority concentration. Historical transitions toward dictatorship often emerge not through abrupt institutional destruction, but progressive weakening of stabilizing mechanisms like parliamentary oversight, judicial autonomy, political pluralism, decentralized governance. Adolf Hitler's power consolidation following the Weimar Republic's collapse provides a historical example in which democratic counterbalances lost corrective capacity under economic crisis, institutional fragility, coordinated mass mobilization, etc. While the historical causes of authoritarian transitions have been extensively studied, the dynamics governing the failure of democratic stabilizing mechanisms is less characterized. We introduce a dynamical systems framework aimed at identifying early-warning signals associated with democratic destabilization and executive power concentration. We represented democratic governance as a multidimensional attractor stabilized by negative feedback mechanisms generated by institutional independence, distributed authority, informational plurality, constitutional constraints. Using historical data from Germany between 1928 and 1934, we built a composite systemic stress index integrating economic instability, war trauma, ideological vulnerability, institutional fragility, political polarization and Nazi mobilization. Simulations based on nonlinear response functions and state-space trajectories showed threshold-like transitions in which progressive stress accumulation was followed by accelerated concentration of political authority, once stabilizing feedbacks became insufficient. Democratic collapse could be interpreted as a loss of systemic resilience associated with attractor deformation and feedback amplification. Potential applications of nonlinear approaches include comparative analysis of institutional fragility, quantitative assessment of democratic resilience and development of early-warning frameworks for detecting conditions associated with excessive concentration of political power in contemporary political systems.

Keywords: bifurcation; resilience; synchronization; damping; polarization; Nazi

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The transition from democratic governance to authoritarian concentration of power has been extensively examined across political science, modern history, sociology and institutional theory. Existing studies on the collapse of the Weimar Republic and the consolidation of power by Adolf Hitler emphasize a broad range of contributing factors, including the consequences of the Versailles Treaty, the psychological legacy of the First World War, polarization between extremist movements, anti-liberal ideological currents, economic destabilization during the Big Depression and the organizational and propagandistic capacity of the NSDAP together with Hilter's charismatic figure (Feldman 1997; Fergusson 2010; Kershaw 1998; McDonough 2024; Sebestyen 2026; Straumann 2019; Tooze 2006). Additional analyses have focused on institutional fragility, presidential emergency powers, erosion of parliamentary legitimacy, paramilitary violence, the role of conservative elites and the progressive weakening of constitutional counterbalances after 1933 (Abraham 1988; Overy 1996; Welch 2002). Studies addressing ideological and cultural dimensions have further examined nationalism, propaganda, anti-liberal currents and the interaction between culture and authoritarian

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politics (Koonz 1992; Speer 1970; Ullrich 2016; Ullrich 2020). While these interpretations have provided detailed descriptions of authoritarian transition, they usually rely on linear causal schemes that only partially capture the nonlinear interactions between institutional, economic, ideological and social processes. As a result, the mechanisms through which democratic stabilizing structures progressively lose resilience and converge toward centralized executive authority are insufficiently formalized. In particular, the identification of threshold conditions preceding the collapse of democratic counterbalances has received comparatively limited quantitative treatment.

We introduce a dynamical systems representation of democratic stability aimed at examining how constitutional counterbalances progressively lose corrective capacity under interacting social, institutional and economic pressures. Rather than interpreting democratic collapse as the direct consequence of a single causal factor or abrupt institutional rupture, we conceptualize political governance as a metastable configuration maintained through distributed negative feedback mechanisms generated by parliamentary oversight, judicial independence, informational plurality, regional autonomy and constitutional constraints. Within this perspective, democratic systems occupy a bounded region of political state space in which destabilizing fluctuations can be partially absorbed without irreversible concentration of authority. Authoritarian consolidation is instead interpreted as a transition toward a competing attractor state characterized by synchronization of institutional structures around a centralized executive node and progressive reduction of corrective feedbacks.

We provide simulations to examine whether cumulative destabilizing processes can progressively deform the geometry of the democratic attractor landscape and produce threshold-like transitions associated with accelerated executive concentration. Our study focuses on the evolution of democratic counterbalance failure rather than exclusively on electoral ascent. Attention is directed toward the interaction between institutional weakening, ideological radicalization, polarization, economic destabilization and organized political mobilization. Instead of assuming linear proportionality between political stress and institutional response, our model allows for nonlinear amplification effects in which gradual increases in systemic pressure may initially produce limited institutional changes, but subsequently trigger rapid transitions once stabilizing mechanisms become insufficient. Therefore, democratic resilience is treated as a dynamic property emerging from the capacity of the political system to dissipate perturbations without converging toward highly centralized configurations. Within this conceptual structure, the late Weimar period provides a historically documented trajectory in which multiple destabilizing processes evolved simultaneously prior to the concentration of power by Adolf Hitler after 1933. By representing political destabilization as a trajectory-dependent process evolving within a multidimensional state space, our simulations aim to identify critical transition regions associated with the weakening of parliamentary and constitutional restraints and the subsequent acceleration of executive consolidation.

We will proceed as follows: first, we describe the historical variables and construction of the systemic stress index. Then, we present the dynamical systems model and simulation procedures. Finally, we analyze the resulting trajectories, threshold behavior and evolution of democratic counterbalance failure.

Methods

We assessed the concentration of political power in Germany between 1928 and 1934, focusing on the progressive weakening of democratic counterbalances after the erosion of parliamentary, institutional and civic restraints. Our simulations were based on six groups of destabilizing variables representing ideological vulnerability, war and Versailles trauma, institutional fragility, political polarization, economic shock and Nazi mobilization capacity. Each group was operationalized through historically interpretable indicators expressed in real units and subsequently transformed into normalized state variables. The normalized variables were combined into a weighted systemic-stress index through a nonlinear logistic response function designed to identify threshold-like transitions between partial democratic resilience and accelerated counterbalance failure. Therefore,

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our simulations aimed to reconstruct how interacting destabilizing processes progressively reshaped the political state space during the late Weimar period and facilitated convergence toward centralized executive authority after weakening of constitutional corrective mechanisms.

Data construction. We defined the observation window as $t \in \{1928,1929,1930,1931,1932,1933,1934\}$, because this interval captures the movement from late Weimar instability to post-accession executive consolidation. For each year t, we assembled six empirical or proxy variables corresponding to the causal blocks retained in our model. We assigned a specific historical indicator to each destabilizing dimension to achieve a quantifiable representation of the underlying political, institutional, economic and ideological processes.

1. Ideological vulnerability incorporated: the broader ideological climate associated with anti-liberal and anti-rational currents, including selective reinterpretations of pre-war philosophical milieu, antisemitism, racial Darwinism, irrationalism, and support for authoritarian, nationalist and collectivist alternatives (Bullock 1962; Fest 1974; Haffner 1981; Koonz 1992; Ullrich 2016). These dimensions were treated as long-term cultural susceptibilities facilitating receptivity to anti-democratic narratives rather than as isolated doctrinal components. Ideological vulnerability was approximated by the combined anti-system vote share, expressed as percentage of valid votes.
2. War and Versailles trauma incorporated: the social and psychological consequences of the First World War and Ruhr occupation, including widespread collective trauma, normalization of violence among veterans, persistence of paramilitary culture, symbolic appeal of national revival, the perception of national humiliation associated with the Versailles settlement and anti-Western nationalism (Evans 2004; McDonough 2024; Sebestyen 2026; Straumann 2019). These factors were interpreted as long-duration destabilizing conditions contributing to resentment against democratic institutions and postwar political arrangements. War and Versailles trauma was quantified by the reparations settlement, fixed at 132 billion gold marks, treated as a persistent structural burden rather than as a rapidly varying yearly driver.
3. Institutional fragility integrated: structural weaknesses inherited from Wilhelmine authoritarian traditions, the limited historical consolidation of parliamentary democracy in Germany, skepticism toward parliamentary pluralism and Weimar representative democracy, chronic governmental instability, progressive reliance on presidential emergency authority and erosion of Reichstag centrality (Abraham 1988; Evans 2005). The resulting variable denoted the weakening of institutional counterbalances and the reduced corrective capacity of constitutional governance. Institutional fragility was quantified by the number of Article 48 emergency decrees or decree-laws per year.
4. Polarization incorporated: anti-system political movements, intensifying conflict between ideological poles, fear of communist revolution, increasing delegitimization of political compromise and growing fragmentation between democratic and anti-democratic actors (Kershaw 1998; Shirer 1960). The variable represented the progressive reduction of cooperative political coupling within the Weimar political system. Polarization was approximated by the combined vote share of extreme parties.
5. Economic shock incorporated: the cumulative consequences of postwar hyperinflation, widespread material insecurity, declining middle-class stability and general social discontent associated with the Great Depression (Feldman 1997; Fergusson 2010; Overy 1996; Tooze 2006; Turner 1985). Economic destabilization was interpreted as a major amplifier of institutional stress because abrupt deterioration in living conditions increased receptivity to centralized and anti-system political alternatives. Economic shock was quantified by registered unemployment in millions of individuals.
6. Nazi mobilization capacity incorporated: Hitler's personal charisma, his frequent underestimation by conservative elites, the organizational strength and youthful image of the Nazi Party, the of modern propaganda techniques, the expansion of paramilitary organizations and political violence and the promise of national rebirth (Kershaw 2000; Longerich 2019; Speer

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1970; Welch 2002). These elements were treated as mechanisms transforming preexisting systemic instability into coordinated political consolidation and executive concentration. Nazi mobilization capacity was quantified by NSDAP membership in millions.

The empirical input vector for each year was therefore written as $X_t = (x_{1t}, x_{2t}, x_{3t}, x_{4t}, x_{5t}, x_{6t})$, where the six components denote ideology, war-Versailles burden, institutional fragility, polarization, economy and mobilization, respectively. Because the variables have different units, direct summation was avoided. Each variable was first transformed into a dimensionless state coordinate before entering the dynamical calculation.

Variable scaling. For each time-dependent variable x_{it}, we applied min-max normalization,

$$z_{it} = \frac{x_{it} - \min_t (x_{it})}{\max_t (x_{it}) - \min_t (x_{it})},$$

so that $z_{it} \in [0,1]$. This operation preserved the temporal ordering of the data while making heterogeneous measurements comparable. For a constant structural variable, such as the Versailles reparations burden, min-max scaling would be undefined because $\max_t(x_{it}) = \min_t(x_{it})$. We therefore assigned a fixed normalized value $z_{2t} = 0.55$, representing a persistent background load rather than an observed annual fluctuation. This choice ensured that the Versailles-related component contributed continuously to systemic stress without producing artificial year-to-year dynamics. The normalized state vector was then written as

$$Z_t = (I_t, W_t, D_t, P_t, E_t, M_t),$$

where I_t is ideological vulnerability, W_t is war and Versailles trauma, D_t is institutional fragility, P_t is polarization, E_t is economic shock and M_t is mobilization capacity. Each component was constrained to the interval $[0, 1]$, allowing later interpretation as partial activation of a destabilizing factor.

Factor weights. We assigned weights to the six groups to construct a composite stress index. The weights were not interpreted as definitive historical causal coefficients, but as explicit modeling parameters used to encode the relative contribution of each factor block in the simulation. The sum of weights was constrained to one:

$$\sum_{i=1}^{6} w_i = 1.$$

The systemic stress index was then calculated as

$$S_t = w_1 I_t + w_2 W_t + w_3 D_t + w_4 P_t + w_5 E_t + w_6 M_t.$$

The selected weights were:

Factor group	Symbol	Weight
Ideological vulnerability	w_1	0.15
War and Versailles trauma	w_2	0.15
Institutional fragility	w_3	0.25
Political polarization	w_4	0.15
Economic shock	w_5	0.20
Nazi mobilization capacity	w_6	0.10

Institutional fragility received the largest weight because the simulations focused on the failure of democratic counterbalances rather than on electoral growth alone. Indeed, our primary objective was not simply to reproduce the electoral expansion of the NSDAP, but examine the progressive failure of democratic counterbalances and the subsequent concentration of executive authority. Economic shock was given the second-largest weight because unemployment and material insecurity strongly altered the political state space. Ideology, war-Versailles burden and polarization were treated as intermediate contributors, while mobilization capacity was modeled as a conversion mechanism translating systemic stress into organized power.

Stress index. After normalization and weighting, S_t was expressed as a percentage for visualization:

$$S_t^{(\%)} = 100 S_t.$$

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The yearly contribution of each factor to systemic stress was also computed as

$$C_{it} = 100 w_i z_{it},$$

so that

$$S_t^{(\%)} = \sum_{i=1}^{6} C_{it}\,.$$

This allowed the model to display both the total stress trajectory and the internal decomposition of that stress. In the figures, stacked bars represented C_{it}, while the dotted line represented $S_t^{(\%)}$. This decomposition was useful because two years may have similar total stress but different internal compositions. For example, a given value of S_t may be driven mainly by economic deterioration, institutional weakening or mobilization expansion. The decomposition therefore preserved causal heterogeneity while still permitting a one-dimensional threshold analysis.

Outcome variable. We defined the dependent variable as executive power concentration or democratic counterbalance failure, denoted by Y_t. This variable was expressed as a percentage between 0 and 100, where lower values indicate partial preservation of parliamentary and institutional counterweights, while higher values indicate increasing concentration of authority. We used historically anchored values rather than direct survey measurements, because the relevant outcome is institutional configuration rather than public preference alone. The anchor series was written as

$$Y_t \in [0,100].$$

For computational fitting, the outcome was rescaled to unit interval:

$$y_t = \frac{Y_t}{100}.$$

The anchor values were chosen to reflect the progressive weakening of Weimar counterbalances between 1928 and 1932, the major consolidation of executive authority in 1933 and the near-complete concentration of power by 1934. These values should be understood as ordinal-calibrated institutional anchors, not as direct measurements from a single archival source.

Response function. We linked systemic stress to counterbalance failure using a logistic response function:

$$\hat{y}_t = b + \frac{L}{1 + \exp\left[-k(S_t - S_0)\right]}.$$

Here, $\hat{y}_t$ is the simulated counterbalance-failure value, b is the lower baseline, L is the response amplitude, k is the steepness of the transition and S_0 is the critical threshold. The logistic form was selected because it captures three relevant properties: low response below threshold, rapid acceleration near the inflection region and partial saturation at high stress. The estimated value S_0 corresponds to the systemic-stress level at which the response curve has maximal slope. Mathematically, the inflection condition occurs when

$$\frac{d^2\hat{y}}{dS^2} = 0,$$

which is satisfied at

$$S = S_0.$$

The slope at the threshold is

$$\frac{d\hat{y}}{dS}\Big|_{S=S_0} = \frac{Lk}{4}.$$

Thus, S_0 identifies the critical transition region, while k quantifies how abrupt the loss of counterbalances becomes once the threshold is approached.

Parameter fitting. We estimated the logistic parameters $\theta = (L, k, S_0, b)$ by nonlinear least squares. The residual for each year was defined as

$$\varepsilon_t = y_t - \hat{y}_t.$$

The objective function minimized during fitting was

$$\mathcal{J}(\theta) = \sum_t \left(y_t - b - \frac{L}{1 + \exp\left[-k(S_t - S_0)\right]}\right)^2.$$

Parameter bounds were imposed to avoid noninterpretable solutions:

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$$0.3 \le L \le 1.0, 0.1 \le k \le 50, 0 \le S_0 \le 1, 0 \le b \le 0.2.$$

The lower and upper bounds on L constrained the maximum simulated change in counterbalance failure. The bounds on k allowed both gradual and steep transitions while avoiding numerical instability. The threshold S_0 was restricted to the normalized systemic-stress interval. The baseline b allowed nonzero pre-crisis counterbalance weakness. After fitting, all simulated values were reconverted into percentages using $\hat{Y}_t = 100\hat{y}_t$.

Dynamical interpretation. We interpreted the normalized state vector Z_t as a point in a six-dimensional political state space. The democratic configuration was represented as a region in which stabilizing feedbacks still counteract executive concentration. The authoritarian configuration was represented as a region toward which trajectories converge after stabilizing mechanisms lose corrective strength. In compact form, the yearly evolution can be written as

$$Z_{t+1} = F(Z_t, \eta_t),$$

where F denotes the historical update map and η_t represents unmodeled disturbances. Since the aim was not to estimate every component of F, we projected the six-dimensional state onto the scalar stress coordinate S_t. Counterbalance failure was then modeled as

$$Y_t = G(S_t) + \epsilon_t,$$

where G is the logistic response and ϵ_t is the residual discrepancy. In this reduced representation, threshold crossing occurs when $S_t$approaches S_0. The transition is therefore not defined by a single historical factor, but by the weighted convergence of multiple destabilizing variables.

Simulation sequence and software tools. We followed a fixed technical sequence. First, we defined the annual time vector from 1928 to 1934. Second, we entered the six real-unit variables corresponding to the factor groups. Third, we normalized each time-varying series into a dimensionless coordinate. Fourth, we assigned the Versailles-war structural factor a constant normalized background value. Fifth, we multiplied each normalized coordinate by its predefined weight. Sixth, we summed the weighted components to obtain S_t. Seventh, we fitted the logistic response linking S_t to the institutional concentration anchor Y_t. Eighth, we extracted the threshold S_0, steepness k, baseline b and amplitude L. Ninth, we generated simulated values $\hat{Y}_t$.

All calculations were performed using Python. Numerical arrays were managed with NumPy. Nonlinear least-squares fitting was performed with SciPy, specifically the curve_fit routine from scipy.optimize.

Results

We report on the temporal evolution of the six destabilizing factor groups, the resulting systemic-stress trajectories and the nonlinear transition associated with democratic counterbalance failure between 1928 and 1934. Our analyses quantify the interaction between institutional weakening, ideological radicalization, polarization, economic destabilization and Nazi mobilization capacity. Also, we examine the emergence of a threshold region associated with accelerated executive concentration.

Factor dynamics. Our six-factor reconstruction revealed partially synchronized but temporally heterogeneous trajectories across the destabilizing variables incorporated into the simulations. Ideological vulnerability progressively increased during the late Weimar period through expanding electoral support for anti-system parties (Figure 1, upper-left panel). War and Versailles trauma remained persistently elevated throughout the simulated interval, since the reparations burden was treated as a long-duration structural destabilizing background rather than a rapidly varying annual process (Figure 1, upper-middle panel). Institutional fragility showed a marked escalation after 1930 through increasing reliance on Article 48 emergency decrees and decree-laws, pointing towards progressive weakening of parliamentary corrective mechanisms (Figure 1, upper-right panel). Political polarization intensified through the growing electoral strength of extremist parties and increasing fragmentation between democratic and anti-democratic actors (Figure 1, lower-left panel). Economic shock simultaneously expanded through worsening unemployment and widespread

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social destabilization, in particular after the Great Depression (Figure 1, lower-middle panel). Nazi mobilization capacity also increased progressively through expansion of party organization, propaganda and coordinated political activity (Figure 1, lower-right panel).

Summarizing, the coordinated escalation of ideological, institutional, economic and organizational variables generated a coherent temporal shift toward increasing systemic destabilization and weakening democratic resilience.

Threshold transition. Our nonlinear simulations identified a threshold-like transition linking composite systemic stress to democratic counterbalance failure (Figure 2). The weighted integration of ideological vulnerability, war-related destabilization, institutional fragility, polarization, economic shock and Nazi mobilization capacity generated a progressively increasing systemic-stress trajectory. The stacked decomposition indicated that institutional weakening and economic destabilization became increasingly dominant contributors during the transition toward executive consolidation, while ideological and mobilizational components amplified the overall trajectory once destabilization accelerated. The trajectory representing historically anchored executive concentration remained relatively moderate during the early phase of the simulations, but subsequently approached the threshold region associated with rapid weakening of democratic corrective mechanisms. Above the estimated threshold, our simulated nonlinear response showed accelerated concentration of authority and progressive convergence toward an authoritarian attractor configuration.

Further, logistic response analysis showed that lower levels of systemic stress were associated with comparatively limited institutional transformation, whereas higher stress levels produced abrupt increases in executive concentration and reduced corrective capacity (Figure 3). The inflection region of the fitted nonlinear trajectory corresponded to the critical transition zone separating partial institutional resilience from accelerated counterbalance failure.

Together, nonlinear simulations suggest that authoritarian consolidation emerged through cumulative amplification among interacting destabilizing processes, rather than isolated linear causal mechanisms.

Overall, the weakening of democratic counterbalances during the late Weimar period could be represented as a nonlinear transition emerging from interacting institutional, economic, ideological and organizational processes. Our simulations identified a critical stress region associated with accelerated executive concentration and progressive loss of stabilizing feedbacks. This supports the interpretation of authoritarian consolidation as a cumulative dynamical transformation rather than a single abrupt institutional rupture.

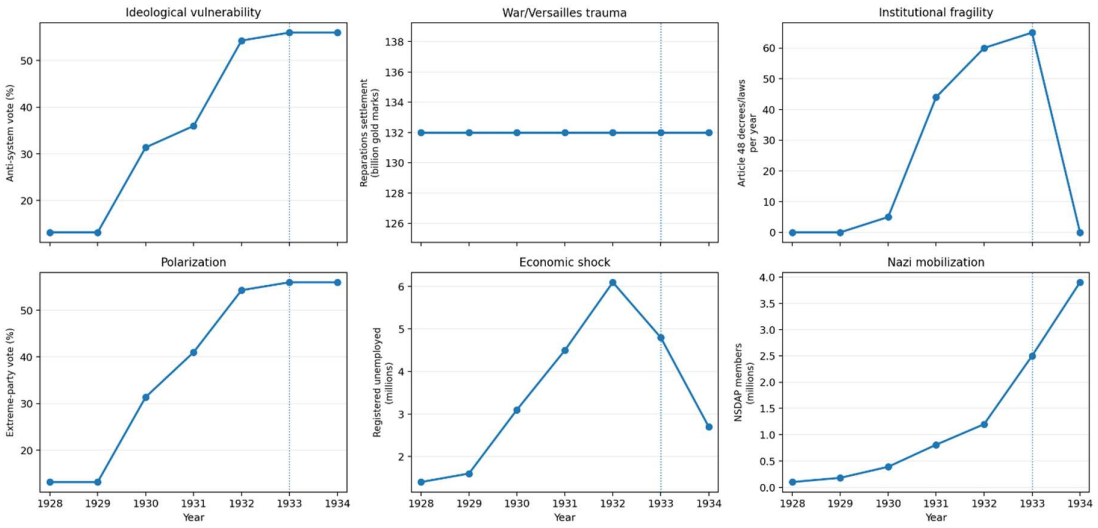


Figure 1. Six factor groups included in our simulation of democratic counterbalance failure in Germany between 1928 and 1934. The vertical dotted line indicates 1933, corresponding to the transition from parliamentary instability to executive consolidation.

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The upper-left panel reports ideological vulnerability through the electoral support for anti-system parties, expressed as percentage of valid votes.

The upper-middle panel shows the Versailles settlement burden represented through reparations commitments in billion gold marks, treated as a persistent structural destabilizing background.

The upper-right panel reports institutional fragility through the annual number of Article 48 emergency decrees and laws.

The lower-left panel illustrates political polarization through the combined electoral support for extremist parties.

The lower-middle panel reports economic shock using registered unemployment in millions of individuals.

The lower-right panel represents Nazi mobilization capacity through NSDAP membership in millions.

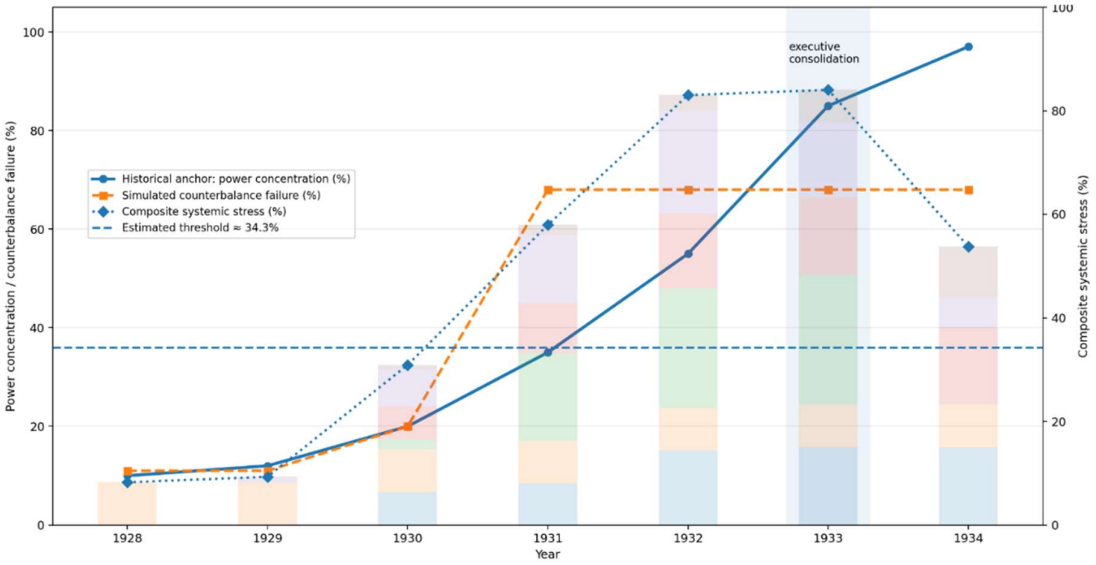


Figure 2. Simulated threshold dynamics of democratic counterbalance failure between 1928 and 1934. The solid blue curve represents historically anchored estimates of executive power concentration and weakening of democratic counterbalances, while the dashed orange curve reports the corresponding nonlinear simulation output generated from the composite systemic stress index. The dotted blue trajectory indicates the temporal evolution of systemic stress derived from the weighted integration of the six factor groups included in our model. The horizontal dashed line marks the estimated critical threshold at approximately 34.3% systemic stress, above which the simulated dynamics transition from gradual institutional weakening to accelerated concentration of executive authority. Semi-transparent stacked bars represent the relative weighted contributions of different factor groups. The shaded vertical region identifies the phase of executive consolidation around 1933, when democratic stabilizing feedbacks progressively lose corrective capacity and the trajectory converges toward an authoritarian attractor state.

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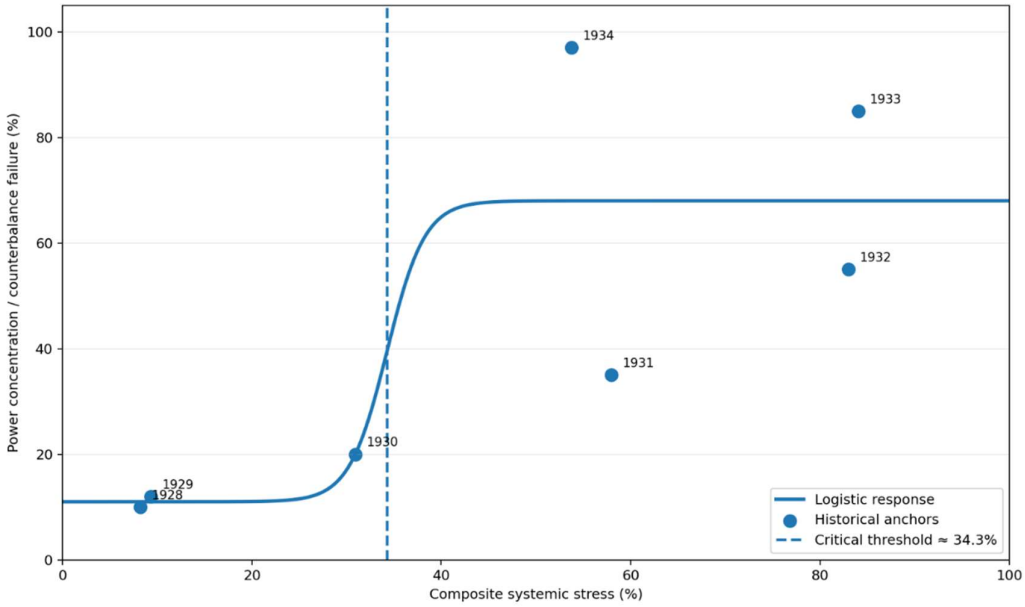


Figure 3. Logistic response function relating composite systemic stress to the estimated degree of democratic counterbalance failure and executive power concentration. Each point corresponds to a historically anchored yearly state between 1928 and 1934. The continuous curve represents the fitted nonlinear response function used in our dynamical systems model. The vertical dashed line identifies the estimated critical threshold at approximately 34.3% systemic stress, corresponding to the inflection region of the logistic trajectory. Below this threshold, increases in stress are associated with relatively limited changes in executive concentration, indicating partial institutional resilience. Above the threshold, the system undergoes rapid acceleration toward high levels of centralized authority, reflecting loss of stabilizing democratic feedbacks and convergence toward an authoritarian regime configuration.

Conclusions

We asked whether collapse of democratic counterbalances and concentration of executive authority can be represented as a nonlinear transition emerging from the interaction of multiple destabilizing variables rather than from isolated historical events. To address this question, we reconstructed the political evolution of Germany between 1928 and 1934 using six groups of factors associated with ideological vulnerability, war-related and Versailles-related destabilization, institutional fragility, polarization, economic crisis and mobilization capacity linked to the NSDAP. Our simulations compared the temporal evolution of these variables with a modeled trajectory of executive concentration and weakening of democratic corrective mechanisms. We found that the combined escalation of institutional weakening, economic destabilization, extremist polarization and organized mobilization generated a progressive increase in systemic stress followed by a threshold-like transition associated with accelerated concentration of political authority. The estimated response curve suggested that democratic resilience was partially preserved below a critical stress region, whereas above this interval the system rapidly converged toward centralized executive control. Therefore, our reconstructed trajectories are consistent with the interpretation that authoritarian consolidation in the late Weimar Republic emerged through cumulative feedback amplification involving interacting institutional, economic and social processes. Our results also suggest that the weakening of democratic systems may be more appropriately described as a progressive loss of stabilizing feedbacks than as a single abrupt constitutional rupture.

Compared with conventional historical narratives, linear causal models and descriptive institutional analyses, our approach explicitly integrates heterogeneous variables into a unified dynamical representation. Several accounts examine economic crisis, propaganda, political violence, constitutional fragility or elite decisions separately, whereas our simulations treated these variables

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as interacting components of a coupled system evolving within a shared state space. In contrast to standard regression-based interpretations, the use of nonlinear response functions allowed the identification of threshold regions and accelerated transitions rather than assuming proportional linear responses. Unlike qualitative political analyses, our model incorporated weighted temporal trajectories expressed in historical units and represented the evolution of systemic stress quantitatively. Our approach also differs from traditional equilibrium-based political models, because it interprets democratic stability as a metastable condition maintained by negative feedback mechanisms rather than as a fixed institutional state. Our representation allows executive concentration to be examined as a trajectory-dependent process associated with deformation of the political attractor landscape and progressive reduction of institutional corrective capacity.

Our study has limitations. The outcome variable describing executive power concentration and democratic counterbalance failure was manually anchored from institutional interpretation rather than derived from a standardized quantitative historical dataset. Similarly, the weights assigned to the six factor groups were heuristic parameters chosen to preserve internal interpretability and dynamical consistency, but they were not inferred through large-scale statistical optimization or independently validated against external historical benchmarks. The Versailles-related component was treated as a constant structural background term, which stabilizes the simulations mathematically but simplifies the temporal complexity of postwar political dynamics. Several ideological variables, including irrationalism and pre-war philosophical accounts, were incorporated into broader latent dimensions without direct operationalization, limiting causal specificity. The estimated threshold near 34.3% systemic stress should therefore be interpreted cautiously because the temporal dataset is sparse and based on only seven yearly observations, making nonlinear fitting sensitive to perturbations. The dynamical-systems interpretation is conceptual, since no explicit differential-equation system, empirical attractor reconstruction, Lyapunov analysis or formal bifurcation analysis was computed. Additional unresolved questions concern the transferability of the modeled thresholds to other political systems, the temporal dependence of factor weights and the extent to which institutional resilience can be represented through measurable dynamical observables.

Experimentally testable hypotheses and predictions can be drawn. A first prediction is that democratic destabilization should exhibit nonlinear scaling rather than proportional response, such that modest increases in systemic stress below the estimated threshold produce limited institutional effects, whereas equivalent increases above the threshold generate accelerated concentration of executive authority. This prediction can be tested longitudinally by quantifying constitutional restrictions, emergency executive actions, judicial independence scores or media-centralization indices across historical and contemporary political systems.

A second prediction is that institutional fragility and economic destabilization should display stronger coupling with executive concentration than isolated ideological variables alone. Quantitatively, our model predicts that simultaneous increases in unemployment and emergency decree frequency should produce steeper transitions than either variable independently.

A third prediction is that increasing synchronization among political subsystems, including media alignment, party concentration and weakening of regional autonomy, should precede rapid executive consolidation by measurable temporal intervals. This may be tested through network analyses of parliamentary voting coherence, media ownership concentration or regional-governance homogenization.

A fourth prediction is that systems approaching threshold conditions should display increased variance and slower recovery after perturbation, analogous to critical slowing-down phenomena observed in nonlinear systems.

Future research may incorporate higher-resolution temporal datasets, comparative historical trajectories, adaptive time-dependent weights, stochastic perturbation terms, empirical attractor reconstruction and explicit differential-equation formulations. Additional directions include interdisciplinary integration of computational social modeling, agent-based political dynamics,

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social-network propagation models and information-theoretic measures of institutional diversity, in order to examine whether threshold behavior is stable under alternative representations of democratic resilience.

Our approach could also assess institutional robustness under complex social stress conditions. Weighted dynamical representations may assist comparative studies of constitutional resilience by integrating economic, political and organizational variables within a common temporal structure. Quantitative state-space representations could complement traditional political-risk methodologies by introducing threshold-sensitive indicators rather than exclusively linear trends. Similar procedures may also contribute to longitudinal archival studies, examining how emergency legislation, institutional centralization and polarization evolve jointly across different regimes.

In conclusion, we reconstructed the progressive weakening of democratic counterbalances in Germany between 1928 and 1934 through a dynamical representation integrating institutional, economic, ideological and organizational variables. Our simulations identified threshold-like transitions associated with accelerated executive concentration, suggesting that destabilization emerged from interacting processes rather than isolated events. Our data support the interpretation of authoritarian consolidation as a nonlinear transformation of political-system dynamics involving cumulative amplification of weakening stabilizing mechanisms.

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Availability of data and materials. All data and materials generated or analyzed during this study are included in the manuscript. The Author had full access to all the data in the study and took responsibility for the integrity of the data and the accuracy of the data analysis.

Declaration of generative AI and AI-assisted technologies in the writing process. During the preparation of this work, the author used ChatGPT 5.3 to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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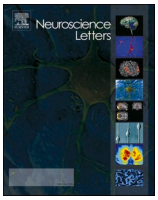
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Embedding EEG trajectories in a Möbius-like manifold: An exploratory study

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A R T I C L E I N F O

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Attractor
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A B S T R A C T

Time–frequency decompositions and nonlinear dynamical methods analyze electroencephalographic (EEG) signals as time series evolving in Euclidean state spaces. We explore an alternative representation of EEG dynamics in which neural activity evolves within a Möbius-like state space. While conventional amplitude–phase descriptions represent oscillatory activity within a cylindrical state space, we additionally consider that a shift of half an oscillatory cycle reverses the sign of the waveform, transforming positive amplitudes into negative ones and vice versa. This symmetry introduces a twist into the cylindrical representation, yielding a non-orientable topology analogous to a Möbius strip in which EEG activity evolves as a continuous cyclic trajectory. Using normalized signal amplitude and instantaneous phase derived from the Hilbert transform, we reconstructed three-dimensional trajectories from EEG recordings of a healthy young adult. Our Möbius-like approach describes the geometry of the embedded EEG trajectory in terms of cyclic evolution, phase-dependent symmetry, winding number and torsion. The winding number quantifies cumulative oscillatory phase progression by measuring the number of rotations performed by the trajectory around the manifold, whereas torsion captures local changes in amplitude–phase organization by characterizing how strongly the trajectory twists in three-dimensional space. Together, these descriptors provide complementary assessment of global and local neural dynamics that are not represented by conventional EEG measures based solely on temporal, spectral or statistical properties. Potential applications include characterization of physiological and pathological brain activity, trajectory-based EEG feature extraction, integration with brain–computer interface approaches and comparative analysis of neural dynamics across cognitive and behavioral conditions.

1. Introduction

Electroencephalographic (EEG) signal analysis employs time–frequency decompositions, spectral and statistical descriptors and nonlinear methods such as attractor reconstruction and entropy metrics [35,17,6,14,23,24,21]. Also, geometric and topological approaches like delay embeddings and topological data analysis have been used to capture global structure in neural dynamics beyond local signal features [18,27,29,2,22,23,12,28]. These approaches, representing signals as trajectories in Euclidean spaces or orientable manifolds like cylinders or tori derived from phase variables, may fail to capture intrinsic symmetries and phase-dependent inversions.

We explore an alternative geometric representation in which EEG trajectories are embedded in a Möbius-like manifold. EEG recordings reflect the collective electrical activity of large neuronal populations, characterized by oscillations varying in both amplitude and phase. At every instant, an oscillatory signal can be described by two quantities: its instantaneous magnitude and its position within the oscillatory cycle. A natural way to represent amplitude–phase dynamics is through a geometric space in which phase defines a cyclic coordinate and amplitude determines the distance from the central axis. Because phase evolves periodically and returns to equivalent states after complete rotations, this construction generates a cylindrical state space. Nevertheless, oscillatory signals possess an additional symmetry: a phase shift of half a cycle transforms a waveform into its sign-reversed counterpart, meaning that positive and negative peaks represent the same oscillatory process observed at different phases of the cycle. By identifying amplitude–-phase states that differ by a half-cycle phase shift and an inversion of amplitude orientation, the cylindrical representation acquires a twist and becomes topologically equivalent to a Möbius-like manifold [10,13,25].

Based on this observation, we embedded EEG trajectories in a

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Möbius-like manifold, aiming to provide a scale-independent measure of electrical activity in which the instantaneous phase was extracted through the Hilbert transform, whereas the signal amplitude was normalized. Within this representation, each time point of the recorded electrical activity corresponds to a position along a three-dimensional trajectory whose evolution reflects the progression of neural oscillations through successive cycles. Continuous motion along the manifold represents the temporal evolution of oscillatory activity, whereas changes in trajectory geometry arise from variations in amplitude-phase relationships. The cumulative number of rotations around the manifold is quantified by the winding number, which provides a geometric description of phase advancement through time. In turn, local deviations of the trajectory from a planar path are quantified by torsion, which captures changes in the organization of amplitude–phase relationships across successive oscillatory states[30,1]. Accordingly, the Möbius-like embedding is not intended as a physical model of neural tissue, but rather as a geometric representation that explicitly incorporates cyclic evolution and phase-dependent symmetry in oscillatory EEG signals.

We will proceed as follows. First, we describe the methods used to reconstruct the Möbius-like embedding and compute geometric descriptors of the trajectory. Then, we present trajectory representations and derived measures from EEG recordings. Finally, we discuss the conceptual and practical implications for the analysis of EEG dynamics.

2. Methods

We evaluated an EEG recording acquired from a healthy 20-year-old subject, analyzing the first 4000 ms of activity from channel *C*3 sampled at 1 ms intervals. We used signal normalization, Hilbert-transform phase extraction, a Möbius-like geometric embedding and trajectory-based differential geometry to reconstruct the temporal evolution of the EEG trace as a path in a three-dimensional non-orientable manifold. Then, we computed topological and geometric descriptors, including cumulative winding number, curvature, torsion and recurrence structure. We aimed to identify discriminable trajectories' signatures, namely local twisting, global rotational accumulation and repeated visits to nearby regions of the embedded space.

Data acquisition and temporal discretization. The analyzed signal consisted of a univariate EEG time series from channel *C*3 of a healthy 20-year-old subject. Let the raw trace be denoted by s_n, where $n = 0, 1, \cdots, N-1$ indexes discrete samples. Because each sample corresponded to 1 ms, physical time was defined by

$$t_n = n\Delta t, \Delta t = 1 \text{ ms} = 10^{-3} \text{ s}.$$

The first portion of the recording used for the trajectory analysis contained $N = 4000$ samples, corresponding to a total duration

$$T = N\Delta t = 4000 \text{ ms} = 4 \text{ s}.$$

The signal was treated as a discrete realization of an underlying continuous function $s(t)$, with the approximation

$$s(t_n) \approx s_n.$$

All subsequent operations were performed on this finite sequence. The present section focuses on the single-channel analysis of *C*3. No temporal smoothing, band-pass filtering, artifact rejection or epoch averaging was applied before the Möbius-like embedding, so that the embedding procedure operated directly on the recorded sequence after normalization.

Amplitude standardization and analytic signal construction. The raw series s_n was transformed into a dimensionless standardized amplitude x_n to remove scale dependence and center the data. If

$$\mu = \frac{1}{N}\sum_{n=0}^{N-1} s_n$$

denotes the sample mean and

$$\sigma = \left(\frac{1}{N}\sum_{n=0}^{N-1}(s_n - \mu)^2\right)^{1/2}$$

denotes the population-standard deviation, the normalized signal was defined as

$$x_n = \frac{s_n - \mu}{\sigma}.$$

This transformation preserved waveform shape while rescaling the signal to zero mean and unit variance. Instantaneous phase was then extracted from the analytic signal. For a continuous signal $x(t)$, the Hilbert transform is

$$H[x](t) = \frac{1}{\pi}\,\text{p.v.}\int_{-\infty}^{\infty}\frac{x(\tau)}{t-\tau}\,d\tau,$$

where p.v. denotes the Cauchy principal value [5,11]. The corresponding analytic signal is

$$z(t) = x(t) + iH[x](t).$$

In discrete form, numerical estimation of the analytic signal yielded

$$z_n = x_n + i\widetilde{x}_n,$$

where $\widetilde{x}_n$ is the discrete Hilbert transform of x_n. From z_n, the instantaneous amplitude and phase were

$$a_n = \left|z_n\right| = \sqrt{x_n^2 + \widetilde{x}_n^2}, \phi_n = \arg(z_n).$$

Because phase returned by the argument function is defined modulo 2π, an unwrapped phase sequence Φ_n was computed to eliminate artificial jumps of approximately 2π. The wrapped phase used in the embedding was then

$$\theta_n = \Phi_n mod 2\pi, \theta_n \in [0, 2\pi).$$

This step assigned to every EEG sample both a normalized scalar amplitude x_n and a cyclic phase coordinate θ_n. These two variables formed the basis of the subsequent manifold embedding. Their pairing created a phase-amplitude representation in which the same time series was mapped into a space with explicit cyclic organization.

Topological identification and Möbius-like state space. A phase shift of π transforms a sinusoidal waveform into its sign-reversed version, since

$$\sin(\theta + \pi) = -\sin(\theta), \cos(\theta + \pi) = -\cos(\theta).$$

For this reason, amplitude-phase states related by a half-cycle shift can be identified through

$$(x, \theta) \sim (-x, \theta + \pi).$$

Equivalently, if one starts from a cylindrical phase-amplitude space with periodic identification $(x, 0) \sim (x, 2\pi)$, the additional pairing

$$(x, \theta) \sim (-x, \theta + \pi)$$

produces a non-orientable quotient space topologically equivalent to a Möbius strip. This identification was not interpreted as an anatomical property of the brain or scalp, but as a mathematical rule imposed on the signal representation. The construction associates states of opposite amplitude sign that differ by half an oscillatory cycle, thereby preserving phase relations while reducing redundancy in the representation of cyclic motion.

To achieve three-dimensional coordinates, the phase-amplitude pair (x_n, θ_n) was mapped to a standard Möbius-strip parametrization. Let $R >$

0 denote the radius of the central circular axis and w_n denote a bounded transverse coordinate derived from amplitude. We used

$$w_n = \mathrm{clip}\left(\frac{x_n}{3}, -0.8, 0.8\right),$$

where $\mathrm{clip}(u, \alpha, \beta)$ truncates u to the interval $[\alpha, \beta]$. This bounded scaling prevented very large standardized amplitudes from generating excessive excursions away from the strip. With $R = 1.5$, the embedding coordinates were defined by

$$X_n = \left(R + w_n \cos\frac{\theta_n}{2}\right)\cos\theta_n,$$

$$Y_n = \left(R + w_n \cos\frac{\theta_n}{2}\right)\sin\theta_n,$$

$$Z_n = w_n \sin\frac{\theta_n}{2}.$$

The discrete EEG series therefore generated a trajectory

$$\mathbf{r}_n = (X_n, Y_n, Z_n) \in \mathbb{R}^3.$$

Because the parametrization contains half-angle terms, one full traversal in θ reverses local transverse orientation, matching the Möbius identification. Therefore, the embedded EEG evolved as an ordered set of points on a twisted surface rather than as a curve in an untwisted cylindrical phase space.

Trajectory reconstruction and differential-geometric observables. The ordered sequence $\mathbf{r}_n$ was interpreted as a discrete trajectory approximating a differentiable curve $\mathbf{r}(t)$. Numerical derivatives were obtained using finite differences implemented through centered discrete gradients. Writing

$$\mathbf{r}_n = \begin{pmatrix} X_n \\ Y_n \\ Z_n \end{pmatrix},$$

the first derivative was approximated by

$$\mathbf{r}_n' \approx \frac{\mathbf{r}_{n+1} - \mathbf{r}_{n-1}}{2\Delta t},$$

the second derivative by

$$\mathbf{r}_n'' \approx \frac{\mathbf{r}_{n+1} - 2\mathbf{r}_n + \mathbf{r}_{n-1}}{\Delta t^2},$$

and the third derivative by discrete differentiation of $\mathbf{r}_n''$. In practice, the numerical gradient operator provided consistent discrete estimates at all interior points, with one-sided approximations at boundaries. Curvature was computed from the classical Frenet formula

$$\kappa_n = \frac{\|\mathbf{r}_n' \times \mathbf{r}_n''\|}{\|\mathbf{r}_n'\|^3 + \varepsilon},$$

where $\times$ denotes the vector cross product, $\|\cdot\|$ is the Euclidean norm and ε is a small regularization constant introduced numerically to avoid division by zero at points with very small speed. Curvature measures the instantaneous bending of the path. Torsion was computed as

$$\tau_n = \frac{(\mathbf{r}_n' \times \mathbf{r}_n'')\cdot \mathbf{r}_n'''}{\|\mathbf{r}_n' \times \mathbf{r}_n''\|^2 + \varepsilon},$$

where $\cdot$ denotes the scalar product. Torsion measures the local deviation of the path from planarity, that is, the extent to which the trajectory twists in three dimensions. Since all derivatives were estimated with $\Delta t = 1$ ms, the independent variable of κ_n and τ_n was directly physical time.

The embedded path was also inspected through its arclength increment,

$$\Delta \ell_n = \|\mathbf{r}_{n+1} - \mathbf{r}_n\|,$$

and cumulative arclength

$$L_m = \sum_{n=0}^{m-1} \Delta \ell_n.$$

Although arclength was not the primary descriptor shown in the figures, it served as an internal consistency check that the trajectory evolved continuously over the manifold. Together, $\mathbf{r}_n$, κ_n, τ_n and L_m provided a differential-geometric description of the EEG trace after the Möbius-like transformation.

Winding number estimation and oscillatory accumulation. Global rotation around the Möbius-like manifold was quantified through the cumulative winding number. In continuous form, if $\theta(t)$ is an unwrapped phase, the winding accumulated over an interval $[t_0, t_1]$ is

$$N_w(t_0, t_1) = \frac{1}{2\pi}\int_{t_0}^{t_1} \frac{d\Phi}{dt}\, dt = \frac{\Phi(t_1) - \Phi(t_0)}{2\pi}.$$

For the discrete EEG trajectory we used the unwrapped phase sequence Φ_n, so the cumulative winding up to sample m was

$$W_m = \frac{\Phi_m - \Phi_0}{2\pi}.$$

Equivalent numerical estimation can be written in incremental form as

$$W_m = \sum_{n=1}^{m} \frac{\Phi_n - \Phi_{n-1}}{2\pi}.$$

Because one full oscillatory cycle corresponds to an increase of approximately 2π in unwrapped phase, W_m measures the accumulated number of rotations executed by the trajectory around the strip's central loop. Its temporal slope,

$$\frac{dW}{dt} = \frac{1}{2\pi}\frac{d\Phi}{dt},$$

is directly related to instantaneous frequency. In discrete form,

$$f_n \approx \frac{\Phi_{n+1} - \Phi_{n-1}}{4\pi\Delta t},$$

with $\Delta t = 10^{-3}$ s when frequency is expressed in hertz. In our analysis, the winding curve was retained as a cumulative topological marker rather than reduced to a frequency estimate. This preserved its interpretation as a monotone measure of how the embedded EEG trajectory progresses around the non-orientable manifold. Since the trajectory was built from the first 4000 ms, the winding number was evaluated for the same interval as torsion and curvature. Plotting W_m against physical time yielded a direct visualization of rotational accumulation in the embedded state space. This quantity complemented torsion because it described a global rotational property rather than a local twisting property.

Recurrence structure and distance-based trajectory organization. To characterize repeated visits of the EEG trajectory to nearby regions of the Möbius-like manifold, pairwise Euclidean distances were computed between all embedded points. For samples i and j,

$$D_{ij} = \|\mathbf{r}_i - \mathbf{r}_j\| = \sqrt{(X_i - X_j)^2 + (Y_i - Y_j)^2 + (Z_i - Z_j)^2}.$$

The resulting matrix $D \in \mathbb{R}^{N\times N}$ is symmetric, nonnegative and has zeros on the diagonal. Visualization of D_{ij} as an image yields a recurrence-distance map in which darker values indicate closer proximity of trajectory states. A thresholded recurrence matrix may also be defined as

$$R_{ij} = \Theta(\epsilon - D_{ij}),$$

where Θ is the Heaviside step function and $\epsilon > 0$ is a chosen spatial threshold. The full distance matrix rather than a binarized recurrence map was used for visualization, because it retains graded information about state similarity without introducing threshold dependence. Diagonal continuity reflects temporal continuity of the curve; off-diagonal dark structures indicate returns to similar regions of the manifold at separated times.

The analysis was implemented in Python using the libraries pandas for spreadsheet import, numpy for numerical array operations, scipy.signal for analytic-signal estimation via the Hilbert transform, scikit-learn for principal component analysis when a multichannel summary was inspected, and matplotlib for visualizations.

3. Results

We report the geometric and topological properties obtained after embedding the EEG signal from channel C3 into a Möbius-like manifold using amplitude–phase coordinates derived from the Hilbert transform. The analysis focused on trajectory organization, rotational accumulation, local twisting and recurrence patterns measured directly from the embedded path.

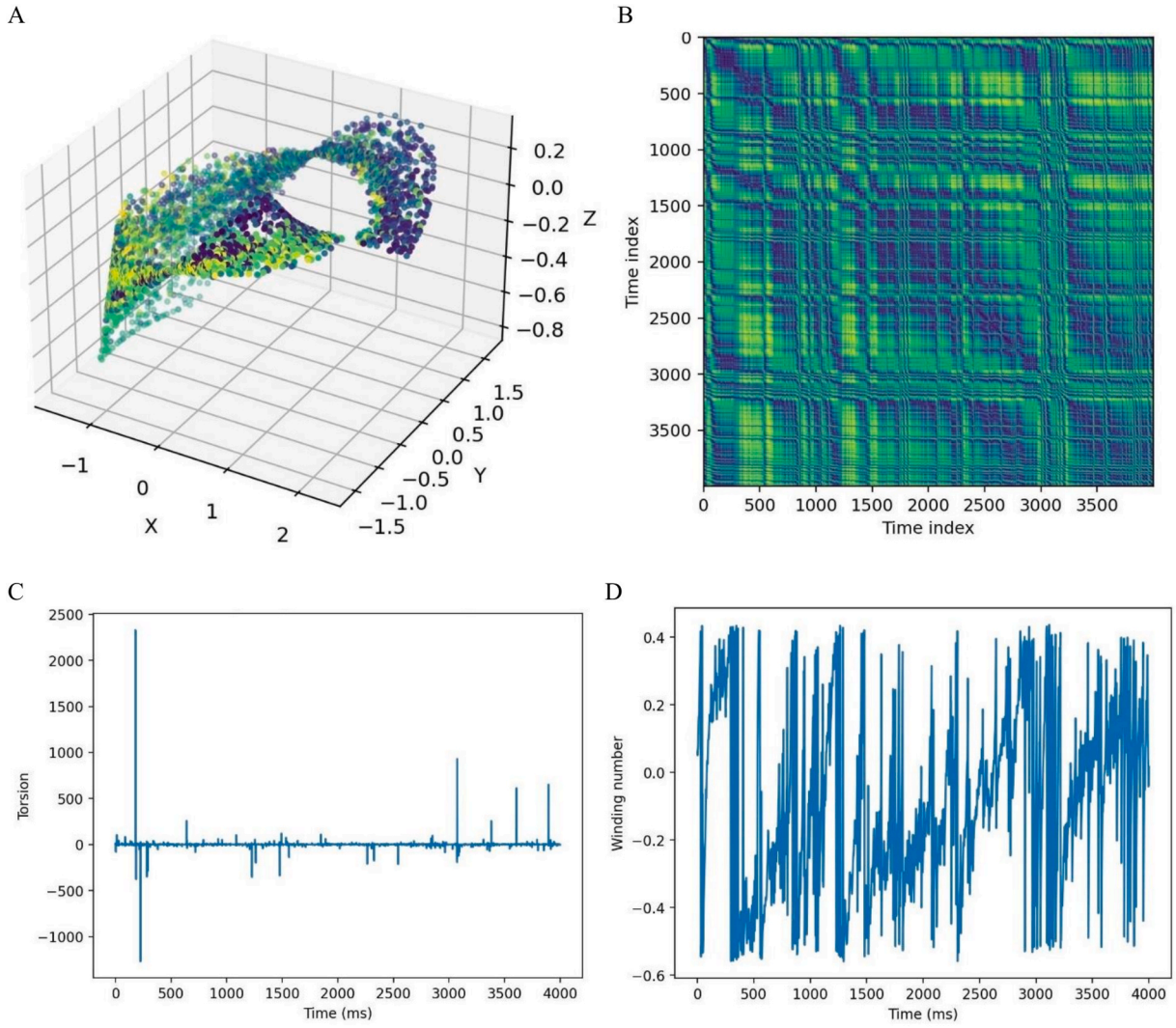


Fig. 1. Geometric and topological characterization of EEG dynamics in a Möbius-like manifold (first 4000 ms). The electrical signal recorded from channel C3 of a healthy 20-year-old subject (sampling interval 1 ms, corresponding to 1 kHz) was embedded in a Möbius-like manifold constructed from the normalized signal amplitude and its instantaneous phase obtained via the Hilbert transform. **(A)** Möbius-like trajectory map of EEG activity. Each EEG sample is mapped to coordinates (X, Y, Z) using a Möbius parametrization, producing a continuous trajectory that represents the temporal evolution of neural electrical activity. The blue curve shows the ordered trajectory of the signal on the manifold. The colored dots correspond to individual sampled points along the trajectory and are color-coded according to time progression (milliseconds): darker colors indicate earlier samples and progressively lighter colors indicate later samples within the 0–4000 ms interval. **(B)** Möbius trajectory density map. Each point represents one EEG sample mapped onto the Möbius-like manifold. Points are colored according to time progression during the first 4000 ms of the recording, with darker colors corresponding to earlier samples and progressively lighter colors indicating later samples. Regions where many points accumulate indicate preferred dynamical channels of the EEG trajectory, reflecting recurrent oscillatory states of neural electrical activity. **(C)** Torsion of the EEG trajectory over the first 4000 ms. Torsion measures the degree to which the path twists in three-dimensional space and is computed from the derivatives of the trajectory vector $r(t)$. Changes in torsion reflect variations in the local geometric structure of the trajectory and may correspond to transitions between oscillatory regimes or modifications in the temporal organization of the EEG signal. Because the data are sampled every 1 ms, the horizontal axis represents physical time in milliseconds. **(D)** Cumulative winding number illustrating the accumulated number of rotations of the trajectory around the Möbius-like manifold over the same time interval. The winding number is derived from the temporal evolution of the phase variable and indicates how many times the trajectory wraps around the embedded space. Since one oscillatory cycle corresponds approximately to a full rotation around the manifold, the slope of the curve reflects the instantaneous oscillatory activity of the EEG signal.

Geometric structure of the embedded EEG trajectory. The Möbius-like embedding transformed the discrete EEG sequence into an ordered three-dimensional trajectory $\mathbf{r}(t) = (X(t), Y(t), Z(t))$ whose evolution reflects the temporal progression of the signal. Over the analyzed interval of 4000 samples, the distribution of points along the surface revealed non-uniform occupancy patterns, with visually identifiable bands where successive trajectory segments accumulated more densely. The trajectory followed smooth loops around the central axis of the strip, reflecting the cyclic structure of oscillatory neural activity encoded by the phase variable. Visual inspection of the density representation confirmed that the trajectory revisited similar regions of the manifold repeatedly over time (Figure A). These regions correspond to clusters of nearby coordinates in the embedded space and indicate recurrent dynamical states of the signal. The recurrence distance matrix further revealed repeated patterns through symmetric off-diagonal structures that indicate temporal intervals where the trajectory approached previously visited states (Figure B). Because the distance matrix compares every pair of embedded points, the appearance of these structured patterns indicates that the reconstructed trajectory does not behave as a random walk in the manifold but instead evolves through geometrically organized paths.

Taken together, these observations establish that the Möbius embedding converts the EEG time series into a structured trajectory whose geometry reflects repeated oscillatory regimes.

Topological descriptors of the trajectory. Quantitative descriptors derived from the embedded path characterize both local geometric deformation and global rotational structure (Fig. 1). Torsion measures the instantaneous twisting of the trajectory in three dimensions and was computed from the first three derivatives of $\mathbf{r}(t)$. The resulting torsion time series displays continuous fluctuations over the 4000 ms interval, indicating that the trajectory does not evolve within a single planar subspace but instead undergoes repeated twisting as it moves across the manifold (Figure C). These variations correspond to local changes in trajectory geometry occuring on the millisecond time scale of the EEG signal. The cumulative winding number captures the global rotational behavior of the trajectory by integrating the unwrapped phase variable. The winding curve increases monotonically across the analyzed interval, indicating repeated rotations of the trajectory around the central loop of the Möbius surface (Figure D). Because each full oscillatory cycle corresponds approximately to one rotation in the embedded space, the slope of this curve reflects the temporal progression of oscillatory activity in the signal. The monotonic increase in winding number confirms that the trajectory advances continuously along the cyclic coordinate defined by the phase variable. Torsion reflects local twisting of the path, whereas the winding number quantifies the cumulative number of rotations executed by the trajectory.

The coexistence of these descriptors demonstrates that the EEG signal contains both local geometric variability and persistent global cyclic structure when represented in the Möbius-like manifold.

Overall, the embedding of the EEG signal in a Möbius-like manifold produced a continuous trajectory with organized geometric and topological patterns. The trajectory exhibited repeated visits to specific regions of the manifold, persistent rotational accumulation and time-dependent twisting captured by torsion. Therefore, representing EEG signals as trajectories in a non-orientable manifold could provide measurable geometric descriptors of neural dynamics.

4. Conclusions

We asked whether the temporal evolution of an EEG signal can be represented as a trajectory evolving on a non-orientable geometric manifold and whether this representation yields measurable descriptors of neural electrical activity. The EEG trace recorded from channel C3 of a healthy subject was transformed into amplitude–phase coordinates derived from the Hilbert transform and embedded into a Möbius-like manifold. Our simulations showed that the embedded signal forms a continuous path progressing around the manifold, while exhibiting local twisting and repeated returns to nearby regions of the space. The winding number captured the cumulative rotations associated with oscillatory cycles, whereas torsion quantified local three-dimensional deformation of the path. Recurrence and density representations indicated that the trajectory repeatedly occupied restricted regions of the manifold. These observations indicate that EEG dynamics can be represented as a geometric trajectory in a non-orientable state space whose properties can be measured through topological quantities.

Our approach should be viewed in relation to other approaches for EEG characterization. Time-frequency methods quantify the spectral content of neural activity and provide direct estimates of oscillatory power[7,8,36], whereas our Möbius-like embedding describes the geometric organization of amplitude–phase evolution. Functional brain-network approaches focus on interactions among channels and the topology of connectivity patterns[19,31,3,26,16], while we examine the geometry of the trajectory generated by single signals. Periodic–aperiodic decompositions separate oscillatory and non-oscillatory contributions to EEG activity[4], whereas we transform amplitude and phase variables into a continuous geometric path whose properties can be quantified through torsion and winding number. Recent neural-manifold and representation-learning approaches seek low-dimensional latent structures underlying neural activity and decoding performance[20,9,33,15], while our Möbius-like representation imposes an explicit non-orientable geometric organization derived from phase symmetry and sign inversion.

Still, EEG activity has been tackled using representation-learning methods and variational autoencoders, both during spontaneous brain activity and under externally induced stimulation[32,34]. These approaches seek compact latent representations optimized for reconstruction, classification or decoding tasks, while our Möbius-like representation is not learned from data, but explicitly built from amplitude-phase relationships and a predefined topological identification reflecting oscillatory symmetry. Compared with Riemannian manifold approaches, which characterize signals through distances and covariance structures on curved but orientable spaces, a Möbius-like manifold introduces a non-orientable topology that explicitly incorporates the equivalence between amplitude sign inversion and half-cycle phase shifts. Within this representation, oscillatory neural activity is associated with continuous rotations around the manifold, while torsion quantifies changes in the local organization of amplitude-phase relationships along the trajectory. Overall, rather than replacing existing EEG descriptors, our approach provides an additional geometric perspective to compare and integrate with the currently-available spectral, network-based, periodic–aperiodic and neural-manifold analyses of brain activity.

Our study has limitations. The Möbius-like manifold is introduced as a representational device rather than as a structure derived from established electrophysiological principles. The identification of amplitude–phase states through the relation $(x, \theta) \sim (-x, \theta + \pi)$ follows from oscillatory symmetry arguments but has not been empirically demonstrated for electroencephalographic signals. Torsion, curvature and winding number were computed from the reconstructed trajectory, but their numerical values depend entirely on the adopted mapping and parameterization. The analysis was performed on a single EEG trace from one individual, which limits generalizability. It remains to be determined whether similar geometric structures appear consistently across different EEG channels, individuals and behavioral states and whether longer recordings produce stable trajectory patterns over extended time scales.

Experimentally testable hypotheses can be drawn. First, the cumulative winding number should scale linearly with time during periods of stable oscillatory rhythms. The slope dW/dt should approximate the instantaneous oscillatory frequency extracted from standard spectral analysis, providing a quantitative correspondence between rotational accumulation and classical frequency measures.

Second, transitions between oscillatory regimes should appear as localized increases in torsion magnitude. This prediction can be evaluated by comparing torsion peaks with independently detected changes in dominant frequency bands or phase resets, with the expectation that larger torsion amplitudes coincide with sharper transitions in the underlying EEG dynamics.

Third, recurrence maps derived from the embedded trajectory should reveal repeated visits to similar regions of the manifold during stable rhythmic activity. Quantitatively, the density of recurrence points within a fixed spatial threshold should increase during sustained oscillatory epochs and decrease during irregular or broadband activity.

Fourth, if the Möbius-like identification captures phase symmetry between half-cycle–shifted states, pairs of trajectory points separated by approximately π in phase should cluster spatially within the manifold, a prediction that can be tested by measuring pairwise distances between amplitude–phase states satisfying $\theta_j \approx \theta_i + \pi$.

Future research may also examine how trajectory geometry changes under different physiological conditions, whether alternative non-orientable embeddings produce comparable descriptors and how geometric measures interact with established nonlinear metrics in neural signal analysis.

Our approach introduces computational observables that may support new forms of signal characterization. Because our embedding converts a time series into a spatial trajectory, quantities like trajectory morphology, spatial recurrence and rotational progression can be incorporated into automated pipelines for signal segmentation, where different portions of a recording are classified according to trajectory morphology rather than spectral content alone. Our approach may also support visualization of neural dynamics in compact geometric spaces that preserve temporal ordering while reducing dimensional redundancy in cyclic signals. In addition, trajectory-based descriptors can be integrated with existing signal-processing techniques to generate hybrid representations.

A Möbius-like representation could also be incorporated into future brain–computer interface pipelines as a source of additional trajectory-based features. Torsion and winding number could be treated as geometric descriptors complementary to conventional spectral, temporal and spatial EEG measures in order to describe the global organization of oscillatory trajectories and their evolution within an amplitude–phase state space. These descriptors could be combined with existing feature sets in machine-learning and neural-decoding algorithms to assess whether geometric properties of EEG dynamics contribute additional discriminative information. Our approach may be relevant in applications requiring continuous monitoring of brain-state transitions, including motor imagery, attention tracking and adaptive neurofeedback paradigms. Future studies could evaluate whether trajectory-based geometric features improve classification performance, robustness to inter-subject variability or sensitivity to subtle changes in neural dynamics that are not readily captured by conventional EEG descriptors.

In conclusion, we examined whether transforming an EEG recording into a trajectory on a twisted geometric surface yields measurable descriptors of neural signal organization. Our continuous representation introduces a different analytical viewpoint in which temporal activity is interpreted through the geometry of its reconstructed path, providing a complementary perspective for investigating the organization of neural electrical recordings.

Significance Statement.

Möbius-like embedding of EEG enables trajectory-based topological descriptors, adding a geometric perspective to neural dynamics.

Ethics approval and consent to participate. This research does not contain any studies with human participants or animals performed by the Author.

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During the preparation of this work, the author used ChatGPT 5.2 to assist with data analysis and manuscript drafting and to improve spelling, grammar and general editing. After using this tool, the author reviewed and edited the content as needed, taking full responsibility for the content of the publication.

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Authors' contributions. The Author performed: study concept and design, acquisition of data, analysis and interpretation of data, drafting of the manuscript, critical revision of the manuscript for important intellectual content, statistical analysis, obtained funding, administrative, technical and material support, study supervision.

Declaration of competing interests

The Author does not have any known or potential conflict of interest including any financial, personal or other relationships with other people or organizations within three years of beginning the submitted work that could inappropriately influence or be perceived to influence their work.

Data availability

No data was used for the research described in the article.

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